The Design of Combinatorial Auctions for Procurement: An Empirical Study of the Chilean School Meals Auction*

Marcelo Olivares, Gabriel Y. Weintraub
Columbia Business School

Rafael Epstein, Daniel Yung
University of Chile

January, 2011

Abstract

In this paper we conduct an empirical investigation of a large-scale combinatorial auction (CA); the Chilean auction for school meals in which the government procures half a billion dollars worth of meal services every year. Our empirical study is motivated by two fundamental aspects in the design of CAs: (1) which packages should bidders be allowed to bid on; and (2) diversifying the supplier base to promote competition. We use bidding data to uncover important aspects of the firms' cost structure and their strategic behavior, both of which are not directly observed by the auctioneer. Based on these estimates we analyze and suggest important improvements to the auction design. Our results indicate that package bidding should be flexible enough so that firms can express their cost synergies due to economies of scale and density. However, we also found evidence that firms can take advantage of this flexibility by discounting package bids for strategic reasons and not driven by cost synergies. Because this behavior can lead to inefficiencies, we identify certain specific combinations that perhaps should be prohibited in the bidding process. Our results also suggest that market share restrictions and running sequential auctions promote competition in the long-run, without significantly increasing the short-run cost for the government due to unrealized cost synergies. Our results highlight that the simultaneous consideration of the firms' operational cost structure and their strategic behavior is key to the successful design of a CA. More broadly, our paper is the first to provide an econometric study of a large-scale CA, providing novel and substantive insights regarding bidding behavior in this type of auctions.

Keywords: combinatorial auctions, procurement, auction design, supply chain management, empirical, public sector applications.

*Acknowledgments: We thank Cristián Martínez, former Vice-Minister of Education of Chile, for his collaboration and the valuable insights he provided to us throughout this work. In addition, executives at JUNAEB and Jaime Catalán provided valuable data, information, and feedback. Sang Won Kim provided superb research assistance. We are thankful for the helpful comments received from Gérard Cachon, Estelle Cantillon, Fangruo Chen, Francesco Decarolis, Awi Federguren, Nicolás Figueroa, Ken Hendricks, Ananth Iyer, Elena Katok, Anthony Kwasnica and seminar participants at Carnegie Mellon, Columbia, Stanford, Penn State, Michigan, MIT, Cornell, INSEAD, LBS, the Workshop in Empirical Research in OM at Wharton, INFORMS conference, and the MSOM Supply Chain SIG. We also thank the Social Enterprise Program at Columbia Business School for their financial support.

1 Introduction

Auctions are increasingly becoming a common mechanism for supply chain procurement. Corporations and government use auctions to procure billions of dollars in inputs and services (Rothkopf and Whinston (2007) and Elmaghraby (2000)). Auction mechanisms are also becoming more sophisticated. In particular, combinatorial auctions (CAs), multi-unit auctions in which suppliers can bid for packages of items, have generated much interest in procurement applications.[1] The use of CAs has also been a widely debated topic in other contexts as well, such as the allocation of spectrum by the Federal Communications Commission (FCC). The main advantage of CAs is that they allow suppliers (who we also refer to as firms or bidders) to express cost synergies among units in the bidding process, which often results in a lower procurement cost for the buyer (who we also refer to as the auctioneer).

Practical experience and academic research have shown that the design of an auction may have an important impact on its outcome; good designs can result in large savings, while poor designs may lead to large losses for the auctioneer (Milgrom, 2004). Typically, the performance of an auction is critically determined by the firms' cost structure and their strategic behavior, both of which are usually not directly observed by the auctioneer. The main objective of this work is to conduct an empirical study of bidding data to uncover important aspects of the firms' cost structure and their strategic behavior in a large-scale procurement CA. Based on this information we evaluate potential changes to the current auction design. While we study a particular CA – the Chilean auction for school meals in which the government procures half a billion dollars worth of meal services every year – our method and analysis could be used more broadly in other CAs.

In this work we study two fundamental aspects of the design of procurement CAs. We do it in the context of single-round sealed-bid first-price CAs, because this is the format used in our application and several CAs in practice share the same format. When evaluating the auction design, we consider two commonly used criteria: (1) *optimality*, that is, minimizing the expected total payments to the bidders; and (2) *efficiency*, that is, maximizing social welfare by assigning the units to the set of bidders that achieves the most cost efficient allocation. We consider both objectives when evaluating improvements to the auction design.[2]

The first design issue we study is how to determine which packages should bidders be allowed to bid on. On one hand, the auctioneer should provide enough flexibility so that bidders can express their synergies in the bids. This mitigates the so called *exposure problem*: if package bidding is not allowed, a firm that exhibits synergies among two units may not bid below the costs for the individual items because of the risk of not getting the package. Indeed, under suitable conditions, allowing package bidding is a necessary condition for the efficiency (Rosenthal and Wang (1996), Rothkopf et al. (1998), Bykowsky et al. (2000)) and optimality (Levin, 1997) of the auction.

[1] See Elmaghraby and Keskinocak (2003), Hohner et al. (2003), Metty et al. (2005), Caplice and Sheffi (2006), Bichler et al. (2006), Sandholm et al. (2006), and Sandholm (2007) for several recent applications.

[2] In many government procurement auctions, like the one we study here, the declared objective is efficiency. In contrast, optimality is the primary objective in auctions run by private firms.

However, allowing too much flexibility on the package bids can also hurt the efficiency and optimality of the auction, while complicating the bidding process unnecessarily. A bidder in a CA may have incentives to submit package bids and offer discounts even in the absence of synergies, a phenomenon we refer to as *strategic bundling*. In this way a bidder may win a combination of units for which it is not the most cost efficient provider. In addition, due to the so called *threshold problem*, in which local bidders free ride on each other to outbid a global bidder in a combination, package bidding can also result in a more expensive allocation (Milgrom, 2000).

The previous discussion suggests that to minimize the negative impact of strategic behavior, the auctioneer should only allow package bidding among units for which synergies are sufficiently large. Because the precise cost structure of firms is often unknown to the auctioneer, we seek to estimate the magnitude of cost synergies among different units using bidding data. (Cantillon and Pesendorfer (2006b) analyze a similar problem in an auction for public bus routes and provide further details on this issue).

The second design issue we study is how to promote diversification and competition among bidders in a context in which cost synergies are important. This usually involves the following trade-off for the auctioneer: if cost synergies are significant, it may be efficient and optimal in the short-run to allocate all units to one or few firms. On the other hand, this could depress competition in the bidders' market for future auctions, as inactive firms may find it hard to compete head-to-head with incumbents, hurting efficiency and increasing expected payments in the long-run. We investigate two mechanisms that together help to intensify competition: (1) imposing market share restrictions for bidders; and (2) awarding the units in multiple sequential auctions. The latter could intensify competition if incumbent firms that won units in previous auctions bid more aggressively due to cost advantages given by their installed base in nearby units. If cost synergies are important, these mechanisms can hurt the efficiency and optimality of the auction, though, because they may prevent bidders to submit package bids containing a large number of units and fully expressing those synergies. Therefore, to study the effectiveness of these measures we compare the intensity of cost synergies with the impact that competition has on bid prices.

Analyzing the two design issues discussed above calls for an empirical analysis to measure the intensity of cost synergies as well as the effect of incumbency and competition. Although CAs have received considerable attention from different academic communities including management science/operations research, economics, and computer science,[3] to the best of our knowledge there are no other empirical studies that use field data on CAs, except for the notable work of Cantillon and Pesendorfer (2006b). However, the application studied by Cantillon and Pesendorfer (2006b) exhibits few package bids and their estimations do not suggest the presence of synergies. Therefore, as far as we know, our paper is the first to provide an econometric study of a real-world CA that exhibits an important amount of package bidding and synergies, providing novel and substantive insights into bidding behavior in CAs. In that sense, we believe our work constitutes an important contribution to the literature.

[3]For surveys on work in CAs, see Pekec and Rothkopf (2003), de Vries and Vohra (2003), Hoffman (2006), and Blumrosen and Nisan (2007), and the recently edited volume by Cramton et al. (2006)

The auction we study is the Chilean auction for school meals (see Epstein et al. (2002), Epstein et al. (2004), and Catalán et al. (2009) for a detailed description). The Chilean government provides breakfast and lunch for 2.5 million children daily in primary and secondary public schools during the school year. In a developing country where about 14 percent of children under the age of 18 live below the poverty line, many students depend on these free meals as a key source of nutrition. Since 1999 the contracts are awarded through a single-round, sealed-bid, first-price CA. Meal services are standardized and firms compete in prices. Chile is divided into territorial units and firms can submit bids on any package of units defining the combinatorial character of this auction. Around 20 firms participate in each auction; each firm submit many bids (hundreds or even thousands) ranging from just one to several units. The CA has been used every year since its inception awarding more than $3 billion of contracts (US$577 million were awarded in 2008).

Our data set contains bids for packages of different sizes that contain units in different locations. Our empirical strategy is based on using variation in bid prices for different combinations to quantify the discounts associated to two types of cost synergies: (1) *economies of scale* that depend on the total number of units served and, for example, are generated by volume discounts in purchasing inputs; and by (2) *economies of density* that arise when serving nearby units, for example, due to the use of common logistics infrastructure. There are three important challenges in the estimation. First, firms are heterogeneous and may have local cost advantages, some of which are not observable in our data. To mitigate a potential omitted variable bias, we exploit the combinatorial character of our data to control for unobserved local cost advantages. A second challenge arises from the endogeneity of competition, which can be related to the costs of providing service. We exploit the panel structure of our dataset to address this issue. The third challenge is that the discounts for combinations cannot be entirely attributed to cost synergies; potentially, they could also be explained by markup adjustments due to strategic bundling. To evaluate this alternative explanation, we conducted a detailed analysis to empirically test for the presence of incentives that lead to strategic bundling and how firms respond to them.

Our analysis of the Chilean auction indicates that cost synergies due to economies of scale and density are important, together they can be as high as 8% of the average bid price. Interestingly, our estimates indicate that cost synergies get practically exhausted after combining seven or more units. While we do find evidence of strategic bundling, on average, they only explain a small fraction of the discounts in our data. In addition, the threshold problem appears to be small. However, we do identify specific units for which strategic bundling is more severe and could be leading to inefficient allocations. We also find that incumbent firms bid more aggressively for units in which they have nearby operations and, as a consequence, all firms (including firms that are not local incumbents) bid more aggressively as local competition intensifies. The effect of competition is comparable in magnitude to the discounts from cost synergies.

We use these insights to evaluate and recommend improvements to the auction design along the two dimensions previously discussed. These recommendations are being considered by the Chilean government to redesign future auctions. First, our results indicate that package bidding that allows firms to express their cost synergies due to economies of scale and density seems appropriate. However, for some specific units where strategic bundling appears to be severe, we recommend to evaluate running separate auctions. This could also serve as a field-experiment to evaluate the potential inefficiency of strategic bundling behavior in CAs.

Our second recommendation is that market share restrictions and running sequential auctions are desirable in the Chilean school meals CAs. Together, these foster supplier base diversification and promote competition in the long-run, without significantly increasing the short-run cost for the government due to unrealized cost synergies. More broadly, our results highlight that the simultaneous consideration of the firms' operational cost structure and their strategic behavior is key to the successful design of a CA.

The rest of the paper is organized as follows. Section 2 reviews related literature. Section 3 describes the Chilean auction for school meals and our data set. The regression analysis of bidding data is presented in Section 4 and results are presented in Section 5. In Section 6 we study strategic bundling. We evaluate potential redesigns to the current auction in Section 7 and conclude in Section 8.

2 Related Literature

Our paper is related to streams of literature in operations management, empirical industrial organization, and experimental economics as we now describe.

First, our paper is related to the recent literature in operations management that studies procurement and on-line auctions.[4] Most of these papers develop models that introduce novel and important operational aspects to traditional auction settings, providing a theoretical or computational analysis. Among these papers the most related to our work are Elmaghraby (2005) that studies the effect of economies of scale and bidders' heterogeneity in production capacity on auctions' performance, and Chen et al. (2005) that studies auctions in a supply chain network explicitly considering transportation costs. None of these papers, however, provide an empirical analysis as we do here.

Our work is related to the extensive empirical auctions' literature in economics (see Hendricks and Porter (2007) Athey and Haile (2006), and Paarsch and Hong (2006) for several surveys). There are several papers that study multi-unit auctions to learn about cost synergies. Ausubel et al. (1997) and Moreton and Spiller (1998) estimate synergies for wireless providers in spectrum auctions by the FCC. Gandal (1997) estimates synergies in an auction for cable television licenses. Marshall et al. (2006) estimates synergies on bidding for the Georgia school milk market using a structural model of simultaneous first-price auctions for multiple homogeneous units. However, in all these papers the auction format do not allow for package bidding. The paper most related to ours is Cantillon and Pesendorfer (2006b) that estimates a model for the CA for London buses routes. While our approach is reduced form, their approach is structural. Because the auctions for London buses routes are small (a maximum of three units are auctioned in each one), a structural approach similar to approaches previously used in single-unit auctions is computationally feasible.

[4]For some references see Keskinocak and Tayur (2001), Pinker et al. (2003), Tunca and Zenios (2006), Gallien and Gupta (2007), Chen (2007), Kostamis et al. (2009), Wan and Beil (2009), and Chu (2009).

Our work is also related to other articles that use field data to test theoretical predictions of equilibrium behavior in multi-unit auctions, such as Hortacsu (2002) and Hortacsu and Puller (2008). List and Lucking-Reiley (2000) study multi-unit auctions in field experiments. These articles, however, do not study combinatorial package bidding.

There is also previous work studying bidding behavior in multi-unit auctions using controlled laboratory experiments. Katok and Roth (2004) compare different formats of multi-round auctions and, consistent with theoretical predictions, their results suggest that auctions formats which permit package bidding tend to mitigate the exposure problem at the expense of reinforcing the threshold problem. Kwasnica et al. (2005) conduct experiments to compare two ascending auction formats used by the FCC to allocate spectrum, which also involve a trade-off between the exposure and threshold problems. Kagel and Levin (2005) conduct experiments to compare bidding behavior in sealed-bid versus ascending-bid uniform price multi-unit auctions. Note that the school meals auction we analyze is a single-round, sealed-bid, first-price CA, which is different from the auction formats studied in these papers. In that sense, our work is more directly related to Chernomaz and Levin (2009) that studies a similar auction format for two homogeneous units with and without package bidding. They show examples for which if cost synergies are small, allowing for package bidding can lead to a less efficient outcome.

3 The Chilean Auction for School Meals

In this section we describe the Chilean auction for school meals, we specify the design issues raised in Section 1 in this context, and we describe our data set. We also describe some interesting patterns observed in package bids.

3.1 Description of the Chilean Auction for School Meals

We present a brief description of the auction process (see Epstein et al. (2002), Epstein et al. (2004), and Catalán et al. (2009) for a more detailed description). Junta Nacional de Auxilio Escolar y Becas (JUNAEB) is a government agency in Chile that provides breakfast and lunch for 2.5 million children daily in primary and secondary public schools during the school year.

Since 1999 JUNAEB assigns its school meals service contracts through a single-round, sealed-bid, first-price CA, that was fully implemented for the first time that year and has been used ever since. For the purposes of the auction, Chile is divided into approximately 100 school districts or territorial units (TUs). Firms can submit bids on various groupings of TUs defining the combinatorial character of this auction. This mechanism is motivated by the belief that firms are subject to cost synergies that arise from operational advantages when serving multiple TUs. JUNAEB holds auctions in one-third of the TUs every year, awarding three-year contracts. Figure 1 presents a map of Chile with the TUs auctioned each year.

The auction process begins when JUNAEB contacts and registers potential vendors. The agency then evaluates the companies from a managerial, technical and financial point of view, and eliminates those that do not meet minimum reliability standards. Qualifying vendors are classified according to two characteristics: their financial capacity (based on data from the firms' balance sheets), and their managerial competence. Usually, firms below a minimum level of managerial competence are not allowed to participate in the auction. Potential vendors then submit their bids through an online system. Meal plans are standardized and firms compete in prices. Upon winning a contract, the firm is responsible for managing the entire supply chain associated to all meal services in the corresponding TUs, starting from sourcing food inputs going all the way to cooking and serving the meals in the schools.

