A comparative study of structural models of corporate bond yields: An exploratory investigation ☆

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Abstract

This paper empirically compares a variety of firm-value-based models of contingent claims. We formulate a general model which nests versions of the models introduced by Merton (1974), Leland (1994), Anderson and Sundaresan (1996), and Mella-Barral and Perraudin (1997). We estimate these using aggregate time series data for the US corporate bond market, monthly, from August 1970 through December 1996. We find that models fit reasonably well, indicating that variations of leverage and asset volatility account for much of the time-series variations of observed corporate yields. The performance of the recently developed models which incorporate endogenous bankruptcy barriers is somewhat superior to the original Merton model. We find that the models produce default probabilties which are in line with the historical experience reported by Moodys.

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1. Introduction

In this paper we estimate structural models of corporate bond yields using monthly observations of yield indices of US investment grade corporate bonds. Following the tradition established by Merton (1974) structural models of corporate bonds treat these as contingent claims on the assets of the firm. Variations in yield are explained by variations in leverage, asset value volatility, and the riskless interest rate. Structural models of the liabilities of the firm are attractive on theoretical grounds, as they link the valuation of financial claims to economic fundamentals. Further motivation for structural models of corporate liabilities is provided by past empirical work which has found that corporate yield spreads over government bonds is related to stock market returns and macroeconomic business cycle indicators (Jaffee, 1975; Duffee, 1998).

Despite the appeal of structural models, they have proved difficult to implement successfully. One problem is that the theoretical models relate yields to fundamental determinants in a highly non-linear way. Furthermore, structural models have greater data requirements than other approaches. Past serious attempts to implement the Merton model on US corporate bonds proved disappointing (Jones et al., 1984, 1985). The models did not fit very well and tended to systematically underestimate observed yields when plausible values of asset volatility were employed. Nevertheless, simple structural models have been the basis of a number of tools used by practitioners in the evaluation of credit-risky instruments.

The alternative approach of practitioners has been to infer fair yields from market yields of other traded instruments that are comparable with respect to rating and maturity. In the simplest application this gives rise to "matrix pricing" where the yield of a given issue is derived from a set of yields of traded benchmarks using ad hoc rules for interpolation. Recently a number of advances have been made which give a rigorous statistical basis for inferring issue yields from market benchmarks. Important studies of these so-called "reduced form" models of corporate yields include Litterman and Iben (1991), Jarrow and Turnbull (1995) and Duffie and Singleton (1997, 1996). This approach introduces a variety of flexible functional forms giving the conditional probability of default. The results have been encouraging, and these reduced form models are useful in some practical applications. However, there are important limitations to this approach.

A first issue is the abundance of possible functional forms which may be calibrated to a given set of benchmarks but which can imply significantly different values when pricing some other issue. Which model is to be believed? A second problem with reduced form models is that for many pricing problems there are no reliable benchmarks. In this case one would like to establish values from first principles. A third limitation of reduced form models is that they tend to ignore systematic risks in a bond portfolio. In fact, it is likely that default events of diverse firms are correlated and coincide with cyclical downturns.

For all of these reasons we feel there is a need for further empirical study of structural models of corporate bonds. An additional reason for undertaking this is that in recent years there has been renewed theoretical work in this area designed to address the limitations encountered with past contingent claims models. It has been recognized that the Merton model is unrealistic in modeling financial distress as an exogenously fixed absorbing barrier [2]. This shortcoming has given rise to new models which determine the lower reorganization boundary of the model endogenously. One approach has been to introduce a game-theoretic model of the bankruptcy process and in this way address the determinants of deviations from absolute priority (see Anderson and Sundaresan, 1996; Anderson et al., 1996; Anderson et al., 1997; Fan and Sundaresan, 1997). A closely related approach treats the liquidation decision as an option (see Mella-Barral and Perraudin, 1997). A third line of modeling follows Black and Cox (1976) in assuming that debt service is paid by issuing new equity; the endogenous bankruptcy point is the value of the firm such that the market price of equity drops to zero. (See Leland, 1994; Leland and Toft.) While there are similarities across models, their implications for pricing can differ significantly. To date there has been no attempt to discriminate among these models on empirical grounds.

