Abstract

This paper introduces a model of household consumption and savings in which household members have imperfectly aligned altruistic preferences. Specifically, member A values his own consumption more than member B values A’s consumption. Each period, members individually choose the amount of household wealth to consume as Nash best responses. At each point in time, the household consumes a higher fraction of wealth than under the full commitment Pareto optimum. Ex-ante Pareto optimal household consumption plans are not subgame perfect because both members wish to deviate to increase their own consumption. As a result the household is willing to pay for a technology that commits them to an optimal lifetime consumption plan. Despite both members individually having time consistent exponential discount rates, equilibrium household consumption dynamics are captured by a single representative agent with hyperbolic time preferences.
Are households able to carry out optimal consumption and savings plans? Recent evidence shows that households value technologies that allow them to commit to increase savings and that these raise savings rates (Thaler and Benartzi 2004; Ashraf, Karlan, and Yin 2006). This is inconsistent with standard models of consumption and savings based on individual maximization. One explanation is that individuals have hyperbolic discount factors or self control problems that render optimal savings plans time inconsistent so that individuals will, ex-post, wish to save less than planned (for example: Thaler Shefrin 1981; Laibson 1997; Laibson, Repetto, and Tobacman 1998; Harris and Laibson 2001). This paper takes a different approach and shows that the same under-saving and time inconsistency arises endogenously in a household where the individual members place more weight on the utility from their own consumption than their partner does. This occurs despite the individual members of the household being fully rational and having time consistent exponential discount rates.

There is abundant evidence that household members do not have perfectly aligned preferences. For example, household consumption decisions are different when money is received by one partner or the other (Lundberg, Pollak, and Wales 1997, Phipps and Burton 1998, Ashraf 2009). In the model I propose here the household is comprised of two members who each choose how much of the combined household wealth to spend on their own private consumption. In addition they can both purchase (non-negative) amounts of a non-rival common consumption good which contributes directly to the utility of both members. The crucial assumption I make is that household members have imperfectly aligned altruistic preferences. Specifically, member A cares more about the utility from his consumption than B cares about A’s consumption. Both members have the same exponential time preferences and agree on the optimal savings rate for the household. I characterize the household’s equilibrium consumption path without commitment as a subgame perfect Nash equilibrium in consumption choices. This is the equilibrium that obtains when household members are unable to enforce contracts conditional on their consumption choices. The household is un-
able to carry out the plan ex-post because both members wish to deviate and increase their own consumption. This intuition is closely related to the theoretical literature on dynamic commons problems that has been used to study national underinvestment (Lancaster 1973, Tornell and Velasco 1992), overexploitation of natural resources (Levhari Mirman 1980), and sovergin debt (Amador 2008). I also show that the household spends too little, relative to the Pareto optimum, on common consumption goods because neither member fully internalizes the benefit of common consumption on the utility of their partner.

The household is willing to pay for a technology that allows them commit to any Pareto optimal consumption plan. The model allows me to numerically calculate the value of commitment. I show that it is increasing in the degree to which members value their own utility over their partner’s and decreasing in the importance of common consumption relative to private consumption.

Next I find the preferences of a single representative agent that would achieve the same allocation as the household. This representative agent is shown to have simple time additive preferences and a hyperbolic discount factor despite both household members individually having the same time consistent exponential discount rates. The hyperbolic discount factor is micro-founded in the parameters of the preferences of the underlying household members. It is decreasing (i.e., is “more hyperbolic”) when household members have more divergent interests and when common consumption has a lower weight in their utility.

I extend the model to allow individual members of the household to differ in their concern for each other’s utility and the weight they place on common consumption. I show that the distortion to intertemporal household consumption and savings decisions is driven by the member who cares most for private consumption and is determined by the sum of the weights that both members place on their own utility. This implies that having one member who cares equally for himself and his partner does not undo the problem. Similarly, increasing the selfishness of either the least or most self interested person has the same effect on equilibrium household savings. Equilibrium savings remain inefficient even when one member cares only
about common consumption.

One interpretation of this paper is that it provides a micro-foundation for hyperbolic savings and consumption behavior. However the psychological evidence for hyperbolic discounting is conducted at the level of the individual (Ainslie 2001). To accommodate this, I extend the model to allow the individual members of the household to have hyperbolic time preferences. Not surprisingly, this exacerbates the inefficiency of the household consumption path and further increases the value of commitment technologies. More interestingly, I show that hyperbolic individual preferences amplify the household problem and that the value of commitment when both problems are combined is larger than its value when both problems are considered in isolation. As such, the goal of the paper is not to replace individual hyperbolic preferences as a description of decision making. Rather my purpose is to show that household decision making also naturally renders optimal intertemporal plans time inconsistent and that in combination both channels can produce sizeable distortions to optimal savings plans and create large demands for commitment technologies.

This paper is closely related to the theoretical literature that incorporates misaligned preferences within the household (see Lundberg and Pollack 2007; Browning, Chiappori, and Lechene 2006 for comprehensive surveys of the literature). In these papers, household decision making is modeled as the outcome of a Nash bargaining process and the focus is directed to determining what determines the threat points and bargaining weights of each household member. As a result, allocations are typically Pareto optimal both within any period and over time. However, it is not obvious that households are able to enforce Nash bargained outcomes ex-post. This is supported by recent evidence (Mazzocco 2007). The equilibrium I study in this paper is the one that obtains when no such commitment is possible and I use this to fully characterize the household’s demand for intertemporal commitment. It is not my objective to argue that households suffer the time inconsistency problem without taking actions to mitigate it. Rather the goal is to show that households have an inherent tendency to undersave and provide a framework for assessing the types of strategies that households
may employ to overcome this problem.

The paper proceeds as follows. Section I sets up the model. Section II solves the simplest version where household members are assumed to have symmetric preferences, computes the value of commitment, and characterizes the preferences of the household’s representative agent. Section III generalizes the model to allow household members to have different levels of concern for each other and different taste for common consumption. In Section IV extends the model to allow the individual members of the household to have hyperbolic time preferences. Section IV concludes.

I. Model of Household Consumption

The household has two members indexed by \( i \) labeled \( A \) and \( B \). Time is discrete and index by \( t \). The household is formed at the beginning of period \( t = 1 \). Both household members live for \( T \) periods and I assume that they remain married for their entire lives with certainty.

A. Preferences

Each period household member \( i \) consumes two types of goods. The first, \( C_{i,t} \), is the private consumption of household member \( i \). This consumption does not provide any direct utility to the other household member. One advantage of being in a household is that it allows the household members to share non-rival common consumption goods such as housing, children, and consumer durables. This is captured by the second good, \( H_t \), that provides utility directly to both household members. The total level of common consumption is the sum of the amount purchased in each period by both household members

\[
H_t = H_{A,t} + H_{B,t}
\]
where $H_{i,t}$ is the amount of the common consumption good purchased by member $i$ in period $t$. The period utility of member $i$ is

$$u_{i,t} = \mu \ln C_{i,t} + (1 - \mu) \ln H_t$$

where $\mu$ captures the relative weight placed on private consumption relative to common consumption. Both household members discount utility from future consumption using exponential discount factor $\beta \in (0, 1)$. The individual discounted utility of household member $i$ is period $t$ is

$$U_{i,t} = \sum_{x=0}^{T-t} \beta^x u_{i,t+x}.$$  

This is the utility of household member $i$ absent any concern for the utility of the other household member. Household members care about the utility of each other. I capture this by supposing that each member places weight $\delta \in (0, 1)$ on their own utility and weight $1 - \delta$ on the utility of the other member. In general, I will focus on the case where household members care more about themselves than the other member of the household $\delta \geq \frac{1}{2}$ but the framework can also be used to study the case where the reverse is true. The total discounted utility of member $i$ at $t$ is

$$V_{i,t} = \delta U_{i,t} + (1 - \delta) U_{j,t}.$$  

This is the object each household member will maximize when taking actions at $t$.

B. Decision Making

Household members cannot commit to a path of consumption. As a result, household members are unable to enforce mutually agreed levels of consumption, either in the present or the future. Household members non-cooperatively decide how much of the household wealth $W_t$ to spend on their own private consumption $C_{i,t} \geq 0$ and common consumption $H_{i,t} \geq 0$ each period. The non-negativity assumption on $H_{i,t}$ implies that household members are
unable to reverse the consumption choice of the other members. In equilibrium this implies that the level of common consumption in each period will be determined by the household member who wants the most common consumption that period. The dynamic equilibrium path of consumption will be the Nash subgame perfect solution to the consumption game between these two members.

To avoid multiple equilibria in the final period, assume that at the start of $t = T$ the remaining household wealth $W_T$ is divided evenly between the two household members.\footnote{With log utility, the exact ratio with which $W_T$ is divided does not affect any of the equilibrium consumption dynamics prior to $T$. What is crucial is that the proportion of wealth received by each member is unaffected by the actions of each household member.} Thus the decision of each household member in the final period is how to divide their share of remaining household wealth between private and common consumption.

### C. Household Budget Constraint

The present value of all combined household wealth at the beginning of $t = 1$ is $W_1$. I set aside household labor supply decisions and take $W_1$ as given. Both household members have full access to the remaining combined household wealth in each period. Let the price of member $i$’s private consumption be $p_i$ and normalize the price of common consumption to unity. The household saves at a gross risk free interest rate of $R$. Household wealth evolves according to the following:

$$W_{t+1} = R \left( W_t - p_A C_{A,t} - p_B C_{B,t} - H_{A,t} - H_{B,t} \right). \quad (1)$$

The wealth of the household at $t$ can be interpreted as the present value of lifetime earnings less the present value of all consumption prior to period $t$. In effect, I assume that both household members can borrow and lend against the combined lifetime income of the household in a frictionless capital market at gross interest rate $R$. As a result in any period, total household consumption can be any non-negative value up to the total value of remaining
household wealth:

\[ W_t - p_{A_t}C_{A,t} - p_{B,t}C_{B,t} - H_{A,t} - H_{B,t} \geq 0. \] (2)

In equilibrium this condition will only bind in the final period because the marginal utility of future consumption becomes infinitely high as savings approach zero.

**II. Equilibrium Consumption Path**

The equilibrium consumption path is solved in the appendix. The equilibrium level of expenditure on private consumption by member \( i \) in period \( t \) is

\[ p_{i,t}C^{*}_{i,t} = \frac{\delta\mu}{\mu(2\delta - 1) + \sum_{x=0}^{T-t} \beta^x} W_t \]

and the equilibrium level of common consumption in each period is

\[ H^{*}_t = \frac{1 - \mu}{\mu(2\delta - 1) + \sum_{x=0}^{T-t} \beta^x} W_t. \]

Since both household members are identical their desired choice of common consumption is the same in every period. Define the equilibrium level of total household expenditure in period \( t \) as

\[ X^{*}_t \equiv p_{A_t}C^{*}_{A,t} + p_{B,t}C^{*}_{B,t} + H^{*}_t. \]

The equilibrium share of expenditure on each members private and common consumption is

\[
\begin{align*}
p_{i,t}C^{*}_{i,t} &= \frac{\delta\mu}{1 + \mu(2\delta - 1)}, \\
H^{*}_t &= \frac{1 - \mu}{1 + \mu(2\delta - 1)}.
\end{align*}
\]

Both household members spend a larger fraction of total expenditure within a period on themselves when they place less weight on the utility of their partner. Holding all else equal,
increasing $\delta$ raises the marginal utility of private consumption while the marginal utility to member $i$ from common consumption remains unchanged.

The dynamics of household expenditure is described by

$$\frac{X_{t+1}}{X_t} = R_\beta \left( \frac{\sum_{x=0}^{T-(t+1)} \beta^x}{\mu (2\delta - 1) + \sum_{x=0}^{T-(t+1)} \beta^x} \right).$$

Since equilibrium consumption shares are fixed over time then each type of consumption will evolve in the same way. Two comparative static results come from this expression. First, increasing $\delta$ reduces the slope of the equilibrium consumption path. This occurs because when each household member decides how much to spend on private consumption in a period they weigh this by $\delta$. However the weight they place on the value of wealth that is carried over to next period is unaffected. This is because household wealth is shared and hence the increase in concern for their own future is offset by the reduced relative concern for their partner. As a result the relative weight on the utility from future consumption is always unity.

This effect is mitigated by the importance of common consumption to each household member as captured by $1 - \mu$. If both household members value themselves more than the other $\delta > \frac{1}{2}$ then decreasing $\mu$ raises the slope of the consumption path. Since common consumption is shared its marginal utility today relative to the value of wealth in the future is unaffected by the degree to which household members care about themselves relative to the other. In the extreme if the household only had common consumption $\mu = 0$ then the dynamic path of consumption would be unaffected by $\delta$.

