We consider a supply chain whose members are divisions of the same firm. The divisions are managed by different individuals with only local inventory information. Both the material and information flows in the supply chain are subject to delays. Under the assumption that the division managers share a common goal to optimize the overall performance of the supply chain (i.e., they act as a team), we characterize the optimal decision rules for the divisions. The team solution reveals the role of information leadtimes in determining the optimal replenishment strategies. We then show that the owner of the firm can manage the divisions as cost centers without compromising the systemwide performance. This is achieved by using an incentive-compatible measurement scheme based on accounting inventory levels. Finally, we investigate the impact of irrational behavior on supply chain performance and demonstrate that it is important for the upstream members of the supply chain to have access to accurate customer demand information.

(Supply Chain Management; Information Delays; Teams; Cost Centers; Incentive Compatibility; Irrational Behavior; The Beer Game)

1. Introduction

A large organization usually has many divisions managed by different individuals. Based on different information, the division managers make their decisions, which jointly determine the overall performance of the organization. If the division managers share a common goal to optimize systemwide performance of the organization, then they can be modeled as a team. This model is reasonable under, for example, a profit/cost sharing plan whereby each manager’s objective function is a fixed, positive proportion of the overall profit/cost of the system. The team theory in the economics literature deals with such models (Marchak and Radner 1972). On the other hand, the division managers may not share the organization’s objective. In this case, one prevalent solution is to manage the divisions as profit/cost centers. The head of the organization determines how to measure the performance of the division managers. For a given measurement scheme, the problems facing the division managers can be modeled as a multiperson game since the decision at one division often affects the performance at another. The task for the organization’s head is then to design a game for the division managers to play so that they behave as if they were a team.

This paper considers a firm with $N$ divisions arranged in series. Customer demand arises at Division 1, Division 1 replenishes its inventory from Division 2, etc., and Division $N$ orders from an outside supplier. The demands in different periods are independent draws from the same probability distribution. When the demand in a period exceeds the on-hand inventory, the excess is backlogged. Information is transmitted along the supply chain from downstream to upstream in the form of replenishment orders. This triggers material flow in the opposite direction. Due to order processing, there are delays in the transmission
of information, i.e., an order placed by a division reaches the upstream division after an information leadtime. Similarly, the material flow is subject to delays due to transportation or production. Inventory holding costs are incurred at every division, and backorder penalty costs are incurred at Division 1. The sum of these costs over all the divisions is the system-wide cost. The firm’s objective is to minimize the long-run average systemwide cost. The divisions are, however, managed by different individuals. The division managers only have local inventory information, partly due to the information delays. The demand distribution and the cost/leadtime parameters are known to the division managers as well as the owner of the firm. The paper considers two models of the firm: a team model and a cost centers model.

For the team model, we identify optimal decision rules for the division managers. It is optimal for each division to follow an installation base-stock policy. The installation stock at a division is its on-hand inventory minus backlogged orders from the downstream division (or backlogged customer demands in the case of Division 1), plus its outstanding orders. Note that the installation stock at a division is local information, accessible to the division manager. The optimal decision rule for a division manager is to place orders so as to keep its installation stock at a constant level. The constant base-stock levels for the divisions are easy to compute. The above result for the team model can be used to understand the costs of information leadtimes. For example, it is found that information leadtimes play exactly the same role as the transportation/production leadtimes in the determination of the optimal decision rules. But the former is less costly than the latter since inventories on paper (i.e., orders being processed) do not incur holding costs. Moreover, numerical examples show that it is beneficial to shift information leadtimes from downstream to upstream. This observation is useful when a firm is allocating its order-processing capacity along the supply chain.

Now suppose the divisions are managed as cost centers. A measurement scheme is said to be incentive compatible if it induces the division managers to behave as if they were a team. We provide such a measurement scheme. It is based on the so-called accounting inventory levels at the divisions. A division’s accounting inventory level is its on-hand inventory minus backlogged orders from the downstream division (or backlogged customer demands in the case of Division 1) under the assumption that the upstream division is perfectly reliable (i.e., has ample stock). Therefore, the accounting inventory level at a division is typically different from the actual one, and it is tracked for accounting purposes only. Each division manager is charged a holding cost if his accounting inventory level is positive, and a penalty cost otherwise. We show how the owner of the firm can choose a set of holding and penalty cost rates for the divisions to induce the team behavior. Numerical examples show that the choice of these cost rates is quite insensitive to the owner’s knowledge of the demand distribution. In other words, the owner does not require perfect information about the demand distribution in order to achieve near optimal costs.

The paper also studies the impact of irrational behavior of the division managers on the systemwide performance. Our results indicate that irrational decisions are very costly, especially the downstream ones. The system, however, becomes much more robust when the upstream managers are able to respond to customer demands instead of downstream orders. This part of the paper is exploratory. The purpose is to draw research attention to irrational behavior in supply chain management.