The purpose of the current study is to see to what extent these new models are able to account for broad time series variations in observed yields of defaultable bonds. Secondarily, it attempts to identify which of the models fits the data best. We formulate a general model which nests versions of the models introduced by Merton (1974), Leland (1994) Anderson and Sundaresan (1996), and Mella-Barral and Perraudin (1997). We estimate these using aggregate time series data for the US corporate bond market, monthly, from August 1970 to December 1996.

Overall the results suggest that the recent efforts to modify the contingent claims model to allow for the endogenous determination of the default barrier based on economic fundamentals have led to an improvement of structural models. These models correlate with observed yield spreads more highly than does the Merton model. Furthermore, their parameter estimates are more

[2] This point and the related empirical literature are discussed at length in Anderson and Sundaresan (1996).

plausible. We also find under plausible assumptions on the market risk-premium for levered firms that the models produce default probabilities for 5 years or more which are inline with the historical experience reported by Moody's.

The remainder of the paper is organized as follows. In Section 2 we describe the data. Section 3 develops the general theoretical model and shows how known models from the literature are nested as special cases. Section 4 presents the empirical estimates. Section 5 is devoted to our conclusions.

2. Data

In this paper we study the time series behavior of yield indices of investment grade corporate bonds as related to aggregate indicators of leverage and asset volatility. We choose this approach over the alternative of working with specific bond issues for three reasons. First, indices of corporate bond yields should be more tightly linked to common economic factors than are individual issues since by averaging the impact of firm-specific factors will be eliminated. Second, with yield indices liquidity premia are likely to be fairly constant and therefore can be modeled relatively simply [3]. Third, it is difficult to acquire all the firm-specific information that may be relevant to the pricing of a specific issue.

We use observations of the generic corporate, on-the-run bond yields for industrial corporations as reported in the Salomon Brothers Book of Analytical Yields. This reports monthly observations of yields on 30 year bonds with S&P ratings of AAA, A, and BBB for a relatively long time-period, from August 1970 to December 1996. It should be noted that by construction these yields are averages of relatively liquid, newly issued bonds trading close to par.

In the first three lines of Table 1 we present descriptive statistics for our sample of yields on AAA, A and BBB bonds, respectively. It should be noted that both the level of the yield and the volatility of yields vary inversely with credit quality. Note that the autocorrelation coefficients with 1, 2, 3, and 12 month lags are all quite high raising the possibility that the yield series are not stationary. In fact, much of the variation of corporate yields over this period of more than 26 years is likely accounted for by changes in the default-free rate.

To verify this we have calculated the spreads of the corporate index over the 30-year US Treasury yield, which we denote as r_t. Summary statistics are presented in lines 4–6 of Table 1. In the sample AAA spreads average 71 basis

[3] It has long been recognized that part of the premium of corporate bond yields over Treasuries reflects their relatively lower liquidity. (See Fisher, 1959; Grinblatt, n.d.) However, modeling of liquidity effects for corporate liabilities is relatively recent and has not yet settled on the determinants of liquidity differences across firms.

Table 1
Descriptive statistics, 30-year, risky yields and spreads over treasuries; 8/70–12/96

	Mean	Std. Dev.	Min	Max	AR-1	AR-2	AR-3	AR-12
Yaaa	0.0938	0.0213	0.0665	0.1650	0.9818	0.9608	0.9404	0.7730
Ya	0.0991	0.0232	0.0700	0.1763	0.9826	0.9638	0.9451	0.7829
Ybbb	0.1051	0.0246	0.0725	0.1850	0.9831	0.9647	0.9455	0.7587
Spaaa	0.0071	0.0036	−0.0005	0.0193	0.9027	0.8629	0.8229	0.5116
Spa	0.0124	0.0048	0.0030	0.0259	0.9165	0.8811	0.8444	0.4608
Spbbb	0.0184	0.0070	0.0081	0.0375	0.9386	0.8988	0.8546	0.3667

points (b.p.'s). The comparable figures for A and BBB bonds are 124 b.p.'s and 184 b.p.'s. The volatility of BBB spreads is roughly twice that of AAA spreads. Indeed, over the sample the BBB varied over a range of almost 300 b.p.'s. The autocorrelation coefficients of the spreads are lower than those of the yields suggesting that there may be a stationary (cointegrating) relationship between corporate and default risk-free yields. It is a basic premise of structural models that the spread should depend upon such factors as leverage and asset volatility as well as the default-free term structure.