A. Comparison to Full Commitment Consumption Path

To evaluate the optimality of the non-cooperative equilibrium consumption path, I compare it to the consumption path that would be achieved if the household was able to fully commit to consumption choices at the start of $t = 1$. Consider the problem the household would face in setting a full commitment path. Whenever $\delta \neq \frac{1}{2}$ household members will
disagree over the optimal allocation. However any allocation that they would choose must be Pareto optimal and hence I characterize the solution to the following full commitment pareto problem:

$$\max_{\{C_{i,t}, H_{i,t}\}_{t=1}^{T}} \Pi = \eta V_{A,1} + (1 - \eta) V_{B,1}$$

subject to

$$W_1 - \sum_{x=0}^{T-1} R^{-x} \left[ p_A C_{A,1+x} + p_B C_{B,1+x} + H_{1+x} \right] \geq 0$$

$$\{C_{A,t}, C_{B,t}, H_t\}_{t=1}^{T} \geq 0$$

where $\eta \in [0, 1]$ is the pareto weight placed on the objective of member $A$. This problem is solved in the appendix. Any pareto efficient full commitment consumption path has the following dynamics of total household expenditure

$$\frac{X_{t+1}^{**}}{X_t^{**}} = R\beta.$$  

The optimal evolution of household expenditure is independent of the pareto weight $\eta$. Notice that since they have the same time preference, both household members agree on the optimal slope of the household’s consumption path. The expenditure shares for any pareto efficient plan are

$$\frac{p_A C_{A,t}^{**}}{X_t^{**}} = \left[ \eta \delta + (1 - \eta) (1 - \delta) \right] \mu$$

$$\frac{p_B C_{B,t}^{**}}{X_t^{**}} = \left[ (1 - \eta) \delta + \eta (1 - \delta) \right] \mu$$

$$\frac{H_t^{**}}{X_t^{**}} = 1 - \mu.$$  

The pareto weight $\eta$ only determines the way in which expenditure will be allocated between the private consumption of each household member. In the case where equal pareto weight is given to each household member their optimal share of household expenditure each period
will be

\[
\frac{P_{AC}^{**}}{X_t^{**}} = \frac{P_{BC}^{**}}{X_t^{**}} = \mu = \frac{1}{2}.\]

The equilibrium path of consumption achieved by the household is inefficient in two ways relative to the full commitment optimum. First, the household without commitment saves too little relative to the full commitment solution.

**Proposition 1** The slope of the non-cooperative equilibrium consumption path is strictly below the full commitment consumption path whenever \( \delta > \frac{1}{2} \) and \( \mu > 0 \).

Proposition 1 follows immediately by observing that

\[
\frac{X_{t+1}}{X_t} < \frac{X_{t+1}^{**}}{X_t^{**}}\]

whenever \( \delta > \frac{1}{2} \) and \( \mu > 0 \). Household savings are effectively a public good that is shared by both household members. When \( \delta > \frac{1}{2} \), household members do not fully internalize the benefit of savings on the utility of their partner. As a result savings are underprovided in equilibrium. This effect is only present when \( \mu > 0 \). This is because the distortion arises whenever household members trade off their own utility against the combined utility of the household. When deciding on the current level of private consumption both household members act unilaterally, taking the action of the other as given while recognizing that any wealth left to the future is share with their partner. As a result, both household members place weight \( \delta \) on private consumption relative to a combined weight of unity for the combined discounted value of future savings. Thus, in total, the household acts as though it places weight \( 2\delta \) on it’s current self relative to the combined future interest of the household. The social planner, for any pareto weight \( \eta \) always places the same weight on the discounted utility of the household in each period. There is no distortion to savings when household members care about each other as much as themselves (\( \delta = \frac{1}{2} \)). In this case both members act as if they are there is no distortion to savings and the non-cooperative growth rate of consumption is identical to the full commitment equilibrium.

Figure 1 shows the amount of total household expenditure for the optimal full commitment consumption path \( X_t^{**} \) and compares it to the path achieved by the household absent
commitment \( X_t^* \) for different parameters. As emphasized in Proposition 1, the household saves too little relative to the full commitment optimum. Consumption is higher than the full commitment solution early in the life of the household and as a result lower consumption than under full commitment later in life. Panel A compares the equilibrium consumption path for two otherwise identical households where the members place weight \( \delta = 0.6 \) and \( \delta = 0.75 \) on their own private utility. A household in which members care less about the utility of each other has a more pronounced undersavings problem. Panel B make a similar comparison varying the weight that each member places on private consumption relative to common consumption. The undersavings problem is more severe when household members place more weight on private consumption.

The second inefficiency displayed by the household relative to full commitment optimum is that conditional on the level of expenditure within any given period, too little is directed toward common consumption.

**Proposition 2** The share of household expenditure each period on common consumption is strictly lower in the non-cooperative equilibrium relative to the full commitment consumption path whenever \( \delta > \frac{1}{2} \) and \( \mu > 0 \).

Proposition 2 follows by observing that

\[
\frac{H_t^*}{X_t^*} = \frac{1 - \mu}{1 + \mu (2\delta - 1)} < \frac{H_t^{**}}{X_t^{**}} = 1 - \mu.
\]

whenever \( \delta > \frac{1}{2} \) and \( \mu > 0 \). As with savings, common consumption is a public good that is underprovided because both household members don’t fully take into account it’s benefit on the utility of their partner.

**A-1. The Value of Commitment**

To quantify the welfare loss incurred by the household I calculate how much the household would be willing to pay at \( t = 1 \) for a technology that allowed them to commit to an opti-
mal consumption path. Let $V_{i,1}^*(W_1)$ be the discounted lifetime utility that will be achieved by household member $i$ absent commitment as a function of initial household wealth. Let $V_{i,1}^{**}(W_1 - \phi, \eta)$ be the counterpart for the case where the household has purchased commitment at price $\phi$ and the full commitment plan places weight $\eta$ on the preferences of member $A$. The value of commitment $\phi^{**}$ is defined as the most that the household will pay while ensuring that there exists a weight $\eta$ so that the purchase is a pareto improvement for both members. Formally $\phi^{**}$ solves:

$$\phi^{**} = \max_{\phi, \eta} \phi$$
subject to $V_{i,1}^{**}(W_1 - \phi, \eta) \geq V_{i,1}^*(W_1)$ for $i \in \{A, B\}$, and
$$\eta \in [0, 1].$$

An analytical solution for $\phi^{**}$ is intractable in most cases. Instead I solve for $\phi^{**}$ numerically in Figure 2 which shows how the value of commitment varies with the parameters of the model. In each case the value of commitment is expressed as a percentage of initial household wealth $W_1$. Due to the log additive structure of both household members’ utility functions this ratio is invariant to the scale of $W_1$.² Panel A shows that the value of commitment increases monotonically with the weight that household members place on their own utility relative to the utility of the other. This is the central force which leads household members to undervalue savings in the no-commitment equilibrium Panel B shows that the value of commitment increases when private consumption has a larger weight in the utility of each household member. In the extreme, when the household has only common consumption, there is no distortion to savings decisions and thus a commitment technology has no value.

Panel C shows that the value of commitment varies non-monotonically with the discount rate of the household members. This stems from the fact that there are two countervailing forces. First, when $\beta$ is larger, both household members care more about the future and

²The log additive utility functions also ensure that value of commitment is also invariant to $R$. 13
hence are willing to pay more to avoid the effect that undersaving will have on their future consumption levels. Conversely, increasing $\beta$ raises both household members desire to save and thus mitigates the problem. Panel C shows that this first force dominates for most values of $\beta$ and is only reversed by the second force when $\beta$ is very close to unity.\(^3\)

B. \textit{Representative Agent}

Typically household savings and consumption decisions are modeled as if they are made by a single optimizing representative agent. This typically presumes that the interests of household members are perfectly aligned. I now study what the representative agent will be for a household in which the interests of its members are not perfectly aligned. To do this, consider the problem of a single representative agent who chooses the level of $C_\text{A,t}$, $C_\text{B,t}$, and $H_t$ each period. The period utility of the representative agent is

$$ u_{r,t} = \mu_{A,r} \ln C_{A,t} + \mu_{B,r} \ln C_{B,t} + (1 - \mu_{A,r} - \mu_{B,r}) \ln H_t $$

where $\mu_{A,r}$ and $\mu_{B,r}$ capture the weight that the representative agent places on each of the private consumption goods relative to common consumption. The discounted utility of the representative agent at time $t$ is

$$ U_{r,t} = u_{r,t} + \Omega_r \sum_{x=1}^{T-t} \beta_r^x u_{r,t+x} $$

where $\beta_r \in (0, 1]$ is a standard exponential discount factor and $\Omega_r \in (0, 1]$ is a hyperbolic discount factor of the type introduced by Laibson (1997). While more general utility functions and discount functions could be considered the results below show that this form is sufficiently flexible to represent the household. The representative agent faces the same budget constraint captured by (1) and (2) that the household faced.

\(^3\)Qualitatively, the same non-monotonic relationship obtains for all other choices of $\mu$, $\delta$, and $T$ that I have tried.
As stressed by Laibson (1997), when $\Omega_\tau < 1$ any optimal path of consumption from the perspective of the representative agent at $t$ will be time inconsistent. When considering the representative agent without commitment, I study the problem where the agent is aware of this time inconsistency and takes it into account when making consumption choices each period. As a result the consumption path chosen by the representative agent will be found by backward induction where consumption choices are subgame perfect best responses given the resulting choices that they will lead to in the future. The consumption path of the representative agent with and without commitment is solved in the Appendix.

**Proposition 3:** The representative agent without commitment has an identical path of consumption as the household without commitment if:

\begin{enumerate}
  \item $\beta_\tau = \beta$,
  \item $\Omega_\tau = \frac{1}{1 - (1 - 2\delta) \mu}$, and
  \item $\mu_{A, \tau} = \mu_{B, \tau} = \frac{\delta \mu}{1 - (1 - 2\delta) \mu}$.
\end{enumerate}

Proposition 3 is proved in the Appendix. The key result in Proposition 3 is that when $\delta > \frac{1}{2}$ and $\mu > 0$ then $\Omega_\tau < 1$. Thus, when household members care more about their own utility than of their partner, the representative agent for the household is a hyperbolic discounter. This captures the central intuition for the household undersavings result documented in Proposition 1. At any point in time, when a household member decides how much to spend on private consumption he places weight $\delta$ on the utility from this consumption relative to unity for the combined marginal utility of an additional dollar of savings. Since both members are doing this, in total they act as though they are currently worth $2\delta$ relative to unity for their combined marginal utility from future savings. In total, despite the fact both members of the household have standard exponential time preferences, the household acts as if it always discounts the entire future with hyperbolic discount factor $\Omega_\tau < 1$. Note that if both household members care about the utility of their partner as much as themselves ($\delta = \frac{1}{2}$) then
only in this case does the representative agent also have standard exponential time preferences ($\Omega_r = 1$). Thus modelling households as having standard exponential time preferences is valid only if we assume that household members have perfectly aligned objectives in which case they are effectively identical and hence aggregation is trivial.

Within each period, the representative agent achieves the same allocation between different types of consumption by overweighting private consumption relative to common consumption. The representative agent acts as though the household places weight

$$1 - \mu_{A,r} - \mu_{B,r} \cdot \frac{1 - \mu}{1 - (1 - 2\delta)\mu}$$

on common consumption. Whenever $\delta > \frac{1}{2}$ and $\mu > 0$ this is strictly less than $1 - \mu$, which is the weight it has in the household’s total welfare. This captures the fact that the household without commitment underprovides common consumption goods.

Next I consider what representative agent would achieve the same consumption path as the household if it was able to fully commit to a Pareto optimal consumption path.

**Proposition 4:** The representative agent with or without commitment has an identical path of consumption as the household with commitment if:

1. $\beta_r = \beta$,
2. $\Omega_r = 1$,
3. $\mu_{A,r} = (1 - \delta - \eta (1 - 2\delta))$, and $\mu_{B,r} = (\delta + \eta (1 - 2\delta))\mu$

where $\eta \in [0, 1]$ is the Pareto weight place on member A’s utility.

Proposition 4 is established in the Appendix. If the household is able to commit to an optimal consumption path then it is represented by an agent who also has standard exponential time preferences ($\Omega_r = 1$). In this case the preferences of the representative agent are time consistent and thus the same path is achieved whether or not it is modelled
as having full commitment.

Comparing Proposition 3 and 4 shows that a different representative agent is required to model the consumption dynamics of the household depending on whether the underlying household is able to commit to a path of consumption. As a result, studying the effect of commitment technologies for the household is not accurately captured by studying the allocation that would be achieved by the associated representative agent with or without commitment. To see this, consider the representative agent defined in Proposition 3. In the appendix I study the consumption path that the representative agent would achieve if it is assumed to have full commitment. That representative agent would commit to a path of consumption that differs from the one that household would achieve with full commitment in two ways. First, the representative agent would commit to consumption dynamics between the first and second period in which

\[
\frac{X_{r}^{**}}{X_{1}^{**}} = \Omega_{r} \beta_{r} R
\]

whereas since household members do not actually have hyperbolic time preferences they would commit to

\[
\frac{X_{2}^{**}}{X_{1}^{**}} = \beta_{r} R.
\]

Thus the representative agent would choose to commit to a drop in consumption after the first period while the household would not wish to do so. The consumption dynamics for all periods after the first period will be the same for the representative agent with full commitment and the household with full commitment.

\[
\frac{X_{r}^{***}}{X_{1}^{***}} = \frac{X_{2}^{**}}{X_{1}^{**}}.
\]

The second difference relates to the allocation of resources between private and common consumption. When the household has full commitment it no longer underprovides common
consumption and thus as a result allocates a share \(1 - \mu\) of expenditure each period to common consumption. However using the representative agent, we would not predict any change in the allocation between common and private consumption. Since this inefficiency is embedded in the preferences of the representative agent the effect of commitment on this allocation would be lost when studying the household by way of the representative agent.

### III. Asymmetric Model of Household

In the previous section household members were assumed to be symmetric in their concern for each other \(\delta\) and the relative weight on private versus common consumption \(\mu\). I now consider a generalized version of the same model where members of the household can vary on both of these dimensions. I maintain that both household members have the same exponential discount rate \(\beta\).