The coordination research in operations is categorized by Whang (1995) based on three perspectives of an organization: the single-person, team, and nexus-of-contract perspectives. Our models fall under the second and third categories. We generalize the classic model of Clark and Scarf (1960) by introducing information delays and decentralized decision making. The role of information in inventory management is studied by, e.g., Milgrom and Roberts (1988), Axsater and Rosling (1993), Hariharan and Zipkin (1995), Lovejoy and Whang (1995), Chen (1998), Gavirneni et al. (1999), and Lee et al. (1997, 1999). Incentive issues are widely studied in economics (see, e.g., Groves 1973, and Groves and Loeb 1979) and are increasingly recognized in the operations literature, see, e.g., Por-
teus and Whang (1991), Cachon and Lariviere (1996), Kouvelis and Lariviere (1996), Corbett (1997), Cachon and Zipkin (1999), and Lee and Whang (1999). Closely related to our paper is research conducted by Lee and Whang (1999), who provided an incentive-compatible measurement scheme for a decentralized version of the Clark-Scarf model. Their scheme relies on transfer payments between the divisions, whereas ours depends on transactions between the owner of the firm and the divisions. The literature on channel coordination also addresses various incentive issues, see, e.g., Jeuland and Shugan (1983), Lal and Staelin (1984), Monahan (1984), Pasternack (1985), Lee and Rosenblatt (1986), Eliashberg and Steinberg (1987), Weng (1995), Donohue (1996), Kandel (1996), and Chen et al. (1997). Finally, the models in this paper are related to the beer game (Sterman 1989) which assumes a non-stationary, deterministic demand process completely unknown to the players, whereas we have a stationary, stochastic demand process, and our players know the demand distribution.

The rest of the paper is organized as follows. Section 2 introduces the models. Section 3 considers the team model. Section 4 deals with cost centers. Section 5 allows irrational behavior. Section 6 concludes.

2. Description of Models
A firm has $N$ divisions arranged in series. Customer demand arises at Division 1, Division 1 replenishes its inventory from Division 2, 2 from 3, etc., and Division $N$ orders from an outside supplier. The demands in different periods are independent draws from the same probability distribution. This sequential production/distribution process is managed by division managers: At the beginning of each period, Manager $i$ determines how much to order from Division $i+1$, $i = 1, \ldots, N$. (For convenience, the outside supplier is sometimes referred to as Division/Manager $N+1$.) Information in the form of replenishment orders flows from downstream to upstream, triggering material flow in the opposite direction. Both flows are subject to delays: The material flow from Division $i+1$ to Division $i$ requires a constant leadtime, and the orders placed by Manager $i$ are received by Manager $i+1$ after a constant order-processing delay, $i = 1, \ldots, N$.

(An order sent by Manager $i$ but not yet received by Manager $i+1$ is being processed.)

The division managers partially observe the inventory status of the supply chain. At any point in time, each manager knows 1) his on-hand inventory, 2) the orders he has placed with the upstream division, 3) the shipments he has received from the upstream division, 4) the orders he has received from the downstream division, and 5) the shipments he has sent to the downstream division. However, he does not exactly know the shipments that are in transit from the upstream division which may not be perfectly reliable. Neither does he know the orders from the downstream division that are currently being processed. One exception is Manager $N$ who knows all the shipments in transit from the outside supplier, which is assumed to have ample stock and thus is perfectly reliable. Of course, the decisions made by each manager can only be based on what he knows.

We will consider two models of the firm. The first model assumes that the division managers behave as a team, i.e., they have a common goal to minimize the long-run average total cost in the system. This is reasonable when, e.g., the owner of the firm has implemented a cost sharing plan whereby each manager’s objective function is a fixed, positive proportion of the overall cost of the system. A key question here is: What are the optimal decision rules for the local managers? The second model assumes that the divisions are managed as cost centers, i.e., each manager is responsible for the long-run average cost incurred in his own division according to a predetermined mechanism for assessing the local costs. The question is now how to measure the local performance so that the individual managers behave as if they were members of a team.

For the remainder of this section, we state assumptions and define basic notation. We assume that each division manager must fill the outstanding orders from the downstream division as much as possible from his on-hand inventory. In case he runs out of stock, the unfilled portion of the orders is backlogged and is treated as an outstanding order from the downstream division. For Division 1, we assume that when the customer demand exceeds the on-hand
inventory, the excess is backlogged. Linear inventory holding costs are incurred at every division, and linear penalty costs are incurred at Division 1 for customer backorders.

For clarity, all the replenishment activities in a period are assumed to occur at the beginning of the period. For Division \( i > 1 \), they occur in the following sequence: i) order from Division \( i - 1 \) is received, ii) order is placed with Division \( i + 1 \), iii) shipment from Division \( i + 1 \) is received, iv) shipment is sent to Division \( i - 1 \). For Division 1, the sequence is i) order is placed with Division 2 and ii) shipment from Division 2 is received. Customer demand arises during the period.

For \( i = 1, \ldots, N \), define

\[
L_i = \text{production/transportation (or just production) leadtime from Division } i + 1 \text{ to division } i, \text{ a nonnegative integer;}
\]

\[
L_i = \text{information leadtime from Division } i \text{ to Division } i + 1, \text{ a nonnegative integer;}
\]

\[
L_i = \text{total leadtime at Division } i = L_i + l_i;
\]

\[
M_i = \text{downstream information leadtime} = \sum_{j=1}^{i-1} l_j.
\]

\[
M_1 = 0;
\]

\[
H_i = \text{holding cost rate at Division } i, \text{ per unit per period;}
\]

\[
h_i = \text{echelon (incremental) holding cost rate at Division } i, \text{ per unit per period} = H_i - H_{i+1} > 0 \text{ with } H_{N+1} = 0;
\]

\[
p = \text{backorder cost rate at Division 1, per unit per period;}
\]

\[
D(t) = \text{customer demand in Period } t, \text{ with cdf } F(\cdot) \text{ and mean } \mu.
\]

Note that the holding cost rate is assumed to be increasing as the product moves down the supply chain, i.e., \( H_N < H_{N-1} < \cdots < H_1 \). This is reasonable when the divisions perform value-adding activities. We assume that the inventories in transit from Division \( i + 1 \) to Division \( i \) incur holding costs at rate \( H_{i+1} \), \( i = 1, \ldots, N - 1 \), and that the inventories in transit to Division \( N \) do not incur any holding costs. We adopt the convention of assessing holding and backorder costs at the end of each period. It is easy to include linear transportation/production costs. We ignore them for brevity. Finally, the demand distribution and the cost/leadtime parameters are common knowledge.