A bid can cover any combination from one to 8 TUs and specifies the price for which the firm would serve all meals included in the TUs in the combination. Vendors can submit many bids and each package bid is either fully accepted or rejected (i.e. the mechanism does not allocate a fraction of a bid); most firms submit hundreds or even thousands of bids.

The allocation is chosen by selecting the combination of bids that supply all of the TUs at a minimum cost. The problem is formulated as an integer linear program that incorporates other considerations and side constraints. An important set of constraints put limits on the maximum number of meals that can be assigned to any given firm, both nationally and in specified geographical regions (to encourage competition in the suppliers' market).

3.2 Design Issues in the Chilean Auction

In this section we discuss the two design issues presented in Section 1 in the context of the school meals procurement auction: (1) which package bids should be allowed; and (2) how to diversify the supplier base to promote competition.

Package Bidding

Package bidding should be sufficiently flexible to let bidders express cost synergies among units. Indeed, in the current design all possible combinations are allowed in the bidding process. However, too much flexibility can hurt efficiency and increase expected total payments if bidders strategically take advantage of package bidding. Therefore, we suggest that the auction mechanism only allows package bidding among units for which cost synergies are sufficiently large. Given the cost structure associated to serving a typical set of TUs we identify two types of cost synergies:

1. **Economies of Scale.** Approximately 50% of the total cost of serving meals is related to food inputs that include perishable and non-perishable items. Most of the food is purchased centrally and firms can get important volume discounts from their providers. These discounts result in economies of scale when serving multiple TUs. Note that these synergies are only a function of the total volume of meals served and are independent of the proximity between the units served.

2. **Economies of Density.** Logistical costs associated to transportation and administration amount to around 9% of total cost. Some of these costs are fixed and can be shared by TUs that are close to each other (e.g., by sharing a local warehouse and a distribution network), resulting in economies of density. Note that these synergies are a function of the proximity of the units served.

The distinction between these two types of cost synergies is important from an auction design perspective. If economies of density are predominant, then it could be preferable to restrict combinations to units that are closely located. If economies of scale are predominant, then there are simpler auction mechanisms that would allow bidders to express them in their bids (e.g. providing prices for each unit and a discount curve). We therefore seek to identify separately these two sources of cost synergies using bidding data.[5]

Promoting Competition

The school meals auction exhibits two specific characteristics that help to diversify the supplier base and promote long-term competition among bidders:

1. The current mechanism imposes strict market share restrictions in the allocation of units to bidders. There are several such restrictions: (1) a maximum number of TUs that each firm can be allocated in any given auction; this maximum is based on the financial evaluation conducted by JUNAEB every year and therefore can be different across firms and auctions, ranging from 2 to 8 TUs; (2) at any point in time, the total standing contracts of any firm cannot exceed 16% of the total number of meals included in all TUs in the country; (3) a local market share constraint that limits the number of TUs a firm can be awarded with in pre-established geographical regions (this limit varies across regions); and (4) a constraint that enforces a minimum number of firms (around 10) included in the allocation of each auction.

2. The TUs are split into multiple sequential auctions which are conducted in consecutive years. Figure 1 shows how the different TUs across the country are grouped on multiple auctions. In general, TUs in adjacent geographic regions are awarded in different years, so that each auction awards units scattered all along the country rather than concentrated in a specific area.

The latter generates *local incumbents* – for most of the TUs in a given auction (except the first one in 1999), there are firms with on-going contracts for nearby TUs awarded in *previous* auctions. Because these local incumbents already have an installed base in the proximity of some TUs, they may have cost advantages and therefore bid more aggressively. If other firms anticipate this, auction theory predicts that a non-incumbent firm should also bid more aggressively (Krishna, 2002). Hence, an increase in the intensity of *local competition*, measured as the number of local incumbents, is likely to reduce bid prices from all participant firms. The market share restrictions would reinforce this local competition effect as it tends to

[5]Approximately 25% of the operating costs is related to labor dedicated to the preparation of the meals at the schools. The number of servers per school and their salaries are heavily regulated and not subject to cost synergies, so we ignore this cost in our analysis.

increase the number of local incumbents as well as the pool of firms that can actively compete against the local incumbents.

These benefits notwithstanding, there are also potential disadvantages of the current design. In particular, the market share restrictions together with running multiple auctions may result in many firms providing service in narrowly defined geographical areas, precluding them from fully realizing their cost synergies. It is therefore useful to compare the magnitude of the local competition effects relative to the cost synergies in order to evaluate the overall effectiveness of the current auction mechanism.[6]

3.3 Description of the Data

Our data set contains all bids presented by all firms in all auctions between 1999 and 2005. We also collected information about the firms and the TUs on each auction. In this section we describe this data and provide some summary statistics.

For each auction, we know the identity of all participant firms, which are around 20 each year.[7] We have data on all the bids presented by each firm. Each bid specifies a set of TUs and the price per meal for which the firm would serve all units in the combination.[8] Table 1 provides summary statistics regarding the number of participant firms and the average number of submitted bids *per firm*. Firms submit from tens to several thousand bids. The total number of bids submitted by all firms was around 4,000 in 1999 and increased substantially since then; in 2003 it was 43,000. In 2004 the process of submitting bids was completely digitalized for the first time; before, bids were submitted in paper. That year the total number of submitted bids jumped to 190,000 and it was 115,000 in 2005.

The financial capacity evaluation made by JUNAEB determines the maximum number of units a firm can be awarded with. Firms cannot submit bids that contain more units than this upper bound. We group firms in four categories according to their classification: L, LM, M and S, that correspond, respectively, to Large, Medium-Large, Medium and Small. Roughly, L firms can win up to 8 units, LM firms up to 6-7 units, M firms up to 4-5 units, and S firms up to 2-3 units (there are some exceptions to this rule). In Table 1 we also provide the number of participant firms and the average number of submitted bids *per firm* disaggregated by firms' classification. Due to these restrictions, large firms submit more bids on average because they are allowed to bid on larger packages. Note that on average firms only submit a small subset of all possible package bids: the number of possible combinations for L firms is about 19.5 million; for LM firms is 3.5 million; for M firms is 165,000; and for S firms is 3,000. In the next section we describe some patterns on how the submitted packages are selected.

In addition, we know the set of winning bids in each auction and, therefore, at every point in time we

[6]Our analysis is focused on the effects of *local* competition. Although more global competitive effects could be important, our application does not have variation in this dimension because the number of bidders on auctions that award the same set of TUs is relatively stable over time.

[7]The average number of firms entering and exiting the market each year is close to three.

[8]The current allocation mechanism uses some criteria to eliminate unrealistically low bids. In our analysis, we do not eliminate bids.

know the identity of the firm serving each TU. On average, in each auction there are 11.5 winning bids distributed among 10 winning firms. The average size of the winning packages is 2.7 units. In Table 2 we describe the number of winning bids over all auctions disaggregated by firms' classification and size of the winning package bid. While Large firms win more frequently, other firms, including the Smallest firms win quite often (15 out of 69 bids). In addition, small packages of 1 or 2 units also win quite often (35 out of 69 bids). Similar patters are observed in each individual auction.

We have detailed data about TUs including the location and population of all schools in them. TUs are heterogeneous in their size and density of school population, factors that strongly affect the procurement cost. For example, there are units in urban locations that cover a small geographic area with high school population, but also units in isolated parts of the country that cover large areas with a low population density. Urban TUs have an average size of 2.8 million meals per year and an average of 71 schools; TUs in rural areas have 2.2 million meals per year and 99 schools on average. The average price per meal over all bids is \$0.75, and the average unit size among all bids is 2.5 million. Typically, each TU is auctioned every three years so that each unit is auctioned at least twice in our data set, which is an important aspect for the estimation method described in Section 4.

3.4 Package Bidding

In this section we describe patterns observed in the data regarding how firms select units in package bids. Not only these patterns are interesting by itself, but they will also inform the regression analysis of bid prices that we do in the next sections.

We start by showing in Figure 2 an histogram of the number of TUs in a bid disaggregated by firms' classification. We observe a significant degree of variation in bid sizes and an important presence of combinatorial bidding. Firms submit bids as small as one TU to large ones with eight TUs. As expected, larger firms tend to submit larger packages. Note, however, that the mode of the package size for Large firms is six; this is below the maximum package size this firms can win. In addition, Figure 3 shows a scatter plot of bid prices (per-meal, in US\$) vs. the size of the package bid (measured in million meals per year). The figure suggests that bidders use volume discounts in their package bids.

The previous analysis suggests how firms select the size of their packages. The rest of this section provides a series of analysis to study how firms select the units in these packages. First, we note that bids from Large firms essentially include all units being auctioned in at least one package. On average, the fraction of units covered is 95%. For the rest of the firms, this number is somewhat smaller, around 80%. The TUs for which these firms do not bid tend to be co-located, suggesting that some smaller firms select specific geographical regions in which they do not compete.

Second, Figure 4 shows the average maximum distance among the TUs contained in a bid, for bids of different sizes.[9] Firms make bids that includes TUs that are close to each other and also TUs that are

[9]The distance between two TUs is the bird-fly distance between the weighted geographic centers of the respective TUs. The weighted geographic center of a TU is calculated as the weighted average latitude and longitude of the schools it contains, weighted

far apart. The figure also shows the expected average maximum distance if the packages were selected randomly. We observe that in the actual bids firms tend to select combinations of TUs that are closer to each other when compared to a random pattern.

Motivated by the previous observation and that we observe significant heterogeneity with respect to how many times a unit appears in a package, we explore in more detail how firms select units in package bids. For this purpose, for each firm-auction pair we constructed a graph where each node represents a different TU and has a value equal to the number of package bids the TU is included on. Each edge represents a pair of TUs and has a value equal to the number of package bids the pair is included on (if the value is zero there is no edge between those units). We call these values the *popularity of a TU* and *the popularity of a pair of TUs*, respectively. We define the *degree of a TU* as the number of different units it is packaged with (or equivalently, the degree of the node representing the TU in the graph).

We separately regress the popularity and degree of firm-auction-TU nodes against several characteristics of the firm and unit. The results are shown in Table 3, reporting standardized coefficients (normalizing all the variables to have a standard deviation of one so that the coefficients can be interpreted as the changes in the dependent variable – measured in standard deviations – when the covariate changes one standard deviation). Both regressions include firm-auction fixed effects and the R-square reports the explained variation within a firm-auction. The results suggest that larger and more dense units (in terms of meals per school) are more popular and have a higher degree; these units tend to be cheaper to serve. In particular, meals per school is among the factors that explain most variation in the popularity and degree of a unit. In addition, a TU is more popular and connected for firms that are incumbents, in the sense that they are trying to renew the contract in that TU or that they have ongoing operations in neighboring TUs.

We also estimated a regression where the dependent variable is the popularity of a pair of TUs for a given firm in a particular auction. An important factor in this regression is distance: increasing distance by two standard deviations decreases the popularity of a pair of TUs by .13 standard deviations. Also, the popularity increases by 0.5 standard deviations for two units in the same region. A pair of units for which the firms is looking to renew contracts tends to be more popular (0.5 standard deviations higher); a smaller effect is also observed when the firm operates nearby TUs in both units.[10]

The previous analysis suggests that firms focus their bidding efforts in certain units and certain combinations of them. Moreover, the following analysis confirms that package bids are built systematically. We say that a bid on a package a by firm f in auction t is "nested" if there exists at least one package bid from the same firm in the same auction for a package a' such that $a \subset a'$. Small packages (say of 1 or 2 units) are likely to be nested. However, given the large number of possible combinations and the relatively small fraction of this set that firms bid on, it becomes more likely that larger packages are not nested. In Figure 5 we show the fraction of nested bids observed in the data as a function of their size and compared them with

by the school populations.

[10]This regression includes firm-auction fixed effects. Due to limited space we do not report the results but they can be obtained from the authors upon request.

the fractions induced by a set of randomly generated bids that is consistent with the total number and sizes of bids submitted by each firm. We observe that firms tend to nest bids much more significantly than in a random sample of bids, suggesting they systematically use a "bottom-up" approach to construct the package bids.

The cost discovery and bid submission process of small firms – who bid on packages of up to 2 units – is relatively simpler, because the total number of packages they can bid on is manageable (around 500 hundred). In contrast, for Large firms that can bid on packages of up to 8 units, this number is around 20 million and it is unrealistic to expect that firms will evaluate all possible combinations. In fact, firms only submit a small fraction of these combinations. Moreover, our analysis suggests that firms are systematic when selecting package bids and focus their efforts on certain type of packages. Perhaps this is not surprising as we have learned anecdotally from conversations with JUNAEB that many of the firms use computer programs to generate the bids (e.g. spreadsheet macros). This process help firms to methodically evaluate the costs of serving the different packages. On top of these costs, firms add a mark-up that can depend on the package. This provides further support that the bidding data that we use in the next sections conveys useful information regarding firms' economic costs and their strategic behavior.

To summarize, the analysis in this section suggests that package bids are built systematically and exhibit some distinguishable patterns. First, combinatorial bidding is widely observed suggesting the presence of economies of scale. Second, firms tend to include TUs that are close to each other in the same bid suggesting the presence of economies of density. Third, firms tend to bid more frequently on units for which they are incumbents, suggesting incumbency advantages. In the next section we test these hypothesis and quantify these effects with an econometric analysis of the bid prices.

4 Econometric Analysis of Bidding Data

In this and the following sections we provide an econometric study to measure the intensity of economies of scale and density as well as the effect of local incumbency and competition; these measurements will shed light on the two design issues discussed in Section 1. In particular, we use an econometric model to study the factors that explain variation in bid prices. Since most of the variation in the prices is explained by the number of meals included in the package, we normalize the total bid price dividing it by the number of meals. Throughout, we use i to index territorial units (hereon units), a to index sets of units (also referred to as combinations or packages), t to index auctions, and f to index firms. The dependent variable in our empirical analysis is the price-per-meal, defined as total bid price (in 1999 Chilean pesos) divided by the number of meals of a combination a submitted by a firm f in a specific auction t, and is denoted by b_{aft} (hereon refereed to as the bid price).[11]

There are two main factors that explain the variation in bid prices. First, there is heterogeneity in the

[11] We used the consumer price index of food to normalize prices to real 1999 Chilean pesos.

individual prices of the units, which arises from differences in costs and markups associated with each unit. This heterogeneity is across units (e.g. some units are more expensive to operate than others), across firms (e.g. some firms could be more efficient in particular regions, or charge different markups) and across auctions (e.g. the price of inputs may vary from year to year). Second, different combinations are subject to different discounts and the data shows that larger combinations tend to have, on average, a lower bid price. We expect that a significant portion of these discounts arises from cost synergies due to economies of scale and density.

To account for these sources of variation, our specification decomposes the bid price into two parts: (1) the stand-alone prices of the units contained in the package, which captures heterogeneity across units; and (2) discounts for combinations, which captures *interactions* among the units contained in the package and is therefore package-specific. Let δ_{ift} be the average price of a unit i for firm f in auction t (we also refer to δ_{ift} as the average unit price). Let v_i be the size of unit i (measured in million number of meals per year), and accordingly define, with a slight abuse of notation, the size of a combination a as $v_a = \sum_{i \in a} v_i$. We model the bid price as:

$$b_{aft} = \sum_{i \in a} \delta_{ift} \cdot \frac{v_i}{v_a} - g_{ft}(a). \tag{1}$$

Equation (1) decomposes the bid price as a weighted average of the average unit prices δ_{ift} minus a discount function $g_{ft}(a)$. The discount function depends on the package a and could potentially be different across firms and auctions.

Before describing in further detail this specification, it is useful to link the two auction design issues we study with equation (1). Recall that the first design issue is concerned about which combination bids should be allowed in the auction. This is related to the discount function $g_{ft}(a)$; combinations should be restricted to packages a for which the discount due to costs synergies is substantial. The second design issue is related to promoting local competition in the auction. An active presence of local incumbents for a unit i should enhance competition and lead to lower average unit prices δ_{ift} for that unit from all firms. The rest of this section describes details on the specifications of the discount function and average unit prices which capture these relevant effects.

