TIME-CONSISTENT INDIVIDUALS, TIME-INCONSISTENT HOUSEHOLDS

ANDREW HERTZBERG*

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Abstract

I present a model of consumption and savings for multi-person households in which members are imperfectly altruistic and share wealth. I show that, despite having standard exponential time preferences, the household is time-inconsistent: members save too little and overspend on private consumption goods. Access to private illiquid durable goods can exacerbate overconsumption by providing a way for members to lock-up wealth from each other. The household remains time-inconsistent, even when it is possible for members to save separately, whenever intra-household relative-wealth shocks create the possibility that one member will choose to transfer wealth to the other in the future.

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*Columbia Business School. E-mail: ah2692@gsb.columbia.edu. I thank Patrick Bolton, Pierre-Andre Chiappori, Daniel Gottlieb, Ulrike Malmendier, Daniel Paravisini, Bryan Routledge, Tano Santos, Daniel Wolfenzon, and seminar participants at Anderson School of Management, Columbia Business School, Carlson School of Management, Dartmouth University, Fuqua School of Business, Rady School of Management, Stockholm School of Economics, as well as conference participants at the American Economic Association Meetings, Corporate Finance NBER Fall Session, Household Finance NBER Summer Institute, Western Finance Association Conference, and World Finance Conference. I thank Tara Philip for outstanding research assistance. All remaining errors are my own.

The standard theory of household financial decision making is based upon the behavior of a single maximizing agent. However, a considerable body of empirical work strongly rejects the unitary model as an adequate description of household decision making (see for example Lundberg et al. (1997); Browning and Chiappori (1998); Phipps and Burton (1998); and, Ashraf (2009)). The key lesson from this literature is that household members have distinct preferences and are self-interested, caring more about the utility from their own consumption than their partners. Taking up the challenge posed by these empirical findings has led to considerable theoretical work that reconsiders the static theory of household demand for goods and labor supply (see for example Chiappori (1988); Browning et al. (1994); Browning and Chiappori (1998); Chiappori et al. (2002)). Little attention has been directed to the question of how financial decision making will be affected by the presence of distinct and imperfectly aligned preferences within a multi-person household.[1] This paper provides a new framework for addressing this question. In particular I ask: are multi-person households time-consistent or are they inherently unable to carry out optimal consumption and savings plans?

I propose a model of the household comprising two members who are connected in three ways. First, wealth is shared. Second, members derive utility both from private consumption, as well as shared non-rival public consumption goods such as children and housing. Third, following the evidence cited above, household members are altruistic but remain self-interested. Specifically, member A cares more about the utility from his consumption than B cares about A's consumption and vice versa. To be stark, I assume that both members have the same exponential time preferences and therefore agree on the ex ante optimal path of total household consumption and savings. This ensures that any time-inconsistency is derived solely from the strategic interaction between household members.

I characterize the household's equilibrium consumption path as a subgame perfect Nash equilibrium in consumption choices. This is the equilibrium that obtains when household members are unable to commit ex ante to their future consumption choices.[2] I show that the household is time-inconsistent: members systematically over-consume relative to the agreed optimal ex ante consumption and savings plan. Time-inconsistency arises because both members wish to unilaterally deviate from the optimal plan and increase their own private consumption at the expense of shared future savings. The central insight is that in a multi-person household, shared savings is subject to a dynamic commons problem and is therefore under-provided in equilibrium.[3]

To measure the cost of the household's time-inconsistency problem I find the fraction of total

[1]Recent exceptions include Mazzocco 2005; Mazzocco 2007; Schaner 2015.

[2]Technically this requires that members are unable to enforce contracts between themselves written contingent on their consumption choices.

[3]Dynamic commons problems have been used to study national underinvestment (Lancaster 1973, Tornell and Velasco 1992), overexploitation of natural resources (Levhari Mirman 1980), and sovereign debt (Amador 2008).

wealth that the household would be willing to pay ex ante to achieve the full commitment consumption path. I show this is monotonically increasing in the degree of self-interest and the importance of private consumption. It is the interaction of these two factors that distorts the intertemporal trade-off between the private benefit of consumption and the shared benefit of saving that both members faces each period.

Next I show how the time-inconsistency of the household is impacted by access to private illiquid durable goods (for example, sports cars, jewelry, and personal consumer electronics). I show that household members have a private incentive to overconsume these goods relative to non-durables because they offer a form of saving which is not shared with the rest of the household and which, due to illiquidity, other members cannot reverse. This can produce an allocative time-inconsistency in which these goods account for a larger fraction of household expenditure than the household would optimally choose with commitment. I also show that access to these goods can further exacerbate the intertemporal inefficiency of consumption as both members strategically rush to lock-up shared liquid wealth for their own private access in the future using these illiquid assets. I show that the time-inconsistency these goods create can be so large that the household would be better off if it did not have access to these goods at all. This finding contrasts sharply with the literature that has stressed that access to illiquid assets can mitigate individual time-inconsistency by allowing an agent to constrain the actions of their future-selves[4]. Here, illiquidity is overvalued as a way to constrain the future actions of other members of the household.

Having shown that the household overconsumes when savings are shared in a joint account, I turn to studying household behavior when members are able to save individually (in separate accounts that the other cannot access). If household members consume and save in complete isolation, they will do so optimally. However, even when unable to directly access the savings of others, household members remain linked by their altruism. As a result, a member with relatively more wealth may decide to transfer savings to the other member. I show that if household members anticipate that one will make a voluntary transfer to the other in the future, they will act as if their savings are shared, and thus exhibit the same time-inconsistency. Initial transfers, to reduce any ex ante wealth imbalance between members, can reduce the need for future interdependence and thereby make savings more efficient. I show the presence of uninsurable intra-household relative wealth shocks, such as shocks to the value of each member's human capital, limit this function since members anticipate that with some probability the member who receives a favorable shock will transfer wealth to the other in the future. Anticipating this possibility creates the same time-inconsistency that occurs when wealth is shared, despite wealth being held individually. This demonstrates that the phenomenon studied in this paper can be applied to extended families (for example, the interaction between adult siblings) where wealth is held separately but where the

[4] See for example Laibson 1998, Laibson Repetto and Tobacman 1998, and Beshears et al 2011.

possibility of future transfers may arise. Finally, I ask whether household members will choose to save in separate individual accounts rather than sharing wealth. I show that the household will only adopt separate accounts when its exposure to relative wealth shocks is sufficiently small. The intuition is as follows: larger relative wealth shocks increase the risk pooling benefit of a shared account and reduce the impact that separate accounts have on the under-savings problem.

This paper makes several contributions to the literature that studies household savings behavior. The first is to show that the intra-household pattern of ownership and control of assets can impact consumption and savings decisions. This contrasts with the unitary model of the household in which only the combined household balance sheet matters, and hence is unable to rationalize the choice between either arrangement. Survey evidence also suggests that both separate and shared savings are popular in developed countries.[5] Evidence from developing countries indicates that household risk sharing is limited, and hence implies that some assets are effectively owned separately (Robinson (2012), Duflo and Udry (2004)). The results in this paper indicate that even with separate ownership, the possibility of transfers between altruistic household members, which includes extended family, will still produce time-inconsistency. Recent papers by Ashraf (2009) and Schaner (2015) show that household savings decisions differ depending on whether assets are shared or held separately by household members.

By showing that time-inconsistency arises naturally in a multi-person household, the framework rationalizes the use of commitment technologies that limit the ability of individual members to unilaterally deviate from jointly agreed consumption and savings plans. As a prime example, in the US, the Retirement Equity Act 1984 mandates that all retirement plans covered by the ERISA 1974 laws (this includes all defined benefit plans, IRA accounts, and all 401(k) plans) require joint approval by both spouses before funds can be withdrawn or loans can be taken against such savings. Aura (2005) shows that the introduction of this law increased household saving. In the context of developing economies, savings commitment technologies such as ROSCAS are motivated by the ability of one spouse to limit the ability of a partner to over-consume out of shared wealth (Anderson and Baland (2002); Collins et al. (2009)).

This paper contributes to a large theoretical literature that studies household decision making when members have misaligned preferences (see Lundberg and Pollak (2007); Browning et al. (2006) for comprehensive surveys). In these papers, static household decision making is often modeled as the outcome of an efficient bargaining process, and the focus is directed to studying what determines the threat points and bargaining weights of each household member.[6] Evidence on

[5]The 2002 General Social Survey (Smith et al. (2011)) finds that 53 per cent of all married households in the US share all financial wealth. Similar survey evidence for the US and the UK is presented by Treas (1993) and Vogler et al. (2006). A 2006 survey of Japanese wives found that fifty percent held secret savings (referred to in Japan as "hesokuri") (see Alexy (2007)).

[6]As an exception non-cooperative decision making within the household has been considered by Lundberg and

the question of whether households are able to enforce Pareto efficient allocations is mixed. Donni and Chiappori (2011) point out that tests for static efficiency (see, for example, Bobonis (2009), Browning and Chiappori (1998), and Chiappori et al. (2002)) find in the affirmative, whereas tests for dynamic efficiency find the opposite (see for example De Mel et al. (2009), Duflo and Udry (2004), Mazzocco (2007), Robinson (2012), Udry (1996)). Motivated by this evidence, I study the equilibrium that obtains when commitment is not possible since the focus of this paper is intertemporal household decision making.[7]

Finally, this paper is also related to the literature that studies individuals with dynamically inconsistent time preferences.[8] The goal of this paper is to stress that self-interest within multi-person households can also produce time-inconsistency, and rationalizes commitment technologies and behavior that cannot be explained by the literature focusing on individual time preferences or self control (see, for example, Thaler and Benartzi (2004); Ashraf et al. (2006); Beshears et al. (2011)).

The paper proceeds as follows. Section I sets up the base model of household consumption. Section II characterizes the equilibrium consumption choices of the household and shows they are time-inconsistent. Section III studies the impact of access to private illiquid durable goods. Section IV studies household decision making when members are able to save in separate accounts. Section V discusses empirical implications of the model including strategies that the household might adopt to mitigate time-inconsistency. All derivations are provided in the Internet Appendix.

I Model of Household Consumption

The household has two members, indexed by i, labeled A and B. Time is discrete and indexed by t. The household is formed at the beginning of period $t = 0$. Both household members live for Y years. I assume that the household remains together for their entire lives with certainty and abstract from endogenous household formation. Each year contains $N \geq 1$ periods.[9] Consumption occurs from $t = 1$ until $T = NY$. The initial period of the household's life ($t = 0$) is used to assess the optimal consumption plan the household would like to achieve over its life and the utility cost of being unable to precommit to this plan.

Pollack 1993; Chen Woolley 2001; and, Lundberg and Pollack 2003.

[7]Lundberg and Pollack (2003) argue that pareto efficient bargaining may breakdown in a dynamic context because current decisions may affect future bargaining power.

[8]For theory see Thaler Shefrin 1981; Laibson 1997; Harris and Laibson 2001; Laibson, Repetto, and Tobacman 2003 and for evidence see Ainslie 1992; Frederick Lowenstein and O'Donoghue 2002; Shapiro 2005.

[9]For the bulk of the analysis $N = 1$. In the Internet Appendix I consider the limiting case as consumption decisions are made in continuous time by letting $N \to \infty$.

A Preferences

Each period $t \geq 1$ member i derives utility from two goods. The first is a private consumption good denoted $C_{i,t}$. The second is a non-rival public consumption good that both members share, denoted H_t. This captures one defining characteristic of being in a household: members share public consumption such as housing, children, and appliances. The utility derived by member i in period t is[10]

$$u_{i,t} = \mu \ln C_{i,t} + (1-\mu) \ln H_t \tag{1}$$

where $\mu \in [0,1]$ is the relative weight that members place on private consumption. Note that member i does not directly derive utility from member j's private consumption. The total level of household public consumption is the sum of the amount purchased by both members in each period: $H_t = H_{A,t} + H_{B,t}$.

Both household members discount utility from future consumption using exponential discount factor $\delta = d^{\frac{1}{N}}$ where $d \in (0,1)$ is the annualized discount factor. The individual discounted utility of household member i in period t is

$$U_{i,t} = \sum_{x=0}^{T-t} \delta^x u_{i,t+x}. \tag{2}$$

Thus $U_{i,t}$ is the discounted utility of household member i absent any concern for the other household member. Note that these are standard time preferences so that, if they were to act in isolation, the optimal consumption plan for each household member would be time-consistent.

The second defining characteristic of the household is that its members are altruistic. I capture this by supposing that the objective of each member places weight $\frac{1+\Delta}{2}$ on their own utility and weight $\frac{1-\Delta}{2}$ on the utility of the other member. I focus on the case where the altruism between household members is imperfect in the sense that each member cares more about their own utility than their partners: $\Delta \in [0,1]$. In words, Δ measures the degree of self-interest within the household.[11] The objective of member i at t is

$$V_{i,t} = \frac{1+\Delta}{2} U_{i,t} + \frac{1-\Delta}{2} U_{j,t}. \tag{3}$$

[10] Since consumption does not occur $t = 0$, I normalize $u_{i,0} = 0$.

[11] The framework can also be used to study the case where members care more about each other than themselves ($\Delta < 0$). Since the evidence on household consumption decisions suggests that this is generally not the case, I will not focus on this scenario.

B Household Budget Constraint

The present value of all combined household wealth at the beginning of $t = 0$ is W_0. For simplicity I assume the household starts life with wealth W_0, which is taken as given, and has no income.[12] The third defining characteristic of the household is that all wealth is shared so that both household members have full access to the remaining combined wealth in each period. This assumption is made to align the framework with the way the budget constraint is treated in any standard unitary model of intertemporal decision making. Moreover the 2002 General Social Survey (Smith et al. (2011)) finds that 53 per cent of all married households in the US share all financial wealth suggesting that this is the most empirically relevant characterization of the household budget constraint. I study household behavior when household members are able to save in separate assets in Section IV. For simplicity I normalize the relative price of the two consumption goods to unity. Any wealth not consumed by the household is saved between periods at a gross risk-free interest rate of $R = \overline{R}^{\frac{1}{N}}$ where $\overline{R} \geq 1$ is the gross effective annual yield on savings. Household wealth evolves according to the following

$$W_{t+1} = R\left(W_t - X_t\right) \tag{4}$$

where

$$X_t = C_{A,t} + C_{B,t} + H_{A,t} + H_{B,t} \tag{5}$$

is total household expenditure in period t.

C Decision Making

Household members cannot commit to a path of consumption. As a result, household members are unable to enforce mutually agreed levels of consumption, either in the present or the future. Household members non-cooperatively and simultaneously decide how much of the household wealth W_t to spend on their own private consumption $C_{i,t} \geq 0$ and on their contribution to public consumption $H_{i,t} \geq 0$ in each period. The dynamic equilibrium path of consumption will be the Nash subgame perfect solution to the consumption game between these two members. Let a single "*" denote the non-cooperative equilibrium quantities $C^*_{i,t}$ and $H^*_{i,t}$.

Since both members make consumption decisions simultaneously it is possible that both members could attempt to spend more than total household wealth. To avoid this problem I assume that

[12]This is identical to assuming that household labor supply decisions are fixed and the household is free to borrow and lend at R each period. In this case W_t is the present value of all future income plus (minus) any savings (debt) that the household has at period t.

both members are able to consume, at most, half the total household wealth in any single period[13]:

$$C_{i,t} + H_{i,t} \leq \frac{W_t}{2}. \tag{6}$$

This condition can be made arbitrarily weak by making N large. For example, (6) implies that within a year one member can withdraw up to $W_t\left(1-\frac{1}{2^N}\right)$. As $N \to \infty$ this implies that all wealth can be withdrawn in any finite period of time. By imposing (6) I ensure $C_{A,t} + C_{B,t} \leq W_t$, and hence have a well defined budget constraint for each household member's consumption problem each period. I show in the Internet Appendix that (6) does not bind in any period $t < T$.[14]

D Full Commitment Problem and the Value of Commitment

To evaluate the optimality of the non-cooperative equilibrium consumption path, I compare it to the consumption path that would be achieved if the household was able to fully commit to consumption choices at $t = 0$. Consider the problem the household would face in setting a full commitment path. Whenever $\Delta > 0$ household members disagree over the optimal allocation. However, any allocation that they would choose must be Pareto optimal, and hence I characterize the solution to the following full commitment Pareto problem:

$$\max_{\left\{\left\{C_{i,t}\right\}_{t=1}^{t=T}\right\}_{i\in\{A,B\}}} \quad \Pi = \eta V_{A,0} + (1-\eta) V_{B,0} \tag{7}$$

$$\text{subject to } W_0 - \sum_{x=1}^{T} R^{-x}\left[C_{A,x} + C_{B,x} + H_x\right] \geq 0 \text{ and} \tag{8}$$

$$\left\{C_{A,t}, C_{B,t}, H_x\right\}_{t=1}^{t=T} \geq 0. \tag{9}$$

where $\eta \in [0,1]$ is the Pareto weight placed on the objective of member A. Let a double "**" denote the full commitment Pareto optimal consumption quantities $C_{i,t}^{**}$ and H_t^{**} that solve this problem.

To quantify the welfare loss incurred by the household's time-inconsistency, I calculate how much the household would be willing to pay at $t = 0$ for a technology that allowed them to commit to an optimal consumption path. Let $V_{i,0}^{*}(W_0)$ be the discounted lifetime utility that will be achieved by household member i absent commitment as a function of initial household wealth. Let

[13] If the model were extended to allow household income then this constraint would impose the condition that credit markets will not allow the household to raise debt in excess of the present value of all future income.