The period utility of member \(i \in \{A, B\}\) is now generalized to:

\[
u_{i,t} = \mu_i \ln C_{i,t} + (1 - \mu_i) \ln H_t
\]

where \(\mu_i \in (0, 1)\) is the weight member \(i\) places on private consumption. The individual discounted utility of household member \(i\) is period \(t\) is

\[
U_{i,t} = u_{i,t} + \sum_{x=1}^{T-t} \beta^x u_{i,t+x}.
\]

The total discounted utility of member \(i\) at \(t\) is

\[
V_{i,t} = \delta_i U_{i,t} + (1 - \delta_i) U_{j,t}
\]

where \(\delta_i \in (0, 1)\) is the weight member \(i\) places on his own utility. The budget constraint and the way in which household decisions are made remains the same. The equilibrium of this generalized model non-cooperative without commitment along with the full commitment Pareto
optimal problem are solved in the Appendix. I compare both, studying how asymmetries in the household affects the difference between both consumption paths.

A. Equilibrium Consumption

Without loss of generality assume that member $A$ places weakly more relative weight on private consumption than $B$: $\mu_A \geq \mu_B$. Since household members now potentially disagree over the desired level of common consumption in each period this will normalize $B$ to be the one who prefers more common consumption and hence, in equilibrium, be the one who determines its level in each period. Without commitment the equilibrium household consumption path can be characterized in the following way. Within any period the share of expenditure on each type of consumption is

$$\frac{p_i C_{i,t}^*}{X_t^*} = \frac{\delta \mu_i}{1 + \mu_A (\delta_A - (1 - \delta_B))} \text{ for } i \in \{A, B\},$$

$$\frac{H_t^*}{X_t^*} = \frac{\delta_B (1 - \mu_B) + (1 - \delta_B)(1 - \mu_A)}{1 + \mu_A (\delta_A - (1 - \delta_B))}.$$  

Observe that as before these are constant over time. Total household expenditure evolves over time according to

$$\frac{X_{t+1}^*}{X_t^*} = R_\beta \left( \frac{\sum_{x=0}^{T-(t+1)} \beta^x}{\mu_A (\delta_A + \delta_B - 1) + \sum_{x=0}^{T-(t+1)} \beta^x} \right).$$  

Since household expenditure shares are time invariant then this is also the law of motion for each type of consumption.

Equation (4) shows that in addition to time preferences and the interest rate, the consumption dynamics of the household are determined by two components of the non-cooperative game between household members. First is the weight on private consumption for the household member who cares most for private consumption, in this case $\mu_A$. This is multiplied by
the gap in altruism of each member:

\[ \delta_i - (1 - \delta_j). \]

In words, this captures the degree to which, household member \( i \) puts more weight on his own utility (\( \delta_i \)) than \( j \) places on \( i \)’s utility. Notice consumption dynamics are determined only by the sum \( \delta_A + \delta_B \) and hence are unaffected by the distribution of selfishness within the household.\(^4\) The intuition for this result comes by examining the term

\[ \mu_A (\delta_A + \delta_B - 1) = \mu_A \delta_A - \mu_A (1 - \delta_B). \]

The right hand side is the difference between household member \( A \)’s weight on his private consumption (\( \mu_A \delta_A \)) and the weight placed by member \( B \) (\( \mu_A (1 - \delta_B) \)). The significance of this difference can be understood in the following way. Currently, since \( \mu_A \geq \mu_B \), member \( B \) determines the level for two out of the three categories of household consumption each period (\( C_{B,t}^* \) and \( H_t^* \)). If \( B \) were also able to choose \( C_{A,t}^* \) then each period she would decide all of the household’s consumption. Since individually \( B \)’s preferences are time consistent, she would implement a Pareto optimal consumption path (the one with Pareto weight \( \eta = 0 \)). There would be no inefficiency in the savings decision of the household. However each period the household spends more than \( B \) would choose if it had control of \( C_{A,t}^* \). This is because \( A \) values his private consumption more than \( B \) does and hence elects to set \( C_{A,t}^* \) higher than the level desired by \( B \). As a result it is the size of the disagreement in the valuation of \( A \)’s private consumption that drives the consumption dynamics of the household.

\(^4\) Holding \( \delta_A + \delta_B \) as constant, the distribution of \( \delta_A \) and \( \delta_B \) does affect the efficiency of the allocation between types of consumption within any period.
B. Comparison to Full Commitment Optimum

The optimal full commitment consumption path in which Pareto weight \( \eta \) is placed on the preferences of member \( A \) has consumption shares in each period of

\[
\frac{p_{A,C_{A,t}}}{X_{t}^{**}} = \left[ \eta \delta_A + (1 - \eta) (1 - \delta_B) \right] \mu_A,
\]
\[
\frac{p_{B,C_{B,t}}}{X_{t}^{**}} = \left[ (1 - \eta) \delta_B + \eta (1 - \delta_A) \right] \mu_B,
\]
\[
\frac{H_{t}^{**}}{X_{t}^{**}} = 1 - \left[ \eta \delta_A + (1 - \eta) (1 - \delta_B) \right] \mu_A - \left[ (1 - \eta) \delta_B + \eta (1 - \delta_A) \right] \mu_B.
\]

Since the time preferences of the individual members of the household have not changed the optimal rate of consumption growth is still described by

\[
\frac{X_{t+1}^{**}}{X_{t}^{**}} = R\beta. \tag{5}
\]

As before the equilibrium achieved by the household is not Pareto efficient.

**Proposition 5**: For the asymmetric household, the slope of the equilibrium consumption path is always lower than the full commitment optimum whenever

\[
\left[ \max \{ \mu_A, \mu_B \} \right] \times (\delta_A - (1 - \delta_B)) > 0. \tag{6}
\]

Proposition 5 follows immediately by comparing (4) and (5) and observing that

\[
\frac{X_{t+1}}{X_{t}} = R\beta \left( \frac{\sum_{x=0}^{T-(t+1)} \beta^x}{\mu_A (\delta_A - (1 - \delta_B)) + \sum_{x=0}^{T-(t+1)} \beta^x} \right) < \frac{X_{t+1}^{**}}{X_{t}^{**}} = R\beta
\]

whenever \( \mu_A (\delta_A - (1 - \delta_B)) > 0 \).

Next, I consider the efficiency of the equilibrium allocation of consumption, conditional on some level of expenditure \( X_{t}^{*} \) within any given period.

**Proposition 6**: For the asymmetric household for a given level of expenditure \( X_{t}^{*} \) in any
period, the allocation of consumption between $C_{A,t}$, $C_{B,t}$, and $H_t$ is Pareto inefficient whenever the following is true

\[(i) \quad \mu_A, \mu_B \in (0, 1), \]
\[(ii) \quad \delta_A > 1 - \delta_B \Leftrightarrow \delta_B > 1 - \delta_A.\]

Proposition 6 is proved in the Appendix. These conditions ensure that there exists no Pareto weight $\eta \in [0, 1]$ that ensures $\frac{p_{C_{i,t}^*}}{X_t} = \frac{p_{C_{i,t}^{**}}}{X_t}$ for $i \in \{A, B\}$. In words, this requires (i) both household members value both private and common consumption, and (ii) for both members to care more about the utility of their own consumption than their partner cares about that consumption.

C. The Value of Commitment

As in the symmetric case I compute the value of commitment at $t = 1$ for the household. Figure 3 shows how the value of commitment (as a fraction of $W_1$) varies with the underlying parameters of the household’s problem. Panel A shows how the value of commitment varies with the weight that $A$ places on his own utility for $\delta_B = 0.5$ and $\delta_B = 0.75$. The value of commitment is increasing and concave in the degree to which household members value their own individual utility over their partner’s.

Panel B draws the same relationship while changing the degree to which the household members disagree over the relative value of private versus common consumption. Comparing one curve to the next shows that for any level of $\delta_A$ and $\delta_B$ the value of commitment is increasing in the disparity between the relative weight that both member’s place on private versus common consumption. Panel B also demonstrates that the marginal effect of increasing $\delta_A$ on the value of commitment is larger when household members disagree more about the relative valuation of private and common consumption.

Panel C holds constant the degree to which both household members value their indi-
individual utility $\delta_A = \delta_B = 0.75$ and varies the degree to which they value private and common consumption. The value of commitment is always increasing in the weight that either member places on private consumption. Intuitively, increased weight on common consumption aligns the interest of the two household members. When $\mu_A < \mu_B$ this relationship is driven by increased allocative inefficiency since the intertemporal efficiency is only driven by $\max \{\mu_A, \mu_B\}$. Conversely, when $\mu_A > \mu_B$ increasing $\mu_A$ drives both the allocative and intertemporal inefficiency. When $\mu_A = 1$ the value of commitment is invariant to $\mu_B$. This is because when $A$ cares only for private consumption the household allocation is Pareto efficient since $B$ can only be made better off by taking private consumption away from $A$. Thus when $\mu_A = 1$ commitment is valued only to solve the intertemporal inefficiency in household consumption. Proposition 5 demonstrates that the magnitude of the intertemporal inefficiency is driven only by $\max \{\mu_A, \mu_B\}$.

D. Representative Agent

As in the symmetric case a representative agent can be found that achieves an identical consumption path as the household.

**Proposition 7:** The representative agent without commitment has an identical path of consumption as the asymmetric household without commitment if:

1. $\beta_r = \beta$,
2. $\Omega_r = \frac{1}{1 + \max \{\mu_A, \mu_B\} (\delta_A + \delta_B - 1)}$, and
3. $\mu_{i,r} = \frac{\delta_i \mu_i}{1 + \max \{\mu_A, \mu_B\} (\delta_A + \delta_B - 1)}$ for $i \in \{A, B\}$.

Proposition 7 is proved in the Appendix. This generalizes the result from the symmetric case that the household is represented by an agent with hyperbolic time preferences. The
representative agent must have hyperbolic time preferences whenever

\[ \{\mu_A, \mu_B\} (\delta_A + \delta_B - 1) > 0. \]

IV. Hyperbolic Household Members

I now allow each member of the household to discount future utility with a hyperbolic discount factor \( \Psi \leq 1 \). This is motivated by the fact that most of psychological evidence for hyperbolic time preferences is done at the level of the individual (Ainslie 1992). This extension allows me to study how time inconsistency in the individual time preferences interacts with the time inconsistency exhibited by the combined household. The only change to the framework is that member \( i \)'s discounted individual utility is now

\[ U_{i,t} = u_{i,t} + \Psi \sum_{x=1}^{T-t} \beta^x u_{i,t+x}. \]

The rest of the analysis remains the same. The equilibrium of the model is solved in the Appendix along with the full commitment optimum. The allocation of consumption within any period is unchanged from the asymmetric equilibrium as described in (3). The dynamics of equilibrium household consumption is now given by

\[ \frac{X_{t+1}^*}{X_t^*} = R \beta \left( \frac{\sum_{x=0}^{T-(t+1)} \beta^x}{\mu_A (\delta_A + \delta_B - 1) + (1 - \Psi)} + \sum_{x=0}^{T-(t+1)} \beta^x \right). \]  

The optimal full commitment consumption path has the following consumption dynamics. Between the first and second period consumption will evolve according to

\[ \frac{X_2^{**}}{X_1^{**}} = R \Psi \beta \]  

(8)
and for every period $t \geq 2$ after according to

$$\frac{X_{t+1}^{**}}{X_t^{**}} = R\beta$$

(9)

which is the same as before.

The suboptimality of the equilibrium consumption path is further exacerbated. This can be seen for $t \geq 2$ by comparing

$$\frac{X_{t+1}^{*}}{X_t^{*}} = R\beta \left( \frac{\sum_{x=0}^{T-(t+1)} \beta^x}{\Psi + \sum_{x=0}^{T-(t+1)} \beta^x} \right) < \frac{X_{t+1}^{**}}{X_t^{**}} = R\beta.$$

For consumption dynamics between $t = 1$ and $t = 2$ the consumption equilibrium consumption path between $t = 1$ and $t = 2$ will have a lower slope than the optimum if the hyperbolic discount factor is sufficiently close to one. Formally, this requires,

$$\Psi \geq 1 - \frac{\mu_A (\delta_A + \delta_B - 1)}{\sum_{x=1}^{T-(t+1)} \beta^x}.$$ 

The equilibrium level of total household expenditure in the first period is

$$X_1^{*} = \left( \frac{1 + [\max \{\mu_A, \mu_B\}] \times (\delta_A + \delta_B - 1)}{1 + [\max \{\mu_A, \mu_B\}] \times (\delta_A + \delta_B - 1) + \Psi \sum_{x=1}^{T-1} \beta^x} \right) W_1$$

while the full commitment optimum has total expenditure in the first period of

$$X_1^{**} = \left( \frac{1}{1 + \Psi \sum_{x=1}^{T-1} \beta^x} \right) W_1$$

and hence whenever (6) holds, the household will consume strictly more in the first period than under the full commitment optimum, for any level of $\Psi \in (0, 1]$.

The representative agent for the household is altered in the following way.