4.1 Estimation of the Discount Function

Discounts for packages can be a consequence of cost reductions arising from synergies, but also perhaps of other factors that lead to adjustments in the markups charged by firms, such as strategic bundling. In this section, we construct discount functions that, in the absence of markup adjustments, would reflect economies of scale and density. In Section 6 we try to quantify what fraction of the discounts are explained by markup adjustments due to strategic bundling, as oppose to by cost synergies.

As mentioned earlier, there are two main forms of cost synergies in the procurement process: economies of scale, which only depend on the total size of the combination (v_a), and economies of density, which depend on the geographic location of the units in a. Accordingly, we define two separate discount functions: (i) $g^{scale}(v_a, \beta^{scale})$, called the *scale discount function*, intends to capture economies of scale and thereby is a sole function of v_a; and (ii) $g^{dens}(a, \beta^{dens})$, called the *density discount function*, intends to capture economies of density and therefore could vary for each set a.[12] Recall that the discount functions represent discounts per *meal*. These functions are parametrized by the vectors β^{scale} and β^{dens} which measure the intensity of the respective discounts. These parameters could vary across firms and auctions, but for simplicity we assume they are common to all firms and auctions (Section 5.3 shows results from a more flexible specification to capture firms' heterogeneity in this dimension). Because we want to measure discounts arising from the interaction among units, both discounts functions g^{scale} and g^{dens} are set to zero for bids with a single unit (i.e., when a is a singleton).

In estimating g^{scale} and g^{dens}, it is important to allow a sufficiently flexible specification to capture a potential saturation effect of these discounts. Note that $g^{scale}(v_a, \beta^{scale})$ is a function of a single variable and is therefore relatively easy to approximate with a flexible, yet parsimonious, specification. In contrast, representing $g^{dens}(a, \beta^{dens})$ compactly is more challenging because the number of possible sets a is enormous. To overcome this difficulty, we make this function depend only on measures that summarize the geographic proximity of the units in a. We describe this procedure next.

For a given set a, we identified *clusters* of units located within a circular perimeter of 150 km. radius.[13] Let $Cl(a)$ be the set of clusters formed by the units in a, which form a partition of a. The size of each cluster $c \in Cl(a)$ can be calculated as the sum of the sizes of its units, $v_c = \sum_{i \in c} v_i$ (with some abuse of notation). Economies of density would imply discounts which are non-decreasing in the size of a cluster: as more demand is concentrated within the limits of the cluster, the cheaper it is to supply each unit. For example, given two packages a and a' of the same size ($v_a = v_{a'}$), economies of density would imply larger discounts for the package with fewer and larger clusters. Accordingly, we define discounts at the level of a cluster as a function of its size, denoted $g^{clust}(v_c, \beta^{dens})$. Again, this discount function is set to zero when the cluster contains a single unit, so that it indeed captures discounts from combining units that are co-located. Given a specification for $g^{clust}(v_c, \beta^{dens})$, the overall density discounts (per meal) are given by:

$$g^{dens}(a, \beta^{dens}) = \sum_{c \in Cl(a)} g^{clust}(v_c, \beta^{dens}) \cdot \frac{v_c}{v_a}, \tag{2}$$

that is, a weighted average of the discounts generated from each cluster. The following example helps to illustrate the calculations of equation (2) and the logic behind it.

Consider four units, labeled 1 through 4. To simplify the discussion, we assume all units have the same size equal to one and have similar average unit prices. Due to their location, {1,2,3} form a cluster but unit 4 does not form a cluster with any of the other units. Consider two hypothetical bids: bid A for units {1,2,4},

[12] In this sense our approach is related to Caves et al. (1984) that measures and distinguishes between economies of scale and density for the airline industry.

[13] In section 5.3 we discuss results with alternative cluster definitions. We describe the clustering algorithm in Appendix A.

and bid B for {1,2,3}. Both bids have the same size, so scale discounts will be similar. But the clusters formed within each bid are different. Bid A has two clusters, {1,2} and {4}, one of which is a singleton and therefore does not get any discounts. Bid B has a single cluster of size 3. Let $g_v = g^{clust}(v, \beta^{dens})$ be the discount per meal for a cluster of size v. The density discount for bid A is equal to $\frac{2}{3}g_2$. For bid B, the discount is equal to g_3, which is larger than the discount on bid A because: (1) g_v is increasing in the size of the cluster; and (2) all three units in bid B benefit from the density discounts, compared to only two units in bid A. Also note that a bid for package $\{1, 2\}$ – which is a subset of bid A – could have a larger discount per meal than bid A if density discounts are sufficiently large (relative to scale).

To incorporate scale and density discounts into the bid price equation (1), we specify the discount function as:

$$g_{ft}(a) = g^{scale}(v_a, \beta^{scale}) + g^{dens}(a, \beta^{dens}) - \varepsilon_{aft}, \tag{3}$$

where the error term ε_{aft} captures idiosyncratic discounts (or charges) for combination a. The specific parametric forms of g^{scale} and g^{dens} used in the estimation are described in Section 5. Replacing (2) and (3) into (1) gives the regression equation:

$$b_{aft} = \sum_{i \in a} \delta_{ift} \frac{v_i}{v_a} - g^{scale}(v_a, \beta^{scale}) - \sum_{c \in Cl(a)} g^{clust}(v_c, \beta^{dens}) \cdot \frac{v_c}{v_a} + \varepsilon_{aft}. \tag{4}$$

We seek to estimate the parameters $(\beta^{dens}, \beta^{scale})$ that characterize scale and density discounts. In order to estimate cost synergies with regression (4), there are two important challenges that need to be addressed. First, discounts for combinations cannot be entirely attributed to cost synergies if firms adjust their markups when submitting package bids. Second, firms may have local cost advantages that are not observable in the data which could confound the effect of economies of density. We now discuss how we address each of these identification issues.

The first issue, as we discussed before, is whether we can interpret the estimated discount functions g^{scale} and g^{dens} as cost synergies. Recall that firms may have incentives to adjust their markups and provide discounts for packages for pure strategic reasons even in the absence of cost synergies. In Section 6 we study whether markup adjustments due to strategic bundling explain part of the observed discounts. There, we conduct an additional investigation which: (1) analyzes the incentives that lead to strategic bundling and study whether these are present in our data; and (2) empirically test how firms respond to these incentives. Our results will suggest that although bidders do engage on strategic bundling, the effects on the total discounts are small. Hence, we will interpret scale and density discounts as mostly been explained by cost synergies.

There is a second identification issue related to economies of density. As noted by Holmes and Lee (2009), distinguishing economies of density from local advantages can be challenging. To explain, Figure 4 suggests that bids are more likely to include units which are located in close proximity, which could be interpreted as evidence of economies of density. However, an alternative explanation is that firms have local cost advantages in specific regions and are more likely to submit bids for units where they have an advantage. Because advantage are local, packages with co-located units are more likely to be submitted by firms with lower costs and therefore have a lower price. Since we do not observe the local cost advantages of firms, using variation across bids submitted by different firms will tend to overestimate economies of density, as we would attribute the lower price of co-located units entirely to this type of synergy.

To eliminate this source of bias in measuring economies of density, regression (4) controls for the average unit prices, δ_{ift}, which are firm-unit-auction fixed effects that capture firm-specific cost advantages in a unit. Hence, scale and density discounts are estimated using variation across different *combinations* submitted by the same firm over the same set of units in the same auction. Consequently, our estimation strategy requires a large number of combination bids submitted by the same firm in order to obtain consistent estimates of the parameters in (4). Note that the estimation provides estimates of the average unit prices δ_{ift} along with β^{dens} and β^{scale}.

4.2 Analysis of Average Unit Prices

The average unit price δ_{ift} captures the average price charged for supplying a unit before accounting for the interactions that may arise with other units. It is affected by the firm's cost in supplying the unit –net of any cost synergies with other units in a combination– plus a markup. The unit's cost is affected by the characteristics of the unit (e.g. size and location of the schools) and other specific costs advantages that a firm may have. An example of firm specific advantages is when a bidder has a lower cost of supplying a unit because it has a nearby warehouse used to serve other related businesses in the area. Consequently, the average unit price should be lower for firms that are local incumbents; recall that we defined local incumbents as firms with ongoing contracts for *nearby* TUs awarded in a previous auction (consistent with our cluster definition we say that two TUs are near if the distance between their weighted geographical centers is less than 150 km.). As discussed in section 3.2, the presence of local incumbents could generate additional price reductions from all firms (including non-incumbents) through a local competition effect.

For the estimation, we define a binary variable $LocIncumb_{ift}$ indicating if firm f is a local incumbent for unit i in auction t. Local competition is measured as the number of rival firms that are nearby incumbents, defined as $LocComp_{ift} = \sum_{f' \neq f} LocIncumb_{if't}$. We seek to estimate the following regression:

$$\delta_{ift} = \beta_1 LocIncumb_{ift} + \beta_2 LocComp_{ift} + \beta_x X_{ift} + u_{ift}, \tag{5}$$

which captures the effect of local incumbents and competition on the average unit price δ_{ift}. The vector X_{ift} includes other controls (described shortly), and the error term u_{ift} captures other unobservable factors.

Note that the dependent variable in regression (5), δ_{ift}, is not directly observed in our data. Therefore, we follow a two-stage estimation approach. The *first-stage* regression (4) (which has bid price as the dependent variable) provides estimates of the stand-alone average prices, $\hat{\delta}_{ift}$ (along with the parameters characterizing discounts). The *second-step* regression replaces δ_{ift} by its estimate $\hat{\delta}_{ift}$ in equation (5) to estimate the coefficients of *LocIncumb* and *LocComp* (as well as β_x). If $\hat{\delta}_{ift}$ is a consistent estimator of δ_{ift} and assuming the usual orthogonality conditions ($E(u_{ift}|LocIncumb, LocComp, X) = 0$), $(\beta_1, \beta_2, \beta_x)$ can be estimated consistently through Ordinary Least Squares. However, the covariates *LocIncumb* and *LocComp* are endogenous and its effect could be confounded by other factors not captured in the regression. To mitigate endogeneity bias, we include several controls in X_{ift}, which we discuss in detail in what follows.

LocComp measures the number of rival firms that are incumbent to a unit and is therefore affected by the unit's location. Units located in urban areas tend to be smaller and have more "neighboring" units, and thereby tend to have more incumbent firms. Because these territories are also more densely populated and have better transportation infrastructure, they tend to be cheaper to supply. Hence, unobservable unit costs could confound the effect of competition, generating a negative bias on the coefficient on *LocComp.* To mitigate this source of bias, we include unit fixed-effects which control for time-invariant characteristics of a unit. Note that the panel structure of our dataset –with two or more auctions observed for each unit – is essential for the identification of local competition effects. We also include the following controls to capture costs associated to a unit which can vary across auctions: (1) the size of the unit (*Size,* measured as the number of meals), which reduces the cost of serving the unit due to scale economies; and (2) the fraction of "*Special Meals*" supplied, which increase the costs of serving the unit.

The *LocIncumb* covariate could also lead to endogeneity bias. Similar to the bias regarding economies of density described earlier, a local incumbent firm that was previously awarded units in the "neighborhood" of unit i is a potential indication that it had a local cost advantage in the vicinity. If this advantage is persistent over time, the firm would have a lower cost of serving unit i in the *current* auction and therefore could submit a lower bid. In this alternative explanation, *LocIncumb* is a proxy for permanent local cost advantages, which would bias the estimate of the *LocIncumb* coefficient. Note that unit fixed effects do not control for this source of bias because local advantages are firm-specific. In absence of good instrumental variables for *LocIncumb,* we introduce controls into the regression which capture permanent firm local advantages. The idea is to use pre-defined regions, which can contain one or more units, and introduce firm-regions specific fixed effects which capture unobservable firm advantages on each area.[14] With these controls, the identification of *LocIncumb* relies in the variation across units' average unit prices from the same firm within the pre-defined areas and the variation across auctions. Defining these pre-defined areas is somewhat subjective. We used the political regions of Chile as our areas, which contain on average 7.7 units (see Figure 1 for a map describing the regions). We also did some analysis with other pre-specified areas to assess the robustness of the results. In addition, we also include the following firm characteristics that can change over time as additional controls: (1) an indicator variable on whether a firm is attempting to renew a previously awarded contract for the unit, to capture possible sunk costs in the service provision (*Renew*); (2) a firm performance measure assigned by JUNAEB each year, which captures the managerial competence of firms (*Performance*); (3) a financial grade assigned by JUNAEB based on firms' financial classification in

[14]Since the objective is to control for *permanent* cost advantages, this fixed-effects do not vary across auctions.

the range 1 to 7, 1 being the best grade (*FinGrade*); and (4) an indicator for firms that are participating in the auction for the first time (*NewFirm*). Finally, we include auction fixed-effects to capture temporal price trends.

In summary, our estimation is a two-stage regression where the first-stage estimates equation (4) and the second stage uses the estimates of the average unit price $\{\delta_{ift}\}$ as the dependent variable to estimate equation (5) via OLS. This two-stage procedure provides estimates of the parameters characterizing scale and density discounts (β^{dens} and β^{scale}) and the coefficients of *LocIncumb* and *LocComp*. These together with our analysis in Section 6 are useful to study the design issues discussed in Sections 1 and 3.2. The next section describes the main estimation results.

5 Results

This section describes the main results of the two-stage estimation and a sensitivity analysis to assess their robustness.

5.1 First-Stage Regressions: Estimates of Discounts

We begin by providing details on the specifications for the discount functions $g^{dens}(v_c, \beta^{dens})$ and $g^{scale}(v_a, \beta^{scale})$ in the regression equation (4). Recall, the cluster size v_c and package size v_a are measured in million meals per year. The scale discount function $g^{scale}(.)$ is estimated by a step function with 10 equally spaced intervals of size three that cover all the range of bids. Similarly, the density discount function $g^{clust}(.)$ is estimated with a step function with 13 equally spaced intervals of size two. The step functions for g^{clust} and g^{scale} are implemented with binary variables indicating the size level, excluding the smallest level, so that the coefficient represents the average change in bid price at that level relative to the excluded level. For example, the results in Table 4, right column (under scale discounts), indicate that a combination a of size v_a in the range [6,9] is Ch$17.49 cheaper than one of size $v_a \in$[0,3].

Table 4 shows the estimates of the first stage regression (4), which includes covariates measuring scale and density discounts and a set of firm-unit-auction average unit prices $\{\delta_{ift}\}$ (the estimates of average unit prices are not shown). Robust standard errors are reported in parenthesis. All the coefficients are estimated with precision, which is expected given the large sample size. The explanatory power of the regression is remarkably high (R-square equal to 0.98).

Figure 6 compares the estimates of the discount functions due to scale and density in terms of percent change on the average bid price (the average bid price is 75¢). The results suggest that the magnitude of scale discounts are economically significant (see top of Figure 6): increasing the package size to 20 million (about 8 units) generates discounts of approximately 6%.[15] Interestingly, most of the scale discounts are

[15] Note that typical margins in this industry are between 4% and 8%.

seen for volumes below 18 million (about 7 units), suggesting they get exhausted after that point.

Figure 6, bottom chart, graphs the estimates for the density discounts ($g^{clust}(v_c, \beta^{dens})$). The graph shows the average discount (as a function of the cluster size), *in addition* to any discounts generated by scale. Density discounts appear to be economically significant, but relatively smaller than the discounts due to scale. Similarly to scale, they also get exhausted after 7 units. To illustrate, consider a bid for three units of size 3 million each. Scale discounts are approximately 4.5% of the average bid price. In addition, if the 3 units form a cluster, the additional discount due to density is 1.3%, for a total discount of 5.8%.

