Donald E. Sexton, Jr.*

A Microeconomic Model of the Effects of Advertising

In the study of consumer behavior, economics and marketing may perhaps seem headed on divergent paths. Economic models of man typically appear deterministic, while marketing models of man often are stochastic. This article links the microeconomic theory of demand (in a oligopoly situation) to a simple stochastic model of consumer behavior and, with data for one product, compares the empirical success of that model with those of various other models found in the literature.

I. THEORETICAL DISCUSSION
Consider a product class composed of two products, I and II, that are close substitutes for each other. Suppose that the market for product I can be characterized as oligopoly with product differentiation among brands. The purchasing of one brand of product I, brand A, will be examined. Let

\[ y = \text{unit price of brand A (relative to other brands of product I) during time period } t; \]

\[ S = \text{dollar sales of brand A during time period } t. \]

If firm A sets a price of \( y_1 \), \( S_1 \) is the amount of dollars of brand A it sells in time period \( t \) (fig. 1). The demand curve for brand A, \( D_1 \), corresponds to some level of brand A advertising expenditures cumulative to the start of time period \( t \). Let

\[ x_1 = \text{advertising expenditures for brand A cumulative to the start of time period } t \text{ (corresponds to } D_1). \]

Suppose that, during period \( t \), an outlay for advertising, \( \Delta x_1 \), is made. More advertising informs or persuades more people about brand A and therefore increases the demand for brand A at any price, \( y_1 \). That is, an increase in cumulative advertising expenditures, \( x_1 \), shifts to the right, to varying degrees, the entire length of the demand curve \( D_1 \). For the theoretical discussion below, no assumption need be made regarding the effects of the increase in advertising expenditures upon the slope or shape of the demand curve. Any of the shifts shown in figure 2

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1. In the discussion, consumers are assumed to be homogeneous with respect to advertising response, but this is nonrestrictive, since market segments can be defined by advertising response magnitudes and parameters of the model estimated for each segment.

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can follow an increase in advertising. In all cases, sales level $S_2$ is greater than sales level $S_1$. Let

$$x_2 = \text{cumulative level of advertising expenditures corresponding to demand curve } D_2, \text{ that is, } x_2 \text{ is the sum of } x_1 \text{ and } \Delta x_1.$$

Suppose that $\Delta x_1$ is so large an increase in advertising that the value of $x_2$ can be considered infinite. Then the demand curve $D_2$ has a form similar to that shown in figure 3. The middle portion of $D_2$ is drawn as being quite inelastic since, in this extreme situation, all potential consumers of product I identify product I nearly exclusively with brand A. When they decide to purchase product I, the information stock for brand A seems so large relative to that of the other brands that most, although not necessarily all, purchases of product I within a certain price range are for brand A.

Crucial to this discussion is the width of the price range, $y_a$ to $y_b$. For products with high product loyalty among consumers, this range will be relatively large. The bounds of the range are determined by this loyalty and the prices of substitute products, for example, the price of product II: The higher the brand-A price, the more product I buyers

**Fig. 1.—Demand curve for brand A**

**Fig. 2.—Shift of demand curve due to increase in advertising expenditures**
shift to product II; the lower the brand-A price, the more product II buyers shift to product I (brand A). If the production costs of brand A are assumed to be such that the price of brand A is set between endpoints $y_a$ and $y_b$, then the values of $y_a$ and $y_b$ can be ignored for the remainder of this discussion.

The key point in figure 3 is sales level $T$. That amount represents, operationally, the maximum attainable sales level for brand A. That is, sales level $T$ is the size of the pool of sales practically and plausibly obtainable by all brands of product I.

Consider this question: In any time period, what change in brand-A sales, $\Delta S$, results if advertising expenditures are increased by some amount, $\Delta x$?