**Proposition 8:** The representative agent without commitment has an identical path of
consumption as the asymmetric household with hyperbolic members without commitment if:

\begin{align*}
  i. \beta_r &= \beta, \\
  ii. \Omega_r &= \frac{\Psi}{1 + \max\{\mu_A, \mu_B\} (\delta_A + \delta_B - 1)}, \text{ and} \\
  iii. \mu_{i,r} &= \frac{\delta_i \mu_i}{1 + \max\{\mu_A, \mu_B\} (\delta_A + \delta_B - 1)} \quad \text{for } i \in \{A, B\}.
\end{align*}

When the individual members of the household also have hyperbolic time preferences (i.e. \(\Psi < 1\)) the hyperbolic discount rate for the for the combined household is magnified. This amplification can be seen by computing the value of commitment when \(\Psi < 1\). Figure 4 considers a symmetric household and shows how the value of commitment varies with \(\delta = \delta_A = \delta_B\) for \(\Psi = 0.85\) and \(\Psi = 1\). When household members place the same weight on each other’s utility (\(\delta_A = \delta_B = 0.5\)) the value of commitment with \(\Psi = 0.85\) is 1.19% of household wealth. This is the force that Laibson (1997) documents showing that hyperbolic individuals will value commitment to resolve the time inconsistency in their optimal consumption plans. Conversely, when \(\delta_A = \delta_B = 0.65\) and \(\Psi = 1\) is 1.78% the household values commitment to solve their combined inability to implement their optimal consumption path due to the divergence in their personal objectives. A household which combines both forces, so that \(\delta_A = \delta_B = 0.65\) and \(\Psi = 0.85\) will pay 5.10% of household wealth for commitment. This is 2.13% higher than the total value of commitment when both of these forces are considered in isolation. Having hyperbolic individuals magnifies the household’s problem because it further distorts each member’s value of the future combined wealth of the household which they trade off against the current value of their own private utility.

V. Conclusion

This paper introduces a model of household consumption and savings in which household members have imperfectly aligned altruistic preferences. I show that the household is unable
to achieve the optimal consumption path without commitment. I have not addressed the specific strategies that the household will employ to mitigate this problem. Some strategies such as investing in illiquid assets have already been studied in the context of individuals with hyperbolic preferences (Laibson 1997). Strategies specific to the household problem studied here will also be effective. For example, separate bank accounts for each member of the household may improve the efficiency of the consumption path if they are credible. In addition, restrictions on borrowing that require the consent of both partners, as is common with 401K loans (Choi Laibson Madrian Metrick 2004), can also alleviate the problem. A detailed consideration of these strategies is left for future work.

References


VI. Appendix

A. Solution to Generalized Household Problem without Commitment

This section of the Appendix solves the generalized household problem presented in Section III. The symmetric model studied in Section I. is a special case of this where $\delta_A = \delta_B$, $\mu_A = \mu_B$, $\beta_A = \beta_B$ and $\Psi_A = \Psi_B = 1$.

A-1. Equilibrium at $t = T$

In the final period $T$ member $i$ solves the following problem:

$$
\max_{C_{i,T}, H_{i,T}} \delta_i [\mu_i \ln C_{i,T} + (1 - \mu_i) \ln (H_{i,T} + H_{j,T})] + (1 - \delta_i) [\mu_j \ln C_{j,T} + (1 - \mu_j) \ln (H_{i,T} + H_{j,T})]
$$

subject to

$$
p_i C_{i,T} + H_{i,T} \leq \frac{W_T}{2}
$$

$$
C_{i,T}, H_{i,T} \geq 0
$$

taking the choices of member $j$ as given. The budget constraint will bind with equality and hence can be substituted into the objective. This reduces the problem to taking $H_{j,T}$ and
\( C_{j,T} \) as given and solving

\[
\max_{H_{i,T}} \delta_i \left[ \mu_i \ln \left( \frac{1}{p_i} \left( \frac{W_T}{2} - H_{i,T} \right) \right) + (1 - \mu_i) \ln (H_{i,T} + H_{j,T}) \right] \\
+ (1 - \delta_i) \left[ \mu_j \ln C_{j,T} + (1 - \mu_j) \ln (H_{i,T} + H_{j,T}) \right]
\]

subject to

\[
H_{i,T} \geq 0 \\
H_{i,T} \leq \frac{W_T}{2}.
\]

Start by ignoring the non-negativity constraints. The FOC on \( H_{i,T} \) is

\[
\frac{\delta_i (1 - \mu_i) + (1 - \delta_i) (1 - \mu_j)}{H_{i,T} + H_{j,T}} - \frac{\delta_i \mu_i}{\frac{W_T}{2} - H_{i,T}} = 0
\]

which rearranges to

\[
H_{i,T} = \left[ 1 - \mu_i \delta_i - \mu_j (1 - \delta_i) \right] \frac{W_T}{2} - \delta_i \mu_i H_{j,T} \\
1 - \mu_j (1 - \delta_i)
\]

Since the objective is strictly concave in \( H_{i,T} \), \( \delta_i \)'s unique best response to any possible choice of \( H_{j,T} \geq 0 \) is

\[
H_{i,T}^{BR} (H_{j,T}) = \begin{cases} 
  b_{i,T} \frac{W_T}{2} - m_{i,T} H_{j,T} & \text{if } H_{j,T} \leq \frac{b_{i,T} W_T}{m_{i,T}} \\
  0 & \text{if } H_{j,T} > \frac{b_{i,T} W_T}{m_{i,T}}
\end{cases}
\]

where \( b_{i,T} \equiv \frac{1 - \mu_i \delta_i - \mu_j (1 - \delta_i)}{1 - \mu_j (1 - \delta_i)} > 0 \), and

\[
m_{i,T} \equiv \frac{\delta_i \mu_i}{1 - \mu_j (1 - \delta_i)} \in (0, 1).
\]

Note that \( H_{i,T}^{BR} (0) > 0 \) and hence \( H_{A,T} = H_{B,T} = 0 \) cannot be a Nash equilibrium. If \( b_{i,T} \geq \frac{b_{i,T}}{m_{j,T}} \) then \( H_{i,T} = b_{i,T} \frac{W_T}{2} \) and \( H_{j,T} = 0 \) is a Nash equilibrium. In this case equilibrium
private consumption will be

\[ C_{i,T} = \frac{1}{p_i} (1 - b_{i,T}) \frac{W_T}{2} \quad \text{and} \quad C_{j,T} = \frac{1}{p_j} \frac{W_T}{2}. \]

Since \( m_{i,T}, m_{i,T} < 1 \) then this equilibrium is unique. A symmetric argument applies when \( b_{i,T} \leq m_{i,T} b_{j,T} \). Finally, if \( b_{i,T} \in \left( m_{i,T} b_{j,T}, \frac{b_{j,T}}{m_{i,T}} \right) \) then there is an interior Nash equilibrium. This is found by substituting the interior portion of \( j \)'s reaction function into the reaction function of \( i \):

\[
\begin{align*}
H_{i,T} &= \frac{b_{i,T} W_T}{2} - m_{i,T} b_{j,T} \left( \frac{W_T}{2} - m_{j,T} H_{i,T} \right) \\
H_{i,T} &= \frac{b_{i,T} W_T}{1 - m_{i,T} m_{j,T}} - m_{i,T} b_{j,T}. 
\end{align*}
\]

To total expenditure on common consumption in this interior solution is

\[ H_T = \left( \frac{b_{i,T} (1 - m_{j,T}) + b_{j,T} (1 - m_{i,T})}{1 - m_{i,T} m_{j,T}} \right) \frac{W_T}{2}. \]

The equilibrium level of private consumption in this interior solution is

\[ C_{i,T} = \frac{1}{p_i} \left( 1 - \frac{b_{i,T} - m_{i,T} b_{j,T}}{1 - m_{i,T} m_{j,T}} \right) \frac{W_T}{2}. \]

Thus the equilibrium value of member \( i \)'s objective function is

\[ V_{i,T} = \ln W_T + k_{i,T}. \]
where \( k_{i,T} \) is a constant term that depends on parameters in the following way

\[
\begin{aligned}
k_{i,T} &\equiv \\
&\begin{cases}
\delta_i \mu_i \left( \frac{1}{p_i} (1 - b_{i,T}) \right) + (1 - \delta_i) \mu_j \ln \left( \frac{1}{p_j} \right) + \delta_i \mu_i \left( \frac{b_{i,T}}{m_{i,T}} \ln (b_{i,T}) \right) + \ln 2 & \text{if } b_{i,T} \leq m_{i,T} b_{j,T} \\
(1 - \delta_i) \mu_j \ln \left( \frac{1}{p_j} (1 - \frac{b_{j,T} - m_{j,T} x_{i,T}}{1 - m_{i,T} m_{j,T}}) \right) + \delta_i \mu_i \ln \left( \frac{1}{p_i} \left( 1 - \frac{b_{i,T} - m_{i,T} x_{j,T}}{1 - m_{i,T} m_{j,T}} \right) \right) + \ln 2 & \text{if } b_{i,T} \in (m_{i,T} b_{j,T}, \frac{b_{j,T}}{m_{j,T}}) \\
\delta_i \mu_i \ln \left( \frac{1}{p_i} \right) + (1 - \delta_i) \mu_j \ln \left( \frac{1}{p_j} (1 - b_{j,T}) \right) + \delta_i \mu_i \left( \frac{b_{i,T}}{m_{i,T}} \ln (b_{j,T}) \right) + \ln 2 & \text{if } b_{i,T} \geq \frac{b_{j,T}}{m_{j,T}}
\end{cases}
\end{aligned}
\]

A-2. Solve for Subgame Perfect Consumption path by Induction

Conjecture the following form for the subgame perfect household allocation.

**Conjecture 3** The subgame perfect equilibrium household allocation from \( t \) until \( T \) is proportional to \( W_t \). That is, the subgame perfect equilibrium levels of private and common consumption can be written as \( C^*_t x + x = g_{i,t+x} W_t \) and \( H^*_t x = h_{i,t+x} W_t \) for \( x \in \{0, 1, ..., T - t\} \) where \( g_{i,t+x} \) and \( h_{i,t+x} \) are strictly positive constants independent of \( W_t \).

I will establish this conjecture by induction. As the first step, note that Conjecture 3 is verified for \( t = T \) above. Next, consider the problem that each household member faces in
period $t < T$. Member $i$ takes $C_{j,t}$ and $H_{j,t}$ as given and solves the following

$$\max_{C_{i,t}, H_{i,t}} \delta_i \left[ \mu_i \ln C_{i,t} + (1 - \mu_i) \ln (H_{i,t} + H_{j,t}) \right]$$

$$+ (1 - \delta_i) \left[ \mu_j \ln C_{j,t} + (1 - \mu_j) \ln (H_{i,t} + H_{j,t}) \right]$$

$$+ \delta_i \Psi_i \sum_{x=1}^{T-t} \beta_i^x \left[ \mu_i \ln C_{i,t+x}^* + (1 - \mu_i) \ln (H_{t+x}^*) \right]$$

$$+ (1 - \delta_i) \Psi_j \sum_{x=1}^{T-t} \beta_j^x \left[ \mu_j \ln C_{j,t+x}^* + (1 - \mu_j) \ln (H_{t+x}^*) \right]$$

subject to

$$W_{t+1} = R \left( W_t - p_i C_{i,t} - p_j C_{j,t} - H_{i,t} - H_{j,t} \right)$$

$$C_{i,t}, H_{i,t} \geq 0$$

Conjecture 3 implies that

$$\delta_i \Psi_i \sum_{x=1}^{T-t} \beta_i^x \left[ \mu_i \ln C_{i,t+x}^* + (1 - \mu_i) \ln (H_{t+x}^*) \right]$$

$$+ (1 - \delta_i) \Psi_j \sum_{x=1}^{T-t} \beta_j^x \left[ \mu_j \ln C_{j,t+x}^* + (1 - \mu_j) \ln (H_{t+x}^*) \right]$$

$$= Y_{i,t+1} \ln W_{t+1} + k_{i,t}$$

where

$$Y_{i,t+1} \equiv \delta_i \Psi_i \sum_{x=1}^{T-t} \beta_i^x + (1 - \delta_i) \Psi_j \sum_{x=1}^{T-t} \beta_j^x > 0$$

$$= \delta_i \Psi_i \left( \frac{\beta_i^x - \beta_i^{T+1-t}}{1 - \beta_i} \right) + (1 - \delta_i) \Psi_j \left( \frac{\beta_j^x - \beta_j^{T+1-t}}{1 - \beta_j} \right)$$

and $k_{i,t}$ is a constant. In equilibrium the budget constraint will bind. Log utility will ensure that $C_{i,t} > 0$ in equilibrium and hence can be ignored for now and verified later. Ignoring
terms that \( i \) takes as given in \( t \), the problem can be written as

\[
\max_{C_{i,t},H_{i,t}} \delta_i \mu_i \ln C_{i,t} + \left[ \delta_i (1 - \mu_i) + (1 - \delta_i) (1 - \mu_j) \right] \ln (H_{i,t} + H_{j,t}) \\
+ Y_{i,t+1} \ln \left( W_t - p_i C_{i,t} - p_j C_{j,t} - H_{i,t} - H_{j,t} \right)
\]

subject to

\[ H_{i,t} \geq 0 \]

Start by ignoring the non-negativity constraint on \( H_{i,t} \). The first order conditions are

\[
C_{i,t} : \frac{\delta_i \mu_i}{C_{i,t}} - \frac{p_i Y_{i,t+1}}{W_t - p_i C_{i,t} - p_j C_{j,t} - H_{i,t} - H_{j,t}} = 0
\]
\[
H_{i,t} : \frac{\delta_i (1 - \mu_i) + (1 - \delta_i) (1 - \mu_j)}{H_{i,t} + H_{j,t}} - \frac{Y_{i,t+1}}{W_t - p_i C_{i,t} - p_j C_{j,t} - H_{i,t} - H_{j,t}} = 0
\]

Suppose that \( H_{j,t} = 0 \). Then \( i \) will set

\[
H_{i,t} = h_{i,t} [W_t - p_i C_{i,t} - p_j C_{j,t}]
\]

where \( h_{i,t} \equiv \frac{\delta_i (1 - \mu_i) + (1 - \delta_i) (1 - \mu_j)}{Y_{i,t+1} + \delta_i (1 - \mu_i) + (1 - \delta_i) (1 - \mu_j)} \in (0, 1) \).