[14] In the Internet Appendix, I solve an otherwise identical model in which household members make consecutive consumption decisions. In that setting (6) is replaced with the standard budget constraint $C_{i,t} + H_{i,t} \leq W_t$. I show that the equilibrium studied here is arbitrarily close to the unique equilibrium from the consecutive move model as $N \to \infty$ thus demonstrating that (6) does not drive the results studied below.

$V_{i,1}^{**}(W_0(1-\phi),\eta)$ be the counterpart for the case where the household has spent a fraction ϕ of their initial wealth W_0 to achieve the full commitment plan that places weight η on the preferences of member A. The value of commitment ϕ^{**} is defined as the most that the household will pay while ensuring that there exists a weight η so that the purchase is a Pareto improvement for both members. Formally ϕ^{**} solves:

$$\phi^{**} = \max_{\phi,\eta} \phi \tag{10}$$

$$\text{subject to } V_{i,0}^{**}(W_0(1-\phi),\eta) \geq V_{i,0}^{*}(W_0) \text{ for } i \in \{A,B\}, \text{ and } \eta \in [0,1]. \tag{11}$$

An analytical solution for ϕ^{**} is intractable in most cases so this will be solved for numerically.

II Consumption Choices and time-inconsistency

A *Example: Two Periods*

To illustrate the source of time-inconsistency within the household, I sketch a solution to the two period version of the model ($T=2$). If the household was able to commit to an optimal consumption plan at $t=0$ then it would satisfy the standard Euler equation

$$\frac{1}{X_t^{**}} = R\delta \frac{1}{X_{t+1}^{**}}. \tag{12}$$

Under this plan total consumption would be allocated between the public and private goods each period according to their preference weights:

$$C_{i,t}^{**} = \frac{\mu}{2} X_t^{**} \text{ and } H_t^{**} = (1-\mu) X_t^{**} \tag{13}$$

where, without loss of generality, I have assumed a symmetric Pareto weight in the planning problem ($\eta = 0.5$). I show that this plan is time-inconsistent by illustrating that both members have an incentive to unilaterally deviate from this consumption allocation at $t=1$. To do this, observe that with log utility the value function that each member will face for the second period can be summarized as

$$V_{i,2} = \ln X_2 + \text{Constant} \tag{14}$$

where the second term is a constant invariant to X_2 and hence irrelevant for the problem at $t=1$. Using (14), consider a deviation by member i from the optimal consumption path at $t=1$ whereby she increases her private consumption above $C_{i,1}^{**}$ by a small amount $\varepsilon_C > 0$, recognizing that this will lower combined household savings and hence consumption at $t=2$ by $R\varepsilon_C$. The marginal benefit from such a deviation is[15]

$$\frac{\partial}{\partial \varepsilon_C}\left\{\frac{1+\Delta}{2}\mu \ln\left(C_{i,1}^{**}+\varepsilon_C\right)+\delta \ln\left(X_2^{**}-R\varepsilon_C\right)\right\}\Big|_{\varepsilon_C=0} = \underbrace{\frac{1}{X_1^{**}}}_{\text{[MU at } t=1]} - \underbrace{\left(\frac{R}{1+\Delta}\right)}_{\text{[PRTSS]}} \underbrace{\delta \frac{1}{X_2^{**}}}_{\text{[MU at } t=2]} \geq 0 \tag{15}$$

where the first equality follows using (13). Comparing (15) to the Euler equation that governs the optimal allocation (12), yields that member i strictly prefers to make such a deviation from the optimal consumption plan whenever altruism is imperfect: $\Delta > 0$. The incentive to deviate stems from the difference between the social return to saving, R, and each member's private return to shared saving, $\frac{R}{1+\Delta}$ (the term denoted $PRTSS$). When considering the trade-off between consumption and saving, each member recognizes that every dollar saved will be shared and, since altruism is imperfect, they do not internalize the full combined household benefit of those shared savings. Thus, a trade-off between the benefit of *private* of consumption and the return to *shared* saving produces an incentive for each member to deviate from the ex ante optimal plan. As the degree of self-interest within the household rises, as measured by Δ, the wedge between the social and private return to saving increases.

To highlight the importance of private consumption in generating time-inconsistency, consider a second possible deviation from the optimal household allocation given by (12). Suppose that member i contemplates a deviation whereby she increases expenditure on public consumption above H_1^{**} by a small amount $\varepsilon_H > 0$, recognizing that this will lower combined household savings, and hence consumption at $t = 2$ by $R\varepsilon_H$. The marginal benefit from such a deviation is[16]

$$\frac{\partial}{\partial \varepsilon_H}\left\{(1-\mu)\ln\left(H_1^{**}+\varepsilon_H\right)+\delta \ln\left(X_2^{**}-R\varepsilon_H\right)\right\}\big|_{\varepsilon_H=0} = \frac{1}{X_1^{**}} - R\delta\frac{1}{X_2^{**}} = 0 \tag{16}$$

where, as before, the first equality follows using (13). In this case there is no intertemporal distortion. Each member trades off the *shared* benefit of consuming the public good today with the *shared* benefit of saving. Since the same concern for the combined household is present in both terms, there is no distortion to the relative intertemporal tradeoff.

B General Case

I now characterize household time-inconsistency by characterizing the household consumption path for any T. With full commitment, the optimal consumption path that the household would

[15] I have omitted all terms unaffected by the deviation from i's objective: $\frac{1-\Delta}{2}\mu \ln C_{j,1}^{**} + (1-\mu)\ln H_1^{**}$.
[16] I have omitted all terms unaffected by the deviation from i's objective: $\frac{1+\Delta}{2}\mu \ln C_{i,1}^{**} + \frac{1-\Delta}{2}\mu \ln C_{j,1}^{**}$.

choose at $t = 0$ would specify the following level of total expenditure each period

$$X_t^{**} = \frac{1}{1 + \sum_{x=1}^{T-t} \delta^x} W_t. \tag{17}$$

The allocation of X_t^{**} remains according to (13).[17]

Without commitment, the equilibrium level of private and public consumption by member i in period $t < T$ is[18]

$$C_{i,t}^{*} = \frac{\frac{1+\Delta}{2}\mu}{1 + \mu\Delta + \sum_{x=1}^{T-t} \delta^x} W_t \text{ and } H_t^{*} = \frac{1-\mu}{1 + \mu\Delta + \sum_{x=1}^{T-t} \delta^x} W_t. \tag{18}$$

Total equilibrium household expenditure in period t is

$$X_t^{*} = \frac{1}{1 + \frac{1}{1+\mu\Delta} \sum_{x=1}^{T-t} \delta^x} W_t. \tag{19}$$

Comparing (17) and (19) yields the following proposition.

Proposition 1. *: Whenever $\mu\Delta > 0$ the household is time-inconsistent. Each period the fraction of remaining wealth that the household consumes is strictly higher in the non-cooperative equilibrium than under the full commitment pareto optimum.*

Proposition 1 highlights that the household will be time-inconsistent whenever altruism is imperfect and members derive utility from private consumption. Whenever this is true, household savings is subject to a dynamic commons problem. As a result the household will exceed the optimal level of consumption early in life and, through the intertemporal budget constraint, consume below the optimal level later in life. For example, if $T = 50$, $\delta = 0.95$, $R = \frac{1}{0.95}$ and $\Delta = \mu = 0.5$ the household will spend more than 33% above the optimal level in the first year and consume less than 50% of the level that the household would like to commit to for each of the last five years of its life.

C Quantifying the Inefficiency

Having shown that the allocation of consumption achieved in the non-cooperative solution is inefficient, I now turn to quantifying this inefficiency. To do this I ask what fraction ϕ^{**} of the household's initial wealth would both household members agree to spend in order to achieve a

[17] For any pareto weight η, $C_{A,t}^{**} = \mu\eta X_t^{**}$ and $C_{B,t}^{**} = \mu(1-\eta) X_t^{**}$.

[18] By assumption the equilibrium consumption in period $t = NT$ is $C_{i,t}^{*} = \frac{W_t}{2}$. Since it doesn't matter who buys a given unit of the public consumption good, the individual choices of $H_{A,t}$ and $H_{B,t}$ are not uniquely determined in equilibrium.

Pareto efficient allocation. Figure 1 shows that the value of commitment increases monotonically with the degree of self-interest within the family (Panel A) and the weight that members place on private consumption (Panel B).[19] A household in which $\Delta = \mu = 0.5$, incurs a utility cost of time-inconsistency that is equivalent to giving up 4.4% of household wealth at $t = 0$. The direct conclusion is that time-inconsistency is more pronounced in households with members who have more misaligned objectives: either due to altruism or shared concern for public consumption. The comparative statics also suggest that when households experience a change in these attributes, the tendency to over-consume will be affected accordingly.

III Durable Goods

So far I have shown that a multi-person household will display time-inconsistency: saving less in equilibrium than household members would optimally choose to if commitment at the start of their life was possible. I now show that, in the context of a multi-person household, access to illiquid durable goods may further exacerbate consumption and savings inefficiencies. I augment the baseline model by supposing members can also purchase an illiquid durable good and highlight two additional sources of potential inefficiency this can give rise to: 1) creating allocative inefficiency within any period whereby the household spends too much on illiquid durable goods relative to non-durables, and 2) further exacerbating the intertemporal inefficiency of consumption.

A Setup with Durable Goods

I adapt the benchmark model introduced in Section I by assuming that household members can derive utility from the services of durable and non-durable goods in any period. To capture the utility from durable consumption, extend the period utility function in (1) to

$$u_{i,t} = \ln\left(C_{i,t} + D_{i,t}\right) \tag{20}$$

where $D_{i,t}$ denotes member i's stock of durable goods in period t. $D_{i,t}$, is a private good consumed only by member i. Having established that time-inconsistency stems only from private consumption, I focus on the case where there are no public goods in the household ($\mu = 1$). A fraction $\kappa \in [0,1]$ of $D_{i,t}$ is lost to depreciation each period. Let p denote the price of the durable good relative to the non-durable consumption good which remains the numeraire. The intertemporal budget constraint of the household remains as in (4) where the definition of total expenditure is

[19]Unreported numerical simulations show that the value of commitment is non-monotonic in δ peaking when $\delta = \frac{1}{R}$.

now expanded to

$$X_t = C_{A,t} + C_{B,t} + p\left[D_t - (1-\kappa)D_{t-1}\right] \tag{21}$$

where $D_t \equiv D_{A,t} + D_{B,t}$ is the household's total consumption of durable consumption at t. The household starts life with no durable goods: $D_{A,0} = D_{B,0} = 0$. Define Λ_t as

$$\Lambda_t \equiv W_t + (1-\kappa)D_{t-1} \tag{22}$$

the total assets of the household at the start of period t held in the form of either shared savings or private depreciating durable goods purchased in the past. To capture the illiquidity of the durable good the expenditure limit of each member in any period (formerly given by (6)) is amended to[20]

$$C_{i,t} + pD_{i,t} \leq \frac{W_t}{2} + p(1-\kappa)D_{i,t-1}. \tag{23}$$

In words, new expenditure on durable or non-durable goods can only come from the shared liquid wealth of the household W_t. Once member i has committed household resources to durable consumption, neither member is able to reverse that decision by selling or borrowing against the future remaining stock of that good

$$D_{i,t} \geq (1-\kappa)D_{i,t-1}. \tag{24}$$

The rest of the framework remains the same as the base line model presented in Section I. I study two special cases of the model that highlight additional sources of inefficiency that can arise as a result of access to illiquid durable goods.

B Durable Goods and Allocative Inefficiency $(p \geq 1)$

I start by considering the case where the illiquid durable good is more expensive than the non-durable: $p \geq 1$. To keep the solution tractable I concentrate on the case where $T = 2$ which is sufficient to understand the allocative inefficiency created by an illiquid non-durable good. The additional intertemporal inefficiency created by the presence of an illiquid non-durable good is studied later, so this simplification comes at less cost.[21]

In the final period $t = 2$, the durable and non-durable good are perfect substitutes, and hence both members will each spend their half of W_2 on the non-durable good since $p \geq 1$. This is true also in the optimal full commitment allocation. Thus the question of efficiency depends on

[20] If instead the durable good were fully liquid then this constraint would be $C_{i,t} + pD_{i,t} \leq \frac{1}{2}\left[\Lambda_t + p(1-\kappa)R^{-1}D_t\right]$. In words, each member could consume up to half of the value of all of the households assets in period t. If the durable good were liquid, then the equilibrium is identical to the one studied with only non-durable goods, where wealth is adjusted for the effective rental price of the durable good: $p\left[1-(1-\kappa)R^{-1}\right]$.

[21] The distortions that drive the allocative inefficiency highlighted here where $T = 2$ carry over, and are stronger, with longer horizons.

how much, and which type of good, the household consumes at $t = 1$. In the optimal allocation the household will consume a non-zero quantity of the durable good ($D_1^{**} > 0$) if and only if the following condition is met:

$$p \leq \overline{p}^{**1} \equiv 1 + \frac{1-\kappa}{R}. \tag{25}$$

In words, (25) requires that one unit of the durable good costs no more than the present value of the same stream of consumption delivered by the non-durable good[22].

For the household without commitment the trade-off between each type of consumption good is different. To provide intuition for this difference suppose that $p > \overline{p}^{**1}$ and the household begins at a symmetric Pareto optimal allocation with only non-durable consumption at $t = 1$. Now consider the private incentive for member i to deviate from this allocation by substituting $\varepsilon > 0$ units of the non-durable good with durable consumption at $t = 1$. Doing so would, by construction, leave consumption and the utility of both members at $t = 1$ unchanged. The deviation will alter the consumption of both members at $t = 2$ in the following way. Member i will enjoy the services of the remaining durable good: $\varepsilon(1-\kappa)$. Both members will have $\varepsilon R(p-1)$ less shared liquid wealth at $t = 2$ to spend on consumption. Crucially, this cost is borne by both members. From i's perspective, for any concave period utility function u, making such a deviation is privately optimal if

$$\frac{\partial}{\partial \varepsilon}\left[\frac{1+\Delta}{2}u\left(\varepsilon(1-\kappa)+C_{2,i}^{**}-\varepsilon\frac{R}{2}(p-1)\right)+\frac{1-\Delta}{2}u\left(C_{2,j}^{**}-\varepsilon\frac{R}{2}(p-1)\right)\right]\Bigg|_{\varepsilon=0} \geq 0. \tag{26}$$

Simplifying (26) yields that i will make such a deviation as long as the price of the durable good satisfies the following condition: [23]

$$p \leq 1 + \left(\frac{1-\kappa}{\frac{R}{1+\Delta}}\right). \tag{27}$$

Comparing (25) to (27) shows that household members are prepared to pay more for the durable good than the present value of the services that it renders whenever altruism is imperfect: $\Delta > 0$. The logic is as follows. When a household member buys the illiquid durable good she receives the full benefit of the services of the durable good at $t = 2$. The illiquidity ensures that this wealth is locked in for her own exclusive use. Conversely, if the member buys the non-durable good and saves the remaining $p-1$ for consumption at $t = 2$ then these additional savings are shared equally with the other household member. Thus the effective *private* return on saving a dollar of liquid

[22] If (25) holds the household may also optimally chose to consume the non-durable good at $t = 1$ or $t = 2$ depending on p and whether $R\delta$ is greater or less than one. Exact conditions are given the Internet Appendix.

[23] The simplification makes use of the assumption that the full commitment allocation is symmetric so that $u'\left(C_{2,i}^{**}\right) = u'\left(C_{2,j}^{**}\right)$. Notice that the argument applies for any concave and differentiable period utility function.

wealth is $\frac{R}{1+\Delta}$ which is lower than the household's *combined* return of R. This leads each member to privately undervalue liquid savings relative to durable goods, which creates the incentive for an allocative inefficiency whereby the household overconsumes illiquid durable goods relative to non-durables.

For the remainder of the analysis I focus on the case where $\Delta = 1$ for simplicity[24]. Equilibrium decisions, characterized fully in the Internet Appendix, are such that household members will consume the durable good at $t = 1$ ($D_1^* > 0$) if and only if:

$$p \leq \overline{p}^{*1} \equiv 1 + \left(\frac{1-\kappa}{\frac{R}{2}} \right). \tag{28}$$

Observe that $\overline{p}^{**1} < \overline{p}^{*1}$. When $p \in \left[\overline{p}^{**1}, \overline{p}^{*1}\right]$ the household members choose to consume the durable good even though the same consumption services could be obtained by consuming the non-durable good at strictly lower cost. Formally:

Proposition 2. *: In equilibrium the household will direct a higher fraction of expenditure at $t = 1$ towards the illiquid durable good ($\frac{pD_1^*}{X_1^*} > \frac{pD_1^{**}}{X_1^{**}}$) for intermediate values of p. In the case where $T = 2$ this occurs if and only if $p \in \left[\overline{p}^{**1}, \overline{p}^{*1}\right]$. When the price of the durable good is in this range, the degree of household over-expenditure at $t = 1$ is worse than when no illiquid durable good is available ($\frac{X_1^*}{X_1^{**}} > \frac{1+\delta}{1+\frac{\delta}{2}}$).*

Proposition 2[25] highlights that in addition to allocative inefficiency, the increased incentive to purchase durable goods, in order to lock in shared household wealth for private consumption, can also lead the household to consume earlier in life in a strategic race to capture a larger share of the combined resources. I return to this inefficiency later in this section. When the durable good is prohibitively expensive, $p \geq \overline{p}^{*1}$, the pattern of consumption and inefficiency is identical to the original setup where the durable good is not available and there is trivially no impact of having it available.