3. The Team Solution
This section considers the team model. The division managers have local information and make local decisions, but their common goal is to minimize the total cost in the system. We identify optimal decision rules that achieve this goal. We first define a set of variables to characterize the material and information flow in the system. We then observe that the current model is equivalent to an existing one with known optimal solutions. Toward the end of the section, we discuss the costs of information leadtimes, the strategies for reducing these costs, and implementation issues.

Let \( Q_i(t) \) be the quantity ordered by Division \( i \) in Period \( t, i = 1, \ldots, N \). Let \( S_i(t) \) be the quantity shipped by Division \( i \) in Period \( t \) to Division \( i - 1, i = 2, \ldots, N + 1 \). Note that in Period \( t \), Division \( i \) receives a shipment of Size \( S_{i-1}(t - L_i) \) from Division \( i + 1 \) and an order of Size \( Q_{i-1}(t - l_{i-1}) \) from Division \( i - 1 \) or if \( i = 1 \), an order of Size \( D(t) \) from the customers. The shift in time is due to the production and information leadtimes.

For any Periods \( t_1 \leq t_2 \), write \([t_1, t_2] \) for Periods \( t_1, \ldots, t_2 \), \((t_1, t_2) \) for Periods \( t_1 + 1, \ldots, t_2 \), and \([t_1, t_2] \) for Periods \( t_1, \ldots, t_2 - 1 \). Therefore, for example, \( Q_i[t_1, t_2] \) denotes the total orders placed by Division \( i \) in Periods \( t_1, \ldots, t_2 \).

As mentioned earlier, all the replenishment activities (i.e., order placement and fulfillment) in a period occur at the beginning of the period. In what follows, “the beginning of Period \( t \)” is used to refer to the time epoch immediately after those replenishment activities.

A division’s net inventory is its on-hand inventory minus the backlogged orders from the downstream division (or backlogged customer demands in the case of Division 1). Let \( IN_i(t) \) be the net inventory at Division \( i \) at the beginning of Period \( t, i = 1, \ldots, N \). Since the net inventory at a division is increased by the shipments it receives from the upstream division and decreased by the orders it receives from the downstream division, we have the following inventory balance equations:
Decentralized Supply Chains Subject to Information Delays

\[ IN_i(t + \mathcal{L}_i) = IN_i(t) + S_\gamma(t - L_i, t + l_i) - D[t, t + \mathcal{L}_i] \]  

(1)

and

\[ IN_i(t + \mathcal{L}_i) = IN_i(t) + S_{i+1}(t - L_i, t + l_i) - Q_{i-1}(t - l_{i-1}, t + L_i - l_{i-1}) \]

\[ i = 2, \ldots, N. \]  

(2)

For cost accounting purposes, it is useful to also measure net inventories at the end of each period. Let \( t^+ \) be the end of Period \( t \). Thus \( IN_i(t^+) \) is the net inventory at Division \( i \) at the end of Period \( t \). Notice that for Division \( i (>1) \), nothing happens from the beginning of Period \( t \) to the end of the period, and that for Division 1, the only event during a period is the customer demand. Therefore,

\[ IN_i(t^+) = IN_i(t) - D(t) \quad \text{and} \quad IN_i(t^+ - t) = IN_i(t), \]

\[ i = 2, \ldots, N. \]  

(3)

An important concept in the team model is the so-called installation stocks. Division \( i \)'s installation stock is equal to its net inventory plus its outstanding orders, which include orders in transit to the upstream division (i.e., being processed), shipments in transit from the upstream division, and orders backlogged at the upstream division. Recall that Manager \( i \) knows the orders he has placed as well as the shipments he has received. The difference between the two is the outstanding orders. Therefore, installation stock is local information, and any decision rule that is based on it is feasible. Let \( IS_i(t) \) be the installation stock at Division \( i \) at the beginning of Period \( t, i = 1, \ldots, N \).

Again for cost accounting purposes, we define \( IS_i(t^+ - t) \) to be the installation stock at Division \( i \) at the end of Period \( t \). As in (3),

\[ IS_i(t^+) = IS_i(t) - D(t) \quad \text{and} \quad IS_i(t^+ - t) = IS_i(t), \]

\[ i = 2, \ldots, N. \]  

(4)

The installation stock at a division evolves over time in a simple manner. Take any Periods \( t_1 \leq t_2 \). Since the installation stock at Division 1 is increased by its orders and decreased by the customer demands,

\[ IS_i(t_2) = IS_i(t_1) + Q_i(t_1, t_2) - D[t_1, t_2]. \]  

(5)

Similarly,

\[ IS_i(t_2) = IS_i(t_1) + Q_i(t_1, t_2) - Q_{i-1}(t_1 - l_{i-1}, t_2 - l_{i-1}), \]

\[ i = 2, \ldots, N. \]  

(6)

Note that the evolution of installation stocks is independent of the shipments.