Thus, in order to fit structural contingent claims models to monthly time series of yields on generic US corporate bonds, we construct monthly variables to serve as proxies for leverage and asset volatility. Our leverage measures are based on data contained in the annual aggregate balance sheets of non-financial corporations contained in the US Federal Reserve's Flow of Funds Accounts. Debt figures are reported as book values and equity figures are reported as market values. In addition, we have incorporated in our leverage indicator flow-based information about credit quality. In particular we calculate interest burden from the US. National Income Accounts as the ratio of interest to total profits. Our leverage proxy is a linear combination of these stock-based and flow-based measures,

$$\text{LEV} = \phi \text{ Total Liabilities/(Total Liabilities+Equity)} + \omega \text{ Interest/Profits.}$$

Monthly series of this measure were constructed assuming total liabilities and the interest/profits ratio follows constant trends within a year. We construct a monthly series of the market value of equity by assuming that the monthly deviation from the one-year growth trend is the same for the Flow of Funds equity index as for the S&P 500 index. We denote our leverage observation in month t as P_t.

Our proxy for asset value volatility is derived from the market prices of equity. The volatility of equity returns is calculated as the standard deviation of monthly returns on the S&P 500 over 12 months ending in month t. We denote this by v_t. For simplicity we take the ratio of asset volatility to equity volatility to be a constant, a, so that the implied proxy for asset volatility is av_t. The parameter a is determined from the data.

3. A general structural model of corporate bonds

In order to compare models and to provide a framework for testing, we introduce a general framework that gives rise to closed form solutions for the case of perpetual coupon bonds and which nests the classic model of Merton (1974) as well as some recent contributions as special cases. This framework includes the model of Leland (1994). Furthermore, it incorporates a special case of the game theoretic model of Anderson and Sundaresan (1996). In particular, Anderson et al. (1996) study the continuous-time limit of the discrete-time Anderson/Sundaresan model. They solve the model in closed-form for the case of perpetual coupon debt and find a formula which is a special case of the solution previously presented by Mella-Barral and Perraudin (1997). We refer to this model as AST below but it should be borne in mind that it is a special case of both the models of Anderson and Sundaresan and of Mella-Barral and Perraudin.

All the models we consider are contingent on a single state variable, V,which is the value of the firm. They have solutions for perpetual coupon debt which take the form

$$B = \frac{cP}{r}(1 - P_d) + P_d \max(\theta V^* - K, 0),$$

where B is the value of the bond, c the coupon rate, P the principal, and r the risk-free rate. Here financial distress costs in case of default are represented by the proportional recovery rate, θ, and the fixed bankruptcy cost K. We can interpret P_d as a probability of default and V^* as the default barrier. The structural models can be viewed as all stating that the value of a risky bond equals the value of a riskless bond times the probability of no default plus the value of the collateral times the probability of default. Structural models of perpetual coupon debt differ with respect to how they assess the probability of default and the value of collateral upon default [4]. We now discuss how special cases of this model correspond to the perpetuity models of Leland, AST, and Merton. [5]

In both the Leland and AST frameworks the value process follows a geometric Brownian motion with drift $(\mu - \beta)V$ where β is a cash flow payout rate. For these models, the default probability is given by

[4] Note, however, that this gives a risk-neutral probability of default. In general it will not directly comparable to historical default rates. This issue is discussed below when we interpret our estimates.

[5] A fuller development of this can be found in the working paper upon which the current article is based (Anderson and Sundaresan, 1999).

$$P_d = \left(\frac{V}{V^*}\right)^{\gamma_2}, \tag{2}$$

where $\gamma_2 < 0$ so that $0 < P_d < 1$.[6] As V approaches V^* from above, this probability of default approaches unity. The Leland and AST differ in the way they determine the trigger point V^*.