Alternately, access to an illiquid durable good can also improve efficiency.

Proposition 3. *: In equilibrium household members will achieve the full commitment allocation if the illiquid durable good is sufficiently durable and inexpensive. In the case where $T = 2$ this*

[24] The qualitative results will extend to the case where altruism takes on intermediate values.

[25] *Note that if the illiquid durable good were not available, the degree of overspending at $t = 1$, captured by the ratio of equilibrium to full commitment expenditure would be $\frac{X_1^*}{X_1^{**}} = \frac{1+\delta}{1+\frac{\delta}{2}}$.*

occurs if and only if

$$(i)\ p \leq \overline{p}^{**1} \text{ and } \kappa \leq 1 - R\delta, \text{ or}$$
$$(ii)\ p \leq \overline{p}^{**3} \text{and } \kappa \in \left[1 - R\delta,\ 1 - \frac{R\delta}{2}\right].$$

where $\overline{p}^{**3} \equiv \left(\frac{1+\delta}{\delta}\right)(1-\kappa)R^{-1}$*noting that* $\overline{p}^{**3} < \overline{p}^{**1}$ *when* $\kappa \in \left[1 - R\delta,\ 1 - \frac{R\delta}{2}\right]$.

In words, this requires that the price of the durable good be below the present value of the price of the same services from the non-durable good. Note also, $R\delta$ is usually close to unity in general equilibrium, and so Proposition 3 either requires the durable good to be extremely durable or priced well below the equivalent set of non-durable services. Under these conditions full intertemporal efficiency stems from the fact that illiquidity prevents the household borrowing against the future depreciated stock of the durable good. Thus, if it is optimal to devote all wealth to durable consumption at $t = 1$, household members cannot consume even more than this, promising to sell the good at $t = 2$ to fund the deficit.

The relationship between the efficiency of consumption and the price of the durable good p, is shown in Figure 2.[26] Panel A and B show how the allocative and intertemporal efficiency of consumption vary with the price of the illiquid durable good. Both are improved when the price is low and exacerbated with the price is high (i.e. below or above $\overline{p}^{**1}$). Panel B shows that when $p \in \left[\overline{p}^{**1}, \overline{p}^{*1}\right]$ the ovexpenditure at $t = 1$ is larger than when the durable good is not available. Panel C shows how access to the durable good impacts the degree of time-inconsistency, as measured by the value of commitment. It is reduced when p is low and exacerbated for higher values. Panel D shows that access to the durable good improves welfare when it is inexpensive. However, at intermediate prices, the reverse is true and household welfare would be strictly higher if access to durable goods were not possible. This highlights a stark consequence of time-inconsistency in the multi-person household: expanding the set of available goods can lower household welfare.

C Durable Goods and Intertemporal Inefficiency $(p = 1)$

I now consider a different special case of the set-up with durable goods to isolate the intertemporal inefficiencies that access to illiquid durable goods can produce. To do this, I focus on the case where $p = 1$ and $T \to \infty$.[27] This removes the possibility of allocative inefficiency, since the durable good dominates the non-durable and, by studying the case where the household is long-

[26] The Figure is drawn for the case where $\kappa \in \left[1 - R\delta,\ 1 - \frac{R\delta}{2}\right]$.

[27] In the Internet Appendix I solve the model for all possible $T \geq 2$. The results are qualitatively the same as the infinite horizon case considered here. Infinite horizon affords much simpler results by ensuring that equilibrium choices are stationary.

lived, allows the full impact of any intertemporal inefficiency to be seen. I continue to simplify the analysis by focusing on the case where there is no altruism: $\Delta = 1$.

The Pareto optimal consumption plan that household would choose, if it had full commitment, is:

$$D_t^{**} = \left\{ \begin{array}{cc} W_1 (1-\kappa)^{t-1} & \text{if } \kappa \leq \overline{\kappa}^{**} \\ \frac{\Lambda_t}{\left[1-(1-\kappa)R^{-1}\right]\left(1+\frac{\delta}{1-\delta}\right)} & \text{if } \kappa > \overline{\kappa}^{**} \end{array} \right] \text{ where } \overline{\kappa}^{**} \equiv 1 - R\delta. \tag{29}$$

In words, if the durable good is sufficiently durable, $\kappa \leq \overline{\kappa}^{**}$, then, household members will optimally spend all their wealth on the durable good immediately at $t = 1: \ D_1^{**} = W_1$. After this point consumption is only derived from the depreciating stock of these goods. Note that if $R\delta$ is close to unity, as is often the case in general equilibrium, this requires the durable good to not depreciate ($\overline{\kappa}^{**} = 0$). Alternately, if the good is of low sufficiently durability, $\kappa > \overline{\kappa}^{**}$, then the optimal plan calls for household members to incrementally acquire additional amounts of the durable good over time.

Without commitment, the equilibrium time path of consumption of the durable good is[28]

$$D_t^{*} = \left\{ \begin{array}{cc} W_1 (1-\kappa)^{t-1} & \text{if } \kappa \leq \overline{\kappa}^{*} \\ \frac{\Lambda_t}{\left[1-(1-\kappa)R^{-1}\right]\left(1+\frac{1}{2}\frac{\delta}{1-\delta}\right)} & \text{if } \kappa > \overline{\kappa}^{*} \end{array} \right] \text{ where } \overline{\kappa}^{*} \equiv 1 - \frac{R\delta}{2}. \tag{30}$$

The first difference is that, if in equilibrium the household incrementally acquires the durable good over time (which occurs when the durable good depreciates quickly:$\kappa > \overline{\kappa}^{*}$), then, without commitment too much is consumed early. Since these dynamics are identical to the case where only a non-durable good is available, they offer no new insights. Instead, I focus on the other difference: the range of cases for which the household rushes to consume all wealth right away is strictly larger without commitment: $\overline{\kappa}^{**} < \overline{\kappa}^{*}$. Thus, there are two cases to consider depending on the magnitude of κ.

When the good is highly durable so that $\kappa \leq \overline{\kappa}^{**}$, the household spends all wealth on durable consumption at $t = 1$, and this is Pareto optimal. Crucially, this condition implies that the slope of the optimal consumption path, $R\delta$, is smaller than slope generated from consuming the depreciating goods: $1-\ \kappa$. In this case overconsumption is prevented by illiquidity: members cannot borrow against their future stock of depreciated durable goods. But, as observed above, if $R\delta \approx 1$, this scenario requires the durable good to last indefinitely, and is hence unlikely to be realistic.

[28]The stark nature of the equilibrium (all liquid wealth is spent immediately) is a feature of several simplifying assumptions and hence should be taken as indicative of the general phenomenon to rush to consume durable goods in richer setups.These assumptions include: the household can borrow against all lifetime income, the durable good and non-durable good are perfect substitutes, and the household is infinitely lived. I show in the appendix, if the household is finitely lived, the rush to lock up all wealth in illiquid durables can occur during the life of the household, with incremental consumption occurring prior to that.

With intermediate durability, $\kappa \in [\overline{\kappa}^{**}, \overline{\kappa}^{*}]$, the household spends all wealth on the durable good right away, even though the Pareto optimal solution calls for the household to save liquid wealth each period. This stems from the same force behind the allocative inefficiency above: the incentive to save is distorted by the fact that savings are shared whereas wealth committed to the illiquid durable good is used exclusively by the member who purchases it. Further, this force prompts a strategic race to consume. If household members anticipate that next period all remaining wealth will be spent on durable goods, then it is better to move earlier and claim a larger share of the combined resources today. Applying this logic iteratively produces large distortions to the intertemporal path of consumption.

Figure 3 plots the amount the household would be willing to pay to achieve the full commitment solution, comparing scenarios when the durable good is, and is not, available. The time-inconsistency problem is particularly severe when κ is less than, but close to $\overline{\kappa}^{*}$. The household is prepared to pay almost all its wealth for commitment to avoid the rush to spend all liquid wealth immediately. Only when the good is highly durable, and hence comes close to mimicking the desired lifetime path of consumption, does it lower the time-inconsistency problem, and hence reduce the value of commitment relative to the case when the illiquid durable good is not available. As argued above, since $R\delta$ is normally close to unity in general equilibrium, this requires the durable good be extremely slow to depreciate. Since the optimal consumption plan is unchanged for any $\kappa > \overline{\kappa}^{**}$ it follows that the household is made strictly worse off whenever the value of commitment with access to the durable good is higher than without. Figure 3 therefore further indicates that household time-inconsistency is such that access to illiquid private durable goods can make the household strictly worse off.

IV Separate Accounts and Intra-household Transfers

So far the paper has shown that imperfectly altruistic household members, who are linked by saving in a shared asset, will under-save. In this section I study household behavior when members are able to break their direct financial interdependence by saving in separate accounts that the other cannot access. For example, this would correspond to each member of a married couple holding assets such as bank accounts and retirement accounts solely in their own names. For studying extended family settings (for example, adult children interacting with each other and their parents and grand parents) it is more natural to assume that each member has their own privately held wealth that others cannot directly access. If there were no other connection between family members, having separate accounts would trivially eliminate the inefficiency highlighted thus far, by making each member fully autonomous. However, even with separate accounts, altruism among household members may lead one member to anticipate making or receiving transfers from another, thereby making the future consumption of one member depend on the wealth of the other. The purpose of this section is to show that this link is sufficient to generate under-saving. I also show that the endogenous initial transfer of wealth between household members will be set to minimize this possibility and demonstrate that this remedy is of limited use when members are uncertain about shocks to their individual relative wealth in the future. Finally, I show that households will endogenously choose to share all savings when the volatility of the relative wealth shocks they face is sufficiently large.

A Setup with Separate Accounts

To incorporate individual ownership of financial wealth into the baseline model, I introduce a measure of the wealth that each member has at the beginning of a period: $W_{i,t}$. Within each period the timing of events is as follows. First, household members simultaneously choose a non-negative amount $\Psi_{i,t} \in [0, W_{i,t}]$ to transfer to the other member. Transfers are chosen as non-cooperative Nash best responses. Once transfers have been made, each member takes as given their wealth net of transfers, $\widetilde{W}_{i,t} = W_{i,t} - \Psi_{i,t} + \Psi_{j,t}$, and selects a level of private non-durable consumption $C_{i,t}$. I allow for the possibility that $C_{i,t} > \widetilde{W}_{i,t}$ so that members can borrow, at expected gross interest rate R, against transfers they will receive in the future. Implicit in this formulation is that member i must repay any debts carried from the previous period (captured by $W_{i,t} < 0$) before consuming out of the remaining wealth $\widetilde{W}_{i,t}$. Note that neither member is compelled to repay the debt of her partner, although she may effectively choose to do so through a voluntary transfer. No new borrowing is possible at $t = T$ and, in equilibrium, all loans will be repaid with certainty or else i would be forced to have zero consumption at $t = T$ which is infinitely costly. The intertemporal budget constraint of each member is analogous to that in the baseline model

$$W_{A,t} = R\left(\widetilde{W}_{A,t-1} - C_{A,t-1} + \varpi_t\right) \text{ and } W_{B,t} = R\left(\widetilde{W}_{B,t-1} - C_{B,t-1} - \varpi_t\right) \tag{31}$$

where each member's initial wealth endowment $W_{i,0}$ is given. This intertemporal budget constraint includes a stochastic uninsurable shock to the relative wealth of the household members: $\varpi_t \sim N(0, \sigma_t)$ with associated density functions of $F()$ and $f()$. This captures shocks to the market value of the human capital of each member, stochastic inheritances, or an unpredictable component to savings returns. The shock can also be thought of providing a reduced form representation of shocks to relative consumption needs that may arise, for example, from unexpected illness. Consistent with the rest of the paper, I abstract from shocks to the total wealth of the household, which will not meaningfully change the analysis, and focus entirely on shocks to the relative wealth of each member. The presence of a relative wealth shock captures one key benefit to household membership: risk sharing.[29] Shocks to initial wealth are subsumed into the analysis by studying any initial combination of $W_{A,0}$ and $W_{B,0}$ and so, without loss of generality, I set $\sigma_1 = 0$.

The objective function of each member is the same as in the baseline model as per (3). Since the objective of each member is the same and the resources of the combined household are unaffected by the use of separate accounts, it follows that the Pareto efficient full commitment consumption allocation is the same as in the baseline model as per (17).

I limit the analysis to the case where $T = 2$ which is sufficient to capture the key forces in this setting and rule out the availability of durable goods. I discuss how the results generalize to any horizon length where possible.

B *Equilibrium Choices at* $t = 2$

At the end of their lives, both members will consume whatever wealth they have left net of any transfers: $C^*_{i,2} = \widetilde{W}_{i,2}$. The only interesting question is: conditional on their relative wealth, how much will each member transfer to the other? Without loss of generality, I focus only on equilibria that do not involve redundant transfers. This implies that $\Psi_{i,t} > 0$ for at most one member. Thus, taking $W_{i,2}$, $W_{j,2}$, and $\Psi_{j,t} = 0$ as given, the optimal level of any non-zero transfer from member i will maximize the weighted sum of each member's utility, where the weights accord with member i's altruism:

$$\max_{\Psi_{i,2}} \frac{1+\Delta}{2} \ln\left(W_{i,2} - \Psi_{i,2}\right) + \frac{1-\Delta}{2} \ln\left(W_{j,2} + \Psi_{i,2}\right) \text{ subject to } \Psi_{i,2} \in [0, W_{i,t}]. \tag{32}$$

The solution to this problem,

$$\Psi^{BR}_{i,2} = \max\left\{\frac{1-\Delta}{2} W_{i,2} - \frac{1+\Delta}{2} W_{j,2}, 0\right\} \tag{33}$$

characterizes i's best response. Intuitively, (33) implies that i will transfer wealth to j at $t = 2$ if and only if i's relative wealth exceeds her relative concern for j: $\frac{W_{i,2}}{W_{i,2}} \geq \frac{1+\Delta}{1-\Delta}$. The resulting Nash equilibrium in transfers will involve one of three arrangements: A transfers to B, neither A or B transfers at all, or B transfers to A. Formally,

$$\left\{\Psi^*_{A,2}, \Psi^*_{B,2}\right\} = \left\{ \begin{array}{cc} \left\{\left(\frac{1-\Delta}{2}\right) W_{A,2} - \frac{1+\Delta}{2} W_{B,2}, 0\right\} & \text{if } \frac{W_{A,2}}{W_{B,2}} \geq \frac{1+\Delta}{1-\Delta} \\ \{0, 0\} & \text{if } \frac{W_{A,2}}{W_{B,2}} \in \left(\frac{1-\Delta}{1+\Delta}, \frac{1+\Delta}{1-\Delta}\right) \\ \left\{0, \left(\frac{1-\Delta}{2}\right) W_{B,2} - \frac{1+\Delta}{2} W_{A,2}\right\} & \text{if } \frac{W_{A,2}}{W_{B,2}} \leq \frac{1-\Delta}{1+\Delta} \end{array} \right] . \tag{34}$$

[29] Evidence of the importance of risk sharing in households is provided by papers such as Kotlikoff and Spivak (1981), Rosenzweig and Stark (1989), and Hess (2004).

Crucially, household members will refrain from transferring wealth to each other only if their relative wealth is sufficiently equal in the sense that it falls between their relative concern for each other. When this is not the case, the wealthy member will make a transfer of sufficient size to ensure that the relative consumption of the two members matches her relative concern for each. As a result, (34) produces equilibrium consumption choices for member i, as a function of $W_{i,2}$ and $W_{j,2}$ of

$$C^*_{i,2}\left(W_{i,2},W_{j,2}\right)=\left\{\begin{array}{cc}\frac{1+\Delta}{2}\left(W_{A,2}+W_{B,2}\right) & \text{if } \frac{W_{i,2}}{W_{j,2}}\geq\frac{1+\Delta}{1-\Delta}\\ W_{i,2} & \text{if } \frac{W_{i,2}}{W_{j,2}}\in\left(\frac{1-\Delta}{1+\Delta},\frac{1+\Delta}{1-\Delta}\right)\\ \left(\frac{1-\Delta}{2}\right)\left(W_{A,2}+W_{B,2}\right) & \text{if } \frac{W_{i,2}}{W_{j,2}}\leq\frac{1-\Delta}{1+\Delta}\end{array}\right].\tag{35}$$

Note that when there is sufficient intra-household wealth inequality at the start of $t=2$, members will be endogenously financially linked, despite having imperfect altruism and separate accounts. In this case, the consumption of each member is determined purely by the *combined* wealth of the household, as if all wealth were pooled. To see this connection more clearly, let $V'_{i,2}$ be value function[30] of member i at the start of $t=2$ as a function of the realized value of the two state variables $W_{A,2}$ and $W_{B,2}$,

$$V'_{i,2}\left(W_{i,2},W_{j,2}\right)=\left\{\begin{array}{cc}\ln\left(W_{A,2}+W_{B,2}\right)+v_2^{High} & \text{if } \frac{W_{i,2}}{W_{j,2}}\geq\frac{1+\Delta}{1-\Delta}\\ \frac{1+\Delta}{2}\ln W_{i,2}+\frac{1-\Delta}{2}\ln W_{j,2} & \text{if } \frac{W_{i,2}}{W_{j,2}}\in\left(\frac{1-\Delta}{1+\Delta},\frac{1+\Delta}{1-\Delta}\right)\\ \ln\left(W_{A,2}+W_{B,2}\right)+v_2^{Low} & \text{if } \frac{W_{i,2}}{W_{j,2}}\leq\frac{1-\Delta}{1+\Delta}\end{array}\right].\tag{36}$$

Hence, if household members anticipate that one member will be transferring wealth to the other at $t=2$, then the value function of each is solely determined by their combined wealth and thus, wealth is effectively shared. Intuitively, when member i decides how much to transfer, she takes into account the assets held by both members and, thereby, effectively sets $\Psi_{i,2}$ so as to allocate the *combined* wealth of the household according to her preferences. I now turn to studying the impact this has on decisions at $t=1$.