By definition, \( IS_i(t) \) consists of the net inventory \( IN_i(t) \) and the outstanding orders placed by Division \( i \). An order placed by Division \( i \) in Period \( t \), if any, arrives at Division \( i + 1 \) in Period \( t + l_i \). If \( IN_{i+1}(t + l_i) \geq 0 \), then Division \( i + 1 \) has fulfilled all the orders it has received by Period \( t + l_i \). In this case, the outstanding orders of Division \( i \) in Period \( t \) can be written as \( S_{i+1}(t - L_i, t) \) (i.e., shipments in transit at \( t \)) plus \( S_{i+1}(t, t + l_i) \) (i.e., shipments triggered by orders backlogged at Division \( i + 1 \) at \( t \), if any, and orders that are being processed at \( t \)). Thus, \( IS_i(t) = IN_i(t) + S_{i+1}(t - L_i, t + l_i) \). On the other hand, if \( IN_{i+1}(t + l_i) < 0 \), then Division \( i + 1 \) has not satisfied all the orders it has received by Period \( t + l_i \) and the shortfall is \(-IN_{i+1}(t + l_i)\). In this case, \( IS_i(t) = IN_i(t) + S_{i+1}(t - l_{i+1}, t + l_i) - IN_{i+1}(t + l_i) \). Combining the above two cases,

\[ IN_i(t) + S_{i+1}(t - L_i, t + l_i) \]

\[ = IS_i(t) + \min\{0, IN_{i+1}(t + l_i)\} \]

\[ \leq IS_i(t) + IN_{i+1}(t + l_i), \quad i = 1, \ldots, N, \]  

(7)

with \( IN_{N+1}(t + l_n) = +\infty \). We can interpret the left side of the above inequality as the effective installation stock at Division \( i \) at the beginning of Period \( t, i = 1, \ldots, N \), since it excludes the part of \( IS_i(t) \) that does not arrive at the division by Period \( t + \mathcal{L}_i \).

Obviously, Manager \( i \), \( i = 1, \ldots, N - 1 \), does not observe his effective installation stock, while Manager \( N \) is assured that his effective installation stock is always equal to his installation stock since the outside supplier is perfectly reliable.

To establish a linkage to an existing multiechelon inventory model, we require the following additional definitions. For \( i = 1, \ldots, N \),
which, together with (3) and (4), leads to
\[ IL_i(t + \mathcal{L}_i) = IP_i(t) - D[t - M_i, t + \mathcal{L}_i - M_i], \]
\[ i = 1, \ldots, N, \]  
which, together with (3) and (4), leads to
\[ IL_i(t + \mathcal{L}_i) = IP_i(t) - D[t - M_i, t + \mathcal{L}_i - M_i], \]
\[ i = 1, \ldots, N. \]  

We proceed to consider the costs incurred in the system. Recall that holding and backorder costs are assessed at the end of each period. Following Chen and Zheng (1994), we write the holding and backorder costs in a period as
\[ \sum_{i=1}^{N} h_i IL_i(t) + (p + H_i)[IL_i(t)]^- \]  
where \( [x]^-=\max\{-x, 0\} \). There is one caveat, though. Note that \( IL_i(t), i \geq 2 \), includes orders that are being processed. These orders do not represent physical inventories and thus should not incur any holding costs. Fortunately, the long-run average value of such orders contained in \( IL_i(t) \) is constant. To see this, note that each Division \( j (<i) \) has \( l_j \) orders being processed at the end of any period and that the average size per order is \( \mu \), the expected one-period customer demand, under any reasonable policy. Therefore, (11) leads to an overestimate of the long-run average total cost by
\[ C_i = \sum_{i=2}^{N} h_i(l_1 + \cdots + l_{i-1}) \mu = \sum_{i=2}^{N} h_iM_i\mu. \]  

We now show that the team model has the same structure as the Clark-Scarf model. First, we define a sequence of time epochs recursively. Let \( t_N \) be an arbitrary period. Define \( t_i = t_{i-1} + \mathcal{L}_{i-1} - l, \) for \( i = N - 1, \ldots, 1 \). It is easy to verify that \( t_N = M_N \leq t_{N-1} - M_{N-1} \leq \cdots \leq t_1 - M_1 \). Write \( IP_i \) for \( IP_i(t_i) \), \( IL_i^- \) for \( IL_i^-(t_i + \mathcal{L}_i) \), and \( IL_i \) for \( IL_i(t_i + \mathcal{L}_i), i = 1, \ldots, N \). For an illustration of the above time epochs as well as these inventory variables, see Figure 1, which assumes \( N = 4 \). Consider the following cost expression:
\[ \sum_{i=1}^{N} h_iIL_i + (p + H_i)[IL_i^-]. \]  

Clearly, (11) and (13) have the same long-run average value. Moreover, we have from (8)
\[ IP_i \leq IL_i^- \quad i = 1, \ldots, N - 1, \]  
and from (9) and (10)
\[ IL_i^- = IP_i - U_i \quad \text{and} \quad IL_i = IP_i - V_i, \]  
\[ i = 1, \ldots, N, \]  
where \( U_i \) and \( V_i \) are customer demands in nonoverlapping intervals and thus are independent. Finally, we observe that the cost expression in (13) and the relationships in (14) and (15) are precisely
those used by Chen and Zheng (1994) to characterize the Clark-Scarf model. Therefore, one can follow the approach there to obtain an optimal solution to the team model. We briefly outline the solution below.