Let $f(x, y) = S =$ brand-A sales during time period $t$, given price, $y$, and cumulative advertising expenditures, $x$. For example,

$$f(x_1, y_1) = S_1; \quad (1)$$

$$f(x_2, y_1) = S_2. \quad (2)$$

Moreover, from the argument above,

$$\lim_{x \to \infty} f(x, y) = T \quad \text{for} \quad y_a < y < y_b, \quad (3)$$

The magnitude of $S$ must depend upon the following factors: (1) the amount of the increase in advertising expenditures, $\Delta x$; (2) the sales effectiveness per dollar of advertising, $b$; and (3) the maximum possible increase in sales above current level, $S$, that is, $(T - S)$. (Note that this last specification implies advertising has diminishing returns.)

If the simplest possible relationship among these factors, a linear relation is assumed:$^2$

\[ \Delta S = (T - S) \, b \Delta x. \]  
\[ \text{(4)} \]

Rewriting in terms of differentials, at any relative price \( y \) (where \( y_a < y < y_b \)),

\[ dS = (T - S) \, b \, dx, \]
\[ \text{(5)} \]
or:

\[ \frac{dS}{(T - S)} = b \, dx. \]
\[ \text{(6)} \]

After integrating both sides,

\[ -\ln (T - S) + c = bx + a, \]
\[ \text{(7)} \]

where \( c \) and \( a \) are integration constants.

Assume, not implausibly, that if cumulative advertising, \( x \), is zero, then, at any relative price, \( y \), sales of a brand are at some constant level. Then that level of sales is represented by the constant \( a \) and is a function of relative price \( y \). Assume that this function is linear, that is, again assume the simplest situation. That is, let

\[ a = q - sy \quad \text{for} \quad sy < q, \]
\[ = 0 \quad \text{for} \quad sy \geq q. \]
\[ \text{(8)} \]

Sales, \( S \), of the brand are zero when both cumulative advertising, \( x \), is zero, and relative price, \( y \), is sufficiently high so that \( sy \) is greater than or equal to \( q \) (i.e., when the brand is priced “out of the market”). That is, when brand-A sales are zero,

\[ -\ln (T) + c = 0, \]
\[ \text{(9)} \]

implying

\[ c = \ln (T). \]
\[ \text{(10)} \]

Therefore, for any values of \( x \) and \( y \) (where \( y_a < y < y_b \) and \( sy < q \)),

\[ \ln (T - S) = -[-\ln(T) + bx + q - sy], \]
\[ \text{(11)} \]
or

\[ (T - S) = e^{[\ln(T) - (bx + q - sy)]}, \]
\[ \text{(12)} \]
or

\[ (T - S) = T \, e^{-(bx + q - sy)}, \]
\[ S = T \, [1 - e^{-(bx + q - sy)}]. \]
\[ \text{(13)} \]

The variable \( q \) can be interpreted as a measure of the brand’s quality, \( sy \) as the effects of price, and \( bx \) represents the cumulative effects of advertising on sales. The latter quantity can be expressed as the sum of its components:

\[ bx = \sum_{all \, i} b_i x_i, \]
\[ \text{(15)} \]
where $b_i x_i$ = contribution to cumulative effects of advertising on sales by advertising in time period $i$.

Then,

$$S = T \left[1 - e^{-\left(q - sy + \sum_{n=1}^{i} b_n x_n\right)}\right].$$

If both sides of this expression are divided by the maximum obtainable sales, $T$, then

$$\frac{S}{T} = \left[1 - e^{-\left(q - sy + \sum_{n=1}^{i} b_n x_n\right)}\right].$$

This ratio approximates the observed market share of brand A, call it $B$, to the extent total observed sales of product I approximates the sales saturation level, $T$. If values of $T$ can be obtained or estimated, then the ratio, $S/T$, can be interpreted as the probability that a randomly selected potential consumer of product I purchases brand A in time period $T$. Such an interpretation adopted often by marketers yields a simple stochastic model of consumer behavior.