C Equilibrium Choices at $t=1$

At the end of $t=1$, member A takes $\widetilde{W}_{A,1}$, $\widetilde{W}_{B,1}$, and $C_{B,1}$ as given and chooses $C_{A,1}$ to solve

$$\max_{C_{A,1}}\frac{1+\Delta}{2}\ln C_{A,1}+\frac{1-\Delta}{2}\ln C_{B,1}+\delta V_{A,2}\tag{37}$$

where $V_{A,2}$ is the expected value function of A. This is determined by combining the intertemporal budget constraint (31) with the realized value function (36) and taking expectations over possible

[30]Define constant terms $v_2^{Low}\equiv\frac{1+\Delta}{2}\ln\frac{1-\Delta}{2}+\frac{1-\Delta}{2}\ln\frac{1+\Delta}{2}$ and $v_2^{High}\equiv\frac{1+\Delta}{2}\ln\frac{1+\Delta}{2}+\frac{1-\Delta}{2}\ln\frac{1-\Delta}{2}$.

realizations of the relative wealth shock ϖ_2. Formally,

$$V_{A,2} = \int_{-\infty}^{\overline{\varpi}^{Low}} \left[\ln\left(\widetilde{W}_{A,1} + \widetilde{W}_{B,1} - C_{A,1} - C_{B,1}\right) + v_{A,2}^{Low}\right] f(\varpi)\, d\varpi \tag{38}$$
$$+ \int_{\overline{\varpi}^{Low}}^{\overline{\varpi}^{High}} \left[\frac{1+\Delta}{2}\ln\left(\widetilde{W}_{A,1} - C_{A,1} + \varpi\right) + \frac{1-\Delta}{2}\ln\left(\widetilde{W}_{B,1} - C_{B,1} - \varpi\right)\right] f(\varpi)\, d\varpi$$
$$+ \int_{\overline{\varpi}^{High}}^{\infty} \left[\ln\left(\widetilde{W}_{A,1} + \widetilde{W}_{B,1} - C_{A,1} - C_{B,1}\right) + v_{A,2}^{High}\right] f(\varpi)\, d\varpi + \ln R,$$

The limits of the integral are determined by the realized values of the relative wealth shock that place the household at the boundary of the region over which no transfers are made at $t = 2$. Specifically, when

$$\varpi_2 \geq \overline{\varpi}^{High} \equiv \frac{1+\Delta}{2}\left(\widetilde{W}_{B,1} - C_{B,1}\right) - \frac{1-\Delta}{2}\left(\widetilde{W}_{A,1} - C_{A,1}\right) \tag{39}$$

the relative wealth shock is sufficiently favorable to A that she will choose to transfer wealth to B at $t = 2$. The reverse is true when

$$\varpi_2 \leq \overline{\varpi}^{Low} \equiv \frac{1-\Delta}{2}\left(\widetilde{W}_{B,1} - C_{B,1}\right) - \frac{1+\Delta}{2}\left(\widetilde{W}_{A,1} - C_{A,1}\right). \tag{40}$$

The best response function of member A is characterized by the Euler equation that solves (37). This is[31]

$$\underbrace{\frac{\frac{1+\Delta}{2}}{\delta C_{A,1}}}_{[\text{MU at } t=1]} - \underbrace{\left(\frac{1}{\widetilde{W}_{A,1} + \widetilde{W}_{B,1} - C_{A,1} - C_{B,1}}\right)\left[F\left(\frac{\overline{\varpi}^{Low}}{\sigma}\right) + F\left(\frac{-\overline{\varpi}^{High}}{\sigma}\right)\right]}_{\text{Expected MB of Shared Saving: Transfer at 2}} \tag{41}$$
$$-\underbrace{\int_{\overline{\varpi}^{Low}}^{\overline{\varpi}^{High}} \left(\frac{\frac{1+\Delta}{2}}{\widetilde{W}_{A,1} - C_{A,1} + \varpi}\right)\left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{-\left(\frac{\varpi^2}{2\sigma^2}\right)} dw}_{\text{Expected MB of Individual Saving: No Transfer at 2}} = 0.$$

This Euler equation has three terms. Member A trades off the marginal utility of her private consumption at $t = 1$ with the expected marginal benefit of saving, both in the scenario where a transfer does and does not occur, in the next period. Crucially, A's marginal benefit of saving in each scenario reflects her *individual* marginal benefit of additional saving when no transfer is expected, and, conversely, the *combined* marginal benefit of pooled total household savings when a transfer is expected. As such, the marginal incentive to save lies between the classic problem of the unitary household, where agents act alone, and the scenario studied so far in this paper, where all savings

[31]The optimization problem and best response for B is defined analogously, allowing for the asymmetry that ϖ enters with the opposite sign.

are shared.

Analytically characterizing the Nash equilibrium that results from (41) and its counterpart for B is, in general, infeasible. For this reason I proceed by studying analytical solutions for two special cases and then use a numerical solution to describe how the equilibrium varies between these extremes.

C-1 No Intra-Household Risk Sharing: $\sigma_2 = 0$

Equilibrium Consumption Choices I start the analysis by considering the case where there is no shock to the relative wealth of each member at $t = 2 : \sigma_2 = 0$. Define the relative wealth of member i, after transfers have taken place in period 1, as $w_{i,1} \equiv \frac{\widetilde{W}_{i,1}}{\widetilde{W}_{j,1}}$. I solve for the equilibrium consumption choices in the Internet Appendix. Written as a function of $w_{i,1}$, these are

$$C^*_{i,1} = \begin{cases} \left(\frac{\frac{1+\Delta}{2}}{1+\Delta+\delta}\right)\left(\widetilde{W}_{i,1} + \widetilde{W}_{j,1}\right) & \text{if } w_{i,1} \leq w^{'}_{i,1} \\ \text{(B Transfers to A at } t\text{=2)} & \\ \frac{1}{1+\delta}\widetilde{W}_{i,1} & \text{if } w_{i,1} \in \left[w^{'}_{i,1}, w^{''}_{i,1}\right] \\ \text{(No Transfer at } t\text{=2)} & \\ \left(\frac{\frac{1+\Delta}{2}}{1+\Delta+\delta}\right)\left(\widetilde{W}_{i,1} + \widetilde{W}_{j,1}\right) & \text{if } w_{i,1} \geq w^{''}_{i,1} \\ \text{(A Transfers to B at } t\text{=2)} & \end{cases} \tag{42}$$

where $w^{'}_{i,1}$ and $w^{''}_{i,1}$ are cutoffs such that

$$w^{'}_{i,1} \leq \left(\frac{1-\Delta}{1+\Delta}\right)\left(\frac{\delta}{1+\delta}\right) < 1 < \left(\frac{1+\Delta}{1-\Delta}\right)\left(\frac{1+\delta}{\delta}\right) \leq w^{''}_{i,1}. \tag{43}$$

The equilibrium consumption choices at $t = 1$ fall into three possible cases, depending on the relative wealth of each household member. When the wealth of each member is close to the other $w_{i,1} \in \left[w^{'}_{i,1}, w^{''}_{i,1}\right]$, then no transfer is expected at $t = 2$ and the two members effectively operate as separate agents. As a result, the unique equilibrium consumption choice is determined purely by their time preferences and achieves the full commitment optimum. In contrast, when there is sufficient inequality between the wealth of each member so that $w_{i,1} \leq w^{'}_{i,1}$ or $w_{i,1} \geq w^{''}_{i,1}$, then equilibrium consumption choices are identical to Pareto inefficient consumption choices in the baseline model with a shared account as per (18). When a transfer is anticipated, each member trades off the marginal benefit from their own consumption with the *combined* marginal benefit of total household savings next period. This recreates the same commons problem encountered when the household saves in a single combined savings account.

Using the equilibrium consumption choices in (42) I can write the value function of member i

as a function of of the wealth of each member, net of transfers, at $t=1$. This is

$$V_{i,1}=\left\{\begin{array}{cc} V_i^{(1)}=(1+\delta)\ln\left(\widetilde{W}_{i,1}+\widetilde{W}_{j,1}\right)+v_i^{(1)} & \text{if } w_{i,1}\leq w'_{i,1} \\ \text{(B Transfers to A at } t\text{=2)} & \\ V_i^{(2)}=(1+\delta)\left[\frac{1+\Delta}{2}\ln\widetilde{W}_{i,1}+\frac{1-\Delta}{2}\ln\widetilde{W}_{j,1}\right]+v_i^{(2)} & \text{if } w_{i,1}\in\left[w'_{i,1},w''_{i,1}\right] \\ \text{(No Transfer at } t\text{=2)} & \\ V_i^{(3)}=(1+\delta)\ln\left(\widetilde{W}_{i,1}+\widetilde{W}_{j,1}\right)+v_i^{(3)} & \text{if } w_{i,1}\geq w''_{i,1} \\ \text{(A Transfers to B at } t\text{=2)} & \end{array}\right] \tag{44}$$

where $v_i^{(1)}$, $v_i^{(1)}$, and $v_i^{(1)}$ are constants defined in the Internet Appendix. Note that when members anticipate that either one will make a transfer at $t=2$ then the value function of both members is determined solely by total household wealth, as if it was held in a shared account. If, for example, $w_{i,1}\geq w''_{i,1}$ then A anticipates transferring wealth to B at $t=2$. Any reallocation of wealth at the start of $t=1$, that leaves $w_{i,1}\geq w''_{i,1}$ unchanged, will have no impact on the expected utility of either member. Conversely, when members anticipate making no transfer to each other in the future, then a marginal transfer of wealth from i to j at the start of $t=1$ directly changes the relative consumption of each member. These observations highlight the key forces that will drive the equilibrium choice of transfers at $t=1$.

Equilibrium Transfers Stepping back to the start of $t=1$, equilibrium choice of transfers at $t=1$ will be

$$\left\{\Psi^*_{A,1},\Psi^*_{B,1}\right\}=\left\{\begin{array}{cc} \left\{\frac{1-\Delta}{2}RW_{A,0}-\frac{1+\Delta}{2}RW_{B,0},0\right\} & \text{if } \frac{W_{A,1}}{W_{B,1}}\geq\frac{1+\Delta}{1-\Delta} \\ \{0,0\} & \text{if } \frac{W_{A,1}}{W_{B,1}}\in\left(\frac{1-\Delta}{1+\Delta},\frac{1+\Delta}{1-\Delta}\right) \\ \left\{0,\frac{1-\Delta}{2}RW_{B,0}-\frac{1+\Delta}{2}RW_{A,0}\right\} & \text{if } \frac{W_{A,1}}{W_{B,1}}\leq\frac{1-\Delta}{1+\Delta} \end{array}\right]. \tag{45}$$

This equilibrium gives rise to the following proposition.

Proposition 4. *: Suppose there is no intra-household relative wealth shock: $\sigma_2=0$. Equilibrium transfer choices at $t=1$ will ensure $w_{i,1}\in\left[\frac{1-\Delta}{1+\Delta},\frac{1+\Delta}{1-\Delta}\right]$ and hence rule out the possibility of future transfers in the future. As a result equilibrium consumption choices will be identical to the full commitment Pareto optimum.*

The intuition for this Proposition is as follows. First, suppose that the initial allocation of wealth within the household is roughly equal: $\frac{W_{i,1}}{W_{i,1}}\in\left(w'_{i,1},w''_{i,1}\right)$. As a result, both members are assured to remain independent in the future if no transfers are made at $t=1$. Consumption and savings decisions are efficient when made independently. In this case, any transfer made at $t=1$ lowers the wealth of the member who makes it and, if large enough, potentially renders consumption choices inefficient. Thus, the only transfer that will be made in this scenario occurs if $\frac{W_{i,1}}{W_{j,1}}\in\left[\frac{1+\Delta}{1-\Delta},w''_{i,1}\right]$

and is simply made to realign the relative consumption of each member with i's relative concern for herself and her partner.

Now consider the opposite case, where the initial allocation of wealth within the household is substantially unequal $\frac{W_{i,1}}{W_{i,1}} > w^{'}_{i,1}$. In this scenario, small transfers which leave $w_{i,1}$ above $w^{''}_{i,1}$ will be insufficient to prevent a transfer in the future, and hence, will have no impact on the resulting equilibrium consumption choices as per (42). If however, i makes a sufficiently large transfer to render $w_{i,1} = \frac{1+\Delta}{1-\Delta}$ then she will be strictly better off for two reasons. First, such a transfer will decouple the savings decision of each member, and hence, implement the efficient level of household consumption and saving each period. Second, the transfer ensures that the relative consumption of each member matches i's relative concern for herself and her partner, thereby maximizing welfare according to her preferences.

The direct implication of Proposition 4 is that when there are no relative wealth shocks, equilibrium transfers at $t = 1$ will endogenously ensure effective financial separation and efficient consumption and savings from then on. Further, Proposition 4 indicates that if a household member anticipates making a transfer to another member in the future, that it is better to make a sufficiently large transfer right away to avoid the need to do so later. Making the transfer sooner is preferred because it credibly separates each member and therefore avoids time-inconsistency. Further, from the perspective of the member making the transfer, it ensures that the relative consumption of both members matches her relative concern right away and is therefore preferred. The result shows that with separate accounts the allocation of wealth within the household impacts the propensity for overconsumption, with relative equality within the household producing more efficient choices. When $\sigma_t = 0\ \forall t$, this result can be generalized by an argument of induction for any $T \geq 2$. I now show that this conclusion depends crucially on the absence of relative wealth shocks.

C-2 Relative Wealth Shocks are Arbitrarily Large: $\sigma_2 \to \infty$

Equilibrium Consumption Choices Having studied the case where household members do not experience relative wealth shocks, I now turn to the other extreme case where these shocks are arbitrarily large ($\sigma_2 \to \infty$). As before, the savings decisions of each member at $t = 1$ will deterministically set the level of *total* household resources at $t = 2$. But these will have a vanishingly small impact on the realized *relative* wealth of each member at the start of $t = 2$. Instead, the member who receives the beneficial shock will, almost surely, make a transfer to the other member. Thus, irrespective of previous decisions, each member anticipates that she will make or receive a transfer, with probability half each. As such, household members are financial linked with certainty. In this case, the third term in (41) disappears, and the Euler equation that defines the consumption best response of each member at $t=1$ is

$$\frac{\frac{1+\Delta}{2}}{\delta C_{i,1}} - \left(\frac{1}{\widetilde{W}_{A,1}+\widetilde{W}_{B,1}-C_{A,1}-C_{B,1}}\right) = 0. \tag{46}$$

The resulting Nash equilibrium consumption choices are the same outcome obtained with a shared account,

$$C^*_{i,1} = \frac{\frac{1+\Delta}{2}\left(\widetilde{W}_{A,1}+\widetilde{W}_{B,1}\right)}{1+\Delta+\delta}, \tag{47}$$

and hence, generate the same undersavings problem highlighted earlier in the paper. The logic is as follows. When trading-off consumption at $t=1$ with the marginal benefit of savings, each member knows that a dollar saved will, by virtue of its impact on the endogenous voluntary transfer that will arise in the future, be shared. Either a fraction $\frac{1-\Delta}{2}$ of each dollar will be transferred to the other member, or conversely, each additional dollar saved will offset a transfer of $\frac{1+\Delta}{2}$ from the other member.

Equilibrium Transfer Choices Notice that (47) is invariant to the initial distribution of wealth, as are the equilibrium consumption outcomes at $t=2$. It follows that any transfer that members might make at $t=1$ will have no impact on the resulting consumption choices, and hence, by an argument of indifference, any combination of choices forms a Nash equilibrium.

Proposition 5. *: Suppose the standard deviation of intra-household relative wealth shocks is arbitrarily large: $\sigma_2 \to \infty$. The consumption and savings decisions of a household with separate accounts is identical to those with a shared account.*

This highlights a crucial caveat to Proposition 4: transfers at $t=1$ implement the full commitment consumption choices only if they can reduce the probability of transfers in the future to zero. In a perfectly nonstochastic environment ($\sigma=0$) this is possible. In contrast, when $\sigma\to\infty$ transfers at $t=2$ are unavoidable regardless of what allocation of wealth the household elects to adopt at the start of $t=1$. This result extends by an argument of induction to any household lifespan $T\geq 2$ when $\sigma_t\to\infty\ \forall t$.