Given the strict concavity of \( i \)'s problem in \( H_{i,t} \), this is a Nash equilibrium if and only if \( h_{i,t} \geq h_{j,t} \). Thus the unique nash equilibrium level of common consumption will be

\[
H_t = h_t [W_t - p_i C_{i,t} - p_j C_{j,t}]
\]

where \( h_t \equiv \max \{ h_{A,t}, h_{B,t} \} \in (0, 1) \).

The FOC for \( i \)'s choice of private consumption implies

\[
p_i C_{i,t} = g_{i,t} [W_t - p_j C_{j,t} - H_{i,t}].
\]

where \( g_{i,t} \equiv \frac{\delta_i \mu_i}{Y_{i,t+1} + \delta_i \mu_i} \in (0, 1) \).
Substituting \( j \)'s reaction function into \( i \)'s gives

\[
p_i C_{i,t} = \frac{g_{i,t}(1 - g_{j,t})}{1 - g_{i,t}g_{j,t}} [W_t - H_t].
\]

Thus the equilibrium level of common consumption is

\[
H_t = h_t \left[ W_t - \frac{g_{i,t}(1 - g_{j,t}) + g_{j,t}(1 - g_{i,t})}{1 - g_{i,t}g_{j,t}} [W_t - H_t] \right]
\]

\[
H^*_t = \left[ 1 - \frac{1 - h_t}{1 - h_t g_{i,t}(1 - g_{j,t}) + g_{j,t}(1 - g_{i,t})} \right] W_t
\]

note that

\[
g_{A,t}(1 - g_{B,t}) + g_{B,t}(1 - g_{A,t}) = \frac{\delta_{A} \mu_{A} Y_{B,t+1} + \delta_{B} \mu_{B} Y_{A,t+1}}{Y_{A,t+1} Y_{B,t+1} + \delta_{A} \mu_{A} Y_{B,t+1} + \delta_{B} \mu_{B} Y_{A,t+1}} \in (0, 1)
\]

which combined with the fact that \( h_t \in (0, 1) \) ensures \( \frac{H^*_t}{W_t} \in (0, 1) \) thus confirming our conjecture for common consumption. Equilibrium private consumption is

\[
p_i C^*_t = \frac{g_{i,t}(1 - g_{j,t})}{1 - g_{i,t}g_{j,t}} \left( 1 - \frac{1 - h_t}{1 - h_t g_{i,t}(1 - g_{j,t}) + g_{j,t}(1 - g_{i,t})} \right) W_t.
\]

Note that

\[
\frac{g_{i,t}(1 - g_{j,t})}{1 - g_{i,t}g_{j,t}} \in (0, 1)
\]

and so \( \frac{p_i C^*_t}{W_t} \in (0, 1) \) which completes the proof by induction of Conjecture 3. The equilibrium ratio of expenditure on private consumption for \( i \) and \( j \) is

\[
\frac{p_i C^*_t}{p_j C^*_t} = \frac{g_{i,t}(1 - g_{j,t})}{g_{j,t}(1 - g_{i,t})} = \frac{\delta_{i} \mu_{i} Y_{j,t+1}}{\delta_{j} \mu_{j} Y_{i,t+1}} = \frac{\delta_{i} \mu_{i} \left( \psi_{j} \sum_{x=1}^{T-t} \beta_{j}^{x} + (1 - \delta_{j}) \psi_{i} \sum_{x=1}^{T-t} \beta_{i}^{x} \right)}{\delta_{j} \mu_{j} \left( \psi_{i} \sum_{x=1}^{T-t} \beta_{i}^{x} + (1 - \delta_{i}) \psi_{j} \sum_{x=1}^{T-t} \beta_{j}^{x} \right)}.
\]
The total expenditure on all consumption is

\[ p_i C_{i,t}^* + p_j C_{j,t}^* + H_t^* = \left( \frac{g_{i,t} (1 - g_{j,t}) + g_{j,t} (1 - g_{i,t})}{1 - g_{i,t} g_{j,t}} \right) \left( \frac{1 - h_t}{1 - h_t \frac{g_{i,t} (1 - g_{j,t}) + g_{j,t} (1 - g_{i,t})}{1 - g_{i,t} g_{j,t}}} \right) W_t \]

\[ + \left[ 1 - \frac{1 - h_t}{1 - h_t \frac{g_{i,t} (1 - g_{j,t}) + g_{j,t} (1 - g_{i,t})}{1 - g_{i,t} g_{j,t}}} \right] W_t \]

\[ = \left[ 1 - \left( 1 - \frac{g_{i,t} (1 - g_{j,t}) + g_{j,t} (1 - g_{i,t})}{1 - g_{i,t} g_{j,t}} \right) \left( \frac{1 - h_t}{1 - h_t \frac{g_{i,t} (1 - g_{j,t}) + g_{j,t} (1 - g_{i,t})}{1 - g_{i,t} g_{j,t}}} \right) \right] W_t \]

The equilibrium fraction of total household expenditure in period \( t \) on common consumption is

\[ \frac{H_t^*}{p_i C_{i,t}^* + p_j C_{j,t}^* + H_t^*} = \frac{1 - \frac{1 - h_t}{1 - h_t \frac{g_{i,t} (1 - g_{j,t}) + g_{j,t} (1 - g_{i,t})}{1 - g_{i,t} g_{j,t}}}}{1 - \left( 1 - \frac{g_{i,t} (1 - g_{j,t}) + g_{j,t} (1 - g_{i,t})}{1 - g_{i,t} g_{j,t}} \right) \left( \frac{1 - h_t}{1 - h_t \frac{g_{i,t} (1 - g_{j,t}) + g_{j,t} (1 - g_{i,t})}{1 - g_{i,t} g_{j,t}}} \right)} \]

The equilibrium fraction of total household expenditure in period \( t \) on \( i \)'s private consumption is

\[ \frac{p_i C_{i,t}^*}{p_i C_{i,t}^* + p_j C_{j,t}^* + H_t^*} = \frac{\frac{g_{i,t} (1 - g_{j,t})}{1 - g_{i,t} g_{j,t}} \left( \frac{1 - h_t}{1 - h_t \frac{g_{i,t} (1 - g_{j,t}) + g_{j,t} (1 - g_{i,t})}{1 - g_{i,t} g_{j,t}}} \right)}{1 - \left( 1 - \frac{g_{i,t} (1 - g_{j,t}) + g_{j,t} (1 - g_{i,t})}{1 - g_{i,t} g_{j,t}} \right) \left( \frac{1 - h_t}{1 - h_t \frac{g_{i,t} (1 - g_{j,t}) + g_{j,t} (1 - g_{i,t})}{1 - g_{i,t} g_{j,t}}} \right)} \]

Household wealth evolves over time in the following way:

\[ W_{t+1} = R \left( 1 - \frac{g_{i,t} (1 - g_{j,t}) + g_{j,t} (1 - g_{i,t})}{1 - g_{i,t} g_{j,t}} \right) \left( \frac{1 - h_t}{1 - h_t \frac{g_{i,t} (1 - g_{j,t}) + g_{j,t} (1 - g_{i,t})}{1 - g_{i,t} g_{j,t}}} \right) W_t. \]
The equilibrium path of common consumption is

\[
\frac{H_{t+1}^*}{H_t^*} = \frac{1 - \frac{1-h_{t+1}}{1-h_t} \frac{g_{i,t+1} (1-g_{j,t+1}) + g_{j,t+1} (1-g_{i,t+1})}{1 - h_{t+1} g_{j,t+1} + g_{i,t+1} (1-g_{i,t+1})}}{1 - \frac{W_{t+1}}{W_t}}
\]

\[
= R \left( \frac{1 - h_t}{h_t} \right) \left( 1 - \frac{1 - h_{t+1}}{1 - h_{t+1} g_{j,t+1} + g_{i,t+1} (1-g_{i,t+1})} \right).
\]

The equilibrium path of \(i\)'s consumption is

\[
\frac{C_{i,t+1}^*}{C_{i,t}^*} = \frac{g_{i,t+1} (1-g_{j,t+1})}{1 - g_{i,t+1} g_{j,t+1}} \left( \frac{1-h_{t+1}}{1-h_t} g_{i,t+1} (1-g_{j,t+1}) + g_{j,t+1} (1-g_{i,t+1}) \right)
\]

\[
\cdot \frac{W_{t+1}}{W_t}
\]

\[
= R \frac{1 - g_{i,t}}{g_{i,t}} \left( \frac{g_{i,t+1} (1-g_{j,t+1}) (1-h_{t+1})}{1 - g_{i,t+1} g_{j,t+1} - h_{t+1} [g_{i,t+1} (1-g_{j,t+1}) + g_{j,t+1} (1-g_{i,t+1})]} \right).
\]

### A-3. Special Case I: Symmetric Household

The symmetric model studied in Section I. is a special case of the model solved here with \(\delta = \delta_A = \delta_B, \mu = \mu_A = \mu_B, \beta = \beta_A = \beta_B\) and \(\Psi_A = \Psi_B = 1\). In that case

\[
Y_{i,t+1} = \sum_{x=1}^{T-t} \beta^x,
\]

\[
h_t = h_{i,t} = \frac{1 - \mu}{\sum_{x=1}^{T-t} \beta^x + 1 - \mu}, \text{ and }
\]

\[
g_{i,t} = \frac{\delta \mu}{\sum_{x=1}^{T-t} \beta^x + \delta \mu}.
\]
Thus the equilibrium choice of common consumption in each period is

\[
H^*_t = \frac{h_t (1 - g_{i,t})}{1 + g_{i,t} - 2h_t g_{i,t}} \left[ 1 + g_{i,t} - 2h_t g_{i,t} \right] W_t
= \left[ \frac{(1 - \mu)}{\sum_{x=1}^{T-t} \beta^x + 2\delta \mu + (1 - \mu)} \right] W_t.
\]

The equilibrium choice of private consumption in each period is

\[
p_{i}C^*_{i,t} = g_{i,t} (1 - h_t) \frac{1 + g_{i,t} - 2h_t g_{i,t}}{1 + g_{i,t} - 2h_t g_{i,t}} W_t
= \frac{\delta \mu}{\sum_{x=1}^{T-t} \beta^x + 2\delta \mu + (1 - \mu)} W_t.
\]

Thus total equilibrium household expenditure in each period is

\[
X^*_t = \frac{2\delta \mu + (1 - \mu)}{\sum_{x=1}^{T-t} \beta^x + 2\delta \mu + (1 - \mu)} W_t
\]

The fraction of total equilibrium household expenditure that is devoted to common consumption each period is

\[
\frac{H^*_t}{X^*_t} = \frac{1 - \mu}{2\delta \mu + (1 - \mu)}
\]

and the fraction devoted to each member’s private consumption is

\[
\frac{p_{i}C^*_{i,t}}{X^*_t} = \frac{\delta \mu}{2\delta \mu + (1 - \mu)}.
\]

Household wealth evolves in the following way

\[
W_{t+1} = R \left( \frac{(1 - g_{i,t}) (1 - h_t)}{1 + g_{i,t} - 2h_t g_{i,t}} \right) W_t
= R \left( \frac{\sum_{x=1}^{T-t} \beta^x}{\sum_{x=1}^{T-t} \beta^x + 2\delta \mu + (1 - \mu)} \right) W_t.
\]
The growth rate of member \( i \)'s consumption is

\[
\frac{C_{i,t+1}^*}{C_{i,t}^*} = R \frac{1 - g_{i,t}}{g_{i,t}} \left( \frac{g_{i,t+1} (1 - h_{i,t+1})}{1 + g_{i,t+1} - 2h_{i,t+1}g_{i,t+1}} \right)
\]

\[
= R \left( \frac{\sum_{x=1}^{T-t} \beta^x}{\sum_{x=1}^{T-(t+1)} \beta^x + 2\delta \mu + (1 - \mu)} \right)
\]

Since each type of consumption remains in fixed proportion over the life of the household then they must all evolve at the same rate.