5.2 Second-Stage Regression: Effect of Incumbents and Local Competition

Table 5 shows the results from the second-stage regression (equation (5), replacing δ_{ift} with its estimate from the first stage).[16] As is common with panel data, we decompose the error term u_{ift} into a random-effect ξ_{if} plus an idiosyncratic error, and account for this heteroskedasticity in the estimation (typically known as "random-effect" estimator, see Wooldridge (2002)). Specification (1) includes firm, auction and unit dummy variables as controls, plus all the control variables described in section 4.2 (*FinGrade* is included with a linear effect plus a dummy variable indicating firms with the highest financial grade*, FinGrade=1*). The coefficient on *LocIncumb* is negative and significant, revealing that firms which operate in nearby units (awarded in previous auctions) tend to bid, on average, 2.3% lower. This is similar in order of magnitude to the discounts generated by density. The effect of local competition *(LocComp)* is also negative and significant.[17] Increasing the number of local incumbents by 4 (about 1 standard deviation) reduces average unit prices by 3%. The controls *Renew, Size* and *SpecialMeals* all have the expected signs.[18]

Column (2) in Table 5 includes firm-region fixed effects as additional controls (together with auction and unit fixed effects and all the other controls). The coefficient of *LocComp* and *LocIncumb* are similar to those obtained in column (2), although the statistical significance of *LocIncumb* is smaller. Given the small difference across the estimates, it appears that the potential bias on *LocIncumb* due to permanent local firm advantages is small.[19]

[16]The 1999 auction was the first year in which the CA was fully implemented. Hence the measures of local incumbency *(LocIncumb)* and local competition (*LocComp*) are not well defined. For this reason, we chose to exclude this auction from the second-stage estimation. We also note that on average, each δ_{ift} is estimated with 600 observations (which corresponds to the number of packages containing that unit for that firm-auction) and the average relative precision of the estimates – the standard error divided by the point estimate – is 0.7%, which is quite high.

[17]We also estimated additional regressions to test a possible non-linear effect of *LocComp.* Perhaps surprisingly, all of these models suggest a linear effect of *LocComp.*

[18]While the other controls are statistically significant, we do not have theory to predict their sign a-priori.

[19]A Hausman test cannot reject that the coefficients of *LocIncumb* in models (1) and (2) are similar (t-stat=.87). We also ran a regression that treats δ_{if} as fixed effects. The standard errors in this regression are much larger and the coefficients on *LocIncumb* and *LocComp* were no longer statistically significant. However, their magnitude and sign were similar to those obtained in Table 5, and a Hausman test cannot reject that the estimates are equivalent. Finally, we also estimated a regression with firm-auction fixed effects (instead of firm fixed effects) to account for unobservable firm characteristics that change over time beyond the ones we control for as described in Section 4.2; the results were also similar (although the statistical significance for *LocIncumb* was smaller).

5.3 Sensitivity Analysis

We conducted some additional empirical analysis to validate the robustness of these results, which we briefly discuss in this section.

To assess the validity of the estimated δ_{ift} – which capture the *average* price for a unit after accounting for volume and density discounts – we compared them with the *actual submitted* stand-alone bids b_{ift}. The correlation between the two measures is 0.983, and a scatter plot shows a very small dispersion along the identity line. The average ratio b_{ift}/δ_{ift} is 0.9994, with a standard deviation of 0.038. These results suggests that regression (4) is effectively separating average individual unit prices from package discounts.

The second stage regression (5) uses the estimated average unit prices δ_{ift} as a dependent variable, which is an appropriate measure for how aggressive a firm is bidding for a unit on average. For robustness, we also estimated the regression with the actual submitted stand-alone bid prices (b_{ift}) as the dependent variable and the main results were similar.

The CA that is ran by the government eliminates some bids which fall below a given price band. This price band is calculated with the actual bids submitted by all firms, and is therefore unknown to the firms at the time of bidding. For this reason, we did not exclude bids outside the band in our analysis. To check the robustness of the estimates, we re-estimated regression (4) excluding the bids below this price band– that is, using only the bids that were considered in the actual allocation mechanism. The estimated discount curves are practically identical to those reported in Table 4 and Figure 6.

Our definition of clusters –using a 150 km radius – is a reasonable area to capture economies of density in this industry based on the information available to JUNAEB. To analyze the robustness of the estimates of density discounts, we considered alternative cluster sizes, with radiuses of 100, 200, 300 and 400 km. Figure 7 shows these estimates using the different cluster definitions. Density discounts appear to decrease rapidly as the radius of the cluster increases. The estimated discounts with a cluster radius of 150 km. can be as much as 40% larger than the discounts estimated with a cluster radius of 400 km. The discounts estimated with cluster radiuses of 100 km and 200 km are similar to those obtained in our main results. Overall, this analysis suggests that our cluster definition of 150 km radius is reasonable at capturing density discounts.

We also estimated the first step-regression with the logarithm of the bid price as the dependent variable. All the estimated effects were similar in order of magnitude and statistical significance.

As an alternative specification for equation (4), we estimated a more parsimonious regression which replaces the average unit prices δ_{ift} with separate indicator variables for firm, units and auction (with no interactions between them). The estimates of density discounts in this model differ substantially from those reported in our main results. This suggests that controlling for the average unit prices $\{\delta_{ift}\}$, which controls for firm's local advantages, are important to obtain consistent estimates of the discount functions.

We studied heterogeneity of the discount curves for firms of different sizes to see if the observed pattern in the discounts varied. We used a firm classification based on *FinGrade* to group firms into three types: large, medium and small (recall that *FinGrade* is based on firms' financial classification). This is similar to the size classification introduced in Section 3.3. We ran regression (4) separately for the three types of firms. While we found heterogeneity, the magnitude and shape of the scale discounts were somewhat similar across the three types. Small and medium firms tend to exhaust their scale discounts at around 15 million meals, while large firms exhaust their discounts at around 20 million. In all the regressions, the scale discounts where in the order of 4% to 7%. The estimates of economies of density where in the order of 1% to 2%, and medium firms tend to offer higher density discounts. In summary, although we do find some heterogeneity across firms of different size, our main results prevail: (1) discounts are economically significant; (2) scale discounts dominate; and (3) discounts get exhausted at around 7 units.

One possible interpretation for why the scale discounts get exhausted after 7 units is that cost synergies get exhausted after that point. However, there could also be other alternative explanations related to markup adjustments. In particular, it may be possible that firms adjust markups due to the side constraints in the allocation mechanism which limit the maximum number of meals that can be awarded to a single firm. This is specially relevant for large firms for which the average maximum number allowed is around 20 million meals, which is close to the number at which discounts get exhausted. To test this alternative explanation, we allowed for interactions between the maximum number of meals allowed (which may change across auctions for a given firm) and the scale discount function, so that large firms with different maximums may exhaust their discounts at different quantities. The estimates suggest that the restriction on the maximum meals allowed does not change significantly the quantity at which scale discounts get exhausted. We cannot reject the null hypothesis that a one standard deviation change in the maximum restriction does not change the quantity at which the scale discounts are maximized (p-value>0.11).[20] The next section presents a detailed study of a different potential source of markup adjustments in package bids.

6 Strategic Behavior and Markup Adjustments

The regression analysis suggests that discounts for combinations of units are significant. It also shows that scale discounts are larger than density discounts. While this is suggestive that cost synergies are substantial, it is not conclusive because discounts may also arise due to markup adjustments. In fact, the following simple statistics regarding the discounts observed in the bidding data provide some evidence that this may be the case.

Table 6 shows a comparison of the average discounts for packages of two units ($N = 2$), where one of the units is located in Santiago –the centrally located capital with a high a population density– and the other unit

[20] More specifically, for the purpose of this test the discount curve is specified as a quadratic polynomial including main effects for volume and volume square plus their interactions with maximum volume restriction. This analysis is limited to large firms, which are the only firm types which have variation in the maximum volume restriction. For these firms, the restriction is usually determined by the market share constraint over the total standing contracts (see point 1 on page 7 for a description of the different types of market share restrictions). A given firm may have different standing contracts in different auctions. Therefore, the maximum number of meals the firm can be awarded with in a given auction changes. For other types of firms, the maximum number allowed is determined by the financial evaluation, which usually does not change across years.

varies among the following locations: (1) another unit located in Santiago; (2) a unit located in the extreme south (these units are more than 2000 kms. apart from Santiago); (3) any location (including Santiago and extreme south). The package discount (per meal) is defined as $(b_{ift}v_i + b_{aft}v_a - b_{i\cup a,ft}(v_i + v_a)) / (v_i + v_a)$. We only used bids submitted by firms already established and with standing awarded contracts, to control for the effect of fixed costs needed to initiate the operations. The table also provides the average discounts for larger packages of 5 units ($N = 5$), in which 4 units are in Santiago and the fifth unit changes its location.

The table shows that combining two units in Santiago yields larger discounts than average, which is consistent with economies of density. What is surprising is that combining a unit in Santiago with a unit in the extreme south results in even larger average discounts. A similar pattern is observed in larger packages. This discount pattern is unlikely to be generated by cost synergies alone. Arguably, a package formed by two units in Santiago should exhibit larger cost synergies than a package formed by a unit in Santiago and another unit in the extreme south, because only in the former case economies of density can be exploited. Moreover, because of transportation costs, some of the inputs to serve the units in the extreme south are purchased locally, so economies of scale that arise from volume discounts associated to centrally purchasing inputs cannot be exploited as much. Consequently, we conjecture that a fraction of the discounts observed for packages combining units in Santiago with units in the extreme south could be explained by strategic behavior in bidding.

The focus of this section is to formally analyze markup adjustments that arise from such strategic behavior and to assess their magnitude. Section 6.1 revises a theory describing bidders' incentives to submit discounted package bids even in the absence of cost synergies, which we refer to as *strategic bundling*. Based on this theory, we establish testable empirical predictions regarding strategic bundling in a CA (Section 6.2). This requires quantifying the randomness due to asymmetric information that bidders face; Section 6.3 provides a way to do this. Section 6.4 describes an empirical test of the theoretical predictions regarding strategic bundling. Finally, in Section 6.5 we discuss another potential source of strategic behavior: the threshold problem.

6.1 The Bidder's Problem and Strategic Bundling

We start by providing intuition that illustrates why bidders may have incentives to submit discounted package bids even in the absence of cost synergies. To do so, we use the observation by Cantillon and Pesendorfer (2006b) that when consumer valuations are additive (and under other fairly general conditions) the pricing problem of a multiproduct monopolist is equivalent to the bidder's problem in a private value CA. The mapping is as follows. In the auction setting, from the perspective of a firm, bids submitted by opponents are random due to asymmetric information. Similarly, the multiproduct monopolist faces random consumers' valuations. The distribution of consumer valuations for each unit maps to the distribution of the minimum of the opponents' bids for the unit. In addition, the optimal prices for the bundles in the multiproduct monopolist problem correspond to the optimal bid prices of the bidder in each package in the CA setting.

An established literature in multiproduct monopolist pricing has showed that the firm may have incentives to provide discounts for a bundle of products even if consumers' valuations over units are additive and in the absence of production cost synergies (see Adams and Yellen (1976), Schmalensee (1984), McAfee et al. (1989), Fang and Norman (2006), and Chu et al. (2009)). This is observed when the consumers' valuations for the bundle exhibit less variance than the valuations for individual items, because of averaging. Since the bundle exhibits less variance, the monopolist can extract surplus more easily from it than from individual units, providing incentives to sell it at a discounted price or even to just offer the bundle. A typical example of this type of behavior is the selling of bundles of channels by TV cable companies (Crawford, 2008). Given the parallel between the monopoly problem and the bidder's problem in a CA, these results suggest that bidders may have incentives to submit discounted package bids even in the absence of cost synergies. Cantillon and Pesendorfer (2006a) provide a simple example of this behavior, which we refer to as strategic bundling.

More formally, following equation (4), but for now assuming additive bids to draw a parallel with the multiproduct monopolist literature, we consider that a bidder's statistical model of the bids (per meal) of opponent firm f are given by:

$$b_{aft} = \sum_{i \in a} \delta_{ift} \frac{v_i}{v_a}, \tag{6}$$

where for given t, $(\delta_{1ft}, ..., \delta_{N(t)ft})$ are i.i.d. samples across f of a multivariate distribution known to the bidder ($N(t)$ is the number of units in auction t). To win a unit, the bidder needs to underbid all of its competitors; in that sense, he competes against the minimum of the bids submitted by its opponents. Accordingly, let $\underline{\delta}_i = \min_f \delta_{ift}$, where the minimum is taken over all of the bidder's opponents (to simplify notation, we omit the bidding firm under consideration and auction indexes from $\underline{\delta}_i$).

As mentioned above, the main insight of the multiproduct monopolist literature is that bidders may have incentives to submit discounted package bids in any best response if the competitors' bids for the package exhibit less variance than the bids for the individual items. This will be the case if the pairwise correlations across i among the minimum unit bids in the package, $\{\underline{\delta}_i\}_{i \in a}$, is not too high. More generally, the incentives to submit discounted package bids increase as the effect of the reduction in variance of a package becomes larger. For example, because of the law of large numbers, this will be the case if a is a large package and $\underline{\delta}_i$ are independent random variables (Bakos and Brynjolfsson, 1999). Moreover, the reduction in dispersion provides stronger incentives to engage in strategic bundling for firms with relative cost advantages, because they have a higher chance to win the package.

6.2 Testable Predictions

We now elaborate on the previous insights to formulate testable predictions regarding discounts due to strategic bundling in a bidder's best response when competing against the distribution of its opponents' bids.[21] Although the bidder's problem in our setting has similarities to the multiproduct monopolist, there are three issues that limit the applicability of the existing results in the literature in the context of our study:

1. Most of the existing results focus on the case of two units only.

2. Although a few studies analyze settings with more units (Fang and Norman (2006), Bakos and Brynjolfsson (1999)), their focus is to analyze under what conditions it is preferable for a bidder to exclusively submit a package bid covering all units (refered to as pure bundling) as opposed to individual bids for each unit. In contrast, in our data firms often submit bids for individual items as well as package bids. Our focus is to analyze the *magnitude of discounts* due to strategic bundling given a mechanism that allows for both package bids and individual bids.

3. Previous work assumes that bidder's costs and competitors' bids are additive (like in equation (6) above). In contrast, in the CA of school meals package bids exhibit discounts (they are sub-additive) and firms appear to have cost synergies.[22]

We perform a significant set of computational experiments to deal with these challenges and study whether the insights from the multiproduct monopolist literature carry over to our setting. First, to deal with the first two problems mentioned above, we analyzed the extensive numerical experiments in Chu et al. (2009).[23] These results, when translated to our CA setting, provide the optimal bid prices for a firm that can bid on any package in a CA when competing against a distribution of (additive) competitors' bids. The number of packages (and hence of optimal bid price decisions) grows exponentially with the number of units, so the analysis is limited to 5 units. The results are provided in Appendix B.1.

The results in Chu et al. (2009) (like most of the literature in multiproduct monopolist) assume additive opponents' bids and the absence of cost synergies. To deal with this limitation we develop our own set of numerical experiments. The results, reported in Appendix B.2, suggest that the predictions from the first set of experiments described above extend to the case of sub-additive bids and bidder's cost synergies. Due to the computational complexity involved in computing optimal bid prices, the second set of experiments is focused on two units only.

Finally, note that the theoretical predictions from the multiproduct monopolist literature are based on the distribution of the minimum of the opponents' bids. Unfortunately, we do not have enough data to quantify statistical properties of the distribution of the minimum average unit prices directly. Nevertheless, numerical experiments show that, with normally distributed bids, the correlation structure and coefficient of variation of the underlying opponents' bids carries over to the distribution of the minimum of these bids. More specifically, the coefficient of variation of the random variable $\min_f \delta_{ift}$ increases with the coefficient of

[21] Our approach is similar to Crawford (2008) who tests the theoretical predictions for multiproduct bundling in the cable television industry, finding evidence of strategic bundling.

[22] The only exception we are aware of is Salinger (1995) that incorporates cost synergies. However, it only studies a specific numerical example.

[23] We are grateful to the authors for providing us with this data.

variation of the underlying random variable δ_{ift}; the correlation between $\min_f \delta_{ift}$ and $\min_f \delta_{jft}$ increases with the correlation between δ_{ift} and δ_{jft}. In addition, in the second set of experiments mentioned above we explicitly consider the distribution of the minimum of the opponents' bids.