C-3 Relative Wealth Shocks are Finite: $\sigma_2 > 0$

Equilibrium Consumption Choices For the case where relative wealth shocks are positive but finite, I study a symmetric version of the model ($W_{A,0}=W_{B,0}$) numerically. The symmetry of initial wealth ensures that in equilibrium no transfers occur at $t=1$ by focusing on the case where the initial wealth distribution already minimizes the probability of any financial dependence between members at $t = 2$. The only decisions of interest in this symmetric case are date 1 consumption choices. The Euler equations describing each member's optimal consumption choice at $t = 1$ are as per (41). Both members recognize that, with some probability, a sufficiently large relative wealth shock will be realized at $t = 2$ and, as a result, their consumption choices will be endogenously linked by a transfer. Thus, with some intermediate probability, each dollar of savings is effectively shared. Conversely, with some probability, ϖ_2 will take on a sufficiently low absolute value and no transfers will occur. In this case, each member will independently consume every additional dollar of their individual savings. Recognizing that both scenarios are possible recreates an intermediate recurrence of the commons problem whose strength is determined by the probability that a transfer will occur.

Figure 4 Panel A shows the equilibrium consumption choice of household members at $t = 1$ as a function of the normalized standard deviation of this relative wealth shock ($\frac{\sigma_2}{W_{i,1}}$). For comparison, the level of consumption with a joint account (no commitment) and with full commitment are shown for comparison (see dotted and dashed lines respectively) and both are indifferent to σ_2. The level of consumption with separate accounts falls between these two cases. As per Proposition 4, when there is no relative wealth shocks, then separate accounts implement full commitment consumption choices. When the volatility of these shocks increases, members progressively lower their private assessment of the marginal value of a dollar saved, because it is more likely to be shared. Thus equilibrium consumption choices, and hence, the undersaving problem, grows monotonically with this probability. The improvement in the efficiency of savings decisions with separate accounts, relative to those made with a joint savings account, decreases with this probability (indexed by $\frac{\sigma_2}{W_{i,1}}$). The numerical solutions verify the analytical result in Proposition 5: consumption decisions under separate and joint accounts are the same when σ_2 becomes arbitrarily large. The figure demonstrates that the central argument of the paper, household savings will be time-inconsistent, is robust even to allowing savings to occur in separate accounts, provided that there is some risk of relative wealth shocks to individual members.

D The Choice to Use Separate Accounts

Having studied household consumption and savings decisions with separate and joint accounts, I now ask: which savings arrangement will the household select? I assume that the choice occurs at $t = 0$ and, for simplicity, is binding for the life of the household. To abstract from the details of the bargaining problem that might arise to make this decision, I focus on the case where members have ex ante identical wealth: $W_{A,0} = W_{B,0}$. Under this assumption the expected utility of each member at $t = 0$, $V_{i,0}$, is identical. Hence the decision is reduced to finding under which arrangement this value function is largest. I present analytical solutions for each the extreme cases of σ_2 and then numerically study the intermediate cases between them.

D-1 No Intra-Household Risk Sharing: $\sigma_2 = 0$

When there is no intra-household risk, the choice of accounts follows directly from Proposition 8. Since in this case, separate accounts implement the unique Pareto efficient full-commitment consumption decisions, it must be strictly preferred to joint accounts.

D-2 Relative Wealth Shocks are Arbitrarily Large: $\sigma_2 \to \infty$

As per Proposition 9, when the relative wealth shock is of arbitrarily large variance ($\sigma_2 \to \infty$), the initial consumption choice of each household member is the same as when they share a joint account. The difference between the two arrangements arises later in life, when the shock to relative wealth is realized. With separate accounts, the level of consumption of each member at $t = 2$ will depend upon the realization of the shock. If the shock is favorable to A, then she will decide how much to transfer to B and hence will consume a fraction $\frac{1+\Delta}{2}$ of the household's combined remaining wealth. Conversely, if the shock is unfavorable, her fraction will be strictly lower: $\frac{1-\Delta}{2}$. Since both scenarios are equally likely, the expected level of consumption is one half of the households combined wealth, which is the same as what would be consumed, with certainty, if savings were shared. As a result, the difference in the expected value function under each scenario is

$$V_{i,0}^{Shared} - V_{i,0}^{Separate} = \delta^2 \left[\ln \left(\frac{1}{2} \right) - \frac{1}{2} \left[\ln \frac{1+\Delta}{2} + \ln \frac{1-\Delta}{2} \right] \right] > 0. \tag{48}$$

In words, the only difference between the two arrangements is that perfect risk sharing is achieved only under the joint account, and hence, it is strictly preferred.

D-3 Relative Wealth Shocks are Finite: $\sigma_2 > 0$

The analytical solutions for the two polar cases suggest that the household balances two forces when deciding whether or not to share financial assets. Sharing leads to inefficiently low savings, but also pools risk. The resolution of this trade-off depends on the magnitude of the relative wealth shocks. When σ_2 is low, the gain from pooling risk is low and are outweighed by the improved savings decisions made with separate accounts. As σ_2 increases, the value of risk pooling grows. Moreover, as seen already in Figure 4 Panel A, the efficiency gain from separate accounts falls as the exposure to intra-household risk increases. With higher risk, both members recognize that their future finances are, with a higher probability, linked by endogenous transfers. Both effects make the expected utility of sharing wealth rise relative to separate savings. This intuition is borne out in Figure 4 Panel B, which compares the expected utility of a household member under both arrangements. When the relative wealth shock is above a finite threshold, the household will endogenously elect to save in a joint account despite the inefficient consumption decisions this gives rise to. The figure suggests that even small degrees of intra-household risk are sufficient to render joint accounts optimal[32]. Unreported numerical solutions indicate that this threshold level of σ_2 is decreasing in the altruism between household members, so that, all else equal, households with more altruism are more likely to share accounts.[33]

E Additional Considerations

Before concluding the consideration of separate and joint accounts, a few observations are warranted. First, in the model presented in this section, separate accounts do not implement perfectly efficient consumption choices because of the possibility of future financial transfers, which effectively link the wealth of both. Another important factor that will reinforce this conclusion is that members are also likely to be linked through their shared concern for public goods, such as children or elderly parents. In this case, with separate accounts, each member will recognize that each dollar they save individually will increase their contribution to the public good and offset the contribution of the other member. Initial transfers may be able to minimize this problem.[34] However, uncertainty over future public good expenditure needs or over the relative wealth of each member will both have the same effect: limit the consumption efficiency benefit of shared accounts and while bringing about poorer risk sharing.

It should also be noted that the ability of household members to separate their finances is subject to the institutional setting in which they exist. For example, following the federal Uniform Marriage and Divorce Act (UMDA) of 1970, US states moved towards equitable distribution of household assets (see: Golden (1983), Turner (2005), and Voena (2015)). Thus, even with separate accounts, if both members of a married household recognize that divorce is possible with some probability, then their savings in this scenario is shared and hence, subject to the same commons problem. Combined with the results in this section, this reinforces the usefulness of the initial setup of the paper whereby household behavior is studied assuming savings are shared.

[32]Numerical solutions with a range of other parameters also support this. However, given the stylized nature of the model, it is difficult to conclude whether this risk is high or low relative to empirical applications. That is an empirical question left for future work.

[33]In the extreme, a household with no altruism will always prefer separate accounts. Conversely, if members have perfect altruism, $\Delta = 0$, then the household is not subject to either problem, and separate and joint accounts will both render the full commitment solution.

[34]Although, in this case, transfers need to ensure the allocation of wealth is sufficiently unequal to avoid members being linked by a shared responsibility for providing the public good.

V Discussion and Empirical Implications

The canonical theory of household financial behavior supposes that a household can be modeled as a single optimizing agent. Significant evidence rejects this premise: household members have distinct preferences and are self-interested.[35] The paper introduces a model of consumption and saving in a multi-person household in which members have shared wealth, consume both private and public goods, and have imperfectly aligned altruistic preferences. The central finding of the paper is that the household is time-inconsistent, consuming a larger fraction of wealth each period than the optimal plan that the household would like to commit to. The model highlights that in a multi-person household, savings are subject to a dynamic commons problem. The tendency to under-save stems from each member's ability to deviate unilaterally from the optimal household plan and direct more resources to their own private consumption at the cost of shared wealth. The extent of time-inconsistency is larger when members are more self-interested and when they place less weight on shared public consumption goods such as children and housing. The paper shows that access to illiquid durable goods can further exacerbate the degree of time-inconsistency. Household members individually overconsume these goods because they provide a way for shared wealth to be saved in an asset that the other member cannot access.

Unlike standard theories based on the unitary model of the household, this paper is able to rationalize the way that ownership and control of assets within the household can affect consumption and savings decisions. The final section of the paper shows how the ability to separate the wealth of each member can impact consumption and savings decisions. Even if each member saves individually, members are linked by their altruism: a relatively wealthy member may voluntarily decide to transfer funds to the other at some point in time in the future. If such a voluntary transfer is anticipated in the future, then members will act as if they have shared wealth and will therefore exhibit the same time-inconsistency. Endogenous initial transfers, to restore sufficient parity of wealth, can ensure that members with separate accounts are effectively independent from then on, and are, therefore, time-consistent. This relies on the assumption that members do not face any shocks to their relative wealth. If on the contrary, household relative wealth shocks are possible, members are unable to fully separate themselves because they anticipate that one member may choose to transfer wealth to the other in the future. This possibility makes individual wealth effectively shared and thereby introduces the same time-inconsistency created when household members share their savings. Finally I show that when relative wealth shocks are sufficiently large, households will optimally elect to share wealth in order to pool risk even though this creates the time-inconsistency problem stressed above.

[35] see for example Lundberg et al. (1997); Browning and Chiappori (1998); Phipps and Burton (1998); and, Ashraf (2009)

Another strategy suggested by the paper is for household members to save in the form of assets that require joint approval for withdrawals. Joint approval will remove the ability of household members to act unilaterally and will therefore limit the possibility of over-consumption. As such the paper provides a framework that can rationalize why several of the most important household saving assets require joint approval to withdraw or borrow against. As a primary example, the U.S. 1984 Retirement Equity Act revised the rules governing all retirement plans covered by the 1974 Employee Retirement Income Security Act to require exactly this form of joint approval. This covers all assets held by married households in 401(k) plans, IRA accounts, and defined benefit plans, and thus accounts for the bulk of US retirement savings outside of housing. Aura (2005) shows that the passing of these laws did in fact increase savings for households affected by this law change. Similarly, joint ownership of a house prevents a household member borrowing against home equity savings without the approval of his spouse. Joint approval may come at the cost of significant inflexibility: for example limiting the ability to adapt consumption choices to privately observed shocks to the marginal utility of each member and therefore may not be adopted. Further consideration of this trade-off is left for future work.

Finally the model suggests that household members may use punishment strategies to mitigate the temptation to over consume. Evidence that households do not exhibit dynamic efficiency (see, for example, De Mel et al. (2009), Duflo and Udry (2004), Mazzocco (2007), Robinson (2012), Udry (1996)) suggest that in practice these strategies are of limited effectiveness. This may be because shocks to marginal utility are unobserved or because households are unable to credibly commit not to renegotiate planned punishment strategies. A more detailed theoretical consideration of punishment strategies within the household is left for future work.

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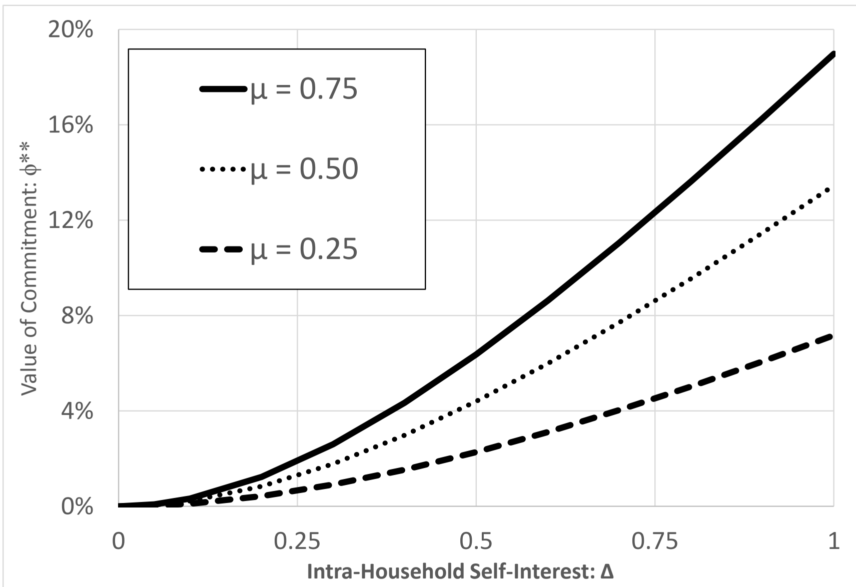
VI Appendix - Figures and Tables

Figure 1: Comparative Statics - The Value of Commitment

This plot shows the fraction of W_0 that the household would be willing to pay at $t = 0$ to achieve the full commitment consumption path. Due to the log additive utility functions this fraction is invariant to the level of W_0. Panel A shows how the value of commitment varies with the degree of self-interest within the household: Δ. Panel B shows how the value of commitment varies with the relative concern for private consumption: μ. The plot is drawn using the following parameters: both household members have exponential discount factor of $\delta = 0.95$, the gross interest rate is $R = 1/0.95$, and the household exists for $T = 50$ years.

(a) Intra-Household Self-Interest and the Value of Commitment



(b) Concern for Private Consumption and the Value of Commitment

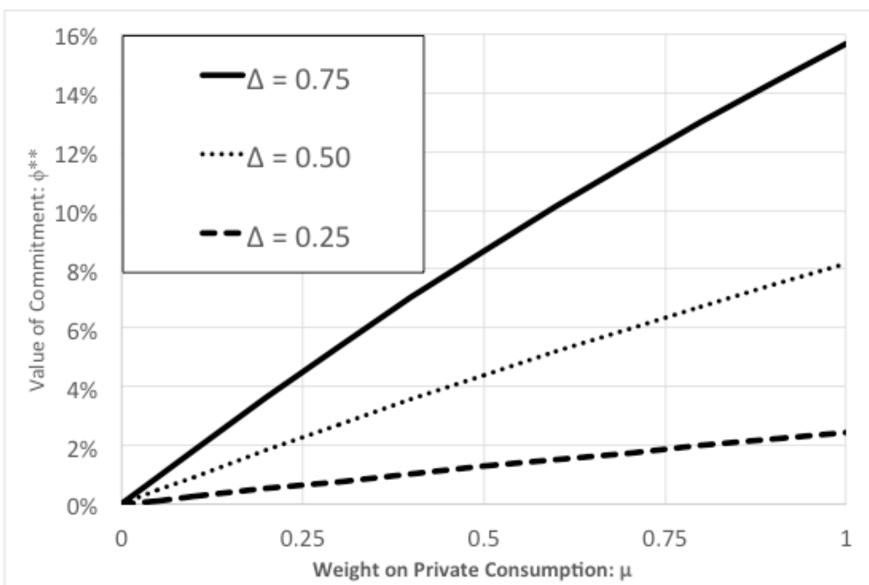
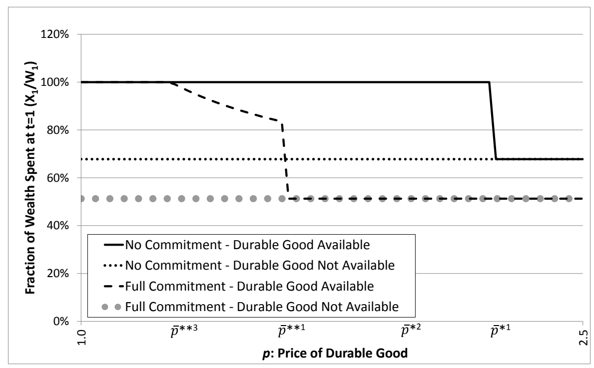


Figure 2: Illiquid Durable Good $p \geq 1$ and $T = 2$

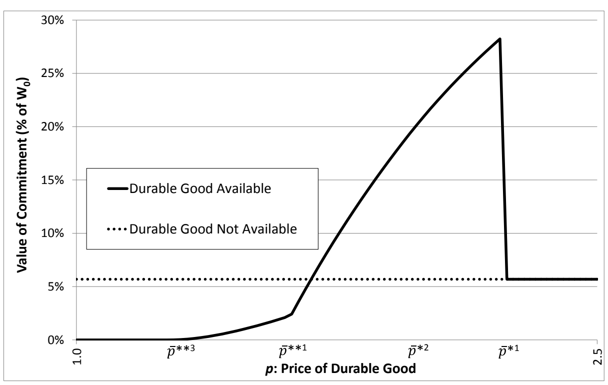
These plots are drawn for the case where the household members can buy an illiquid durable good at $p \geq 1$. The parameters used assume no altruism ($\Delta = 1$), the exponential discount factor is $\delta = 0.95$, the gross interest rate is $R = 1/0.95$, the durable good depreciation is $\kappa = 0.35$, and the household exists for $T = 2$ periods. Panel A shows the misallocation of consumption between goods by comparing the fraction of total household expenditure on the durable good as a fraction at $t = 1$ with and without commitment. Panel B shows the intertemporal misallocation of consumption by comparing the fraction of wealth that the household spends at $t = 1$ both with and without commitment, and also compares to the case where the household does not have access to the illiquid durable good. Panel C compares the value of commitment when the durable good is and is not available. Panel D compares the discounted value of utility for a member when the illiquid durable good is and is not available. The price of the durable good, p, is on the x-axis in all panels.