First, define a sequence of functions recursively. Let

\[ G_1(y) = E[h_1(y - V_1) + (p + H_1)(y - V_1)^-]. \]

Clearly, \( G_1(y) \) is convex and has a finite minimum point, which is denoted by \( Y_1 \). Now suppose that \( G_i(y) \) is defined and that it is convex and minimized at a finite point \( Y_i \). Define

\[ G_{i+1}(y) = E[h_{i+1}(y - V_{i+1}) + G_i(\min(Y_i, y - U_{i+1}))], \]

\[ i = 1, \ldots, N - 1. \]

It can be easily verified that \( G_{i+1}(y) \) is also convex and has a finite minimum point \( Y_{i+1} \). Then, \( C^* = G_N(Y_N) \) is a lower bound on the long-run average value of (13) among all policies. The optimal policy that achieves this lower bound is one whereby Division \( i \) orders to keep its installation stock at the constant level \( s_i^* \), where \( s_1^* = Y_1 \) and \( s_i^* = Y_i - Y_{i-1} \) for \( i = 2, \ldots, N \). (The optimal policy for the Clark-Scarf model is usually expressed in terms of echelon base-stock levels. The conversion of the optimal echelon base-stock levels \( Y_i \) to the installation base-stock levels \( s_i^* \) does not affect the performance of the system; see Axsater and Rosling (1993). This fact was also used by Lee and Whang (1999) in studying a decentralized version of the Clark-Scarf model.)

**Theorem 1.** For the team model, it is optimal for Division \( i \) to follow an installation, base-stock policy with order-up-to level \( s_i^* \), \( i = 1, \ldots, N \). The minimum long-run average system-wide cost is \( C^* = C^0 \).

For the remainder of this section, we discuss implementation issues, the costs of information leadtimes, the strategies for reducing these costs, and a potential coordination problem in the team model. Numerical examples are used for illustration in various places.

The optimal decision rules are easy to implement. As mentioned earlier, each division manager knows the level of his installation stock at any time. Once the installation stock reaches the optimal target level, the implementation becomes straightforward: Each order is equal to the total demand since the last order. Formally, \( Q_i(t) = D(t - 1) \) and \( Q_i(t) = Q_{i-1}(t - l_{i-1}) \) for \( i = 2, \ldots, N \). Thus the demand process at an upstream division is equal to the customer demand.
process shifted in time. Since these two demand processes have the same variance, there is no bullwhip effect (or variance amplification). The bullwhip effect has been studied in multiechelon inventory systems with correlated demands by, e.g., Drezner et al. (1996), Chen et al. (1997a, b), Lee et al. (1997), and Baganha and Cohen (1998).

Notice that the optimal installation base-stock levels only depend on the total leadtimes \( L_i \), which determine the distributions of the random variables \( U_i \) and \( V_i \) (see Figure 1). This suggests that the information leadtimes \( l_i \) play exactly the same role as the production leadtimes \( L_i \) in the determination of the optimal policies. But their effects on the total cost differ. If we fix the total leadtime at a division but increase its information leadtime, the total cost decreases because \( C^* \) remains fixed but \( C^d \) increases (see Equation (12)). Therefore, information leadtimes are not as costly as production leadtimes. This is intuitive since orders being processed do not incur holding costs, whereas in-transit shipments do. Information leadtimes can be reduced by streamlining the order-processing operations, of course. They can also be eliminated by parallel processing: As soon as a division places an order, a signal is immediately sent to inform the upstream division of this order even though the order still has to go through the same administrative process. In this way, as far as the supply chain performance is concerned, the information leadtimes have disappeared. This is similar to a feature of the model considered by Lovejoy and Whang (1995) where the signals are imperfect (i.e., the underlying orders may not materialize).

When there are multiple optimal solutions, the division managers must coordinate their decisions in order to minimize the systemwide cost. Consider the following example with two divisions. Demand in each period is normally distributed with mean 10 and standard deviation 3. The leadtimes are \( l_1 = l_2 = 1 \) and \( L_1 = L_2 = 0 \); and the cost parameters are \( h_1 = 1, h_2 = 30, \) and \( p = 10 \). The installation, base-stock policies with order-up-to levels \( (s_1, s_2) = (29, -3), (28, -2), (27, -1), \) and \( (26, 0) \) are all optimal. The minimum cost is 66.77. These solutions are on a diagonal in Table 1. The table also provides the systemwide cost when the base-stock levels are mismatched. Note that mismatched base-stock levels can be costly. This coordination problem can easily be solved if the managers communicate or if there is an implicit understanding between them that nobody is going to choose a negative base-stock level, in which case the solution is \( s_1 = 26 \) and \( s_2 = 0 \). It also helps if the owner of the firm intervenes. Since \( s_2 \leq 0 \) in all cases, Division 2 does not hold any inventory and thus the system functions essentially as a one-division firm. (The effective base-stock level for the one-division firm is \( s_1 + s_2 \). This is why the optimal solutions are on the diagonal.) If the firm removes Division 2, i.e., making it a trans-shipment point, then the supply chain becomes a single location with a unique base-stock level. Alternatively, the firm can be organized as cost centers, in which case the decisions can be coordinated through an appropriate performance metric (see the next section).