Leland (1994) assumes that asset liquidation is costly and that partial liquidations of assets are not possible. Furthermore, he follows Black and Cox (1976) in assuming that contractual debt service is made through issues of new equity. In the version of the model we follow here, he assumes that the firm will pay contractual service on outstanding debt until the asset value falls to the point where the value of equity is zero. At this point the firm is liquidated, and the bondholders receive the available collateral. He makes the further parametric assumptions that $\beta = 0$ and $K = 0$. Under these assumptions he shows that the liquidation (default) barrier is V_L^* given by the expression

$$V_L^* = \frac{cP}{(r + 0.5\sigma^2)}. \tag{3}$$

Note that this default barrier (i) is proportional to the contractual debt service, (ii) is independent of the current value of the assets of the firm, (iii) does not depend upon the marginal recovery rate parameter θ, and (iv) is decreasing in level of volatility of assets. Furthermore, note that in this formulation the shareholders do not default on the contract until equity falls to zero at which point the firm is liquidated. In other words, in bankruptcy there are no deviations from absolute priority of claims.

Anderson and Sundaresan (1996) argue that in modeling of corporate liabilities the starting point should be a serious consideration of the bankruptcy regime that applies to the firm. They show that this feature leads to deviations from absolute priority. In much the same spirit, Mella-Barral and Perraudin (1997) allow for value to be extracted from creditors through a strategic default by shareholders. Based on these insights Anderson et al. (1996) characterize the bankruptcy trigger as

$$V_{\mathrm{AST}}^* = \frac{(cP/r + K)}{\theta(1 - 1/\gamma_2)}. \tag{4}$$

This default barrier is increasing in c and can be shown to be decreasing in asset volatility, σ^2. However, unlike the default barrier found by Leland, V_{AST}^* is decreasing in the marginal recovery rate, θ, and increasing in the deadweight liquidation cost, K. The economic interpretation of this is that the greater costliness of liquidation, the greater is the shareholder ability to extract

[6] Specifically, $\gamma_2 = 0.5 - (r - \beta)/\sigma^2 - \sqrt{((r-\beta)/\sigma^2 - 0.5)^2 + 2r/\sigma^2}$.

debt-service concessions from creditors. The consequence of this feature is that the bond yields for firms for which liquidation is a very remote prospect may still reflect a significant premium to compensate for possible partial reductions in debt service.

In his analysis of perpetual coupon bonds Merton (1974) assume that contractual debt service is met through asset liquidation and continues until such time that all assets are exhausted. Furthermore, he assumes $\beta = 0$, $\theta = 0$, and $K = 0$. Then the default probability is given by

$$P_d^M = \frac{(2cP/\sigma^2 V)^{(2r/\sigma^2)}}{\Gamma(2 + 2r/\sigma^2)} M\left(\frac{2r}{\sigma^2}, 2 + \frac{2r}{\sigma^2}, \frac{-2cP}{\sigma^2 V}\right), \tag{5}$$

where $\Gamma(\cdot)$ is the gamma function and $M(\cdot)$ is the confluent hypergeometric function. Note that this formulation assumes that assets may be liquidated freely and that such partial liquidations do not involve a loss of value. These assumptions may be faulted as being counter-factual: bond covenants typically do restrict asset sales, and distressed sales of assets may well involve a loss of value. While this criticism is important, still Merton's formulation might be adequate for modeling bonds of firms far from financial distress.

4. Results

As discussed in Section 2, we are interested in finding a stable relationship between corporate yields and our aggregate measures of leverage and volatility. The structural models of Section 3 provide alternative forms of this possible relationship. We implement them empirically using the following estimable form:

$$\begin{aligned} y_t &= \lambda + y(P_t, v_t, \bar{r}_t; a; K, \theta, \beta) + u_t, \\ u_t &= \rho u_{t-1} + \epsilon_t. \end{aligned}$$

Here y_t is the observed market yield for corporate bonds rated BBB. We have focused on this rating category because of the greater volatility of BBB spreads relative to higher grade issues and because we expect that structure models of contingent claims are likely to be most useful for relatively lower grade issues.