**A-4. Special Case II: Symmetric Discounting**

I now consider the solution for the case where both household members have the same time preferences so that \( \beta = \beta_A = \beta_B \) and \( \Psi = \Psi_A = \Psi_B \). In that case

\[
Y_{i,t+1} = Y_{t+1} = \Psi \sum_{x=1}^{T-t} \beta^x
\]

\[
h_{i,t} = \frac{\delta_i (1 - \mu_i) + (1 - \delta_i) (1 - \mu_j)}{Y_{i+1} + \delta_i (1 - \mu_i) + (1 - \delta_i) (1 - \mu_j)}
\]

and so in each period the level of common consumption will be set by member \( i \) if and only if \( h_{i,t} \geq h_{j,t} \) which requires

\[
(\delta_i + \delta_j - 1) (1 - \mu_i) \geq (\delta_j + \delta_i - 1) (1 - \mu_j)
\]

which holds if and only if \( \mu_j \geq \mu_i \) and \( \delta_i + \delta_j \geq 1 \). Without loss of generality let member \( A \) be such that \( \mu_A \geq \mu_B \) and so the level of common consumption will be set by member \( B \). Thus

\[
h_t = h_{B,t} = \frac{\delta_B (1 - \mu_B) + (1 - \delta_B) (1 - \mu_A)}{Y_{i+1} + \delta_B (1 - \mu_B) + (1 - \delta_B) (1 - \mu_A)}
\]

Note also that
The equilibrium level of common consumption in each period is thus

\[ H_t^* = \left[ 1 - \frac{1 - h_t}{\frac{1 - h_t}{\delta_{i\mu_i} + \delta_{j\mu_j}}} \right] W_t \]

The equilibrium level of expenditure on member \( i \)'s private consumption is

\[ p_i C_{i,t}^* = \frac{g_{i,t} (1 - g_{j,t})}{1 - g_{i,t} g_{j,t}} \left( 1 - \frac{1 - h_t}{1 - h_t g_{i,t} + g_{i,t} (1 - g_{i,t})} \right) W_t \]

Total equilibrium expenditure in period \( t \) is therefore

\[ X_t^* = \left( \frac{\delta_B (1 - \mu_B) + (1 - \delta_B) (1 - \mu_A) + \delta_A \mu_A}{Y_{t+1} + \delta B + (1 - \delta_B) (1 - \mu_A) + \delta A \mu_A} \right) W_t. \]

The equilibrium share of total expenditure on each type of consumption in each period is

\[ \frac{p_i C_{i,t}^*}{X_t^*} = \frac{\delta_{i\mu_i}}{\delta_B + (1 - \delta_B) (1 - \mu_A) + \delta_A \mu_A} \]

\[ \frac{H_t^*}{X_t^*} = \frac{\delta_B (1 - \mu_B) + (1 - \delta_B) (1 - \mu_A)}{\delta B + (1 - \delta_B) (1 - \mu_A) + \delta A \mu_A} \]
remembering that we have assumed that member $B$ cares more about common consumption than member $A$. Note that these fractions are time invariant. Wealth evolves according to

$$W_{t+1} = R \left( \frac{\Psi \sum_{x=1}^{T-t} \beta_x}{\Psi \sum_{x=1}^{T-t} \beta_x + \delta_B + (1 - \delta_B) (1 - \mu_A) + \delta_A \mu_A} \right) W_t$$

and consumption grows according to

$$\frac{X_{t+1}}{X_t} = R \left( \frac{\Psi \sum_{x=1}^{T-t} \beta_x}{\delta_B + (1 - \delta_B) (1 - \mu_A) + \delta_A \mu_A + \Psi \sum_{x=1}^{T-(t+1)} \beta_x} \right)$$

which is the same rate of growth for each type of consumption.

**A-5. Special Case III Asymmetric Discounting**

I now study the case where the only difference between the household members is their rate of time preference $\beta_A \neq \beta_B$. I assume that they place equal value on each other’s utility $\delta_A = \delta_B = 0.5$ and the same weight on private versus common consumption $\mu = \mu_A = \mu_B$. Suppose also that individually they have standard exponential discount rates $\Psi_A = \Psi_B = 1$. In this case

$$Y_{A,t+1} = Y_{B,t+1} = \frac{1}{2} \sum_{x=1}^{T-t} (\beta_A^x + \beta_B^x),$$

$$h_{A,t} = h_{B,t} = h_t = \frac{1 - \mu}{\frac{1}{2} \sum_{x=1}^{T-t} (\beta_A^x + \beta_B^x) + 1 - \mu},$$

$$g_{A,t} = g_{B,t} = \frac{\mu}{\sum_{x=1}^{T-t} (\beta_A^x + \beta_B^x) + \mu}.$$

Note that

$$\frac{g_A(t) (1 - g_B(t)) + g_B(t) (1 - g_A(t))}{1 - g_A(t) g_B(t)} = \frac{2\mu}{\sum_{x=1}^{T-t} (\beta_A^x + \beta_B^x) + \mu} \left( \frac{\sum_{x=1}^{T-t} (\beta_A^x + \beta_B^x)}{\sum_{x=1}^{T-t} (\beta_A^x + \beta_B^x) + \mu} \right)^2$$

$$= \frac{2\mu}{\sum_{x=1}^{T-t} (\beta_A^x + \beta_B^x) + 2\mu}$$

$\text{41}$
\[
g_{i,t} \left(1 - g_{j,t}\right) \frac{1}{1 - g_{i,t}g_{j,t}} = \frac{\mu}{\sum_{x=1}^{T-1}(\beta_A^x + \beta_B^x) + \mu} \left( \frac{\sum_{x=1}^{T-1}(\beta_A^x + \beta_B^x)}{\sum_{x=1}^{T-1}(\beta_A^x + \beta_B^x) + \mu} \right)^2 - 1
\]

\[
= \frac{\sum_{x=1}^{T-1}(\beta_A^x + \beta_B^x) + 2\mu}{\sum_{x=1}^{T-1}(\beta_A^x + \beta_B^x) + 2\mu}
\]

and

\[
1 - h_t \frac{g_{i,t}(1-g_{j,t}) + g_{j,t}(1-g_{i,t})}{1 - g_{i,t}g_{j,t}} = \frac{\mu}{2} \frac{\sum_{x=1}^{T-1}(\beta_A^x + \beta_B^x)}{\sum_{x=1}^{T-1}(\beta_A^x + \beta_B^x) + 1 - \mu}
\]

\[
= \frac{\frac{1}{2} \sum_{x=1}^{T-1}(\beta_A^x + \beta_B^x) + \mu}{\frac{1}{2} \sum_{x=1}^{T-1}(\beta_A^x + \beta_B^x) + 1}
\]

\[
H_t^* = \left[1 - \frac{1 - h_t}{1 - h_t \frac{g_{i,t}(1-g_{j,t}) + g_{j,t}(1-g_{i,t})}{1 - g_{i,t}g_{j,t}}} \right] W_t
\]

\[
= \left[\frac{1 - \mu}{\frac{1}{2} \sum_{x=1}^{T-1}(\beta_A^x + \beta_B^x) + 1}\right] W_t
\]

\[
p_i C_{i,t}^* = \frac{g_{i,t}(1-g_{j,t})}{1 - g_{i,t}g_{j,t}} \left(1 - h_t \frac{g_{i,t}(1-g_{j,t}) + g_{j,t}(1-g_{i,t})}{1 - g_{i,t}g_{j,t}}\right) W_t
\]

\[
= \left(\frac{\mu}{\sum_{x=1}^{T-1}(\beta_A^x + \beta_B^x) + 2\mu}\right) \left(\frac{1}{2} \sum_{x=1}^{T-1}(\beta_A^x + \beta_B^x) + \mu\right)
\]

\[
= \left(\frac{\mu}{\frac{1}{2} \sum_{x=1}^{T-1}(\beta_A^x + \beta_B^x) + 1}\right)
\]

Note that the consumption of each household member is identical in every period.

\[
X_t^* = \left(\frac{1}{1 + \frac{1}{2} \sum_{x=1}^{T-1}(\beta_A^x + \beta_B^x)}\right) W_t.
\]
Equilibrium expenditure shares in each period are

\[
\frac{H_t}{X_{i,t}^*} = 1 - \mu,
\]

\[
\frac{p_t C_{i,t}^*}{X_{i,t}^*} = \frac{\mu}{2}.
\]

Note that these are constant and hence both members of the household will have an identical path of consumption in each period despite the difference in their discount rates.

Wealth evolves according to

\[
W_{t+1} = R \left( \frac{\frac{1}{2} \sum_{x=1}^{T-t} (\beta^x_A + \beta^x_B)}{1 + \frac{1}{2} \sum_{x=1}^{T-t} (\beta^x_A + \beta^x_B)} \right) W_t
\]

and total expenditure evolves according to

\[
\frac{X_{i,t+1}^*}{X_{i,t}^*} = R \left( \frac{\frac{1}{2} \sum_{x=1}^{T-(t+1)} (\beta^x_A + \beta^x_B)}{1 + \frac{1}{2} \sum_{x=1}^{T-(t+1)} (\beta^x_A + \beta^x_B)} \right).
\]

This implies that at the end of the life of the household that

\[
\frac{X_{i,T}^*}{X_{i,T-1}^*} = R \frac{1}{2} (\beta_A + \beta_B).
\]

B. Solution to Household Allocation with Full Commitment

This section of the Appendix solves the generalized household problem presented in Section III. with full commitment. The symmetric model studied in Section I. is a special case of this where \(\delta_A = \delta_B, \mu_A = \mu_B, \beta_A = \beta_B\) and \(\Psi_A = \Psi_B = 1\). The problem is to solve

\[
\max_{\{C_{A,t}, C_{B,t}, H_t\}_{t=1}^{T}} \Pi = \eta V_{A,1} + (1 - \eta) V_{B,1}
\]

subject to

\[
W_1 - \sum_{x=0}^{T-1} R^{-x} \left[ p_A C_{A,1+x} + p_B C_{B,1+x} + H_{1+x} \right] \geq 0
\]

\[
\{C_{A,t}, C_{B,t}, H_t\}_{t=1}^{T} \geq 0
\]
Log utility ensures that we can ignore the non-negativity constraints on $C_{A,t}$, $C_{B,t}$, and $H_t$. It is easy to verify that the solution satisfies these constraints. The objective of this problem can be written as

$$
\Pi = \eta V_{A,1} + (1 - \eta) V_{B,1} \\
= \eta (\delta_A U_{A,1} + (1 - \delta_A) U_{B,1}) + (1 - \eta) (\delta_B U_{B,1} + (1 - \delta_B) U_{A,1}) \\
= (1 - \theta) U_{A,1} + \theta U_{B,1}
$$

where $\theta \equiv \delta_B + \eta (1 - \delta_A - \delta_B)$

using the expressions for $U_{A,1}$ and $U_{B,1}$ this becomes

$$
\Pi = (1 - \theta) \mu_A \left[ \ln C_{A,1} + \Psi_A \sum_{x=1}^{T-1} \beta_A^x \ln C_{A,1+x} \right] \\
+ \theta \mu_B \left[ \ln C_{B,1} + \Psi_B \sum_{x=1}^{T-1} \beta_B^x \ln C_{B,1+x} \right] \\
+ [(1 - \theta) (1 - \mu_A) + \theta (1 - \mu_B)] \ln H_1 \\
+ \sum_{x=1}^{T-1} [(1 - \theta) (1 - \mu_A) \Psi_A \beta_A^x + \theta (1 - \mu_B) \Psi_B \beta_B^x] \ln H_{1+x}.
$$

Writing the Lagrangian for this problem with $\Gamma \geq 0$ being the multiplier on the budget constraint we have

$$
\max_{\{C_{A,t}, C_{B,t}, H_t\}_{t=1}^{T}} \left( (1 - \theta) \mu_A \left[ \ln C_{A,1} + \Psi_A \sum_{x=1}^{T-1} \beta_A^x \ln C_{A,1+x} \right] \\
+ \theta \mu_B \left[ \ln C_{B,1} + \Psi_B \sum_{x=1}^{T-1} \beta_B^x \ln C_{B,1+x} \right] \\
+ [(1 - \theta) (1 - \mu_A) + \theta (1 - \mu_B)] \ln H_1 \\
+ \sum_{x=1}^{T-1} [(1 - \theta) (1 - \mu_A) \Psi_A \beta_A^x + \theta (1 - \mu_B) \Psi_B \beta_B^x] \ln H_{1+x} \\
+ \Gamma \left[ W_1 - \sum_{x=0}^{T-1} R^{-x} [p_A C_{A,1+x} + p_B C_{B,1+x} + H_{1+x}] \right].
$$
The first order conditions give the optimal level of expenditure on each type of consumption in every period as a function of $\Gamma$:

\[
\begin{align*}
C_{A,1} & : \quad p_{A}C_{A,1}^{**} = \frac{(1 - \theta) \mu_{A}}{\Gamma} \\
C_{A,1+x} & : \quad p_{A}C_{A,1+x}^{**} = \frac{(1 - \theta) \mu_{A} \Psi_{A} \beta_{A}^{x}}{\Gamma R^{-x}} \\
C_{B,1} & : \quad p_{B}C_{B,1}^{**} = \frac{\theta \mu_{B}}{\Gamma} \\
C_{B,1+x} & : \quad p_{B}C_{B,1+x}^{**} = \frac{\theta \mu_{B} \Psi_{B} \beta_{B}^{x}}{\Gamma R^{-x}} \\
H_{1} & : \quad H_{1}^{**} = \frac{(1 - \theta) (1 - \mu_{A}) + \theta (1 - \mu_{B})}{\Gamma} \\
H_{1+x} & : \quad H_{1+x}^{**} = \frac{(1 - \theta) (1 - \mu_{A}) \Psi_{A} \beta_{A}^{x} + \theta (1 - \mu_{B}) \Psi_{B} \beta_{B}^{x}}{\Gamma R^{-x}}
\end{align*}
\]

where $x \in \{1, 2, ..., T - 1\}$ and the double asteriks indicates solution to the full commitment problem. Start by examining the optimal allocation within any period. In the first period, the optimal level of total expenditure is

\[
X_{1}^{**} = \frac{1}{\Gamma}
\]

where $X_{1}^{**} \equiv p_{A}C_{A,1}^{**} + p_{B}C_{B,1}^{**} + H_{1}^{**}$.