**II. ALTERNATE MODELS**

In addition to this negative exponential model, sales-effects models of advertising and pricing whose parameters most commonly have been estimated are linear (18) and loglinear (Cogg-Douglas) (19) in advertising and pricing:

$$S = q + bx - sy,$$

(18)

$$S = q x^b y^{-a}.$$

(19)

Often brand share rather than brand sales is used as the dependent variable in these models. For example, brand-share formulations of (18) and (19) are

$$B = \frac{S}{T} = q + bx - sy,$$

(20)

$$B = \frac{S}{T} = q x^b y^{-a}.\text{ (21)}$$

Microeconomic reasoning supporting any of the above four models has not appeared in the literature. The argument usually and implicitly advanced for them, particularly (19) and (21), concerns the relative convenience of determining the elasticities corresponding to policy vari-

3. For widely used food and drug products, the sales saturation level $T$ may be approximated by total sales of all brands. Also, the purchase decision among brands of these products is likely to be trivial for consumers—an argument for describing their behavior with a stochastic model.

ables when the response functions are assumed to be linear or loglinear. The theoretical weakness of each of these models can be seen if their underlying assumptions are examined.

Equation (18) assumes advertising has constant returns to scale, a view rarely held in the advertising literature. Equation (19) assumes constant elasticity for advertising outlays, but this is an unrealistic and needless imposition on an advertising-effects model. Equations (20) and (21) meet with the same objections as do equations (18) and (19), respectively. In addition, when transformed into a sales model (by multiplying both sides by \( T \)), equations (20) and (21) each have \( T \), total product sales, as a factor in the right-hand side. For equation (20), the theoretical sense of weighting advertising and relative price by total sales of all brands is unclear. For equation (21), if \( T \) is constant, the product \( qT \) plays the same role as \( q \) alone does in equation (19), and little is gained by using brand share as the dependent variable. If \( T \) is not constant, then this formulation may be an improvement over (19), since an increasing market for a product may imply increasing productivity of advertising.

III. E M P I R I C A L  V A L I D A T I O N

To evaluate the relative worth of the negative exponential, linear, and loglinear models, for a leading brand of a nonseasonal, frequently purchased grocery product, the parameters of several alternative sales advertising models including those described above were estimated.\(^5\) The data

base included all the purchases of the grocery product by 600 households in the Chicago area over a two-year interval. Although the Chicago Tribune panel is a highly respected source of marketing data, it should be noted that panel members may differ in various ways from the general population (e.g., they may possibly be more sensitive to prices). Advertising outlay data were obtained from several marketing research firms.

The following alternate models were considered:

\[ S_t = c_1 + c_2 S_{t-1} + u_t, \quad (22) \]
\[ B_t = c_1 + c_2 B_{t-1} + u_t, \quad (23) \]
\[ S_t = c_1 + c_2 t + u_t, \quad (24) \]
\[ B_t = c_1 + c_2 t + u_t, \quad (25) \]
\[ B_t = 1 - e^{-(c_1 + c_2 t + u_t)}, \quad (26) \]

\[ B_t = 1 - (1 - B_{t-1})^\rho e^{-[c_1 + c_3 A_{t-1}^4 + c_4 A_{t-1}^2 + c_5 A_{t-1}^3]} + e_16(p_{t-1} - p_t) \]
\[ -c_16(p_{t-1} - p_t) - c_17(\bar{d}_t - d_t) - c_18(\bar{d}_{t-1} - d_{t-1}) + u_t, \quad (27) \]

\[ B_t = 1 - (1 - B_{t-1})^\rho e^{-[c_1 + c_3 A_{t-1}^4 + c_4 A_{t-1}^2 + c_5 A_{t-1}^3 + c_6 A_{t-1}^4 + c_7 A_{t-1}^1]} \]
\[ + c_8 A_{t-1}^2 + c_9 A_{t-1}^3 + c_{10} A_{t-1}^4 + c_{11} A_{t-2}^1 + c_{12} A_{t-2}^2 + c_{13} A_{t-2}^3 + c_{14} A_{t-2}^4 + c_{15}(p_{t-1} - p_t) \]
\[ + c_{16}(p_{t-1} - p_t) - c_{18}(d_{t-1} - d_{t-1}) + u_t, \quad (28) \]