Overall our computational study confirms the insights from the multiproduct monopolist literature. Based on them we establish testable predictions for the presence of strategic bundling. From the perspective of a bidder in a multi-unit auction, bids submitted by opponent firms (indexed by f) are random due to asymmetric information. We assume that the bidder's statistical model of its opponent bids is given by equation (4) (so they are sub-additive), where for given t, the bid price vector of each firm f, $(\delta_{1ft}, ..., \delta_{N(t)ft})$, is viewed as an i.i.d. sample of a multivariate distribution.

We formulate the following testable hypotheses:

- **Hypothesis H1:** The magnitude of the discounts due to strategic bundling when combining a unit i with a package a (possibly a singleton) increases, as the correlation among the competitors' average unit bid prices among the individual unit (δ_{ift}) and the package ($\{\delta_{jft}\}_{j\in a}$) decreases. (We define the correlation between a unit and a package as the average correlation: $AvgCorr(i, a) = \sum_{j\in a} Corr(i, j)\frac{v_j}{v_a}$.)

- **Hypothesis H2:** The magnitude of the discounts due to strategic bundling when combining a unit i with a package a (possibly a singleton) increases, as the coefficient of variation of the competitors' bid price for the individual unit, δ_{ift}, increases.

- **Hypothesis H3:** Firms with a relative cost advantage in the combined package have stronger incentives to engage in strategic bundling. Hence, for this set of firms the effects described in H1 and H2 are more pronounced.

In what follows we describe an empirical strategy to test H1, H2, and H3 in our data. Finding empirical support of these hypotheses is suggestive that firms' bidding behavior is consistent with strategic bundling.

6.3 Quantifying Bid Randomness and Cost Advantages

In this section, we describe how to come up with estimates of statistical measures of the joint distribution of average unit bid prices $(\delta_{1ft}, ..., \delta_{N(t)ft})$ and of firms' cost advantages that are useful to test H1, H2, and H3.

First, we use the estimates of the average unit prices $\hat{\delta}_{ift}$ (obtained from the first step regression (4)) to estimate statistical measures of the joint distribution of the average unit prices $(\delta_{1ft}, ..., \delta_{N(t)ft})$. Recall that from the perspective of a bidder, the average unit prices of another firm f in auction t, $\hat{\delta}_{ft} = (\hat{\delta}_{1ft}, ..., \hat{\delta}_{N(t)ft})$, are viewed as an i.i.d. random vector sampled from a common multivariate distribution which we seek to characterize.[24] Specifically, using the estimates $\{\hat{\delta}_{ift}\}$ we calculate the sample average

[24]Variation in average unit prices is not a perfect proxy for uncertainty due to asymmetric information if firms can anticipate

($E(i,t)$), sample standard deviation ($SD(i,t)$) and coefficient of variation ($CV(i,t)$) for unit i in auction t as:

$$
\begin{aligned}
E(i,t) &= \frac{1}{F_{it}} \sum_{f} \hat{\delta}_{ift} \\
SD(i,t) &= \left(\frac{1}{F_{it}} \sum_{f} (\hat{\delta}_{ift} - E(i,t))^2 \right)^{\frac{1}{2}} \\
CV(i,t) &= SD(i,t)/E(i,t),
\end{aligned}
$$

where F_{it} is the number of firms for which we could estimate average unit prices for unit i in auction t.[25] Similarly, we calculate the sample correlation between units. To get more precise estimates, we pooled all the auctions to estimate one correlation coefficient for each pair as:

$$
Corr(i,j) = \frac{1}{SD(i)SD(j)\sum_t F_{ijt}} \sum_{f,t} (\hat{\delta}_{ift} - E(i))(\hat{\delta}_{jft} - E(j)),
$$

where $E(i)$ and $SD(i)$ are the sample average and sample standard deviation for unit i, respectively, pooling data across all auctions.[26] F_{ijt} is the number of firms with estimated average unit prices for both units i and j in auction t.[27]

In addition, testing H3 requires a measure of firms' cost advantages. The results from Section 5.2 suggest that local incumbents – those operating nearby TUs to a unit – tend to offer lower prices, possibly due to cost advantages from economies of density. Therefore, the previously defined indicator variable *LocIncumb* is used to identify firms with a relative cost advantage in a unit.

6.4 Empirical Tests of Discounts Due to Strategic Bundling

The correlation among units is a key incentive that drives discounts due to strategic bundling: the higher the correlation, the lower the benefit of combining units in terms of reducing dispersion on the distribution

some of the variation in δ_{ift}. In particular, the set of local incumbents and other firm characteristics are known to all firms and they may help anticipating some variation in bid prices across firms. As a robustness check, we also did all of the subsequent analysis using the *residuals* of the regression of δ_{ift} on the following observable firm characteristics: *LocIncumb, Renew, Performance, NewFirm, FinGrade* and *FinGrade=1*. All the main results that follow in this section were similar.

[25] Some firms did not submit bids for all units, which precluded the estimation of those average unit prices. A few average unit prices were not identified (about 1%); for example, when two units are never observed in separate package bids. We also note that we consider all firms to compute statistics of the joint distribution of bids; these provide a good approximation to statistics of the joint distribution of *opponents'* bids.

[26] We also estimated different correlation coefficients for each auction, but some correlation pairs had few observations (as low as two) and were very noisy.

[27] For robustness, we also did the following analysis using the stand-alone bid prices (b_{ift}) instead of the average unit prices (δ_{ift}) to compute all the statistics that describe the joint distribution bids. The results obtained were similar. Because some firms do not submit stand-alone bids for all units, we decided to use the average prices instead. Moreover, firms compete against all bids submitted by other firms, not just stand-alone bids. For this reason, we beleive average unit prices are a better proxy to summarize the distribution of opponent firms.

of competitors' bids. In the data, more than 90% of the sample correlations are positive, and the median is about 0.6. The relatively high correlations suggest that on average the incentives to do strategic bundling are perhaps not too strong.

To test H1 and H2 empirically, we seek to estimate the effect of variability (captured by *CV*) and correlation on the discounts observed in the bids (measured by *AvgCorr*). To isolate the discount effect of adding a unit i to a package of units a, we looked at "nested" bids submitted by a firm, for which we observe a stand-alone bid for i, a bid for package a (which could also be a single unit) and a bid for the package $a' = a \cup \{i\}$. The discount, that we normalize per meal, is calculated as:

$$D_{ft}(i, a) = \frac{b_{aft}v_a + b_{ift}v_i - b_{a'ft}(v_a + v_i)}{v_a + v_i}. \tag{7}$$

Hence, discounts could only be calculated for a subset of combinations for which we observe nesting. Because we want to isolate the effect of discounts generated by strategic bundling, we also limited our sample to combinations which contained units which are not co-located –units which do not form clusters and are not located in the same geographic region– to remove the effect of density discounts (we revise this point later).

We estimated the following linear regression with $D_{ft}(i, a)$ as the dependent variable, where i is an individual unit and a is a package of units:

$$D_{ft}(i, a) = \beta_1 CV(i, t) + \beta_2 AvgCorr(i, a) + \beta_3 Size_i + \phi_f + \tau_t + \chi_a + e_{ftia}, \tag{8}$$

where e_{ftia} is an error term, and ϕ_f and τ_t are firm and auction fixed effects, respectively. The package fixed-effect χ_a controls for effects from scale and other factors associated with package a that may affect the discount. Similarly, $Size_i$, which measures the number of meals of unit i, is included to control for scale. Given the controls included in equation (8), the estimation uses variation with respect to the different units that were combined with a given package a.

Table 7 shows the results. Each column in the table shows the estimates of regression (8) over a different sample. Column (1) reports the estimates over a sample of small firms which can only bid on packages of one or two units.[28] In this case, the stand-alone and two unit package bids of one of these firms do not compete against its own bids for larger packages (because they cannot submit bids for larger packages). Hence, this setting resembles more closely the multiproduct monopolist problem analyzed in the literature (which we used to motivate our hypotheses) and the numerical experiments in Appendix B.2. Here, *AvgCorr* measures the sample correlation between the two units in the package. The results suggest that discounts increase as the coefficient of variation (*CV*) increases and as the correlation between the units decreases,

[28] To increase the sample size, we also included in this sub-sample packages with co-located units and controlled for the distance between the units (not reported in the table). In all other specifications we restrict the sample to packages with units which are not co-located.

providing support for hypotheses H1 and H2.

Column (2) shows the results in a sample including packages of 2 units from all firms. Here, the coefficient of *CV* and correlation are significant and their directional effect provides further support for H1 and H2, although the magnitude seems somewhat smaller than in Column (1). To test H3, Column (3) uses a sub-sample of regression in Column (2) selecting those bids where the bidder is an incumbent in both units. The coefficients of the *CV* and *AvgCorr* increase in magnitude, although the statistical significance of the correlation coefficient is smaller (p-value 0.07). The larger magnitude of the coefficients is suggestive that the incentives for strategic bundling are larger for firms with a cost advantage.[29]

The results show a similar pattern when the sample includes packages of up to 4 units, reported in Column (4) of Table 7. Discounts increase with *CV* and decrease as the average correlation between a unit and the other units in the package increases. Analyzing the effect of relative cost advantages is harder in this sample because as packages become larger, it is unlikely that a firm will have a relative cost advantage in all units. We have done some analysis limiting the sample to firms that are incumbents in a significant portion of the units (say, 60%) and also found that the effects of correlation and CV were larger in this sample.

Our empirical analysis suggests that the firms' discounts are consistent with strategic bundling. However, this strategic behavior explain a small fraction of the discounts on average. For packages of four units, based on the estimates of column (4), a two-standard deviation change in *CV* and *AvgCorr* increases the average discounts only by 2.7% and 1.0%, respectively. For packages of 2 units, based on the estimates of column (2), a two standard deviation increase (decrease) in *CV (AvgCorr)* increases discounts by less than 3.5% (4.5%) of the average discount. For packages of 2 units submitted by low cost firms, the effect is larger (as predicted by theory), but still reasonably low: 14.7% for *CV* and 10% for *AvgCorr*. We also tested other specifications which included more controls into regression (8) and the effects were similar.[30]

Recall that to remove the effect of density discounts, the empirical analysis is limited to packages including units which are not co-located. However, the correlation between two units tends to increase as the distance between them becomes smaller.[31] Therefore, in light of our results, discounts due to strategic bundling between co-located units should be smaller than for the sample of packages we analyzed.

Overall, the results suggests that although bidders do engage on strategic bundling, these markup adjustments explain on average only a small fraction of scale and density discounts. Nevertheless, our initial motivating example and the analysis suggests that perhaps for some specific units strategic bundling could be more severe and could affect the efficiency of the allocation mechanism. We discuss this effect together with other design recommendations in Section 7. Before, the next section describes another strategic effect that could affect discounts.

[29] Further restricting the sample to small firms that can bid on packages of one or two units only and are incumbents on both units revealed similar results.

[30] We estimated regressions which include a firm-auction fixed effect (instead of the separate fixed effects ϕ_f and τ_t). All results were similar, although in a few cases the coefficients were not statistically significant.

[31] Increasing the distance between two units by 400 km. lowers the correlation by 0.2 on average.

6.5 The Threshold Problem

In this section we discuss another potential source of strategic behavior that could affect markups, namely, the threshold problem. In a nutshell, the threshold problem arises when two local bidders each interested in a single unit, say A and B respectively, need to underprice a large global firm that submits a package bid for the package $A \cup B$ (the "threshold" bid). In this case, a local bidder lowering its bid benefits the other local bidder. Because local bidders do not internalize these benefits, their bids may not be as competitive as they could have been if package bidding was not allowed. A severe threshold problem could bias the auction towards large bidders to the disadvantage of small bidders, potentially creating inefficiencies and increasing the cost allocation (see Milgrom (2000) and Pekec and Rothkopf (2003) for a more detailed discussion; Chernomaz and Levin (2009) and Maréchal and Morand (2009) provide concrete examples of the threshold problem in equilibrium bidding strategies)

Unfortunately, the equilibrium theory of CAs is not well developed. Moreover, there is not a recognized clear mapping between the threshold problem in CAs with another well studied economic problem, like in the case of strategic bundling. Hence, we are not able to directly test in our data whether bidders' behavior is consistent with the threshold problem. However, we provide some evidence that the threshold problem may not be too severe in our application.

One of the negative consequences of the threshold problem is that package bidding creates a disadvantage for small bidders, making their bids less aggressive and thereby depressing competition in the auction. Our analysis of the winning bids suggests this problem is not severe in the combinatorial auction for school meals: small packages (and firms) do win frequently. About 50% of the winning bids are for packages of 2 units or less; and 22% of the winning bids are from small firms (firms that can bid up to or 3 units; they constitute about 28% of the firms), suggesting that small bidders are not at a large disadvantage in the auction (see Table 2).

One possible explanation for why small firms win is that the market share constraints force the allocation to award units to these firms. However, this could be inefficient and costly for the government if bids from small firms are not competitive, so we also study this potential negative effect. To do it, we compared the average bid prices (δ_{ift}'s) across different types of firms. For each unit, we ranked the average prices across firms. Small firms rank in the lowest 10 percentile in units across all regions (except region two, where the cheapest small firm ranks in the lowest 12.5 percentile). Inspection of the average prices reveals that it is not just one small firm that ranks low everywhere – small firms seem to have local advantages so that different small firms are competitive in different regions. This analysis suggest that small firms are, overall, quite competitive due to local advantages.

To further study potential inefficiencies and increases in procurement cost generated by the market share constraints, we re-solved the optimal allocation problem in each auction removing these constraints. In particular, using the same submitted bids, we solved the mathematical program that finds the optimal allocation with and without the market share constraints and compared the allocations and the value of the objective function. Details of the mathematical program can be found in Epstein et al. (2002).We find that even though the optimal allocation can change, the optimal objective function value does not; the average change across all auctions is 0.3%, confirming that small firms seem to be quite competitive.

Finally, the intuition behind the threshold problem suggests that large bidders should make steeper discounts than small firms. The idea is that large firms may inflate their single unit bids relative to the package bids to decrease the chances that a small firm wins a single unit bid by matching it with a large firm's own single unit bid and in that way beat the package. We do not observe this phenomena in our data. In fact, the average discount over all packages of size 2 is actually 20% larger for small firms (that can only win two units maximum) than for large firms (that can win up to 8 units). This difference is still observed after correcting for the type of units included in the packages, which may be different on average for small and large firms.

7 Recommendations for Design

In this section, we use the results in Sections 5 and 6 to evaluate the current mechanism and suggest potential changes for the Chilean schools meals auction along the dimensions discussed in Sections 1 and 3.2. These recommendations are being considered by the Chilean government to redesign future auctions.

Package Bidding

Our results show that scale and density discounts are important (together they can be as high as 8% of the average bid price). Moreover, the results suggest that, on average, only a small fraction of the discounts can be explained by strategic bundling. In addition, other strategic effects that may arise in package bidding – in particular, the threshold problem – appear not to be too severe in the context of this CA. Therefore, allowing package bids seems appropriate. Furthermore, firms seem to take advantage of the flexibility of the current mechanism that allows them to express cost synergies due to scale and density, both of which are present.

Although on average the discounts due to strategic bundling seem to be relatively small, our data analysis suggests that for specific combinations the effect could be relevant. To identify these combinations, we looked at some "influential points" of regression (8) in the sample of two-unit packages and we found the following patterns:[32]

- The two units in the isolated regions of the south of the country (regions 11 and 12) have discounts which tend to be above average when combined with other units located far away. We observed this pattern in Table 6. In addition, the average discount of combining the unit in region 11 with units in regions 3 or 6 (which are more than 1200 kms. appart) is three times higher than the average discount. As previously discussed, given the location of these isolated units, we believe it is implausible that all of these discounts can be explained by cost synergies. We believe that strategic bundling behavior is

[32]Influential points were defined as observations that when removed from the dataset had a large impact in the estimated coefficients of *AvgCorr* and *CV*.

particularly severe in these packages. In fact, it is interesting to observe that consistent with strategic bundling theory, the units in regions 11 and 12 tend to have a high coefficient of variation of average unit prices δ_{ift} (in the top 20% among all units). Moreover, the unit in region 11 has a very low correlation with units in Santiago, region 3 and 6 (the average correlation is 0.02, 0.28 and 0.34, respectively, compared to the national median of the pairwise correlations which is 0.6). The units in region 12 had low correlation with several units in Santiago, in some cases as low as -0.18.