The function $y(\cdot)$ is the yield implied by the bond value, $B(\cdot)$ calculated according to the formula, $y = cP/B$. The variables P_t and av_t are our proxies for leverage and asset volatility. The volatility multiple, a, the fixed bankruptcy cost, K, the marginal recovery rate, θ, and the cash flow rate, β, are all treated as constant parameters to be estimated. The models of Section 3 are one-state models which treat the default-free rate as non-stochastic. In reality, treasury rates vary considerably over time so that the expected risk-free rate is likely to vary as well. We have allowed for this by taking as a proxy for the risk-free rate, $\bar{r}_t$, the twelve-month moving average of 30-year treasuries. [7]

A number of considerations lead us to include an additive constant, λ, in the model. First, this may reflect a premium for the illiquidity of corporate bond markets relative to Treasuries. Second, it may reflect a tax effect, since interest on treasuries is tax deductible for many investors whereas interest on corporates is not. Finally, if the specification of $y(.)$ is biased in some respect, the additive constant will correct for this. In the absence of a model of the determinants of liquidity, it would be difficult to decompose a given estimate of λ into these separate effects.

We have specified the residuals of the yield equation as following a first-order autoregressive process. This may capture autocorrelation of yield spreads not modeled with our proxies for leverage and asset volatility. Alternatively, it may reflect autocorrelation of liquidity premia.

We estimate the parameters λ, a, K, θ, β, and ρ by nonlinear least-squares. The estimates seem well-determined: experimentation with alternative initial guesses of the parameters always converged to the same parameter estimates.

The results of the estimates are presented in panel A of Table 2. Looking first at the Merton model, it is seen that all the parameters are highly significant. The volatility multiple, a, is 0.9, indicating that in the model asset volatility must be close to the volatility of the S&P 500 in order to be consistent with BBB yields. This supports the view that the Merton model requires implausibly high volatility levels in order to fit corporate yields observed in the market. The estimate of λ is 1.8% which compares to an average yield spread of BBB bonds over treasuries of 1.84% (see Table 1). That is, the credit risk variables which enter in the nonlinear portion of the estimated model account on average for only four basis points of observed yield spreads. Nevertheless, these variables are contributing positively to the ability of the model to track time series variations of the observed corporate yields. This is indicated by the fact that the autocorrelation of the residuals of the model, ρ, is 75% as compared to the autocorrelation of BBB yield spreads of 94% that was observed in the sample.

The parameter estimates in the AST model are also highly significant. The volatility multiple is 0.54 indicating that the model requires lower levels of volatility than the Merton model in order to be consistent with observed corporate yields. Furthermore, the estimate of λ is 1.5% implying that the

[7] We have also estimated the models taking the risk-free rate as a constant over the whole sample. In this specification, the models with endogenous bankruptcy barrier (AST and Leland) fit much better than does the Merton model. However, a number of the parameter estimates were economically implausible, suggesting that the constant risk-free rate specification is unacceptable over a long time series. A full discussion of these results is given in Anderson and Sundaresan (1999).

Table 2
Model estimates, 7/71–12/96
Panel A: Parameter estimates and estimated standard errors

	a	K	λ	β	θ	ρ
Merton	0.907		0.018			0.741
	0.100		0.001			0.039
AST	0.545	0.041	0.015	0.068		0.753
	0.099	0.013	0.002	0.019		0.038
Leland	0.801		0.016		0.948	0.756
	0.104		0.001		0.034	0.038

Panel B: Spreads over treasuries

	Mean	Standard Deviation	Correlation with market	1st Quartile	Median	3rd Quartile
Market	0.018	0.007	1	0.0125	0.0169	0.0216
Merton	0.018	0.006	0.479	0.0139	0.0175	0.0217
AST	0.018	0.006	0.522	0.0136	0.0168	0.0219
Leland	0.018	0.006	0.517	0.0135	0.0171	0.0218

credit risk component of the model captures 34 basis points of the average yield spread. The estimate of the cash payout rate β is economically plausible. And the estimated constant bankruptcy cost, K, is 4% of initial asset values which indicates that the model tracks observed corporate yields without assuming implausibly high bankruptcy cost levels. This was precisely one of the motivations which led Anderson and Sundaresan and Mella-Barral and Perraudin to introduce bargaining elements into their modeling of default.

