

# Preface to the Special Issue on Computational Economics

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Economics is the study of how scarce resources are allocated. Operations research studies how to accomplish goals in the least costly manner. These fields have much to offer each other in terms of challenging problems that need to be solved and the techniques to solve them. This was the case after World War II, partly because the individuals who went on to be the leading scholars in economics and operations research worked together during WWII. In fact, the two fields share many early luminaries, including Arrow, Dantzig, Holt, Kantorovich, Koopmans, Modigliani, Scarf, and von Neumann. Unfortunately, as new generations of scholars took charge, those ties weakened, resulting in little interaction between operations research scholars and economists in the past three decades.

In recent years, some economists and applied mathematicians have worked to reestablish ties between these fields. This special issue of *Operations Research* is an important step in that effort. It is not meant to be just a collection of current research that is of interest to both economists and operations researchers. It is an example of what we hope is a future in which we will not need “special issues” to describe computational economics.

The papers in this issue cover a broad range of material, including the introduction of advanced mathematical techniques to computational economics, algorithm development and analysis, examples where advanced computational methods are used to solve important economic problems, and examples of where computational ideas are used to motivate concepts of bounded rationality.

“Tackling Multiplicity of Equilibria with Gröbner Bases,” by Kubler and Schmedders, shows how algebraic geometry methods can be used to address the problem of multiplicity of equilibria. In the past 30 years, the field of algebraic geometry has been an example of where the mathematical theory has been spurred on by the exploding power of computers. The combination of theory, algorithms, and computer power now makes it possible to apply these methods to economic problems.

Another important task of computational theory is to determine how hard problems are. “Sharing Supermodular

Costs,” by Schulz and Uhan, looks at the complexity of computing the least core value in cooperative games with supermodular cost structures. Such games arise in a variety of operations research problems—scheduling, for example. The authors show that such problems are strongly NP hard, but for a special case of single-machine scheduling the Shapley value is in the least core.

“Multigrid Techniques in Economics,” by Speight, is a good example of how advances in computational physics, in this case computational fluid dynamics, can be applied to economic problems. Multigrid methods are a relatively new approach to solving partial differential equations, and this paper explains how they can be applied to dynamic economic problems.

Mechanism design has been an active field in game theory, but there has been relatively little work on developing computational methods that can be used to implement the theory in concrete ways. This issue contains two papers that improve our ability to apply mechanism design theory: “Multidimensional Mechanism Design: Finite-Dimensional Approximations and Efficient Computation” by Belloni, Lopomo, and Wang, and “Endogenous Selection and Moral Hazard in Compensation Contracts” by Armstrong, Larcker, and Su.

Developing methods for computing Nash equilibria of games has been a staple of computational economics, and this issue contains three contributions. “On a Markov Game with One-Sided Information,” by Hörner, Rosenberg, Solan, and Vieille, tackles the difficult computational problems presented by asymmetric information. In “A User’s Guide to Solving Dynamic Stochastic Games Using the Homotopy Method,” Borkovsky, Doraszelski, and Kryukov apply homotopy methods to explore the equilibrium manifold of stochastic games, illustrating a tool that could be used for many types of games. “Intertemporal Pricing and Consumer Stockpiling,” by Su, develops a solution method for the game between a seller and the consumers of durable goods. The key step is to decouple each buyer’s problem into a separate dynamic program.

The next two papers refine current techniques to provide more stable and efficient methods. “Option Pricing Under GARCH Processes Using PDE Methods,” by Breton and de Frutos, advances the literature on using PDE methods to solve option pricing problems. “Monotone Approximation of Decision Problems,” by Chehrazi and Weber, shows how one can impose qualitative information implied by theory concerning shape to help approximate an unknown decision rule.

The ultimate purpose of computational economics is to apply these tools to real problems. “Lumpy Capacity Investment and Disinvestment Dynamics,” by Besanko, Doraszelski, Lu, and Satterthwaite, uses computational examples to explore dynamic questions in oligopolistic competition. “On Cournot Equilibria in Electricity Transmission Networks,” by Downward, Zakeri, and Philpott, analyzes Cournot equilibria in electricity transmission networks. To overcome the complication of transmission congestion, models of such competition often resort to bounded rationality assumptions. Here, the authors derive conditions that ensure the unconstrained Nash–Cournot equilibrium remains a Nash equilibrium in the presence of (lossless) line capacities. “A Single-Settlement, Energy-Only Electric Power Market for Unpredictable and Intermittent Participants,” by Pritchard, Zakeri, and Philpott, considers how uncertainties in electricity demand (variable loads) and supply (renewable energy sources) affect the market for power. They propose an alternative scheduling and dispatch mechanism for intermittent generation that is based on a stochastic programming model and show the scheme is revenue adequate in expectation.

Conventional economics is often criticized for assuming that actors have a clear picture of their problems and an infinite ability to compute the optimal response. Computational economics is a place where scholars explore deviations from these assumptions. In “A Soft Robust Model for Optimization Under Ambiguity,” Ben-Tal, Bertsimas, and Brown develop robust optimization methods for agents facing risks where the probabilities of various events are not known well. “Myopic Solutions of Homogeneous Sequential Decision Processes,” by Sobel and Wei, explores the computational advantages of myopic optima in Markov decision processes (MDPs) and identifies both new classes of MDPs with myopic optima and sequential games that have myopic equilibrium points. “Computational Methods for Oblivious Equilibrium,” by Weintraub, Benkard, and Van Roy, studies the concept of oblivious equilibrium, an

approximation of Markov perfect equilibrium, and provides both algorithms for computing such equilibria and new bounds on the resulting approximation error.

A special issue like this could not happen without the efforts of many people who are not mentioned here. First, we want to thank the referees for their service. Evaluating work that crosses disciplinary lines is often a challenge, and we owe our referees special gratitude. Second, we want to acknowledge the tremendous interest in this issue. We received approximately 125 submissions, roughly equally divided between the economics and operations research communities; this far exceeded the number of papers we could put in one issue. We hope that the authors of papers we could not include continue to develop their work in computational economics.

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