- Units in the extreme north (regions 1 and 2) were discounted above average when combined with units in region 10, which are located more than 3000 kms. apart. Large discounts were also observed for packages that combined regions 1-2 with units in Santiago, located more than 1200 kms apart. Again, we believe it is implausible that all of these discounts can be explained exclusively by cost synergies. Also consistent with strategic bundling theory, the average correlation between units in regions 1-2 with units in region 10 and Santiago is approximately 0.35. For some specific pairs, it can be as low as -0.19. The units in regions 1 and 2 also tend to have a high coefficient of variation (in the top 10% among all units).

The pattern in the observed discounts for these units is unlikely to be driven by cost synergies only and are consistent with strategic bundling. To shed light on whether this strategic behavior could be generating inefficiencies, we revised the winning bids in the years these units where auctioned. In 2000, the unit in region 11 was awarded in a package of 4 units combined with 3 units in region 6, to a medium-sized firm (which won nothing else that year). When we re-solved the allocation problem without including the unit in region 11 (but using the same bids submitted in that auction), this firm did not win any units. This suggests that the actual allocation in 2000 may have not been efficient and that the winning firm may have strategically taken advantage of bundling.

The unit in region 12 exhibits a similar pattern. In 2003, this unit was allocated in a 8-unit package to a large firm that included 1 unit in the capital, 2 in region 4, 2 in region 5 and 2 in region 9 (regions 4, 5 and the capital are adjacent and located in the center of the country; region 9 is 600 km. south of the capital). When we removed region 12 from the auction, the firm won nothing.

Finally, the winning bids in 2004 show that the two units in region 1 were allocated in package of 6 units with 4 units in region 10. In the same year, the two units in region 2 were allocated in a 8-unit package with 2 units from region 10 and the rest in the capital. The bundling of units in the extreme north with units located far away appears to be an effective strategy to win those packages.[33]

Our analysis suggests that strategic bundling behavior may be particularly severe in certain specific combinations creating inefficiencies. Actually, this conclusion confirms similar beliefs by JUNAEB's officials based on intuition and anecdotal evidence (Martinez, 2010). Since our results are not totally conclusive, (for example, we do not know what is the impact of this behavior on total procurement cost), we propose

[33]Removing the four units in regions 1-2 generated major changes in the allocation of units. We conjecture that this would be a major change to the auction and that it would change package bids significantly, so the analysis from removing these units from the auction appears to be less informative.

to further explore prohibiting some specific combinations. A useful field experiment would be to auction the units in the extreme south (regions 11 and 12) and extreme north (regions 1 and 2) in separate auctions. For example, to run a separate CA for the extreme south units and another one for the extreme north units (perhaps with a shorter contract of one year to reduce the risk).

Promoting Competition

Our analysis suggests that market share restrictions together with splitting the TUs into multiple sequential auctions are a useful design element to diversify the supplier base and promote local competition through the presence of local incumbent firms. In fact, the effect of local competition is comparable in magnitude to the discounts from cost synergies. This suggest that it may be worth revising the years at which the TUs are auctioned in order to further increase the benefits from local incumbency in the outcome of the auction.

At the time of writing this paper, the government was evaluating whether to relax the restrictions on bidders' market shares in order to enable them to further exploit cost synergies. In particular, they were evaluating to eliminate the constraint that enforces the total standing contracts to be no larger than 16%. Our results suggest that this change to the design could potentially reduce supplier diversification and the intensity of local competition, which in our analysis shows to be an important factor in lowering bid prices. Moreover, relaxing the market share constraint may not be convenient from a cost efficiency perspective. This constraint imposes a maximum of 19 million meals per year on average (equivalent to approximately 7 units), mainly affecting the large firms. Our analysis suggests that at this point most cost synergies have been exhausted. In fact, as we discussed in section 6.5, when we re-solved the optimal allocation problem without the market share constraints (but with the same bids), there is a minimal decrease in the total cost allocation (0.3% decrease). Hence, the constraint seems to promote competition at a low expense. This suggests that the potential downside of removing market share constraints – reducing competition – outweighs the benefit of further exploiting cost synergies.[34]

8 Conclusions

We have empirically analyzed the procurement combinatorial Chilean auction for school meals. Our empirical analysis provides substantive insight into bidding behavior in CAs, suggesting that firms are sophisticated in their bidding and that they take advantage of the flexibility of the bidding mechanism by expressing cost synergies and by adjusting their markups. These insights can perhaps be extrapolated to the bidding behavior in other CAs in which stakes are high and that are repeated over time (so bidders can learn).

In addition, our results inform important auction design issues, highlighting how the simultaneous consideration of the firms' operational cost structure and their strategic behavior is key to the successful design of a CA. Together with providing estimates of package discounts, we develop testable hypothesis to evaluate

[34]We note that removing market share constraints may also change the submitted bids, which could further change the total allocation costs. However, as we discussed in Section 5.3, changes in the market share restrictions did not change much the shape of the estimated discount curves for large firms nor the point where discounts get exhausted.

how much of the estimated discounts can be explained by strategic bundling as opposed to cost synergies. Moreover, we use our methodology to identify potential sets of units for which strategic behavior can be severe and can lead to inefficiencies in a CA setting. In addition, we estimated the effect of local competition in bid prices. We used our results to evaluate and propose changes to the current auction design.

Our work leaves several interesting directions for future research. First, a significant challenge in our approach was to distinguish whether discounts were determined by cost synergies or markup adjustments. An important factor identified in the literature that affects markups for package bids is strategic bundling; we empirically tested how much of the measured discounts can be explained by this mechanism. In future research, it could be interesting to explore whether there are other important mechanisms that affect markups for package bids and test whether they are present in the data. An alternative approach is to use a structural estimation method that poses a model of bidders' behavior, imposing restrictions on how markups are determined and thereby identifying costs. The main drawback of this approach is that identification relies on strong assumptions on bidders' behavior and that it is computationally demanding. However, we believe it is a useful complement to the work we have presented in this paper and it is being matter of current research.

Our analysis was focused on single-round sealed-bid first-price CAs, because the Chilean government is not planning on changing this format and several CAs in practice have the same format. However, it could be interesting to study the impact of other auction formats (e.g. a multi-round ascending auction) in future research.

Finally, while our focus has been in the Chilean auction for school meals, we believe our method of analysis can be used broadly for many other CAs, enabling the study of a wide range of issues. We hope that our study together with future similar studies in other CAs enhances the understanding of bidders' behavior and, as a consequence, give us insights to improve the design of this type of auctions.

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A Clustering Algorithm

Consider a set of units $a = (a_1, ..., a_{card(a)})$, where $card(s)$ is the cardinality of the set a. Let r be a pre-defined cluster radius (in our main estimation we choose $r = 150\ km.$). For a set of units b, let C_b be the weighted geographic center of b. The weighted geographic center of a set of units is calculated as the weighted average latitude and longitude of all the schools it contains, weighted by the school populations. The distance between two set of units a and b, $dist(a, b)$, is the bird-fly distance between the weighted geographic centers of the respective sets.

We describe the algorithm that builds the set of clusters formed by the units in a, $Cl(a)$.

Algorithm 1 Clustering Algorithm

Let $\mathcal{A}$ be the collection of sets $\{\{a_1\}, ..., \{a_{card(a)}\}\}$, $\underline{d} := 0$
while $\underline{d} < r$ **do**
  Let $\underline{d} := \min_{b,c \in \mathcal{A},\ b \neq c}\ dist(b, c)$. Let $(b^*, c^*) := \underset{b,c \in \mathcal{A},\ b \neq c}{\text{argmin}}\ dist(b, c)$
  if $\underline{d} < r$ **then**
    $\mathcal{A} := \mathcal{A} - (b^* \cup c^*)$, $\mathcal{A} := \mathcal{A} \cup \{b^*, c^*\}$
  end if
end while
$Cl(a) := \mathcal{A}$

B Computational Experiments on Strategic Bundling

As described in Section 6.2 the results in the multiproduct monopolist literature do not quite match our setting, because many of them (1) consider two units only; (2) focus on pure bundling; and (3) assume additive opponents' bids and costs. In Appendix B.1, we show results from numerical experiments that deal with the first two limitations. The numerical experiments in Appendix B.2 deal with the last limitation.

B.1 Several Units, Flexible Package Bidding

In the first set of experiments we consider the computational experiments developed by Chu et al. (2009) to study factors that affect the magnitud of package discounts. These experiments analyze the problem of a multiproduct monopolist that chooses prices for all possible packages in a set of units and sells to consumers with random valuations. Consumers' valuations and costs are assumed to be additive. As mentioned before, in this case Cantillon and Pesendorfer (2006b) show that the bidder's problem in a CA is equivalent to the multiproduct monopolist problem. We study the results in Chu et al. (2009) in the context of a CA.

We assume bidders' statistical model of their opponents' bids are given by equation (6) where $v_i = 1,\ \forall i$. Using the notation introduced right after that equation, our main focus is to study how the following factors affect package discounts:

- The pairwise correlations across i among $\underline{\delta}_i$ in a given package.
- The unit cost of the bidders.
- The variance of $\underline{\delta}_i$.

Chu et al. (2009) find the optimal bid prices in scenarios with different bid distributions, different cost structures, and auctions ranging from 2 to 5 units. Because we are interested in analyzing the effect of correlations and variance of the bids, we focus on the experiments in which $\left(\underline{\delta}_1, ..., \underline{\delta}_{N(t)}\right)$ are assumed to be multivariate normally distributed. We focus on experiments with $N(t)$ equal to 4 or 5 units. These numerical experiments can be grouped into three categories:[35]

1. Experiments for which the $\underline{\delta}_i$'s are all independent random variables, or all pairwise positively correlated, or all pairwise negatively correlated. Their variances are the same across units and experiments.
2. Experiments for which the pairwise correlation of the $\underline{\delta}_i$'s vary across pairs of units; some pairs are independent and others are positively or negatively correlated. Their variances are the same across units and experiments.
3. A third set of experiments that is similar to the first set but the variance of the $\underline{\delta}_i$'s change across units and experiments.

We first identified "nested packages" on every experiment and calculated the discount from adding a unit i to package a as:

$$Disc(i, a) = b_i + b_a - b_{i \cup a}$$

For the first set of experiments, we compared the magnitude of the discounts among the cases with independent, positive correlation and negative correlation, for packages of different sizes. Note that the average is taken over different instances that include auctions of different size, different means of the $\underline{\delta}_i$'s, and different bidders' marginal costs of serving the units; the averages are computed over the same set of instances for each level of correlation. Table A.1 shows this comparison.

Package size	Negative	Indep.	Positive
2	1.65	0.74	0.49
3	2.15	1.12	0.75
4	2.31	1.29	0.94
5	2.39	1.36	1.06

Table A.1: Average discounts for different levels of pairwise correlation.

Table A.1 shows two interesting patterns. First, the discounts tend to increase as the size of the package increases, suggesting that the effect of strategic bundling is observed beyond the two-unit case. Second, as

[35] More details about the model parameters can be found in Chu et al. (2009).

the correlation among the units becomes more negative, the discounts become larger. We also used these experiments to study the effect of costs. The following table compares the discounts for experiments for which the bidder's marginal costs are below or above the mean of the $\underline{\delta}_i$'s distribution. The results show that strategic discounts are mostly relevant when the bidder has relative cost advantages.

Costs	Negative	Indep.	Positive
below mean	3.45	1.56	1
above mean	0.4	0.41	0.32

Table A.2: Average discounts for different levels of pairwise correlation and costs.

To study the second set of experiments, we estimated a regression with $\overline{Disc}(i,a) \equiv Disc(i,a)/(|a|+1)$ as the dependent variable (we normalize discounts by the size of the package), and an independent variable measuring the average pairwise correlation of unit i with units in package a (*AvgCorr*). Each experiment –indexed by k– is specified by: (i) a total number of units in the auction (from 2 to 5); (ii) a multivariate normal distribution with mean μ and variance-covariance Ω that determines the distribution of the minimum of opponents' bids, $\underline{\delta}_i$ (Ω has 0.25 in the diagonal and zeros, negative and positive values off the diagonal ranging from -0.5 to 0.5); and (iii) the marginal costs of each unit (either 0 or 0.2 for all units). The regression is given by:

$$\overline{Disc}_k(i,a) = \beta AvgCorr_k(i,a) + \rho_{ka} + \varepsilon_{kia}.$$

We considered the discounts in different types of packages: (i) packages of two units, with marginal costs for both units below the mean of the respective $\underline{\delta}_i$ (low cost scenario) or both units above the mean (high cost scenario); (ii) similar cases for packages of 3 or more units. For packages of 3 or more, we also include the fixed effect ρ_{ka} to control for the experiment conditions and fixed effects for packages (a).[36]

Table A.3 shows the results from these regressions. The results suggest that when the unit costs are low, discounts tend to be larger when the units are more negatively correlated. This effect is not present when the firm has higher costs. Moreover, the results are similar when considering larger packages of 3 or more units.

[36]Including this fixed effects for the regressions on packages of two units does not change the point estimates but reduces their precision.

	(1) 2 units, low cost	(2) 2 units, high cost	(3) >=3 units, low cost	(4) >=3, high cost
Avgcorr	-0.179***	0.049*	-0.135***	0.173***
	(0.012)	(0.023)	(0.007)	(0.010)
N	10575	1260	55101	11331

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.3: Effect of average correlation in discounts.

	(1)	(2)
	2 units	>=3 units
Var	0.099***	0.103***
	(0.002)	(0.000)
N	11340	51660

Standard errors in parentheses
*** $p < 0.001$

Table A.4: Effect of variance in discounts.

Finally, we used the third set of experiment to analyze the effect of the variance of units in the package discounts. Each experiment –indexed by k– is characterized by a total number of units (from 3 to 5), the variance of each unit (from 0.25 to 1.75) and a pairwise correlation coefficient (0, -0.5 or 0.5).[37] We estimated the following regression:

$$\overline{Disc}_k(i,a) = \beta Var_k(i) + \rho_{ka} + \varepsilon_{kia}.$$

The idea is to estimate how the variance of a unit ($Var_k(i)$) affects the discount of adding a unit to package a, controlling for the package a in each experimental condition k through the fixed-effect ρ_{ka}. The results of this estimation are reported in Table A.4. The results suggest that the discount increases with the variance of the unit added to the package.

In summary, the computational experiments in these section suggest that:

- When firms have a relative cost advantage, they tend to provide larger discounts when combining units which are more negatively correlated.
- The discount of adding a unit to a package increases with the variance of the added unit.[38]

This is consistent with the intuition from the multiproduct monopolist literature that a reduction in the variance from combining units makes it easier for a firm with lower costs to extract surplus, which translates into larger package discounts.

B.2 Sub-additive bids and synergies

The second set of computational experiments addresses a limitation of the previous analysis: in the CA for school meals the bids are subadditive and there are cost synergies. In this section we compute the optimal bid prices of a firm that competes in a CA assuming that its opponents make package discounts and the firm

[37]Marginal costs and mean of the competitors' bid distribution are zero in all the experiments.

[38]In subsequent analysis, because units are heterogeneous in their average bid prices, we measure the variability of the bid price with the coefficient of variation.

exhibits cost synergies. Because of the computational complexity involved in computing optimal bid prices we restrict our analysis to two units. To simplify, both units have the same volume. We assume the firm's costs are given by $c = (c_1, c_2, c_{12})$. We allow for cost synergies, so we can have $c_{12} < c_1 + c_2$. Opponents' bids (per meal) are modeled like:

$$b_{af} = \sum_{i \in a} \delta_{if} \frac{v_i}{v_a} - g_f \mathbf{1}[|a| > 1], \tag{9}$$

where $(\delta_{1f}, \delta_{2f})$ are i.i.d. samples from a multivariate normal distribution with mean vector μ and covariance matrix Ω. The term $g_f \mathbf{1}[|a| > 1]$ allows for opponents' package discounts; g_f are i.i.d. samples from a uniform distribution $[\underline{g}, \overline{g}]$. We assume there are 10 bidders.