\[ S_t = c_1 + c_2 S_{t-1} + c_3 A_{t-1}^1 + c_4 A_{t-1}^2 + c_5 A_{t-1}^3 + c_6 A_{t-1}^4 + c_7 A_{t-1} + c_8 A_{t-1}^1 \]
\[ + c_9 A_{t-1}^2 + c_{10} A_{t-1}^3 + c_{11} A_{t-2}^1 + c_{12} A_{t-2}^2 + c_{13} A_{t-2}^3 + c_{14} A_{t-2}^4 + c_{15}(p_{t-1} - p_t) \]
\[ + c_{16}(p_{t-1} - p_t) + c_{18}(\bar{d}_{t-1} - d_{t-1}) + u_t, \quad (29) \]


6. These data were supplied by the Chicago Tribune through the kind cooperation of Miss Linda Haller and Mr. Richard Heinemann.

\[ B_t = c_1 + c_2 B_{t-1} + c_3 A_t^1 + c_4 A_t^2 + c_5 A_t^3 + c_6 A_t^4 + c_7 A_{t-1}^1 \]
\[ + c_8 A_{t-1}^2 + c_9 A_{t-1}^3 + c_{10} A_{t-1}^4 + c_{11} A_{t-2}^1 + c_{12} A_{t-2}^2 + c_{13} A_{t-2}^3 \]
\[ + c_{14} A_{t-2}^4 + c_{15} (\bar{p}_t - p_t) + c_{16} (\bar{p}_{t-1} - p_{t-1}) \]
\[ + c_{17} (\bar{d}_t - d_t) + c_{18} (\bar{d}_{t-1} - d_{t-1}) + u_t, \]
\[ S_t = e^{c_1 (S_{t-1})} e^{c_2 (A_t^1)} e^{c_3 (A_t^2)} e^{c_4 (A_t^3)} e^{c_5 (A_t^4)} e^{c_6 (A_{t-1}^1)} e^{c_7 (A_{t-1}^2)} e^{c_8 (A_{t-1}^3)} \]
\[ (A_{t-1}^4) e^{c_9 (A_{t-1}^1)} e^{c_{10} (A_{t-2}^1)} e^{c_{11} (A_{t-2}^2)} e^{c_{12} (A_{t-2}^3)} e^{c_{13} (A_{t-2}^4)} e^{c_{14}} \]
\[ \left( \frac{p_t}{\bar{p}_t} \right)^{c_{15}} \left( \frac{p_{t-1}}{\bar{p}_{t-1}} \right)^{c_{16}} \left( \frac{d_t}{\bar{d}_t} \right)^{c_{17}} \left( \frac{d_{t-1}}{\bar{d}_{t-1}} \right)^{c_{18}} e^{u_t}, \]
\[ B_t = e^{c_1 (B_{t-1})} e^{c_2 (A_t^1)} e^{c_3 (A_t^2)} e^{c_4 (A_t^3)} e^{c_5 (A_t^4)} e^{c_6 (A_{t-1}^1)} e^{c_7 (A_{t-1}^2)} e^{c_8 (A_{t-1}^3)} \]
\[ (A_{t-1}^4) e^{c_9 (A_{t-1}^1)} e^{c_{10} (A_{t-2}^1)} e^{c_{11} (A_{t-2}^2)} e^{c_{12} (A_{t-2}^3)} e^{c_{13} (A_{t-2}^4)} e^{c_{14}} \]
\[ \left( \frac{p_t}{\bar{p}_t} \right)^{c_{15}} \left( \frac{p_{t-1}}{\bar{p}_{t-1}} \right)^{c_{16}} \left( \frac{d_t}{\bar{d}_t} \right)^{c_{17}} \left( \frac{d_{t-1}}{\bar{d}_{t-1}} \right)^{c_{18}} e^{u_t}, \]