(a) Fraction of Expenditure on Durable Good at $t = 1$

(b) Fraction of Wealth Spent at $t = 1$



(c) The Value of Commitment



(d) Welfare at $t = 0$

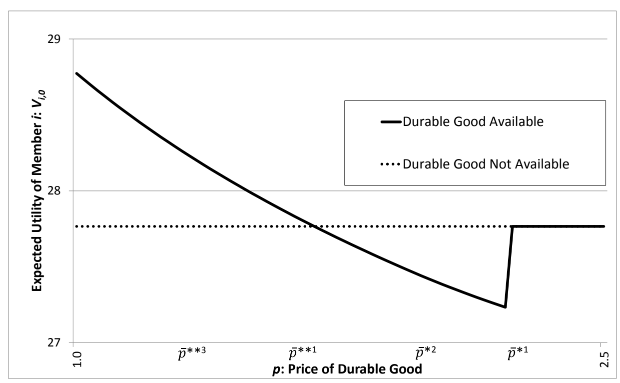


Figure 3: Value of Commitment with Illiquid Durable Goods $p = 1$ and $T \to \infty$

This plot is drawn for the case where household members can buy an illiquid durable good. The parameters used assume no altruism ($\Delta = 1$), $\delta = 0.95$, and $R = 1/0.95$. The figure is drawn for the case where $T \to \infty$ and shows the amount the household would be willing to pay at $t = 0$ (as a fraction of W_0) to achieve the full commitment consumption path. Due to log additive utility functions this fraction will be invariant to the choice of W_0. The figure shows how the value of commitment varies with the rate at which the durable good depreciates κ and compares to the case where the household does not have access to the illiquid durable good.

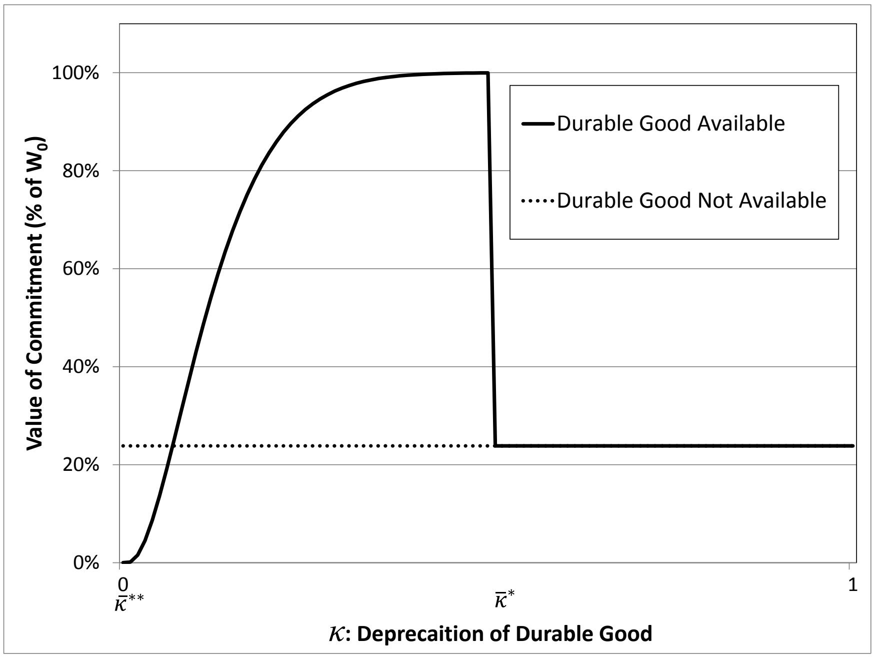
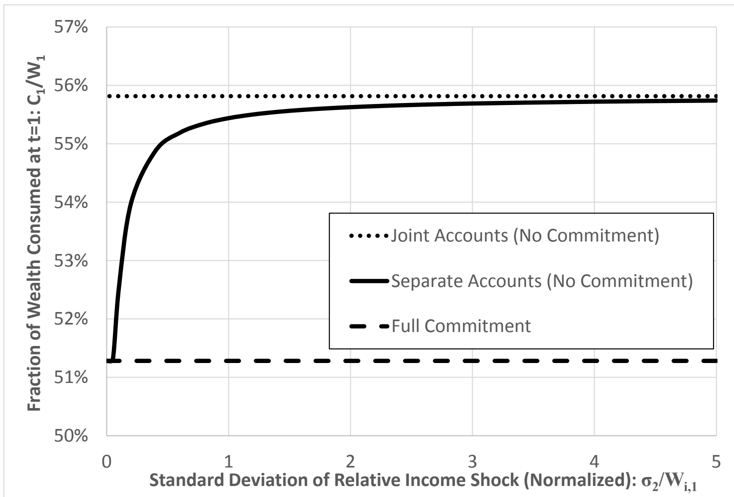


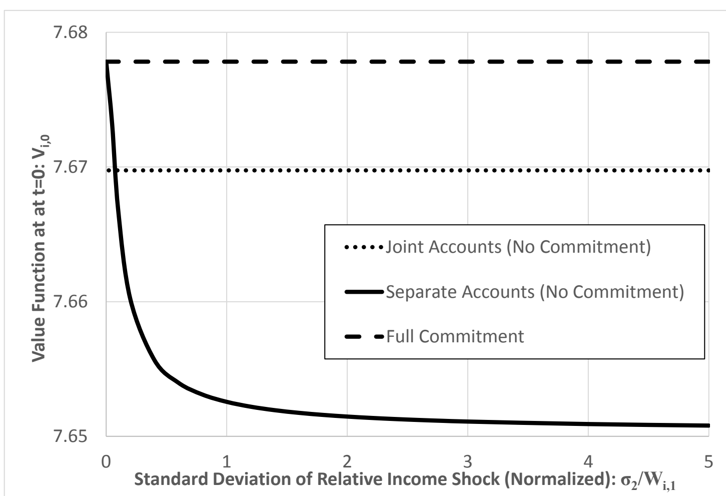
Figure 4: Separate Accounts $\delta_0 > 0$ and $T = 2$

These plots are drawn to compare the case where household members save in separate and joint accounts. Panel A shows equilibrium household consumption choices at $t = 1$ with 1) separate accounts, 2) joint account (absent commitment), and 3) the Pareto efficient full-commitment solution. Panel B compares the resulting value function of each member at $t = 0$ under all three arrangements. Comparative statics are shown with respect to the standard deviation of the relative wealth normalized by the size of each members personal wealth at $t = 1$: $\sigma_2/W_{i,1}$. Each Panel is drawn using the following parameters: altruism is $\Delta = 0.2$, the exponential discount factor is $\delta = 0.95$, the gross interest rate is $R = 1/0.95$, each member begins $t = 0$ with $W_{i,0} = 95$, and the household exists for $T = 2$ periods.

(a) Consumption at $t = 1$ as a Function of Exposure to Relative Wealth Shock



(b) Household Welfare as a Function of Exposure to Relative Wealth Shock



INTERNET APPENDIX

for

Time-Consistent Individuals, Time-Inconsistent Households

This internet appendix presents the formal proof of the results presented in the paper. Each section of the appendix presents the results for the corresponding section in the paper. In addition, Section IV considers a variation on the base model in which household members make consecutive consumption choices. I use this alternative setting to show that the expenditure limit (6) that is assumed in the base model is not crucial for obtaining the results on equilibrium consumption choices.

I Proofs for Section II (Base Model)

This section solves the base model described in Section I of the paper to support the results presented in Section II. I start by solving the ex ante optimal consumption plan and then solve for equilibrium consumption choices without commitment. Finally, by comparing each, I present an analytical solution for value of commitment.

A Optimal Household Allocation with Full Commitment

Ex ante Pareto optimal household allocation solves:

$$\max_{\{C_{A,t},C_{B,t},H_t,\}_{t=1}^{t=T}} \Pi = \eta V_{A,0} + (1-\eta) V_{B,0} \tag{IA.1}$$

$$\text{subject to } W_0 - \sum_{x=1}^{T} R^{-x}\left[C_{A,x} + C_{B,x} + H_x\right] \geq 0 \text{ and} \tag{IA.2}$$

$$\left\{C_{A,t}, C_{B,t}, H_{A,t}, H_{B,t}\right\}_{t=1}^{t=T} \geq 0. \tag{IA.3}$$

The objective of this problem can be re-written as

$$\Pi = (1-\theta) U_{A,0} + \theta U_{B,0} \tag{IA.4}$$

$$\text{where } \theta \equiv \frac{1}{2}\left[1 - (2\eta - 1)\Delta\right] \tag{IA.5}$$

using the expressions for $U_{A,0}$ and $U_{B,0}$ (IA.4) becomes

$$\Pi = (1-\theta)\mu \sum_{x=1}^{T} \delta^x \ln C_{A,x} + \theta\mu \sum_{x=1}^{T} \delta^x \ln C_{B,x} + (1-\mu) \sum_{x=1}^{T} \delta^x \ln H_{1+x}. \tag{IA.6}$$

I will start by ignoring the non-negativity constraints in (IA.3) and verify that these hold later. Writing the Lagrangian for the remaining problem with $\Gamma \geq 0$ being the multiplier on the resource constraint I have

$$\begin{aligned} \max_{\{C_{A,t},C_{B,t},H_t\}_{t=1}^{t=T}} & (1-\theta)\mu \sum_{x=1}^{T} \delta^x \ln C_{A,x} + \theta\mu \sum_{x=1}^{T} \delta^x \ln C_{B,x} \\ & + (1-\mu) \sum_{x=1}^{T} \delta^x \ln H_{1+x} + \Gamma\left[W_0 - \sum_{x=1}^{T} R^{-x}\left[C_{A,x} + C_{B,x} + H_x\right]\right]. \end{aligned} \tag{IA.7}$$

The first order conditions give the optimal level of expenditure on each type of consumption in every period as a function of Γ:

$$C_{A,x} : C_{A,x}^{**} = \frac{(1-\theta)\,\mu\,\delta^{x}}{\Gamma R^{-x}} \tag{IA.8}$$

$$C_{B,x} : C_{B,x}^{**} = \frac{\theta\mu\,\delta^{x}}{\Gamma R^{-x}} \tag{IA.9}$$

$$H_{x} : H_{x}^{**} = \frac{(1-\mu)\,\delta^{x}}{\Gamma R^{-x}} \tag{IA.10}$$

where $x \in \{1,2,...,T\}$ and "**" indicates solution to the full commitment problem. The optimal level of total expenditure in any period is

$$X_{x}^{**} = \frac{\delta^{x}}{\Gamma R^{-x}}. \tag{IA.11}$$

Since the optimal allocation will exhaust the household budget constraint it must be that

$$W_0 = \sum_{x=1}^{T} \frac{X_x^{**}}{R^x} = \frac{1}{\Gamma}\left[\sum_{x=1}^{T} \delta^{x}\right] \tag{IA.12}$$

which implies

$$\Gamma^{**} = \frac{\sum_{x=1}^{T} \delta^{x}}{W_0}. \tag{IA.13}$$

Combining (IA.13) with (IA.11) gives

$$X_t^{**} = \frac{(R\delta)^{t}}{\sum_{x=1}^{T} \delta^{x}} W_0. \tag{IA.14}$$

Under the full commitment allocation household wealth evolves as

$$W_t = R^t W_0 - \sum_{x=1}^{t-1} R^{t-x} X_x^{**} = R^t W_0 \left[\frac{\sum_{x=t}^{T} \delta^{x}}{\sum_{x=1}^{T} \delta^{x}}\right] \tag{IA.15}$$

and so

$$R^t W_0 = W_t \left(\frac{\sum_{x=1}^{T} \delta^{x}}{\sum_{x=t}^{T} \delta^{x}}\right). \tag{IA.16}$$

Hence X_t^{**} can be re-written as

$$X_t^{**} = \frac{1}{\sum_{x=0}^{T-t} \delta^{x}} W_t. \tag{IA.17}$$

This fully describes the total level of consumption each period under full commitment. The optimal levels of $C_{A,t}^{**}$, $C_{B,t}^{**}$, and H_t^{**} follow immediately by using (IA.8), (IA.9), and (IA.10) to get the following constant consumption shares within each period:

$$\frac{C^{**}_{A,t}}{X^{**}_t} = (1-\theta)\,\mu, \tag{IA.18}$$

$$\frac{C^{**}_{B,t}}{X^{**}_t} = \theta\mu, \tag{IA.19}$$

$$\frac{H^{**}_t}{X^{**}_t} = 1-\mu. \tag{IA.20}$$

Note that the optimal solution satisfies (IA.3).

B Equilibrium Choices without Commitment

B-1 Equilibrium at $t = T$

In the final period $t = T$ member i takes $C_{j,T}$ and $H_{j,T}$ as given and solves the following problem:

$$\max_{C_{i,T},H_{i,T}} \frac{1+\Delta}{2}\left[\mu \ln C_{i,T} + (1-\mu)\ln\left(H_{i,T}+H_{j,T}\right)\right] \tag{IA.21}$$

$$+\frac{1-\Delta}{2}\left[\mu \ln C_{j,T} + (1-\mu)\ln\left(H_{i,T}+H_{j,T}\right)\right]$$

subject to

$$\frac{W_T}{2} - C_{i,T} - H_{i,T} \geq 0 \text{ and} \tag{IA.22}$$

$$C_{i,T}, H_{i,T} \geq 0. \tag{IA.23}$$

Since (IA.21) is strictly increasing in $C_{i,T}$ and $H_{i,T}$ it follows that (IA.22) will bind with equality and hence can be substituted into the objective. Ignoring terms which i takes as given I can rewrite her problem as

$$\max_{H_{i,T}} \frac{1+\Delta}{2}\mu \ln\left(\frac{W_T}{2} - H_{i,T}\right) + (1-\mu)\ln\left(H_{i,T}+H_{j,T}\right) \tag{IA.24}$$

subject to

$$\frac{W_T}{2} - H_{i,T} \geq 0 \text{ and} \tag{IA.25}$$

$$H_{i,T} \geq 0. \tag{IA.26}$$

Start by ignoring the boundary conditions (IA.25) and (IA.26) on $H_{i,T}$. The first order condition for the unconstrained problem rearranges to give:

$$H_{i,T} = \frac{(1-\mu)\,W_T - (1+\Delta)\,\mu H_{j,T}}{2-\mu\,(1-\Delta)}. \tag{IA.27}$$

Since the objective is strictly concave in $H_{i,T}$, using the boundary conditions (IA.25) and (IA.26) on $H_{i,T}$ gives that i's unique best response to any possible choice of $H_{j,T} \geq 0$ is

$$H_{i,T}^{BR}\left(H_{j,T}\right) = \left\{ \begin{array}{cc} b_T \frac{W_T}{2} - m_T H_{j,T} & \text{if } H_{j,T} \leq \frac{1-\mu}{\mu}\frac{W_T}{1+\Delta} \\ 0 & \text{if } H_{j,T} > \frac{1-\mu}{\mu}\frac{W_T}{1-\Delta} \end{array} \right\} \tag{IA.28}$$

$$\text{where } b_T \equiv \frac{2(1-\mu)}{2-\mu(1-\Delta)} > 0, \text{ and} \tag{IA.29}$$

$$m_T \equiv \frac{(1+\Delta)\mu}{2-\mu(1-\Delta)} \in (0,1). \tag{IA.30}$$

Note that $H_{i,T}^{BR}\left(H_{j,T}\right)$ is weakly decreasing and hence the most that i will spend on public consumption is

$$H_{i,T}^{BR}(0) = \frac{1-\mu}{2-\mu(1-\Delta)} W_T \tag{IA.31}$$

which is strictly less than the upper bound $\frac{W_T}{2}$ since $\Delta \geq 0$. Thus (IA.25) can be ignored. Note that $H_{i,T}^{BR}(0) > 0$ and hence $H_{A,T} = H_{B,T} = 0$ cannot be a Nash equilibrium. By an argument of symmetry (or formally, since $m_T \leq 1 \leq \frac{1}{m_T}$) there must be an interior Nash equilibrium. This is found by substituting the interior portion of j's reaction function into the reaction function of i:

$$H_{i,T}^{*} = \frac{b_T}{1+m_T}\frac{W_T}{2}. \tag{IA.32}$$

To total expenditure on public consumption is

$$H_T^{*} = \frac{b_T}{1+m_T} W_T. \tag{IA.33}$$

The equilibrium level of private consumption in this interior solution is

$$C_{i,T}^{*} = \left(1 - \frac{b_T}{1+m_T}\right)\frac{W_T}{2}. \tag{IA.34}$$

Thus the equilibrium value of member i's objective function is

$$V_{i,T} = \ln W_T + k_T, \tag{IA.35}$$

where k_T is a constant term defined as

$$k_T \equiv \mu \ln\left(1 - \frac{b_T}{1+m_T}\right) + (1-\mu)\ln\left(\frac{b_T}{1+m_T}\right) - \mu \ln 2. \tag{IA.36}$$

B-2 Solve for Sub-game Perfect Consumption Path by Induction

I conjecture the following form for the subgame perfect household allocation and confirm it by an argument of induction below.