It is possible to have multiple optimal solutions in which all the base-stock levels are positive. Take any two-division firm, and suppose that \( G_{i,c} \) defined above is minimized at \( Y_1 = 10, 11 \) and \( G_{i,c} \) minimized at \( Y_2 = 20, 21 \) (assuming discrete demand). It is clear that the following combinations of echelon base-stock levels are all optimal: \( (Y_1, Y_2) = (10, 20), (10, 21), (11, 20), (11, 21) \). Converting these to installation base-stock levels, we obtain the following optimal solutions for the team model: \( (s_1, s_2) = (10, 10), (10, 11), (11, 9), (11, 10) \). Thus Manager 1 has two possible choices of \( s_1 \) (10 and 11), while Manager 2 has three possible choices of \( s_2 \) (9, 10, and 11). There are 6 combinations, some of which are suboptimal. Note that \( s_2 = 10 \) is an optimal strategy for Manager 2 no matter what Manager 1 does: If \( s_1 = 10 \), then the

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corresponding echelon base-stock levels are \((Y_1, Y_2) = (10, 10 + 10) = (10, 20)\) which is optimal; otherwise, if \(s_1 = 11\), then \((Y_1, Y_2) = (11, 11 + 10) = (11, 21)\) which is still optimal. Moreover, \(s_2 = 10\) is the only choice with such a property. Therefore, it is reasonable to expect that Manager 2 will choose 10 and knowing that, Manager 1 will choose either 10 or 11. In this case, coordination is achieved even without any communication. However, this is not always the case. Consider the same example except now \(G_i(\cdot)\) is minimized at \(Y_1 = 10, 11, 12\). In this case, the optimal strategy for one manager depends on what the other does. As a result, coordination can only be achieved if the managers communicate or the firm is organized as cost centers.

Example 1. Let \(N = 4\). Divisions 1, 2, 3, and 4 are also referred to as retailer, wholesaler, distributor, and factory respectively. The demand in each period is normally distributed with mean 50 and standard deviation 10. The information leadtimes are \((l_1, l_2, l_3, l_4) = (2, 2, 2, 0)\) and the production leadtimes are \((L_1, L_2, L_3, L_4) = (2, 2, 2, 3)\). The echelon holding cost rates are \((h_1, h_2, h_3, h_4) = (\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5})\) and the backorder cost rate (at the retailer) is \(p = 10\).

For this example, the team solution is \((s_1^*, s_2^*, s_3^*, s_4^*) = (295, 210, 206, 152)\). The minimum long-run average cost is 215.48. Suppose the system has the following initial condition: Each division has 50 units of on-hand inventory; no shipments are in transit and no orders are being processed. The optimal strategy for the retailer is to order 295 – 50 = 245 units in the first period and thereafter, order whatever the customer demand was in the previous period. The optimal strategy for the wholesaler is to order 210 – 50 = 160 units in the first period and thereafter, order the request he just received from the retailer; so on and so forth.

Figure 2 illustrates the tradeoff between the information leadtimes at different divisions. It was obtained by varying \(l_1\) while fixing \(l_1 + l_2 = 4\). The minimum systemwide cost increases as \(l_1\) increases. (The same phenomenon has been observed in other examples as well.) Thus it can be beneficial to shift information leadtimes from downstream to upstream. This result is intuitive because the shift decreases the total leadtime at the downstream division. This observation is helpful when a firm allocates its order-processing capacity along the supply chain.

4. Managing Cost Centers

Suppose the owner of the firm decides to manage the divisions as cost centers, i.e., each manager is evaluated based on the costs incurred in his own division. How should the local performance be measured? The challenge is to provide each manager with the right incentive so that his individual objective is compatible with the overall objective of the system. In other words, an incentive-compatible measurement scheme is desired.

The accounting and management literature advocates that individuals should only be evaluated on controllable performance; see, e.g., Horngren and Foster (1991). Therefore, the local inventory level (on-hand inventory minus backorders) at a division is inadequate as a basis for measuring the local performance, since it is also affected by decisions made at the other divisions. This motivates the definition of the accounting inventory levels. The accounting inventory level at a division is its net inventory under the hypothetical scenario that the upstream division always has ample stock. This is hypothetical for all divisions except Division \(N\) because, in reality, the upstream division may run out of stock. Let \(AL_i(t)\) be the accounting inventory level at division \(i\) at the end of Period \(t\), \(i = 1, \ldots, N\). Under the hypothetical scenario, each order by Division \(i\) arrives at the division after a constant leadtime \(L_i\). Thus
It is easy to verify that \( p_1 = p + H_2 \) satisfies the above equation. (When demand is discrete, any value of \( p_1 \) that satisfies \( (h_1 + p_1)F^{x+1}(s^*_i) - p_1 \geq 0 \) and \( (h_1 + p_1)F^{x+1}(s^*_i - 1) - p_1 \leq 0 \) is a solution. The same caveat applies below.)