Thus the optimal share of expenditure on each of the types of consumption is

\[
\begin{align*}
\frac{p_{A}C_{A,1}^{**}}{X_{1}^{**}} & = (1 - \theta) \mu_{A} \\
\frac{p_{B}C_{B,1}^{**}}{X_{1}^{**}} & = \theta \mu_{B} \\
\frac{H_{1}^{**}}{X_{1}^{**}} & = (1 - \theta) (1 - \mu_{A}) + \theta (1 - \mu_{B})
\end{align*}
\]
For any period after the first, the optimal level of total expenditure is

\[ X_{1+x}^{**} = \frac{(1 - \theta) \Psi_A \beta_A^x + \theta \Psi_B \beta_B^x}{\Gamma R^{-x}} \]

where \( X_{1+x}^{**} \equiv p_A C_{A,1+x}^{**} + p_B C_{B,1+x}^{**} + H_{1+x}^{**} \).

Since the optimal allocation will exhaust the household budget constraint it must be that

\[ W_1 = X_1^{**} + \sum_{x=1}^{T-1} \frac{X_{1+x}^{**}}{R^x} \]

\[ = \frac{1}{\Gamma} \left[ 1 + \sum_{x=1}^{T-1} (1 - \theta) \Psi_A \beta_A^x + \theta \Psi_B \beta_B^x \right] \]

which implies that

\[ \Gamma^{**} = \frac{1 + (1 - \theta) \Psi_A \beta_A + \theta \Psi_B \beta_B}{W_1} \]

The optimal share of expenditure on each type of consumption is

\[ \frac{p_A C_{A,1+x}^{**}}{X_{1+x}^{**}} = \frac{(1 - \theta) \mu_A \Psi_A \beta_A^x}{(1 - \theta) \Psi_A \beta_A^x + \theta \Psi_B \beta_B^x} \]

\[ \frac{p_B C_{B,1+x}^{**}}{X_{1+x}^{**}} = \frac{\theta \mu_B \Psi_B \beta_B^x}{(1 - \theta) \Psi_A \beta_A^x + \theta \Psi_B \beta_B^x} \]

\[ \frac{H_{1+x}^{**}}{X_{1+x}^{**}} = \frac{(1 - \theta) (1 - \mu_A) \Psi_A \beta_A^x + \theta (1 - \mu_B) \Psi_B \beta_B^x}{(1 - \theta) \Psi_A \beta_A^x + \theta \Psi_B \beta_B^x} \]

Now consider how the pareto optimal path of consumption evolves over time. Start by comparing periods after the first period \((t > 1)\).

\[ \frac{C_{A,t+1}^{**}}{C_{A,t}^{**}} = R \beta_A \]

\[ \frac{C_{B,t+1}^{**}}{C_{B,t}^{**}} = R \beta_B \]

\[ \frac{H_{t+1}^{**}}{H_t^{**}} = R \frac{(1 - \theta) (1 - \mu_A) \Psi_A \beta_A^{t+1} + \theta (1 - \mu_B) \Psi_B \beta_B^{t+1}}{(1 - \theta) (1 - \mu_A) \Psi_A \beta_A^t + \theta (1 - \mu_B) \Psi_B \beta_B^t} \]

\[ \frac{X_{t+1}^{**}}{X_t^{**}} = R \frac{(1 - \theta) \Psi_A \beta_A^{t+1} + \theta \Psi_B \beta_B^{t+1}}{(1 - \theta) \Psi_A \beta_A^t + \theta \Psi_B \beta_B^t} \]
The dynamics between the first and second period are subject to the household member’s hyperbolic discount factors and hence are different. In this case:

\[
\frac{C_{A,2}^{**}}{C_{A,1}^{**}} = R\Psi_A \beta_A \\
\frac{C_{B,2}^{**}}{C_{B,1}^{**}} = R\Psi_B \beta_B \\
\frac{H_{2}^{**}}{H_{1}^{**}} = R\frac{(1-\theta)(1-\mu_A) \Psi_A \beta_A + \theta (1-\mu_B) \Psi_B \beta_B}{(1-\theta)(1-\mu_A) + \theta (1-\mu_B)} \\
\frac{X_{2}^{**}}{X_{1}^{**}} = R[(1-\theta) \Psi_A \beta_A + \theta \Psi_B \beta_B]
\]

**B-1. Special Case I: Symmetric Household**

The symmetric model studied in Section I. is a special case of the model solved here with \(\delta = \delta_A = \delta_B, \mu = \mu_A = \mu_B, \beta = \beta_A = \beta_B\) and \(\Psi_A = \Psi_B = 1\). In that case the full commitment solution has the following consumption dynamics

\[
\frac{X_{t+1}^{**}}{X_{t}^{**}} = R\beta
\]

with consumption shares in each period of

\[
\frac{p_A C_{A,t}^{**}}{X_{t}^{**}} = (1-\theta) \mu \\
\frac{p_B C_{B,t}^{**}}{X_{t}^{**}} = \theta \mu \\
\frac{H_{t}^{**}}{X_{t}^{**}} = 1 - \mu.
\]
B-2. Special Case II: Symmetric Discount Rates

Now consider the case where the household has symmetric time preferences $\beta = \beta_A = \beta_B$ and $\Psi = \Psi_A = \Psi_B$. The optimal consumption shares are

$$\frac{p_A C_{A,t}^{**}}{X_t^{**}} = (1 - \theta) \mu_A,$$

$$\frac{p_B C_{B,t}^{**}}{X_t^{**}} = \theta \mu_B,$$

$$\frac{H_t^{**}}{X_t^{**}} = 1 - (1 - \theta) \mu_A - \theta \mu_B.$$

Consumption will evolve between the first and second period according to

$$\frac{X_2^{**}}{X_1^{**}} = R\Psi \beta$$

and for every period $t \geq 2$ after that as

$$\frac{X_{t+1}^{**}}{X_t^{**}} = R\beta.$$

B-3. Special Case III: Asymmetric Discount Rates

Consider the full commitment solution when the only difference between the household members is their rate of time preference $\beta_A \neq \beta_B$. I assume that they place equal value on each other’s utility $\delta_A = \delta_B = \frac{1}{2}$ and the same weight on private versus common consumption $\mu = \mu_A = \mu_B$. This implies that $\theta = \frac{1}{2}$. Suppose also that individually each household member has a standard exponential discount rate $\Psi_A = \Psi_B = 1$. In this case

$$\Gamma^{**} = 1 + \frac{1}{2} \left[ \frac{\beta_A - \beta_B}{1 - \beta_A} + \frac{\beta_B - \beta_A}{1 - \beta_B} \right] \frac{W_1}{W_1}.$$
As a result

\[ X_{t+1}^{**} = \frac{(1 - \theta) \Psi_A \beta_A^{t-1} + \theta \Psi_B \beta_B^{t-1}}{\Gamma^{**} R^{-(t-1)}} \]

\[ = \frac{1}{2} \frac{\beta_A^{t-1} + \beta_B^{t-1}}{\Gamma^{**} R^{-(t-1)}} \]

\[ = \frac{1}{2} R^{t-1} \left[ \frac{\beta_A^{t-1} + \beta_B^{t-1}}{1 - \frac{\beta_B - \beta_A}{\beta_A + \beta_B}} \right] W_1 \]

The ratio of total expenditure from one period to the next is

\[ \frac{X_{t+1}^{**}}{X_t^{**}} = \frac{\frac{1}{2} R^t \left[ \frac{\beta_A^t + \beta_B^t}{\beta_A^{t-1} + \beta_B^{t-1}} \right]}{\frac{1}{2} R^{t-1} \left[ \frac{\beta_A^{t-1} + \beta_B^{t-1}}{1 - \frac{\beta_B - \beta_A}{\beta_A + \beta_B}} \right]} \]

\[ = \frac{R \beta_A^t + \beta_B^t}{\beta_A^{t-1} + \beta_B^{t-1}} \]

\[ = \frac{R \beta_B + \beta_A \left( \frac{\beta_A}{\beta_B} \right)^{t-1}}{1 + \left( \frac{\beta_A}{\beta_B} \right)^{t-1}} \]

Note how this changes with \( t \). At the start of the life of the household:

\[ \frac{X_2^{**}}{X_1^{**}} = \frac{R \beta_B + \beta_A}{2} \]

If \( \beta_i < \beta_j \), then

\[ \frac{X_{t+1}^{**}}{X_t^{**}} \to R \beta_j \text{ as } t \to \infty. \]

**B-4. Proof of Proposition 6**

The allocation of expenditure within a period is Pareto optimal if there exists some \( \eta \in [0, 1] \) for which

\[ \frac{p_i C_{i,t}^{**}}{X_t^{**}} = \frac{p_i C_{i,t}^*}{X_t^*} \text{ for } i \in \{A, B\}. \]
Without loss of generality consider an asymmetric household where $\mu_A \geq \mu_B$. For A we require
\[
\frac{\delta_A \mu_A}{1 + \mu_A (\delta_A - (1 - \delta_B))} = \frac{\eta \delta_A + (1 - \eta) (1 - \delta_B)}{1 + \eta \delta_A - (1 - \delta_B)} \mu_A.
\]
This will hold for any $\eta$ if $\mu_A = 0$. When $\mu_A > 0$ then this requires
\[
\eta = \frac{1}{\delta_A - (1 - \delta_B)} \left[ \frac{\delta_A}{1 + \delta_A - (1 - \delta_B)} - (1 - \delta_B) \right]. \tag{10}
\]
By a symmetric argument for B, we require
\[
\frac{\delta_B \mu_B}{1 + \mu_B (\delta_A - (1 - \delta_B))} = \frac{\eta (1 - \delta_A) + (1 - \eta) \delta_B}{1 + \delta_A - (1 - \delta_B)} \mu_B.
\]
This will hold for any $\eta$ if $\mu_B = 0$. When $\mu_B > 0$ this requires
\[
\eta = \frac{1}{\delta_A - (1 - \delta_B)} \left[ \delta_B - \frac{\delta_B}{1 + \delta_A - (1 - \delta_B)} \right]. \tag{11}
\]
For (10) and (11) both to hold requires
\[
\frac{(\delta_A - (1 - \delta_B))(1 - \mu_A)}{1 + \mu_A (\delta_A - (1 - \delta_B))} = 0.
\]
This can only occur if either
\[
\delta_A - (1 - \delta_B) = 0
\]
or if $\mu_A = 1$. This establishes Proposition 6: no such $\eta$ can be found if $\mu_A, \mu_B \in (0, 1)$ and $\delta_A - (1 - \delta_B) > 0$. Note that when $\mu_A = 1$ the equilibrium is pareto efficient and is identical to the within period pareto efficient allocation achieved with $\eta = \frac{\delta_B}{\delta_A + \delta_B}$. 

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C. Solution to Representative Agent Problem

C-1. Representative Agent Without Commitment

Start by solving the problem faced by the representative agent at \( t = T \). He solves

\[
\max_{C_{A,T}, C_{B,T}, H_T} \mu_{A,r} \ln C_{A,T} + \mu_{B,r} \ln C_{B,T} + (1 - \mu_{A,r} - \mu_{B,r}) \ln H_T
\]

subject to
\[
p_A C_{A,T} + p_B C_{B,T} + H_T \leq W_T
\]
\[
C_{A,T}, C_{B,T}, H_T \geq 0
\]

Log utility in each class of consumption ensures that the optimal solution will have the budget constraint bind with equality that the non-negativity constraints will be slack. Hence the problem can be reduced to

\[
\max_{C_{A,T}, C_{B,T}} \mu_{A,r} \ln C_{A,T} + \mu_{B,r} \ln C_{B,T} + (1 - \mu_{A,r} - \mu_{B,r}) \ln (W_T - p_A C_{A,T} + p_B C_{B,T})
\]

The first order conditions give

\[
C_{A,T} : \quad \frac{\mu_{A,r}}{C_{A,T}} - p_A \frac{1 - \mu_{A,r} - \mu_{B,r}}{W_T - p_A C_{A,T} + p_B C_{B,T}} = 0
\]
\[
C_{B,T} : \quad \frac{\mu_{B,r}}{C_{B,T}} - p_B \frac{1 - \mu_{A,r} - \mu_{B,r}}{W_T - p_A C_{A,T} + p_B C_{B,T}} = 0
\]

which gives

\[
C_{A,T}^{*} = \mu_{A,r} W_T
\]
\[
C_{B,T}^{*} = \mu_{B,r} W_T
\]
\[
H_T^{*} = (1 - \mu_{A,r} - \mu_{B,r}) W_T
\]

where "^*" indicates that this is the equilibrium quantity to the representative agent problem.
**Conjecture 2:** The subgame perfect equilibrium household allocation of the representative agent from $t$ until $T$ is proportional to $W_t$. That is, the subgame perfect equilibrium levels of private and common consumption can be written as $C_{i,t+x}^{r,*} = g_{i,t+x} W_t$ and $H_{t+x}^{r,*} = h_{i,t+x} W_t$ for $x \in \{0, 1, ..., T - t\}$ where and are strictly positive constants independent of $W_t$.