To determine the optimal bid prices we use simulation based optimization (Fu et al., 2005). In particular, we construct a grid of bid prices. For each bid price in the grid, (b_1, b_2, b_{12}), expected profits are given by $\sum_{j \in \{1,2,12\}} (b_j - c_j)\, Pr[\text{win j}]$, where the uncertainty involved in computing $Pr[\text{win j}]$ is with respect to the opponents' bids. For each bid price in the grid, we compute expected profits using simulation by sampling opponents' bids from equation (9) (we use a relative precision of 0.2%). The optimal bid prices maximize expected profits among all the grid points.[39]

We computed the optimal bid prices for many different scenarios. We consider the following range of unit costs: $(c_1, c_2) \in [4, 14]^2$. Cost synergies are contained in the interval $c_1 + c_2 - c_{12} \in [0, 4]$. We fixed average opponents' single unit prices $\mu(\delta_{if}) = 15$. The standard deviation of individual bid prices range in $\sigma(\delta_{if}) \in \{1, 2, 4\}$, and the correlation among individual bid prices $\rho(\delta_{1f}, \delta_{2f}) \in \{-0.8, -0.4, 0, 0.4, 0.8\}$. Finally the upper bound for the opponents' package discounts is in the range $\overline{g} \in [0, 6]$ while the lower bound is fixed to $\underline{g} = 0$. We have 8101 scenarios in total. We use the following grid: given the firm's cost vector (c_1, c_2, c_{12}), each candidate of bid price vector, or grid point, which is indexed by j, (b_1^j, b_2^j, b_{12}^j), lies within the following range with incremental size of $h = 0.1$:

$$c_1 \leq b_1^j \leq \mu(\delta_{1f}) + 3\sigma(\delta_{1f})$$
$$c_2 \leq b_2^j \leq \mu(\delta_{2f}) + 3\sigma(\delta_{2f})$$
$$c_{12} \leq b_{12}^j \leq \max_{i,k}\{b_1^i + b_2^k\}$$

The solution in many of the experiments sets prices for stand-alone bids which have zero probability of winning (less than .05%). To focus on package discounts we only consider scenarios for which single-unit bids are relevant (i.e. with positive winning probability), leaving 2114 observations in the sample.[40] We regressed the discount $b_1 + b_2 - b_{12}$ on: (1) the correlation between the stand-alone bids of opponents

[39] To make the comparison of the profits more reliable, we used the same random number seed for all grid points.

[40] In experiments where the optimal solution has irrelevant stand-alone bids, the discount can be infinite. We also analyzed all experiments by fixing the discount at the minimum value at which the stand-alone bids are irrelevant and the observed patterns in the results are similar.

	(1) Low Cost	(2) High Cost
CV	3.388**	1.927**
	(0.359)	(0.272)
Corr	-0.766**	-0.284**
	(0.038)	(0.031)
Observations	944	97

Standard errors in parentheses
* $p < 0.01$, ** $p < 0.001$

Table A.5: Effect of coefficient of variation and variance in discounts.

$(\rho(\delta_{1f}, \delta_{2f}))$; (2) the average coefficient of variation of the two units (defined as $\frac{1}{2}\sum_{i=1,2}\sigma(\delta_{if})/\mu(\delta_{if}))$; (3) the opponents' discount upper bound ($\bar{g}$); and (4) the cost synergies ($c_1 + c_2 - c_{12}$) of the bidder. The regression was estimated separately using experiments where both units' cost are below one standard deviation of mean bid price (*Low Cost*) and where both units's costs are above the mean (*High Cost*). The results are shown in Table A.5.

The results suggest that the patterns suggested by the analysis with additive bids (section B.1) continue to hold under sub-additive bids and cost synergies for $N = 2$: (1) discounts tend to increase when the correlation between units decreases and when the coefficient of variation of the units increase; and (2) the incentives for strategic bundling are more pronounced for firms with a relative cost advantage.

Auction	Firms	Bids	Class. L		Class. LM		Class. M		Class. S	
			Firms	Bids	Firms	Bids	Firms	Bids	Firms	Bids
1999	9	443	4	803	2	304	1	106	2	31
2000	21	572	4	781	4	126	11	754	2	40
2001	22	1,037	6	1,288	5	757	10	1,126	1	44
2002	16	1,466	6	1,145	2	5,298	3	1,485	5	306
2003	20	2,157	6	3,532	3	4,312	4	1,276	7	557
2004	24	7,894	5	16,543	4	11,925	3	9,127	12	2,639
2005	16	7,188	5	10,888	3	11,286	2	12,511	6	280

Table 1 – The second and third column contain the total number of participant firms and the average number of submitted bids *per firm* in each auction. The following columns contain the same information disaggregated by firms' classification.

Firm Class.	Number of TUs in Winning Package								Total
	1	2	3	4	5	6	7	8	
L	8	5	4	4	1	1	0	5	28
LM	4	1	5	1	2	0	0	0	13
M	5	1	1	6	0	0	0	0	13
S	10	1	4	0	0	0	0	0	15
Total	27	8	14	11	3	0	0	5	69

Table 2 – Total number of winning bids over all auctions as a function of winning firm’s classification and number of TUs in winning package.

	(1) Unit Popularity	(2) Degree
Size	0.108**	0.025*
	(6.10)	(2.50)
Meals/School	0.176**	0.115**
	(10.06)	(11.80)
Special meals	0.087**	0.031**
	(5.89)	(3.80)
Nearby unit	0.098**	0.008
	(5.49)	(0.84)
Renew	0.092**	0.028**
	(6.39)	(3.42)
Firm won unit	0.105**	0.034**
	(7.44)	(4.33)
Observations	3328	3328
R-square	0.1360	0.0921

Table 3 – Regression analysis of unit popularity (measured as the number of packages a unit is included on) and degree of a unit (measured as the number of different TUs that are packaged with a unit). Coefficients are standardized and t-statistics are shown in parenthesis. * and ** indicate significance at the %5 and 1% level.

Density Discounts			**Scale Discounts**		
Size	Estimate	SE	Size	Estimate	SE
[2,4]	3.29	(0.17)	[3,6]	12.14	(0.35)
[4,6]	4.01	(0.09)	[6,9]	17.49	(0.35)
[6,8]	4.82	(0.09)	[9,12]	19.78	(0.35)
[8,10]	5.65	(0.10)	[12,15]	21.59	(0.35)
[10,12]	6.06	(0.10)	[15,18]	23.22	(0.35)
[12,14]	6.30	(0.10)	[18,21]	24.99	(0.35)
[14,16]	7.27	(0.11)	[21,24]	26.26	(0.36)
[16,18]	7.72	(0.13)	[24,27]	26.12	(0.37)
[18,20]	7.44	(0.17)	[27,.]	24.07	(1.15)
[20,22]	6.76	(0.23)			
[22,24]	6.78	(0.33)			
[24,.]	7.03	(1.18)			

Table 4 – Results from first step regression (equation (4)). Robust standard errors shown in parenthesis. Size measured in million meals per year. Number of observations is 409,831. Centered R-square is equal to 0.98.

	(1)	(2)
LocIncumb	-13.29**	-10.14*
	(3.876)	(4.555)
LocComp	-3.407*	-3.578*
	(1.504)	(1.545)
Renew	-18.02**	-8.839
	(6.412)	(6.734)
Size	-21.09	-19.06
	(11.53)	(11.08)
Special meals	497.8**	497.7**
	(36.21)	(34.77)
Performance	-18.73*	-32.83**
	(7.925)	(9.986)
NewFirm	-19.67**	-15.94**
	(4.394)	(5.261)
FinGrade	-5.175**	-7.362**
	(1.809)	(2.074)
FinGrade=1	-54.26**	-58.97**
	(9.086)	(11.36)
Observations	2839	2839
R-Square	0.683	0.741

Table 5 – Results from second-stage regression (equation (5)). Both specifications include dummy variables for firm, auction and unit, and the controls specified in Section 4.2. Specification (2) also includes dummy variables for firm-region (which absorb the firm dummies). Standard errors in parentheses. * and ** indicate statistical significance at 0.05 and 0.01 confidence levels.

Sample	Discount N=2	N=5
Extreme south	13.62	7.44
Santiago	11.84	7.17
Any location	11	6.62

Table 6 – Comparison of discounts from combining units in Santiago with other units in different geographic regions.

	(1) 2 units, Small firms	(2) 2 units	(3) 2 units, low cost	(4) <5 units
CV	4.656^{*}	2.640^{*}	15.42^{**}	3.352^{**}
	(2.242)	(1.262)	(5.305)	(0.443)
Correlation	-0.819^{**}	-0.592^{*}	-1.279^{+}	-0.295^{*}
	(0.306)	(0.241)	(0.709)	(0.117)
Cluster	2.264^{**}			
	(0.201)			
Observations	4425	11268	564	77619
R-square	0.266	0.432	0.465	0.635

Standard errors in parentheses
$^{+}$ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$

Table 7 – Results of the regression analysis on strategic bundling (equation (8)). Both regressions include the control variable *Size* and dummies for firm, auction and combination. Standard errors in parentheses. +, *, ** indicate statistical significance at the 0.1, 0.05 and 0.01 confidence levels.

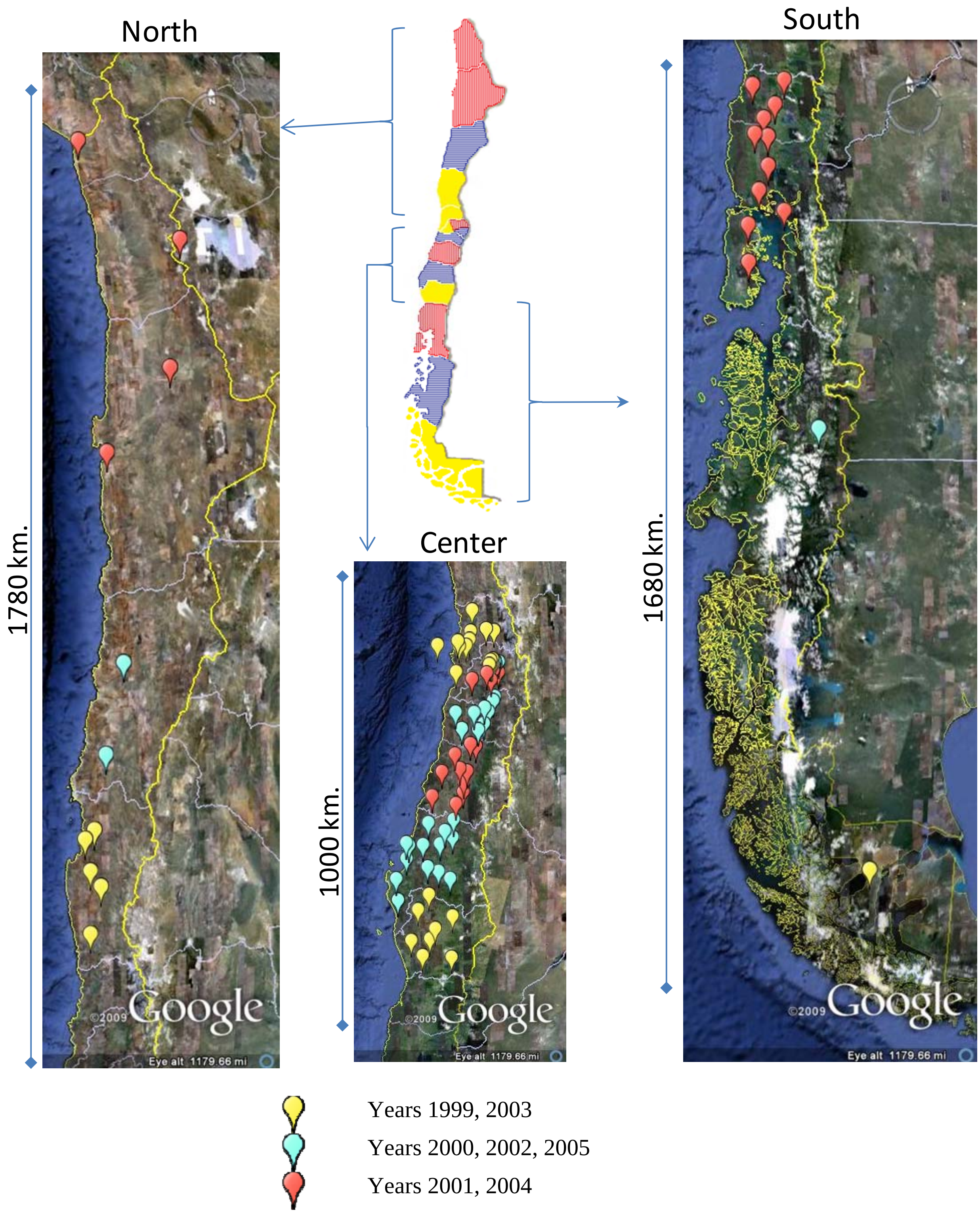


Figure 1 – Map of Chile. Points indicate the centers of the territorial units (TUs); colors indicate the year in which they are auctioned.

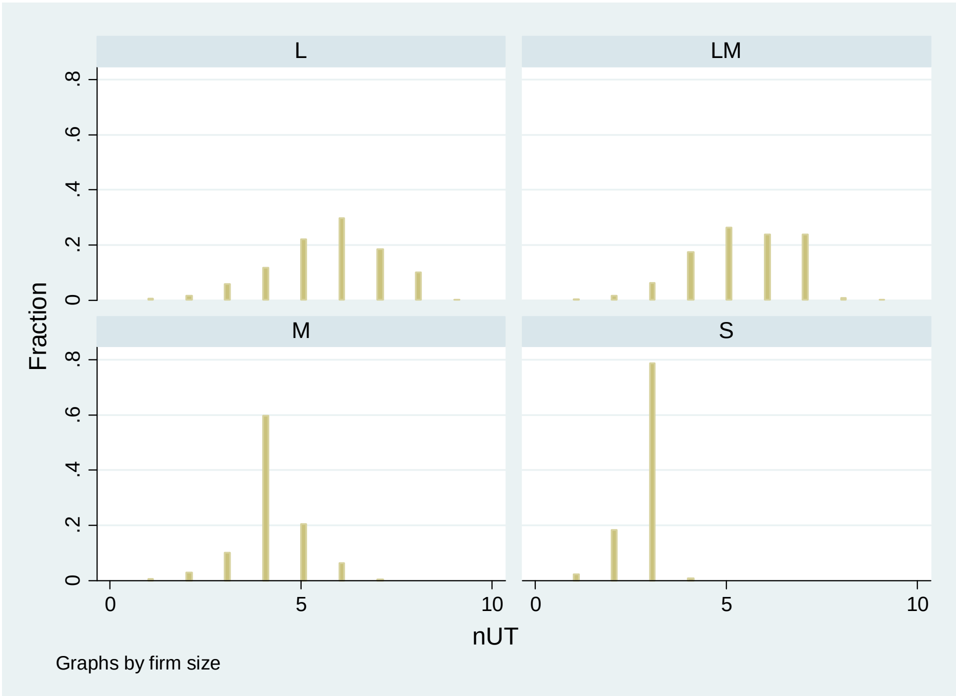


Figure 2 – Histogram of number of units in a bid, by firm size. L denotes large, LM medium-large, M medium, and S small.