Now consider the problem facing Manager 2. Putting himself in Manager 1’s shoes, Manager 2 sees that Manager 1 will follow a base-stock policy (since the contracts are common knowledge). Therefore, the demand at Division 2 in Period \( t \) is just the customer demand in period \( t - l_1 - 1 \), i.e., \( D_2(t) = D(t - l_1 - 1) \). As a result, Manager 2’s problem is again a standard newsboy problem. From (16), the optimal base-stock level \( s_i \) for Division 2 solves \( F^{x}(s_i) = p_2/(h_1 + p_2) \). To have \( s_i = s^*_i \), \( p_2 \) must satisfy \( F^{x}(s^*_i) = p_2/(h_2 + p_2) \). The same analysis can be repeated for the remaining divisions. In summary, the values of \( p_i \) can be obtained from

\[
F^{x}(s^*_i) = \frac{p_i}{h_i + p_i}, \quad i = 2, \ldots, N. \tag{19}
\]

Under the performance metric (17) with \( p_i \) determined by (18) and (19), the division managers are essentially playing an \( N \)-person game, each minimizing his own accounting costs. The above analysis suggests that the strategy combination \( (s^*_1, \ldots, s^*_N) \) is an iterated dominant strategy equilibrium (see, e.g., Rasmussen 1989). That is, Manager 1 has a dominant strategy; given Manager 1’s dominant strategy, Manager 2 has a dominant strategy; etc. It is interesting that Manager \( i, i = 2, \ldots, N \), does not have to know the exact base-stock levels that the downstream managers are using; it suffices to know the form of their contracts.

**Theorem 2.** An incentive-compatible measurement scheme is to evaluate Division Manager \( i \) based on the long-run average value of (17), \( i = 1, \ldots, N \), where the penalty cost rates are determined by (18) and (19). Under this measurement scheme, the team solution \( (s^*_1, \ldots, s^*_N) \) prevails as an iterated dominant strategy equilibrium.

Note that the holding and penalty cost rates given above are not the only ones that achieve incentive compatibility. In fact, multiplying both \( h_i \) and \( p_i \) by any positive number would still lead to the same
decision by Manager \( i \). Also, the problems facing the division managers are simpler than those in the team model; it is the difference between solving a newsboy problem and a multiechelon problem. This simplicity is achieved, though, at the expense of the owner’s effort in identifying an incentive-compatible measurement system.

**Example 2.** Continuing Example 1 of §3, the following cost rates are incentive compatible: \( (h_1, h_2, h_3, h_4) = (\bar{h}, \overline{\bar{h}}, \frac{1}{3}, \frac{1}{3}) \) and \( (p_1, p_2, p_3, p_4) = (10\bar{h}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}) \).

We pause here to note that for a decentralized version of the Clark-Scarf model, Lee and Whang (1999) provided an incentive-compatible measurement scheme. Their scheme relies on transfer payments between the divisions, which occur when a division backlogs a downstream order. Under our scheme, however, transactions occur between the owner of the firm and the divisions. Recently, Cachon and Zipkin (1999) provided linear contracts that achieve incentive compatibility for a two-stage serial system. The two stages in their model are independent firms, whereas the different stages in our serial system are divisions of the same firm.

We close this section with a discussion on the benefits of decentralization. Firms decentralize the control of their operations for many reasons. A key reason is that the local managers are better informed about the local environments than the owner is. Therefore, it makes sense to let the local managers make local decisions. Our model does not capture this benefit due to the common-knowledge assumption.

To understand the benefit of decentralization, suppose the division managers all know the true demand distribution, but the owner does not. All the other system parameters remain common knowledge. Consider the following two scenarios. In one, the owner is a dictator who tells the local managers what to do. Based on her knowledge of the demand distribution, she solves the team model and tells her employees to implement the installation base-stock policies she found. This is going to be suboptimal because the dictator is misinformed. In the other scenario, the owner organizes the divisions as cost centers. She solves the team model for the “optimal” base-stock levels, which are used to determine the penalty cost rates. Then, she tells the division managers how they are going to be evaluated. Using their (accurate) knowledge about the demand distribution, each division manager chooses a replenishment strategy to minimize his accounting costs. In this case, the systemwide performance may still be suboptimal due to, again, the owner’s lack of knowledge. But, at least, the accurate demand distribution plays a role in the determination of replenishment policies thanks to decentralized decision making. The difference in the systemwide performance between these two scenarios can be attributed to decentralization.

Take Example 1 of §3, and now suppose the division managers know the true standard deviation of demand \( \sigma \). Let \( \sigma' \) be the owner’s perceived standard

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deviation. When \( \sigma' \neq \sigma \), the owner is misinformed. All the other data remains common knowledge. Table 2 summarizes the systemwide cost per period for different combinations of \( \sigma' \) and \( \sigma \) under the above two scenarios. The systemwide cost for Scenario one (the dictator or centralized solution) is displayed under that for Scenario two (the cost-centers or decentralized solution). The percentage numbers represent the relative cost increase over the optimal solution. Obviously, the optimal solutions are on the diagonal where \( \sigma' = \sigma \). For the off-diagonal combinations of \( \sigma' \) and \( \sigma \), the decentralized solution virtually remains optimal, whereas the centralized solution becomes significantly suboptimal. The same phenomenon is observed in Table 3, where the only asymmetric information is about the mean of demand. Therefore, 1) when the owner has inferior knowledge about the demand distribution, it can be much better to manage the firm as cost centers than to centralize decision making; and 2) the owner can achieve near optimal costs without having perfect knowledge about the demand distribution. (For a simple newsboy problem with any demand distribution, Pasternack (1985) found an incentive-compatible mechanism that is independent of the demand distribution. As the above examples show, this type of independence is virtually preserved in our multiechelon systems.)