Conjecture 1. *The subgame perfect equilibrium household allocation from t until NT is proportional to W_t. That is, for any period $t \in \{1, ..., NT\}$ the subgame perfect equilibrium levels of private and public consumption can be written as $C^*_{i,t+x} = g_{t+x}W_t$ and $H^*_{t+x} = h_{t+x}W_t$ for $x \in \{0, 1, ..., NT - t\}$ where g_{t+x} and h_{t+x} are strictly positive constants independent of W_t.*

Consider the problem that each household member faces in period $t < T$. Member i takes $C_{j,t}$ and $H_{j,t}$ as given and solves the following:

$$\max_{C_{i,t},H_{i,t}} \frac{1+\Delta}{2}\mu \ln C_{i,t} + (1-\mu)\ln\left(H_{i,t}+H_{j,t}\right) + \frac{1-\Delta}{2}\mu \ln C_{j,t} \tag{IA.37}$$
$$+\sum_{x=1}^{T-t}\delta^x\left[\frac{1+\Delta}{2}\mu \ln C^*_{i,t+x} + \frac{1-\Delta}{2}\mu \ln C^*_{j,t+x} + (1-\mu)\ln\left(H^*_{t+x}\right)\right]$$

subject to

$$W_{t+1} = R\left(W_t - C_{i,t} - C_{j,t} - H_{i,t} - H_{j,t}\right), \tag{IA.38}$$
$$\frac{W_t}{2} - C_{i,t} - H_{i,t} \geq 0, \tag{IA.39}$$
$$C_{i,t} \geq 0, \text{ and} \tag{IA.40}$$
$$H_{i,t} \geq 0. \tag{IA.41}$$

Conjecture 1 implies that

$$\sum_{x=1}^{T-t}\delta^x\left[\frac{1+\Delta}{2}\mu \ln C^*_{i,t+x} + \frac{1-\Delta}{2}\mu \ln C^*_{j,t+x} + (1-\mu)\ln\left(H^*_{t+x}\right)\right] = Z_{t+1}\ln W_{t+1} + k_t \tag{IA.42}$$
$$\text{where } Z_{t+1} \equiv \sum_{x=1}^{T-t}\delta^x \tag{IA.43}$$

and k_t is a constant. In equilibrium the budget constraint will bind. Log utility will ensure $C^*_{i,t} > 0$ in equilibrium and hence (IA.40) can be ignored for now and verified later. Ignoring terms that i takes as given in t and substituting (IA.38) into the objective, i's problem can be rewritten as

$$\max_{C_{i,t},H_{i,t}} \frac{1+\Delta}{2}\mu \ln C_{i,t} + (1-\mu)\ln\left(H_{i,t}+H_{j,t}\right) \tag{IA.44}$$
$$+Z_{t+1}\ln\left(W_t - C_{i,t} - C_{j,t} - H_{i,t} - H_{j,t}\right)$$

subject to

$$\frac{W_t}{2} - C_{i,t} - H_{i,t} \geq 0 \text{ and} \tag{IA.45}$$
$$H_{i,t} \geq 0. \tag{IA.46}$$

Start by ignoring (IA.45) and (IA.46). The first order conditions for the unconstrained problem are

$$C_{i,t}: \frac{(1+\Delta)\mu}{2C_{i,t}} - \frac{Z_{t+1}}{W_t - C_{i,t} - C_{j,t} - H_{i,t} - H_{j,t}} = 0 \tag{IA.47}$$

$$H_{i,t}: \frac{1-\mu}{H_{i,t}+H_{j,t}} - \frac{Z_{t+1}}{W_t - C_{i,t} - C_{j,t} - H_{i,t} - H_{j,t}} = 0 \tag{IA.48}$$

The first order condition for $H_{i,t}$ implies that

$$H_t = H_{i,t} + H_{j,t} = \frac{1-\mu}{1-\mu+Z_{t+1}} \left[W_t - C_{i,t} - C_{j,t}\right]. \tag{IA.49}$$

Hence for any given level of W_t, $C_{i,t}$, and $C_{j,t}$ both members agree on the optimal level of H_t. Since it is funded jointly they are indifferent as to who pays for it. Equation (IA.47) implies

$$C_{i,t} = g_t \left[W_t - C_{j,t} - H_t\right]. \tag{IA.50}$$

$$\text{where } g_t \equiv \frac{(1+\Delta)\mu}{2Z_{t+1} + (1+\Delta)\mu} \in (0,1). \tag{IA.51}$$

Substituting j's analog of (IA.50) into ((IA.50)) gives

$$C_{i,t} = \frac{g_t}{1-g_t}\left[W_t - H_t\right]. \tag{IA.52}$$

Combining (IA.49) and (IA.52) gives the equilibrium level of public consumption

$$H_t^* = \left(\frac{1-\mu}{1+\mu\Delta+\sum_{x=1}^{T-t}\delta^x}\right) W_t. \tag{IA.53}$$

Combining (IA.52) and (IA.53) gives the equilibrium level of private consumption for each member

$$C_{i,t}^* = \left(\frac{\left(\frac{1+\Delta}{2}\right)\mu}{1+\mu\Delta+\sum_{x=1}^{T-t}\delta^x}\right) W_t. \tag{IA.54}$$

Equilibrium total expenditure is thus

$$X_t^* = H_t^* + C_{A,t}^* + C_{B,t}^* = \left(\frac{1}{1+\frac{1}{1+\mu\Delta}\sum_{x=1}^{T-t}\delta^x}\right) W_t. \tag{IA.55}$$

These solutions were derived for the unconstrained problem ignoring (IA.45) and (IA.40). The expression for $C_{i,t}^*$ in (IA.54) demonstrates that (IA.40) is slack. It just remains to show that (IA.45) is not violated for either household member. Note that $X_t^* < W_t$ for any $t < T$ and hence, by an argument of symmetry, (IA.45) must be satisfied. Thus, conditional on Conjecture 1 being true (IA.53), (IA.54) and (IA.55) are the unique subgame perfect equilibrium consumption choices.

The final step of the derivation is to prove Conjecture 1 by induction. As the first step, note that Conjecture 1 is verified for $t = T$ above. Next observe that (IA.53) and (IA.54) give equilibrium consumption levels that are proportional to W_t. Observe also that using (IA.55) I can compute W_{t+1}

as

$$W_{t+1} = R\left(\frac{\frac{1}{1+\mu\Delta}\sum_{x=1}^{T-t}\delta^x}{1+\frac{1}{1+\mu\Delta}\sum_{x=1}^{T-t}\delta^x}\right)W_t, \tag{IA.56}$$

which is also proportional to W_t. By extension of (IA.53) and (IA.54) this implies that $H^*_{t+1}, C^*_{A,t+1}, C^*_{B,t+1}$ are also proportional to W_t. The same argument applies for any period $x > t$. Hence this establishes Conjecture 1 by induction.

C Household Value of Commitment

The value function for member i at $t = 0$ evaluated at the non-cooperative consumption path is

$$V^*_{i,0} = \sum_{t=1}^{T}\delta^t\left\{\frac{1+\Delta}{2}\mu\ln C^*_{i,t} + \frac{1-\Delta}{2}\mu\ln C^*_{j,t} + (1-\mu)\ln H^*_t\right\}. \tag{IA.57}$$

Using (IA.53), (IA.54), and (IA.55) I re-write (IA.57) as

$$V^*_{i,0} = \sum_{t=1}^{T}\delta^t\left[\mu\left(\ln\left(\frac{1+\Delta}{2}\right)+\ln\left(\frac{\mu}{1-\mu}\right)\right)+\ln\left(\frac{1-\mu}{1+\Delta\mu}\right)\right]+\sum_{t=1}^{T}\delta^t\ln X^*_t. \tag{IA.58}$$

Similarly, the value function for each member at $t = 0$ evaluated at the full commitment consumption path using (IA.14), (IA.18), (IA.19), and (IA.20) is

$$V^{**}_{i,0} = \sum_{t=1}^{T}\delta^t\left[\mu\ln\left(\frac{\mu(1-\theta_i)}{1-\mu}\right)+\left(\frac{1+\Delta}{2}\right)\mu\ln\left(\frac{\theta_i}{1-\theta_i}\right)+\ln(1-\mu)\right]+\sum_{t=1}^{T}\delta^t\ln X^{**}_t \tag{IA.59}$$

where $\theta_A = 1-\theta$ and $\theta_B = \theta$. Note that $V^{**}_{A,0}$ ($V^{**}_{B,0}$) is strictly increasing (decreasing) in η while $\eta \in [0,1]$. Note also that $V^*_{i,0}$ is independent of η.

The value of η that solves (**??**) is found by setting $V^{**}_{A,0} - V^*_{A,0} = V^{**}_{B,0} - V^*_{B,0}$. To see this, suppose that η were such that $V^{**}_{A,0} - V^*_{A,0} > V^{**}_{B,0} - V^*_{B,0}$. Then only the constraint $V^{**}_{B,0} \geq V^*_{B,0}$ would bind and η could be reduced to relax this constraint. This is achieved when

$$\eta = \theta = \frac{1}{2}. \tag{IA.60}$$

Since η ensures $V^{**}_{A,0} - V^*_{A,0} = V^{**}_{B,0} - V^*_{B,0}$ I can focus on the value of ϕ that sets $V^{**}_{A,0}(W_0(1-\phi)) = V^*_{A,0}(W_0)$. It follows from (IA.14) and (IA.59) that

$$V^{**}_{A,0}(W_0(1-\phi)) = V^{**}_{A,0}(W_0) + \left(\sum_{t=1}^{T}\delta^t\right)\ln(1-\phi) \tag{IA.61}$$

and so the value of commitment must be such that

$$V^{**}_{A,0}(W_0) + \left(\sum_{t=1}^{T}\delta^t\right)\ln(1-\phi) = V^*_{A,0}(W_0). \tag{IA.62}$$

Re-writing (IA.62) using (IA.58 and (IA.59 evaluated at (IA.60) gives that the value of commitment is

$$\phi = 1 - \left(\frac{(1+\Delta)^{\mu}}{1+\Delta\mu}\right) e^{\left(\sum_{k=1}^{T}\delta^{k}\right)^{-1}\sum_{t=1}^{T}\delta^{t}\ln\left\{\frac{X_t^*}{X_t^{**}}\right\}} \tag{IA.63}$$

where, using (IA.55) and (IA.14) I have that

$$\frac{X_t^*}{X_t^{**}} = \left(\frac{1}{1+\Delta\mu}\right)^{t-1}\prod_{s=1}^{t}\left(\frac{1+\sum_{x=1}^{T-s}\delta^{x}}{1+\frac{1}{1+\Delta\mu}\sum_{x=1}^{T-s}\delta^{x}}\right). \tag{IA.64}$$

II Proofs for Section III (Durable Goods)

Here I solve the model with durable goods as outlined in Section III in the paper.

A *Equilibrium when $p \geq 1$ and $T = 2$*

At $t = 2$ both household members will spend all remaining liquid wealth on the non-durable good so that

$$C_{i,2}^* = \frac{1}{2}W_2 \text{ and } D_{i,2}^* = (1-\kappa)D_{i,1}. \tag{IA.65}$$

Let $X_{i,t} = C_{i,t} + pD_{i,t}$ be the total expenditure of member i at t. In period $t = 1$ i will take $X_{j,1}$ as given and solve the following problem

$$\max_{C_{i,1},D_{i,1}} \ln(C_{i,1}+D_{i,1}) + \delta\ln\left((1-\kappa)D_{i,1} + \frac{R}{2}\left[W_1 - C_{i,1} - pD_{i,1} - X_{j,1}\right]\right) \tag{IA.66}$$

subject to

$$\frac{W_1}{2} - C_{i,1} - pD_{i,1} \geq 0 \tag{IA.67}$$

$$C_{i,1} \geq 0 \tag{IA.68}$$

$$D_{i,1} \geq 0 \tag{IA.69}$$

The Lagrangian for this problem is

$$\max_{C_{i,1},D_{i,1}} \ln(C_{i,1}+D_{i,1}) + \delta\ln\left((1-\kappa)D_{i,1} + \frac{R}{2}\left[W_1 - C_{i,1} - pD_{i,1} - X_{j,1}\right]\right) \tag{IA.70}$$

$$+\Gamma_W\left[\frac{W_1}{2} - C_{i,1} - pD_{i,1}\right] + \Gamma_C C_{i,1} + \Gamma_D D_{i,1}$$

where $\Gamma_W, \Gamma_C, \Gamma_D \geq 0$ are the multipliers associated with (IA.67), (IA.68), and (IA.69) respectively. The first order conditions which characterize the solution are

$$\frac{1}{C_{i,1}+D_{i,1}} - \frac{R\delta}{2(1-\kappa)D_{i,1}+R\left[W_1 - C_{i,1} - pD_{i,1} - X_{j,1}\right]} - \Gamma_W + \Gamma_C = 0 \tag{IA.71}$$

$$\frac{1}{C_{i,1}+D_{i,1}} - \frac{R\delta\left[p-2(1-\kappa)R^{-1}\right]}{2(1-\kappa)D_{i,1}+R\left[W_1 - C_{i,1} - pD_{i,1} - X_{j,1}\right]} - p\Gamma_W + \Gamma_D = 0. \tag{IA.72}$$

Any candidate solution must satisfy (IA.71) and (IA.72) along with (IA.67), (IA.68), and (IA.69) and obey the standard complementary slackness conditions on Γ_W, Γ_C, and Γ_D. The solution to this problem yields a best response piecewise-defined function for i defined over different ranges of the price of the durable good. Define the following cutoffs

$$\overline{p}^{*1} \equiv 1 + 2(1-\kappa)R^{-1}, \tag{IA.73}$$

$$\overline{p}^{*2} \equiv 1 + \delta, \tag{IA.74}$$

$$\overline{p}^{*3} \equiv \left(\frac{1+\delta}{\delta}\right) 2(1-\kappa)R^{-1}. \tag{IA.75}$$

If $p \geq \overline{p}^{*1}$ then

$$C_{i,1}^{BR} = \left\{ \begin{array}{ll} \frac{W_1}{2} & \text{if } X_{j,1} < (1-\delta)\frac{W_1}{2} \\ \frac{W_1 - X_{j,1}}{1+\delta} & \text{if } X_{j,1} \geq (1-\delta)\frac{W_1}{2} \end{array} \right] \text{ and } D_{i,1}^{BR} = 0. \tag{IA.76}$$

If $p \in \left(\overline{p}^{*1} - 1, \overline{p}^{*1}\right)$ then i's best response is

$$D_{i,1}^{BR} = \left\{ \begin{array}{ll} 0 & \text{if } X_{j,1} \leq \overline{X}_1^L \\ \frac{\frac{W_1}{2}\left[\delta(1-\kappa)-(p-1)\frac{R}{2}\right]+(p-1)\frac{R}{2}X_{j,1}}{(p-1)(1+\delta)(1-\kappa)} & \text{if } X_{j,1} \in \left[\overline{X}_1^L, \overline{X}_1^M\right] \\ \frac{W_1}{2p} & \text{if } X_{j,1} \in \left[\overline{X}_1^M, \overline{X}_1^H\right] \\ \frac{W_1 - X_{j,1}}{(1+\delta)\left[P-2(1-\kappa)R^{-1}\right]} & \text{if } X_{j,1} \geq \overline{X}_1^H \end{array} \right] \tag{IA.77}$$

$$C_{i,1}^{BR} = \left\{ \begin{array}{ll} \frac{W_1}{2} & \text{if } X_{j,1} \leq \overline{X}_1^L \\ \frac{\frac{W_1}{2}(1-\kappa)(p-1-\delta)+\left(\frac{W_1}{2}-X_{j,1}\right)p(p-1)\frac{R}{2}}{(p-1)(1+\delta)(1-\kappa)} & \text{if } X_{j,1} \in \left[\overline{X}_1^L, \overline{X}_1^M\right] \\ 0 & \text{if } X_{j,1} \geq \overline{X}_1^M \end{array} \right] \tag{IA.78}$$

where

$$\overline{X}_1^L \equiv \frac{W_1}{2}\left[1 - \frac{2\delta(1-\kappa)}{R(P-1)}\right], \tag{IA.79}$$

$$\overline{X}_1^M \equiv \frac{W_1}{2}\left[1 - \frac{2(1-\kappa)(\delta+1-p)}{R(P-1)p}\right], \tag{IA.80}$$

$$\overline{X}_1^H \equiv \frac{W_1}{2}\left[1 - \delta + \frac{2(1+\delta)(1-\kappa)}{Rp}\right]. \tag{IA.81}$$