where

- \( S_t \) = brand sales in dollars in time period \( t \);
- \( B_t \) = brand share in time period \( t \);
- \( A_t^h \) = advertising outlay by brand in medium \( h \) in time period \( t \) (in dollars for media 1, 2, and 3, lineage for medium 4), where 1 = network television, 2 = spot television, 3 = magazines, 4 = newspapers;
- \( p_t \) = average price\(^8\) of brand in time period \( t \);
- \( \bar{p}_t \) = average price of product in time period \( t \);
- \( d_t \) = average deal magnitude\(^9\) for brand in time period \( t \);
- \( \bar{d}_t \) = average deal magnitude for product in time period \( t \);
- \( c_j \) = indices and coefficients reflecting effects of brand quality, advertising, pricing, and dealing on brand sales or brand share;
- \( u_t \) = random disturbance term.

Models (22), (23), (24), (25), and (26) are naïve autoregressive

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8. For average price variables, from the panel data, the average price per ounce was computed from all purchases in period \( t \) that were not made under deals. For \( \bar{p}_t \), purchases of the brand examined were not included in the average.

9. For the average deal magnitude variables, the average price per ounce was computed from all purchases in the period \( t \) that were made under deals. For \( \bar{d}_t \), purchases of the brand examined were not included in the average.
or time-series models. Model (27) is the negative exponential model derived earlier but cast in distributed lag form. It is necessary to use the distributed lag form due to the serial correlation of advertising expenditures in a given medium. Model (28) is also negative exponential, models (29) and (30) are linear, and models (31) and (32) are loglinear, all also formulated in distributed lag form but with advertising for periods $t - 1$ and $t - 2$ explicitly included.

Parameter estimates were obtained for these models with the data aggregated into both seventy-nine one-week observations and forty-two week observations, so that the effects of aggregation could be observed. Moreover, the last five months of data, twenty-two weeks, were not included in the main sample but were used separately to estimate the models' parameters, so that their stability over time could be investigated. The pairwise simple correlations between the independent variables used in these regressions were almost unanimously below .20. The values of the Durbin-Watson statistic generally suggested that the hypothesis of random disturbances need not be rejected.

The effects of aggregation are instructive. Due to space considerations, the regression coefficients are not shown here, but, in sum, advertising appeared to be less a determinant of brand sales or share when biweekly observations were used than when weekly observations were used. One possible explanation for that result is that the effects of advertising for this product were short term, and that aggregating the data into longer time periods washed out its observed effects. On the other hand, pricing appeared more prominently as a factor explaining brand sale or share variation in the results based on biweekly observations than in those based on weekly observations, suggesting that the effects of pricing were longer term.

The relative values of the alternate models can be examined with table 1. In particular, the models can be compared on the basis of descriptive power—closeness of fit, face validity—plausibility of signs and magnitudes of parameter estimates, and predictive ability—stability of estimates over time.

Models (22)–(26), the naive models, are seen to be very crude. The $R^2$ values for them do not exceed .04. In terms of the coefficient of determination, the brand-share models, models (28), (30), and (32), seem marginally superior to the sales models. In particular, models (28) and (30) have the highest values for $R^2$ in table 1.

Among models (27)–(32), the coefficient estimates of the pricing and dealing variables generally tend to be of logical sign and magnitude.