Finally the parameters of the Leland model are also estimated with small standard errors. The volatility ratio estimate is 0.801, slightly less than that of the Merton model. Similarly, the estimate of λ is intermediate between that found in the Merton and AST models. The estimated marginal recovery rate, θ, is 95% which is broadly sensible. The estimated autocorrelation of the residuals is virtually the same as those found in the Merton and AST estimates.

In order to assess how well the model estimates fit the data we have calculated fitted spreads over treasuries, $s_t = \hat{y}_t - r_t$ and compared them to the observed market spreads. The summary statistics for the sample are reported in panel B of Table 2. It should be noted that the means of the observed BBB spread and those of the models are all approximately 180 basis points so that overall the models are unbiased. However, the fitted spreads are much less dispersed than are observed spreads. This can be judged from the interquartile range which is 91 basis points for the market spreads and 78 basis points, 83 basis points and 83 basis points for the Merton, AST and Leland models, respectively. The simple correlation coefficients with the market spreads are 0.479, 0.522 and 0.517 for the Merton, AST and Leland models, respectively.

This indicates some advantage of the endogenous bankruptcy model in tracking the observed yield spreads; although the difference with respect to the Merton model is not very large.

Model fits can be observed in more detail in Figs. 1–3 which plot the observed market yield spreads ('spbbb') and the fitted yield spreads ('spm') for the three models. The results for the AST model in Fig. 1 indicate that at some time the model tracks the observed spreads quite well. At other times (e.g., observations 100–130) the margin of error is quite substantial. The Leland model (Fig. 2) and the Merton model (Fig. 3) display roughly this same pattern.

Overall these results suggest that the recent efforts to modify the contingent claims model to allow for the endogenous determination of the default barrier based on economic fundamentals have led to an improvement of structural models. This statement is based somewhat on the fact that fitted spreads of the AST and Leland models correlate more highly with observed spreads than does the Merton model. However, more importantly it is based on the parameter estimates which are more plausible for the Leland model and, especially, for the AST model. As mentioned in Section 3, the AST model allows for the possibility of debt renegotiations resulting in deviations from absolute priority;

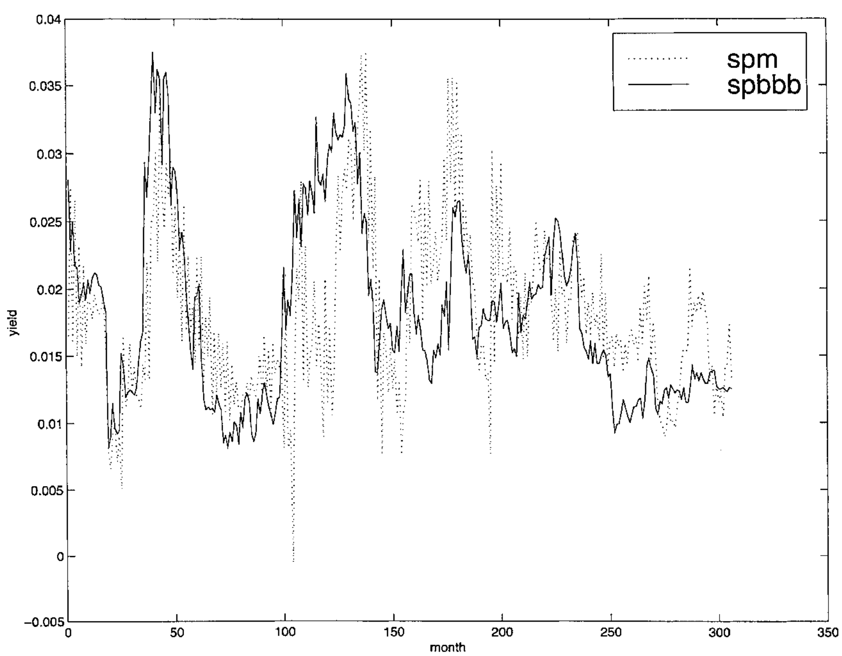

Fig. 1. AST perpetual analysis for 7/71–12/96.