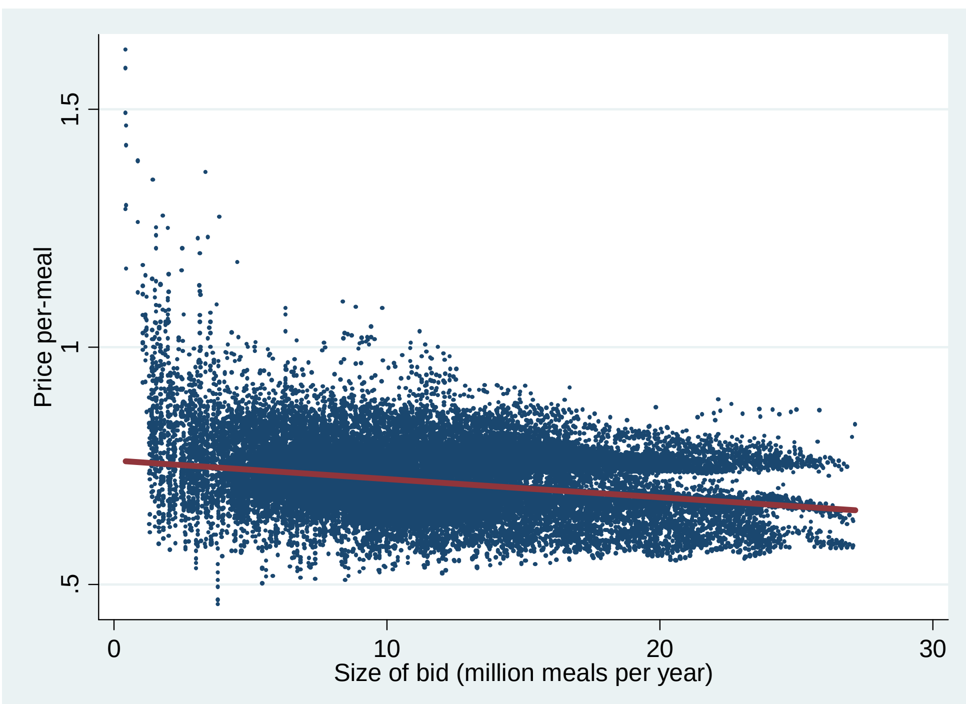


Figure 3 – Bid price as a function of the size of the bid. Includes all bids in auctions 1999, 2000 and 2001.

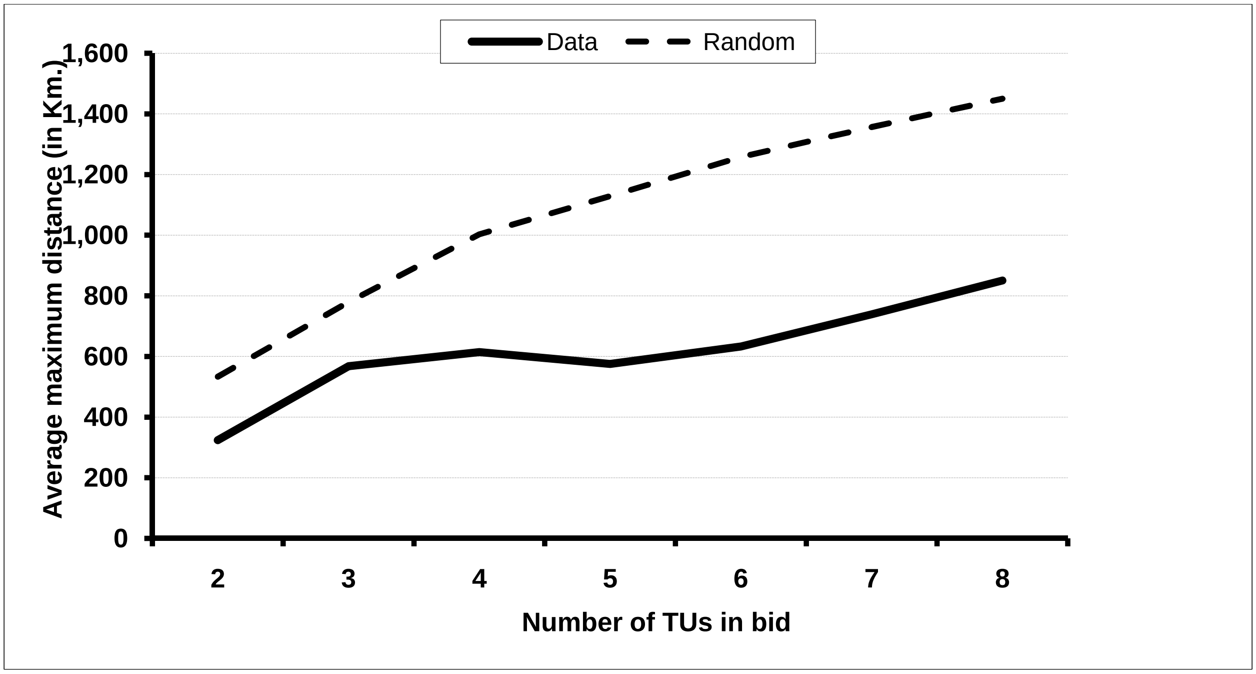


Figure 4 – Average maximum distance among TUs contained in a bid, for bids with different number of units. Dashed line indicates the average maximum distance when the units in the package are sampled at random.

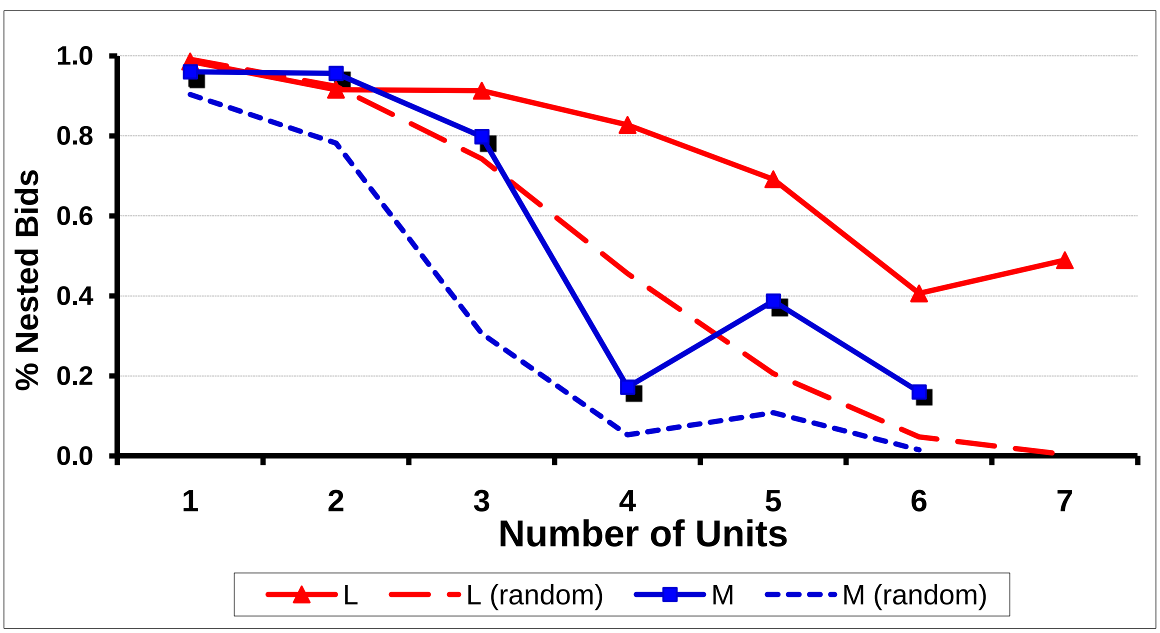
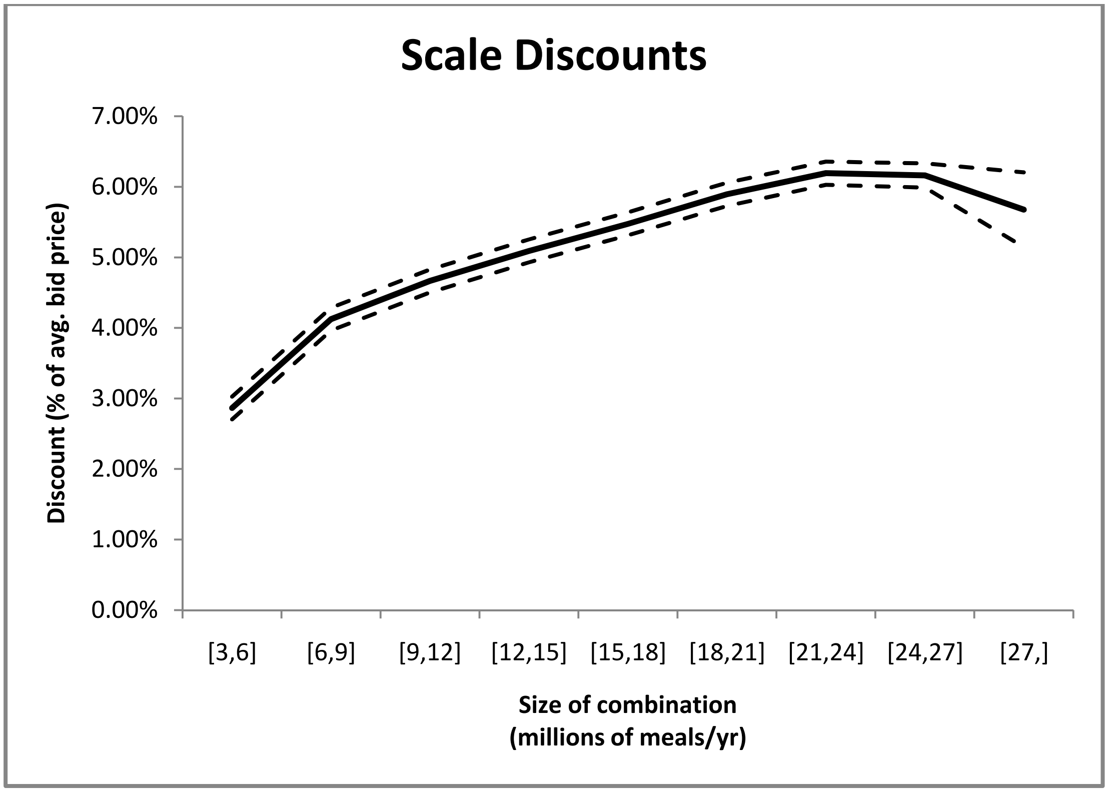


Figure 5 – Percentage of "nested" bids for Large and Medium firms in the data. Dashed lines show percentage of nested bids for a set of *randomly generated bids* that is consistent with the total number and sizes of bids submitted by each firm in the data.



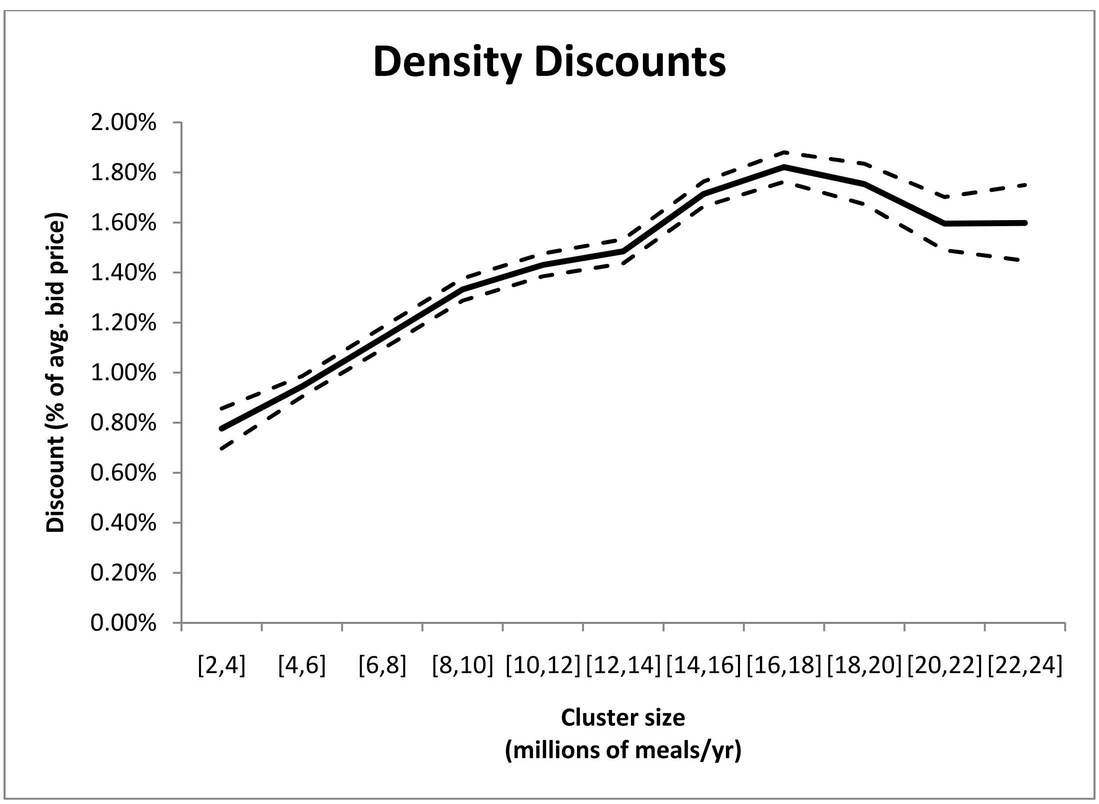


Figure 6 – Estimates of discount curves due to scale (top) and density (bottom). Dotted line indicates the 95% confidence interval of the estimate.

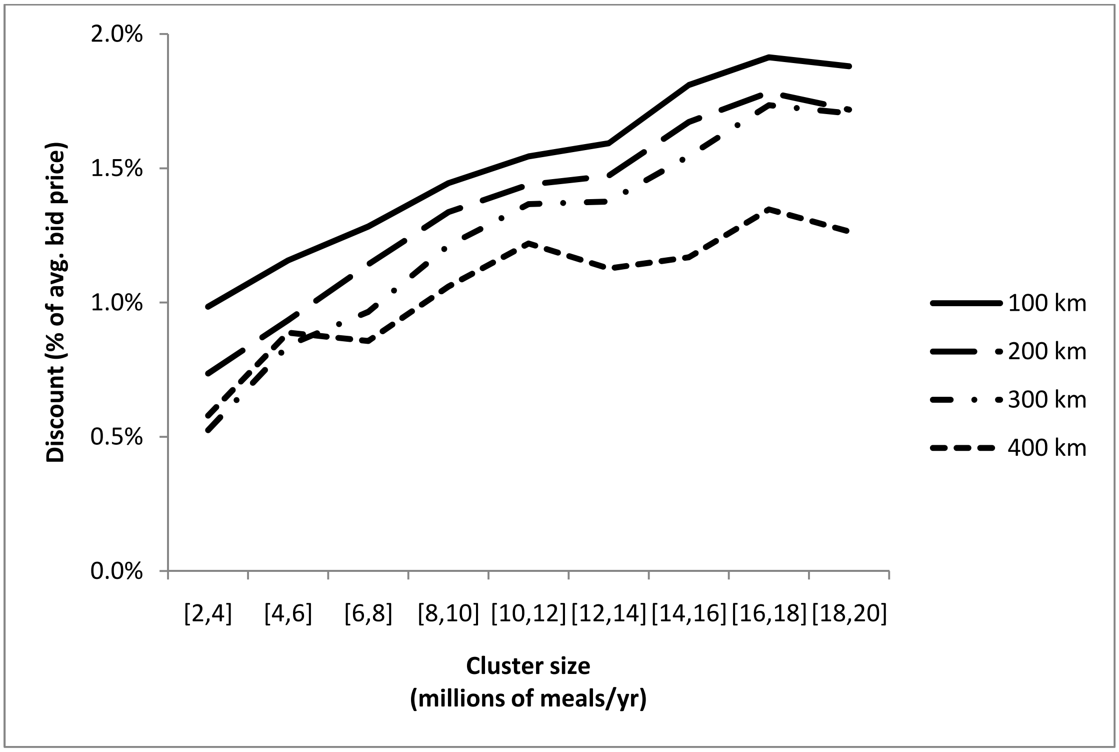


Figure 7 – Estimates of density discounts for different cluster sizes.