11. For regressions that include the lagged dependent variables, the value given for the Durbin-Watson statistic is biased against discovering either positive or negative serial correlation; see Christ, n. 10 above.
Table 1
Summary of Results for Alternate Models Based on First 79 Weeks—Weekly Observations

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<th>Model</th>
<th>Estimated Coefficients</th>
<th>$R^2$</th>
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Table 1 (Continued)

| Model | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 | C13 | C14 | C15 | C16 | C17 | C18 | R²  | F   | D-W |
|-------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|-----|-----|
| 28    |    |    |    |    |    |    |    |    |    | a  | a  | a  |    |    |    |    |    | .42 | 4.35| 1.91 |
| 29    |    |    |    |    |    |    |    |    |    | a  | a  | a  |    |    |    |    |    | .39 | 4.07| 1.96 |
| 30    |    |    |    |    |    |    |    |    |    | a  | a  | a  |    |    |    |    |    | .42 | 4.14| 1.88 |
| 31    |    |    |    |    |    |    |    |    |    | a  | a  | a  |    |    |    |    |    | .27 | 3.30| 2.03 |
| 32    |    |    |    |    |    |    |    |    |    | a  | a  | a  |    |    |    |    |    | .31 | 3.03| 2.00 |

Note.—For each model, 1 = the expected sign of the coefficient, 2 = estimated sign of the coefficient; 3 = significance level of the coefficient: *, significant at .01 level; †, significant at .05 level; ‡ did not contribute .0001 to $R^2$. 


With respect to the advertising variables, the face validities of these models also seem roughly the same. In the negative exponential models, the signs of the advertising variable regression coefficients should be negative. In model (27), there is one perversely signed advertising coefficient; in model (28), five. In the linear and loglinear models, the signs of the advertising variable regression coefficients should be positive. In each of models (29), (30), (31), and (32), perversely signed advertising coefficients, respectively, number five, five, six, and six. Half of the perversely signed advertising coefficients in these six models correspond to advertising outlays made two periods previous to the period whose brand sales or share are being predicted. This evidence further suggests the short-run nature of advertising effects for this brand. It is consistent with the indication of the results, based on biweekly observations, that advertising was not a significant factor in increasing two-week sales or share. Furthermore, it appears that two weeks’ previous advertising may have robbed from current brand sales or share.

When the coefficient estimates for the first seventy-nine weeks were compared with those for the last twenty-two weeks (not shown due to space limitations), the pricing regression coefficients appeared to be of relatively the same magnitude only for models (28) and (30). Generally the correspondence of the advertising regression coefficients for the two time intervals was disappointing. However, for the twenty-two-week results, as for the seventy-nine-week results, half of the coefficients that were perversely signed appeared with the two weeks’ previous advertising variables.

In sum, the model theoretically constructed, models (27) and (28), seemed, with respect to various criteria, no worse than other advertising-sales models in use, but it also seemed not very much better. The justification for the use of this negative-exponential model rather than the alternate models then must rest upon its theoretical foundation which appears relatively stronger than those implicitly supporting the linear and loglinear models.

IV. CONCLUSION

Hypothesizing a naïve model of the effects of brand advertising upon market demand led to a negative exponential model of the effects of advertising upon an individual’s probability of purchasing the brand. In the literature, the negative exponential model, the linear model, and the loglinear model are most often used to describe the effects of pricing, dealing, and advertising on a consumer's purchase. Based on panel data for a frequently purchased grocery product, a comparison was made of different versions of these three and other models.

The results for one brand indicated that, empirically, brand-share models were slightly superior to brand-sales models, but there was little reason to choose among the negative exponential model, linear model, or loglinear model. That is, the results did not strongly confirm the value
of one model over the others. It did appear that the effects of advertising outlays on the sales or share of this particular brand were short term and may have been obscured if the data used were aggregated into too long periods or if the advertising variables were lagged too far behind the period whose brand share or sales were to be predicted.

Further research underway consists of simultaneously estimating the parameters of models corresponding to the various leading brands of this product.\textsuperscript{12} Hopefully, this work will lead to improved estimates and a stronger basis on which to compare the values of models such as these.

12. Based on a technique described in Bass and Beckwith, n. 5 above.