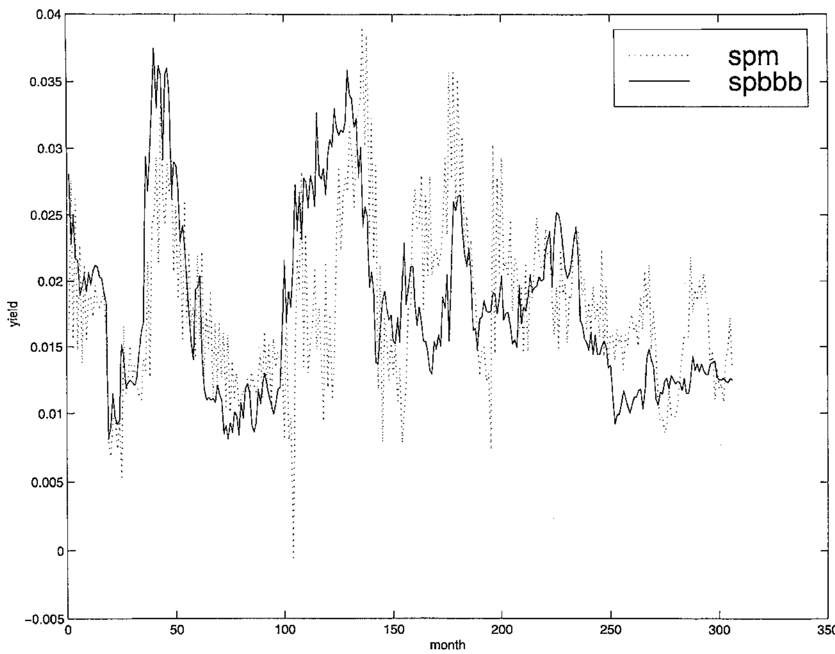

Fig. 2. Leland perpetual analysis for 7/71–12/96.

whereas the Leland model does not. This appears to have contributed to the ability of the AST model to fit the data with a lower assumed asset volatility and with a lower constant spread, λ. Overall, however, the differences in the performance of the AST and the Leland models are relatively minor in this application to BBB bonds. It may that the differences in the models would become more marked when applied to non-investment grade issues.

In order to further explore the models we have studied their implications for the estimated probability of default and have compared these to historically observed default frequencies. Specifically for each set of parameter estimates and for each month's combination of leverage, volatility, and risk-free rate, we have simulated by Monte Carlo 1000 paths of the asset process over 20 years. From this we have calculated the frequency that the model hits the model's default barrier within 1–5, 10, 15, and 20 years. In order to compare these frequencies with the historical default frequencies, we have assumed that the asset drift is given by $\mu_t = r_t + \delta$ where δ is a constant risk premium.

In Fig. 4, the results of the simulations of the AST model estimated assuming time-varying default-free rates are reported for $\delta = 5\%$ and $\delta = 0\%$ (i.e., the risk neutral case). In the same figure, we plot the historical N-year cumulative default probabilities reported for bonds with Moody's rating of Baa at $N = 0$ (Moody's Investor Services, 1997). In the case of a zero risk premium the