I will establish this conjecture by induction. As the first step, note that Conjecture 2 is verified for $t = T$ above. Next, consider the problem that the representative agent faces in period $t < T$:

$$\max_{C_{A,t}, C_{B,t}, H_t} \mu_{A,r} \ln C_{A,t} + \mu_{B,r} \ln C_{B,t} + \left(1 - \mu_{A,r} - \mu_{B,r}\right) \ln H_t$$

$$+ \Omega_r \sum_{x=1}^{T-t} \beta^x_r \left[ \mu_{A,r} \ln C_{A,t+x}^{r,*} + \mu_{B,r} \ln C_{B,t+x}^{r,*} + \left(1 - \mu_{A,r} - \mu_{B,r}\right) \ln \left(H_{t+x}^{r,*}\right) \right]$$

subject to

$$W_{t+1} = R \left(W_t - p_A C_{A,t} - p_B C_{B,t} - H_t\right)$$

$$C_{A,t}, C_{B,t}, H_t \geq 0.$$
The first order conditions are

\[
C_{A,t} : \frac{\mu_{A,r}}{C_{A,t}} - p_A W_t - p_A C_{A,t} - p_B C_{B,t} - H_t = 0
\]

\[
C_{B,t} : \frac{\mu_{B,r}}{C_{B,t}} - p_B W_t - p_A C_{A,t} - p_B C_{B,t} - H_t = 0
\]

\[
H_t : \frac{1 - \mu_{A,r} - \mu_{B,r}}{H_t} - \frac{\Omega_T^{T-t} \beta^T_r}{\sum_{x=1}^{T-t} \beta^T_r} W_t - p_A C_{A,t} - p_B C_{B,t} - H_t = 0.
\]

Adding these three conditions gives that equilibrium total expenditure in each period is

\[
X^{r*}_t = \frac{1}{1 + \Omega_T^{T-t} \beta^T_r} W_t
\]

where \(X^{r*}_t \equiv p_A C^{r*}_{A,t} + p_B C^{r*}_{B,t} + H^{r*}_t\).

Equilibrium expenditure on each type of consumption is a constant fraction of total expenditure in each period

\[
p_A C^{r*}_{A,t} = \mu_{A,r} X^{r*}_t
\]

\[
p_B C^{r*}_{B,t} = \mu_{B,r} X^{r*}_t
\]

\[
H^{r*}_t = (1 - \mu_{A,r} - \mu_{B,r}) X^{r*}_t.
\]

This completes the proof of Conjecture 2 by induction. In equilibrium, household wealth evolves in the following way

\[
W_{t+1} = R \left( \frac{\Omega_T^{T-t} \beta^T_r}{1 + \Omega_T^{T-t} \beta^T_r} \right) W_t.
\]

In equilibrium, the growth of total expenditure is

\[
\frac{X^{r*}_{t+1}}{X^{r*}_t} = R \frac{\sum_{x=1}^{T-t} \beta^T_r}{1 + \sum_{x=1}^{T-(t+1)} \beta^T_r}.
\]

Since they are in fixed proportion, this is the same for all types of consumption.
Equivalence with Household Equilibrium without Commitment and Symmetric Discounting

The equilibrium consumption allocation achieved by the representative agent without commitment is identical to equilibrium of the household under the following parameter restrictions First, time preferences must be equal for both household members because without this the share of expenditure on each type of consumption changes over time. This requires $\beta = \beta_A = \beta_B$ and $\Psi = \Psi_A = \Psi_B$. Consumption growth is identical in each $t$ for all choices of $T$ if and only if the representative agent has the same exponential discount factor $\beta_r = \beta$ and has a hyperbolic discount factor of

$$
\Omega_r = \frac{\Psi}{1 + \max \{\mu_A, \mu_B\} (\delta_A + \delta_B - 1)}
$$

The same consumption shares are obtained if and only if

$$
\mu_{i,r} = \frac{\delta_i \mu_i}{1 + \max \{\mu_A, \mu_B\} (\delta_A + \delta_B - 1)}.
$$

Equivalence with Household with Commitment and Symmetric Discounting

A different configuration of parameters of the representative agent without commitment can capture the full optimal consumption path that the household would commit to if able. Assume that both household members have identical time preferences so that $\beta = \beta_A = \beta_B$ and $\Psi = \Psi_A = \Psi_B$. The representative agent without commitment will have the same consumption growth rate in all as the household if $\beta_r = \beta$ and $\Omega_r = 1$. The same consumption shares are obtained if and only if

$$
\mu_{A,r} = (1 - \theta) \mu_A \quad \text{and} \quad \mu_{B,r} = \theta \mu_B.
$$
C-2. Representative Agent with Commitment

I now solve for the consumption path that the representative agent would achieve with full commitment. In this case the representative agent’s problem is to solve

\[
\max_{\{C_{A,t}, C_{B,t}, H_t\}_{t=1}^{T-1}} \Pi = \mu_{A,r} \ln C_{A,1} + \mu_{B,r} \ln C_{B,1} + (1 - \mu_{A,r} - \mu_{B,r}) \ln H_1 \\
+ \Omega_r \sum_{x=1}^{T-1} \beta^x \left[ \mu_{A,r} \ln C_{A,t+x} + \mu_{B,r} \ln C_{B,t+x} + (1 - \mu_{A,r} - \mu_{B,r}) \ln (H_{t+x}) \right] \\
\text{subject to } W_1 - \sum_{x=0}^{T-1} R^{-x} [p_A C_{A,1+x} + p_B C_{B,1+x} + H_{1+x}] \geq 0 \\
\{C_{A,t}, C_{B,t}, H_t\}_{t=1}^{T} \geq 0
\]

As before the non-negativity constraints on $C_{A,t}, C_{B,t}, H_t$ can be ignored and verified later. Letting $\Gamma_r$ be the Lagrange multiplier on the representative agents budget constraint the Lagrangian for this problem can be written as

\[
\max_{\{C_{A,t}, C_{B,t}, H_t\}_{t=1}^{T-1}} \Pi = \mu_{A,r} \ln C_{A,1} + \mu_{B,r} \ln C_{B,1} + (1 - \mu_{A,r} - \mu_{B,r}) \ln H_1 \\
+ \Omega_r \sum_{x=1}^{T-1} \beta^x \left[ \mu_{A,r} \ln C_{A,t+x} + \mu_{B,r} \ln C_{B,t+x} + (1 - \mu_{A,r} - \mu_{B,r}) \ln (H_{t+x}) \right] \\
\Gamma_r \left[ W_1 - \sum_{x=0}^{T-1} R^{-x} [p_A C_{A,1+x} + p_B C_{B,1+x} + H_{1+x}] \right]
\]
The first order conditions are

\[
C_{A,1} : \frac{\mu_{A,r}}{p_A C_{A,1}} - \Gamma_r = 0
\]
\[
C_{B,1} : \frac{\mu_{B,r}}{p_B C_{B,1}} - \Gamma_r = 0
\]
\[
H_1 : \frac{1 - \mu_{A,r} - \mu_{B,r}}{H_1} - \Gamma_r = 0
\]
\[
C_{A,1+x} : \frac{\Omega_{r}^{\beta_{r}} \mu_{A,r}}{p_A C_{A,1+x}} - R^{-x} \Gamma_r = 0
\]
\[
C_{B,1+x} : \frac{\Omega_{r}^{\beta_{r}} \mu_{B,r}}{p_B C_{B,1+x}} - R^{-x} \Gamma_r = 0
\]
\[
H_{1+x} : \frac{\Omega_{r}^{\beta_{r}} (1 - \mu_{A,r} - \mu_{B,r})}{H_{1+x}} - R^{-x} \Gamma_r = 0
\]

for \( x \in \{1, 2, \ldots, T - 1\} \). The representative agent with full commitment will thus choose expenditure shares in each period of

\[
\frac{C_{r,**}^{A,t}}{X_{r,**}^{t}} = \mu_{A,r}, \quad \frac{C_{r,**}^{B,t}}{X_{r,**}^{t}} = \mu_{B,r}, \quad \frac{H_{r,**}^{t}}{X_{r,**}^{t}} = 1 - \mu_{A,r} - \mu_{B,r}.
\]

Since these are fixed in proportion, the dynamic path of consumption will be the same for total expenditure and each item of consumption. Total expenditure in each period is

\[
X_{1,**}^{r} = \frac{1}{\Gamma_r}
\]
\[
X_{1+x,**}^{r} = \frac{\Omega_{r}^{\beta_{r}}}{R^{-x} \Gamma_r}.
\]
Any optimal allocation will fully exhaust household wealth
\[
W_1 = X_{1}^{\ast*} + \sum_{x=1}^{T-1} \frac{X_{1+x}^{\ast*}}{R^x}
\]
\[
= \frac{1}{\Gamma_r} \left[ 1 + \Omega_r \sum_{x=1}^{T-1} \beta_r^x \right]
\]

As a result
\[
\frac{1}{\Gamma_r} = \frac{W_1}{1 + \Omega_r \sum_{x=1}^{T-1} \beta_r^x}
\]

and
\[
X_{1}^{\ast*} = \frac{W_1}{1 + \Omega_r \sum_{x=1}^{T-1} \beta_r^x}
\]
\[
X_{1+x}^{\ast*} = \frac{\Omega_r \beta_r^x}{R^{-x}} \left( \frac{W_1}{1 + \Omega_r \sum_{x=1}^{T-1} \beta_r^x} \right).
\]

Thus consumption will evolve over time under the representative agent’s plan with full commitment as
\[
\frac{X_{2}^{\ast*}}{X_{1}^{\ast*}} = \Omega_r \beta_r R
\]
\[
\frac{X_{t+1}^{\ast*}}{X_{t}^{\ast*}} = \beta_r R
\]

for \( t \geq 2 \).
Figure 1
Equilibrium and Full Commitment Consumption Path

These plots show the equilibrium level of total household expenditure in every period without commitment $X_t$, and the optimal full commitment consumption path $X^t_\ast$. It is drawn using the following parameters: Initial household wealth is $W_1=3,000,000$, their exponential discount factor is $\beta=0.95$, the gross interest rate is $R=1/0.95$ and the household exists for $T=50$ periods. Panel A is drawn using the relative weight on private consumption of $\mu=0.5$ and compares the scenario where household members place weight on their own utility of $\delta=0.6$ and $\delta=0.75$. Panel B is drawn using the weight on own utility of $\delta=0.6$ and compares the scenario where household members weigh private consumption $\mu=0.25$ and $\mu=0.75$.

Panel A: Consumption Paths with Different Weights $\delta$ on Own Utility

Panel B: Consumption Paths with Different Weights $\mu$ on Private Consumption
Comparative Statics: The Value of Commitment in the Symmetric Household

These plots show the amount the household would be willing to pay at $t=1$ (as a fraction of $W^C$) to achieve the full commitment consumption path. The value of commitment is shown as a fraction of $W^C$ and (due to log additive utility functions) is invariant to the choice of $W^C$. Each Panel shows how the value of commitment varies with: $\delta$ the weight household members place on their own utility (Panel A); $\mu$ the weight household members place on private consumption (Panel B); and, $\beta$ the discount factor of each household member (Panel C). Apart from the variable on the x-axis, each plot is drawn using the following parameters: the weight both household members place on their own utility is $\delta=0.6$, the weight both household members place on private consumption is $\mu=0.5$, their exponential discount factor is $\beta=0.95$, the gross interest rate is $R=1/0.95$ and the household exists for $T=50$ periods.

Panel A: The Value of Commitment and the Weight on Own Utility $\delta$

Panel B: The Value of Commitment and the Weight on Private Consumption $\mu$

Panel C: The Value of Commitment and Household Member Discount Factor $\beta$
Comparative Statics: The Value of Commitment in the Asymmetric Household

These plots show the amount the household would be willing to pay at $t=1$ (as a fraction of $W_t$) to achieve the full commitment consumption path. The value of commitment is shown as a fraction of $W_t$ and (due to log additive utility functions) is invariant to the choice of $W_t$. Panel A shows how the value of commitment varies with $\delta_A$ the weight that member $A$ places on his own utility. It is drawn for both $\delta_B=0.5$ and $\delta_B=0.75$ holding $\mu_A=\mu_B=0.5$ constant. Panel B shows how the value of commitment varies with $\delta_A$ the weight that member $A$ places on his own utility. It is drawn for different levels of disagreement about the relative value of private and common consumption: $\mu_A=\mu_B=0.5$, $\mu_A=0.75$, $\mu_B=0.25$, and $\mu_A=0.9$, $\mu_B=0.1$. It is drawn holding $\delta_B=0.75$ constant. Panel C shows how the value of commitment varies with $\mu_A$ the weight that member $A$ places on private consumption relative to common. It is drawn for $\mu_B=0.2$, $\mu_B=0.5$, and $\mu_B=0.8$ holding the weight both place on their own utility constant at $\delta_A=\delta_B=0.75$. All Panels are drawn with $\beta_A=\beta_B=0.95$, $\Psi_A=\Psi_B=1$, $R=1/0.95$, and $T=50$.

Panel A: The Value of Commitment and the Weight A Places on Own Utility $\delta_A$

Panel B: The Value of Commitment and Differences in the Weight on Private Consumption

Panel C: The Value of Commitment and the Weight A Places on Private Consumption $\mu_A$
Figure 4

Comparative Statics: The Value of Commitment with Hyperbolic Individuals

These plots show the amount the household would be willing to pay at 𝜔=1 (as a fraction of \(W_1\)) to achieve the full commitment consumption path. The value of commitment is shown as a fraction of \(W_1\) and (due to log additive utility functions) is invariant to the choice of \(W_1\). The Figure shows how the value of commitment varies with \(\delta = \delta_B = \delta_B\) (i.e., varying both symmetrically). It is drawn for \(\Psi_A = \Psi_B = 1\) and \(\Psi_A = \Psi_B = 0.85\) holding other parameters constant at \(\mu_A = \mu_B = 0.5, \beta_A = \beta_B = 0.95, R = 1/0.95,\) and \(T = 50.\)

The Value of Commitment for Different Levels of Individual Hyperbolic Discounting

![Graph showing the value of commitment for different levels of \(\delta\). The graph includes two lines: one for \(\Psi = 0.85\) and another for \(\Psi = 1\).](image)