The above numerical results also suggest that decentralized control (implemented with the incentive scheme characterized in Theorem 2) is beneficial even if the owner has perfect information. Suppose the firm faces a fluctuating demand environment. Under centralized control, the owner would have to update the optimal policy for each division every time the demand distribution changes. This can be quite costly due to the frequent optimization of a multiechelon system. With decentralized control, however, the firm can still achieve near optimal costs with an incentive scheme based on an outdated demand distribution. This is supported by the robustness of the penalty cost rates to the exact specification of demand observed in Tables 2 and 3. Although the division managers will still have to adjust their replenishment policies from time to time, it is manageable since the optimization is now done independently for each division. From a managerial perspective, it is beneficial to have a consistent measurement scheme under which the systemwide optimal response to the changing demand environment becomes a simple, local task.

5. Irrational Behavior: An Example and A Remedy

What if the managers make mistakes? What is the impact on the systemwide performance? How can the system be made more robust? These are the questions for this section. It is, however, beyond the scope of this paper to answer these questions in any substantial way. Our approach is exploratory based on numerical examples. Irrational behavior in supply chain manage-
ment has often been overlooked. It is our hope that the following preliminary findings will stimulate research in this area.

Mistakes take many different forms, of course. Let us consider one where Manager \( i \) strives to maintain his net inventory at a constant level \( Y \): If the net inventory is below \( Y \), order the difference; otherwise, do nothing. We call this policy MBSP (Misused Base Stock Policy). It is a mistake because the decision maker ignores the outstanding orders (the optimal strategy is to maintain the installation stock at a constant level). This mistake corresponds to what Sterman (1989) calls the “misperceptions of feedback,” which are prevalent in the beer game. To see the impact of this irrational behavior, we conducted the following simulation experiment. Take Example 1 in §3. Suppose only one manager follows an MBSP. This mistake is unknown to the other managers, who follow the original optimal strategy, keeping their installation stocks at the optimal target levels found in the team solution. Figure 3 depicts the long-run average systemwide cost obtained via simulation. (Ignore the lower curves for now.) The figure indicates that the mistakes are very costly since the minimum long-run average cost is only 215.48. Moreover, a downstream mistake is more costly than an upstream one.

Recall from §3 that if all the managers behave optimally, the customer demand information is transmitted to the upstream managers without any distortion (but with delays). This is no longer true when a manager follows an MBSP. The distorted information prevents the upstream managers from making rational decisions. Now suppose that when the retailer places an order, he is also required to report the demand in the previous period. This demand information is then relayed to the upstream managers.
along with the orders. Let us rerun the above simulation experiment under the assumption that the rational managers place their orders according to the accurate demand information. (Assume the following initial condition: The on-hand inventory at division $i$ is $s^*_i$, $i = 1, \ldots, N$; no shipments in transit; no orders being processed. Then, each rational manager orders the customer demand value he sees in each period.) In this way, when a downstream manager errs, he can no longer corrupt the upstream order decisions. The lower curves in Figure 3 depict the resulting systemwide cost. (When the factory is the only one using an MBSP, the demand information he receives is the same as the order information. This explains why there is only one curve in this case.) The results indicate that by making the accurate demand information accessible to the upstream members of the supply chain, the system becomes much more robust. This is the value of sharing the demand information.

6. Concluding Remarks
We have considered a supply chain whose members are divisions of the same firm. The divisions are managed by different individuals based on local inventory information. Both the material and information flows are subject to delays. Under the assumption that the division managers share a common goal to optimize the overall performance of the supply chain (i.e., they act as a team), we characterized the optimal decision rules for the divisions. It was found that information leadtimes play exactly the same role as the production/transportation leadtimes in the determination of the optimal replenishment strategies, with the former, however, being less costly. The team solution was also used to demonstrate the tradeoff between the information leadtimes at different stages of the supply chain. Numerical examples show that shifting information leadtimes from downstream to upstream is beneficial.

We have also shown how the owner of the firm can manage the divisions as cost centers without compromising the systemwide performance. This is achieved by implementing an incentive-compatible measurement scheme based on accounting inventory levels. Under this measurement scheme, the team solution prevails as an iterated dominant strategy equilibrium. It was found that decentralized decision making is very beneficial when the owner of the firm does not have perfect knowledge about the demand distribution or when the firm faces a fluctuating demand environment.

Our numerical examples quantify, in a particular setting, the impact of irrational behavior on the systemwide performance. They also suggest that a downstream mistake is more harmful than an upstream one. Making accurate customer demand information available to the upstream members of the supply chain seems to make the system more robust.

Finally, this paper has led to several supply chain simulation games. These games are richer in many ways than the traditional beer game, and they allow us to systematically study the impact of different organizational designs (i.e., incentives and information) on supply chain performance. The games can be used either in a classroom setting or as an experimental tool to investigate several empirical questions. Interested readers are referred to Chen and Samroengraja (1997) for details.\footnote{The author would like to thank Hau Lee (the departmental editor), an Associate Editor, and the referees for their valuable comments and suggestions. Thanks also go to the seminar participants at the Wharton School, UCLA, the MIT summer camp, University of Michigan, and Columbia University for helpful discussions. Financial support from the Columbia Business School and the National Science Foundation (SBR-97-0246) is gratefully acknowledged.}

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