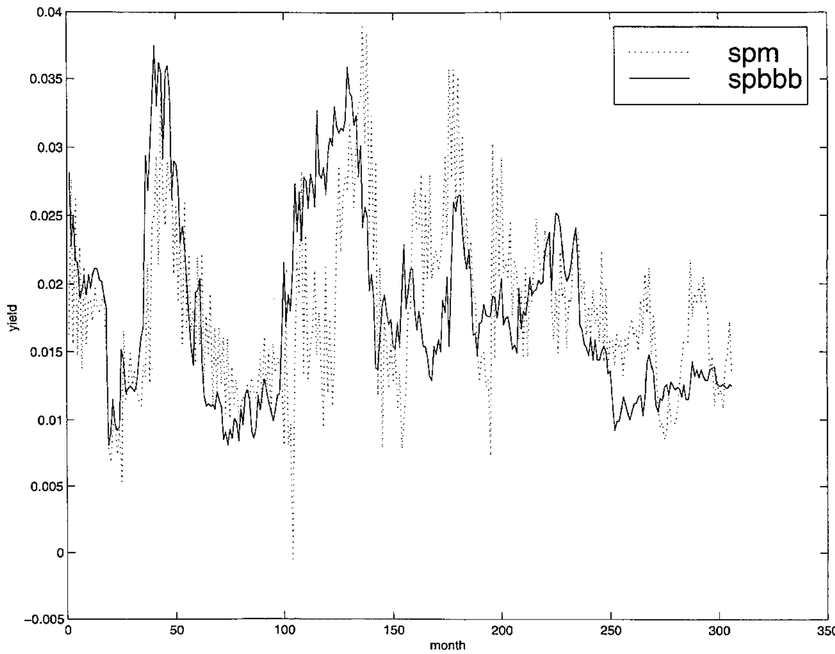

Fig. 3. Merton perpetual analysis for 7/71–12/96.

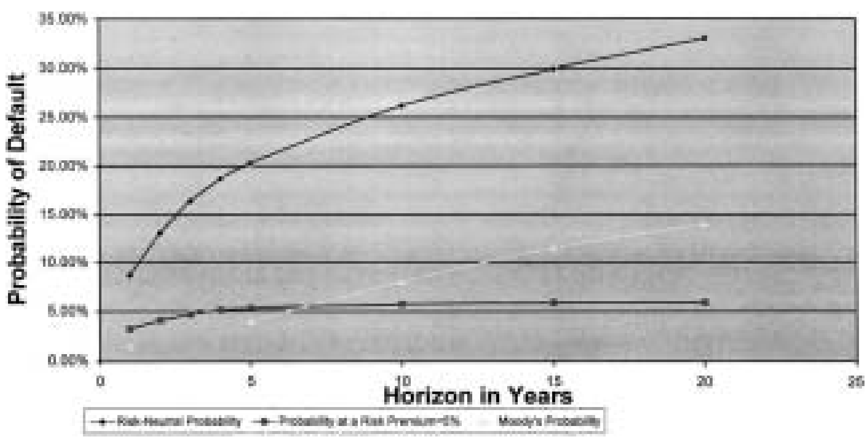

Fig. 4. Implied default probabilities.

AST model implies default probabilities which greatly exceed the historical observations. However, with positive risk-premia the model implies lower default probabilities. We see that assuming a 5% risk premium results in default probabilities which are reasonably close to Moody's figures. However, the fit is certainly not perfect. The AST default probabilities are too high at 1–5 years and too low at 20 years. Stated otherwise, fitted probabilities of default conditional on no-default prior to 5 years are too low compared to historical experience. Delianedis and Geske (1998) in a recent paper have used the Merton (1974) and Geske (1977) models to extract risk-neutral default probabilities. Such an approach can lead potentially to better estimates of the probability of rating migrations and defaults.

Finally, as a further comparison with historical observations we may note that, even though they allow for costly bankruptcy, the AST and Leland models are not entirely satisfactory in that they imply that recovery rates on defaulted bonds are quite high relative to actual experience. For example, Altman and Kishore (1996) report that the recovery rates on investment grade issues of senior unsecured debt are 48% of the principal. In contrast the fitted AST and Leland models imply average recovery rates somewhat over 90%.

5. Conclusion

Overall, these empirical results are fairly encouraging for the prospects of firm-value-based structural models for the pricing of corporate bonds. Much of the movements observed in historical times series of yields on generic corporate bonds can be accounted for in structural models using proxies for leverage and asset volatility. The results suggest that recent modifications of the contingent claims models to allow for endogenous default barriers have significantly improved the performance of the models.

This study is exploratory in nature, and there are clearly many interesting areas for further empirical work. It will be interesting to extend the structural approach by the incorporation of a stochastic risk-free term structure in a model with endogenous bankruptcy barrier and by the more careful modeling of the liquidity premium. Furthermore, the application of endogenous bankruptcy barrier models to specific bond issues using firm measures of leverage and asset volatility would be very interesting.

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