Note that $p \geq 1 \Leftrightarrow \overline{X}_1^L \leq \overline{X}_1^M$ and $p \leq \overline{p}^{*1} \Leftrightarrow \overline{X}_1^M \leq \overline{X}_1^H$. Finally if $p \leq \overline{p}^{*1} - 1$ then then i's best response is

$$D_{i,1}^{BR} = \left\{ \begin{array}{cc} 0 & \text{if } X_{j,1} \leq \overline{X}_1^L \\ \frac{\frac{W_1}{2}\left[\delta(1-\kappa)-(p-1)\frac{R}{2}\right]+(p-1)\frac{R}{2}X_{j,1}}{(p-1)(1+\delta)(1-\kappa)} & \text{if } X_{j,1} \in \left[\overline{X}_1^L, \overline{X}_1^M\right] \\ \frac{W_1}{2p} & \text{if } X_{j,1} \geq \overline{X}_1^M \end{array} \right] \tag{IA.82}$$

$$C_{i,1}^{BR} = \left\{ \begin{array}{cc} \frac{W_1}{2} & \text{if } X_{j,1} \leq \overline{X}_1^L \\ \frac{\frac{W_1}{2}(1-\kappa)(p-1-\delta)+\left(\frac{W_1}{2}-X_{j,1}\right)p(p-1)\frac{R}{2}}{(p-1)(1+\delta)(1-\kappa)} & \text{if } X_{j,1} \in \left[\overline{X}_1^L, \overline{X}_1^M\right] \\ 0 & \text{if } X_{j,1} \geq \overline{X}_1^M \end{array} \right] \tag{IA.83}$$

To solve for the Nash equilibrium at $t = 1$ I can take advantage of the fact that the best response function for both members is symmetric and look for where they cross the 45 degree line. It is easy to show that in all cases the symmetric equilibrium is unique (I omit this for brevity). When $p \geq \overline{p}^{*1}$ the equilibrium at $t = 1$ is

$$C_{i,1}^* = \frac{W_1}{1+\delta} \text{ and } D_{i,1}^* = 0. \tag{IA.84}$$

Next suppose $p \leq \overline{p}^{*1}$ and $p \geq \overline{p}^{*3}$. Note that this space is non-empty if and only if $\kappa \geq 1 - \frac{R\delta}{2}$. In this case

$$C_{i,1}^* = 0 \text{ and } D_{i,1}^* = \frac{W_1}{p+(1+\delta)\left[p-2(1-\kappa)R^{-1}\right]}. \tag{IA.85}$$

Next suppose $p \leq \overline{p}^{*1}$ and $p \geq \overline{p}^{*2}$. This space is non-empty if and only if $\kappa \leq 1 - \frac{R\delta}{2}$. In this case

$$C_{i,1}^* = \left(\frac{p-1-\delta}{(p-1)(1+\delta)}\right)\frac{W_1}{2} \text{ and } D_{i,1}^* = \left(\frac{\delta}{1+\delta}\right)\left(\frac{1}{p-1}\right)\frac{W_1}{2}. \tag{IA.86}$$

Finally, suppose $p \leq \min\left\{\overline{p}^{*2}, \overline{p}^{*3}\right\}$. Note that $\overline{p}^{*2} \leq \overline{p}^{*3}$ if and only if $\kappa \leq 1 - \frac{R\delta}{2}$. In this case

$$C_{i,1}^* = 0 \text{ and } D_{i,1}^* = \frac{W_1}{2p}. \tag{IA.87}$$

B *Full Commitment when $p \geq 1$ and $T = 2$*

The full commitment solution for the household can be found by finding the optimal allocation for each member for some initial wealth allocation where ηW_1 is given to member A and $(1-\eta)W_1$ to B. Without loss of generality I focus on the problem solved by A.

Since $p \geq 1$ any wealth that is carried to $t = 2$ will be spent on the non-durable good

$$C_{A,2}^{**} = R\left[\eta W_1 - C_{A,1} - pD_{A,1}\right] \text{ and } D_{A,2}^{**} = (1-\kappa)D_{A,1} \tag{IA.88}$$

Substituting (IA.88) into A's objective gives us the problem solved by A at $t=1$:

$$\max_{C_{A,1},D_{A,1}} \ln\left(C_{A,1}+D_{A,1}\right)+\delta\ln\left((1-\kappa)D_{A,1}+R\left[\eta W_1-C_{A,1}-pD_{A,1}\right]\right) \tag{IA.89}$$

subject to

$$\eta W_1-C_{A,1}-pD_{A,1}\geq 0 \tag{IA.90}$$

$$C_{A,1}\geq 0 \tag{IA.91}$$

$$D_{A,1}\geq 0 \tag{IA.92}$$

The Lagrangian for this problem is

$$\begin{aligned}&\max_{C_{A,1},D_{A,1}} \ln\left(C_{A,1}+D_{A,1}\right)+\delta\ln\left((1-\kappa)D_{A,1}+R\left[\eta W_1-C_{A,1}-pD_{A,1}\right]\right)\\&+\Gamma_W\left[\eta W_1-C_{A,1}-pD_{A,1}\right]+\Gamma_C C_{A,1}+\Gamma_D D_{A,1}\end{aligned} \tag{IA.93}$$

where $\Gamma_W,\Gamma_C,\Gamma_D\geq 0$ are the multipliers associated with (IA.90), (IA.91), and (IA.92) respectively. The first order conditions which characterize the solution are

$$\frac{1}{C_{A,1}+D_{A,1}}-\frac{R\delta}{(1-\kappa)D_{A,1}+R\left[\eta W_1-C_{A,1}-pD_{A,1}\right]}-\Gamma_W+\Gamma_C=0 \tag{IA.94}$$

$$\frac{1}{C_{A,1}+D_{A,1}}-\frac{R\delta\left[p-(1-\kappa)R^{-1}\right]}{(1-\kappa)D_{A,1}+R\left[\eta W_1-C_{A,1}-pD_{A,1}\right]}-p\Gamma_W+\Gamma_D=0. \tag{IA.95}$$

Any candidate solution must satisfy (IA.94) and (IA.95) along with (IA.90), (IA.91), and (IA.92) and obey the standard complementary slackness conditions on Γ_W,Γ_C, and Γ_D. The solution to this problem is a piecewise-defined function defined over different ranges of the price of the durable good. Define the following cutoffs

$$\overline{p}^{**1}\equiv 1+(1-\kappa)R^{-1}, \tag{IA.96}$$

$$\overline{p}^{**2}\equiv 1+\delta, \tag{IA.97}$$

$$\overline{p}^{**3}\equiv\left(\frac{1+\delta}{\delta}\right)(1-\kappa)R^{-1}. \tag{IA.98}$$

If $p\geq\overline{p}^{**1}$ then

$$C_{A,1}^{**}=\frac{\eta W_1}{1+\delta}\text{ and }D_{A,1}^{**}=0 \tag{IA.99}$$

Next suppose that $p\leq\overline{p}^{**1}$ and $p\geq\overline{p}^{**3}$. Note that this space is non-empty if and only if $\kappa\geq 1-R\delta$. In this case

$$C_{A,1}^{**}=0\text{ and }D_{A,1}^{**}=\frac{\eta W_1}{(1+\delta)\left[p-(1-\kappa)R^{-1}\right]}. \tag{IA.100}$$

Next suppose $p\leq\overline{p}^{**1}$ and $p\geq\overline{p}^{**2}$. This space is non-empty if and only if $\kappa\leq 1-R\delta$. In this case

$$C_{A,1}^{**} = \left(\frac{p-1-\delta}{(p-1)(1+\delta)}\right)\eta W_1 \text{ and } D_{A,1}^{**} = \left(\frac{\delta}{1+\delta}\right)\left(\frac{1}{p-1}\right)\eta W_1. \tag{IA.101}$$

Finally, suppose $p \leq \min\left\{\overline{p}^{**2}, \overline{p}^{**3}\right\}$. Note that $\overline{p}^{**2} \leq \overline{p}^{**3}$ if and only if $\kappa \leq 1 - R\delta$. In this case

$$C_{A,1}^{**} = 0 \text{ and } D_{A,1}^{**} = \frac{\eta W_1}{p}. \tag{IA.102}$$

C Proof of Propositions 2 and 3

Comparing the equilibrium and full commitment allocation with the illiquid durable good when $p \geq 1$ and $T = 2$ requires comparing over four ranges of κ.

C-1 Case 1: $\kappa \leq 1 - R\delta$.

Note that $\kappa \leq 1 - R\delta$ implies $\overline{p}^{*2} \leq \overline{p}^{*1}$ and $\overline{p}^{**2} \leq \overline{p}^{**1}$. Note also that $\overline{p}^{*2} = \overline{p}^{**2}$. In equilibrium total household expenditure at $t = 1$ as a fraction of wealth is

$$\frac{X_1^*}{W_1} = \left\{ \begin{array}{cc} 1 & p \leq \overline{p}^{*1} \\ \frac{1}{1+\frac{\delta}{2}} & p \geq \overline{p}^{*1} \end{array} \right]. \tag{IA.103}$$

In the allocation with full commitment

$$\frac{X_1^{**}}{W_1} = \left\{ \begin{array}{cc} 1 & p \leq \overline{p}^{**1} \\ \frac{1}{1+\delta} & p \geq \overline{p}^{**1} \end{array} \right]. \tag{IA.104}$$

Dividing (IA.103) by (IA.104) gives

$$\frac{X_1^*}{X_1^{**}} = \left\{ \begin{array}{cc} 1 & p \leq \overline{p}^{**1} \\ 1+\delta & p \in \left[\overline{p}^{**1}, \overline{p}^{*1}\right] \\ \frac{1+\delta}{1+\frac{\delta}{2}} & p \geq \overline{p}^{*1} \end{array} \right] \tag{IA.105}$$

noting that $\kappa \leq 1$ ensures that $\overline{p}^{**1} \leq \overline{p}^{*1}$ and that $1 + \delta \geq \frac{1+\delta}{1+\frac{\delta}{2}}$.

Next consider the fraction of total expenditure at $t = 1$ that is dedicated to the durable good. In equilibrium this fraction is

$$\frac{pD_1^*}{X_1^*} = \left\{ \begin{array}{cc} 1 & p \leq \overline{p}^{*2} \\ \left(\frac{\delta}{1+\delta}\right)\left(\frac{p}{p-1}\right) & p \in \left[\overline{p}^{*2}, \overline{p}^{*1}\right] \\ 0 & p \geq \overline{p}^{*1} \end{array} \right]. \tag{IA.106}$$

In the allocation with full commitment the share of expenditure on the durable good is

$$\frac{pD_1^{**}}{X_1^{**}} = \left\{ \begin{array}{cc} 1 & p \leq \overline{p}^{**2} \\ \left(\frac{\delta}{1+\delta}\right)\left(\frac{p}{p-1}\right) & p \in \left[\overline{p}^{**2}, \overline{p}^{**1}\right] \\ 0 & p \geq \overline{p}^{**1} \end{array} \right] . \tag{IA.107}$$

Subtracting (IA.107) from (IA.106) gives

$$\frac{pD_1^{*}}{X_1^{*}} - \frac{pD_1^{**}}{X_1^{**}} = \left\{ \begin{array}{cc} 0 & p \leq \overline{p}^{**1} \\ \left(\frac{\delta}{1+\delta}\right)\left(\frac{p}{p-1}\right) & p \in \left[\overline{p}^{**1}, \overline{p}^{*1}\right] \\ 0 & p \geq \overline{p}^{**1} \end{array} \right] . \tag{IA.108}$$

C-2 Case 2: $\kappa \in \left[1 - R\delta, 1 - \frac{R\delta}{2}\right]$

Note that $\kappa \in \left[1 - R\delta, 1 - \frac{R\delta}{2}\right]$ implies the following ordering $\overline{p}^{**3} \leq \overline{p}^{**1} \leq \overline{p}^{**2} = \overline{p}^{*2} \leq \overline{p}^{*1}$. In equilibrium total household expenditure at $t = 1$ as a fraction of wealth is the same as in (IA.103). In the allocation with full commitment it is

$$\frac{X_1^{**}}{W_1} = \left\{ \begin{array}{cc} 1 & p \leq \overline{p}^{**3} \\ \frac{p}{(1+\delta)\left[p-(1-\kappa)R^{-1}\right]} & p \in \left[\overline{p}^{**3}, \overline{p}^{**1}\right] \\ \frac{1}{1+\delta} & p \geq \overline{p}^{**1} \end{array} \right] \tag{IA.109}$$

Dividing (IA.103) by (IA.109) gives

$$\frac{X_1^{*}}{X_1^{**}} = \left\{ \begin{array}{cc} 1 & p \leq \overline{p}^{**3} \\ 1+\delta - \delta\frac{\overline{p}^{**3}}{p} & p \in \left[\overline{p}^{**3}, \overline{p}^{**1}\right] \\ 1+\delta & p \in \left[\overline{p}^{**1}, \overline{p}^{*1}\right] \\ \frac{1+\delta}{1+\frac{\delta}{2}} & p \geq \overline{p}^{*1} \end{array} \right] \tag{IA.110}$$

noting the following inequalities:

$$p \in \left[\overline{p}^{**3}, \overline{p}^{**1}\right] \Longrightarrow 1+\delta - \delta\frac{\overline{p}^{**3}}{p} \geq 1 \tag{IA.111}$$

$$1+\delta \geq \frac{1+\delta}{1+\frac{\delta}{2}}. \tag{IA.112}$$

Next consider the fraction of total expenditure at $t = 1$ that is dedicated to the durable good. In equilibrium this is the same as in (IA.106). In the allocation with full commitment the share of expenditure on the durable good is

$$\frac{pD_1^{**}}{X_1^{**}} = \left\{ \begin{array}{cc} 1 & p \leq \overline{p}^{**1} \\ 0 & p \geq \overline{p}^{**1} \end{array} \right] . \tag{IA.113}$$

Subtracting (IA.113) from (IA.106) gives

$$\frac{pD_1^*}{X_1^*} - \frac{pD_1^{**}}{X_1^{**}} = \left\{ \begin{array}{cc} 0 & p \leq \overline{p}^{**1} \\ 1 & p \in \left[\overline{p}^{**1}, \overline{p}^{*2}\right] \\ \left(\frac{\delta}{1+\delta}\right)\left(\frac{p}{p-1}\right) & p \in \left[\overline{p}^{*2}, \overline{p}^{*1}\right] \\ 0 & p \geq \overline{p}^{*1} \end{array} \right] . \tag{IA.114}$$

C-3 Case 3: $\kappa \in \left[1 - \frac{R\delta}{2}, 1 - \frac{R\delta}{2+\delta}\right]$

Note that $\kappa \in \left[1 - \frac{R\delta}{2}, 1 - \frac{R\delta}{2+\delta}\right]$ implies the following ordering $\overline{p}^{**3} \leq 1 \leq \overline{p}^{**1} \leq \overline{p}^{*3} \leq \overline{p}^{*1} \leq \overline{p}^{**2} = \overline{p}^{*2}$. Since $p \geq 1$ this implies that $p \geq \overline{p}^{**3}$ in this region. In equilibrium total household expenditure at $t = 1$ as a fraction of wealth is

$$\frac{X_1^*}{W_1} = \left\{ \begin{array}{cc} 1 & p \leq \overline{p}^{*3} \\ \frac{2p}{p+(1+\delta)\left[p-2(1-\kappa)R^{-1}\right]} & p \in \left[\overline{p}^{*3}, \overline{p}^{*1}\right] \\ \frac{1}{1+\frac{\delta}{2}} & p \geq \overline{p}^{*1} \end{array} \right] . \tag{IA.115}$$

In the allocation with full commitment total household expenditure at $t = 1$ as a fraction of wealth is

$$\frac{X_1^{**}}{W_1} = \left\{ \begin{array}{cc} \frac{p}{(1+\delta)\left[p-(1-\kappa)R^{-1}\right]} & p \leq \overline{p}^{**1} \\ \frac{1}{1+\delta} & p \geq \overline{p}^{**1} \end{array} \right] . \tag{IA.116}$$

Dividing (IA.115) by (IA.116) gives

$$\frac{X_1^*}{X_1^{**}} = \left\{ \begin{array}{cc} 1+\delta-\delta\frac{\overline{p}^{**3}}{p} & p \leq \overline{p}^{**1} \\ 1+\delta & p \in \left[\overline{p}^{**1}, \overline{p}^{*3}\right] \\ \frac{1+\delta}{1+\frac{\delta}{2}\left(1-\frac{\overline{p}^{*3}}{p}\right)} & p \in \left[\overline{p}^{*3}, \overline{p}^{*1}\right] \\ \frac{1+\delta}{1+\frac{\delta}{2}} & p \geq \overline{p}^{*1} \end{array} \right] \tag{IA.117}$$

noting the following inequalities:

$$p \in \left[\overline{p}^{**3}, \overline{p}^{**1}\right] \Longrightarrow 1+\delta-\delta\frac{\overline{p}^{**3}}{p} \geq 1 \tag{IA.118}$$

$$1+\delta \geq \frac{1+\delta}{1+\frac{\delta}{2}} \tag{IA.119}$$

$$p \in \left[\overline{p}^{*3}, \overline{p}^{*1}\right] \Longrightarrow \frac{1+\delta}{1+\frac{\delta}{2}\left(1-\frac{\overline{p}^{*3}}{p}\right)} > \frac{1+\delta}{1+\frac{\delta}{2}}. \tag{IA.120}$$

Next consider the fraction of total expenditure at $t = 1$ that is dedicated to the durable good. In equilibrium this fraction is

$$\frac{pD_1^*}{X_1^*} = \left\{ \begin{array}{ll} 1 & p \leq \overline{p}^{*1} \\ 0 & p \geq \overline{p}^{*1} \end{array} \right] . \tag{IA.121}$$

In the allocation with full commitment the share of expenditure on the durable good is the same as in (IA.113). Subtracting (IA.113) from (IA.121) gives

$$\frac{pD_1^*}{X_1^*} - \frac{pD_1^{**}}{X_1^{**}} = \begin{array}{cc} 0 & p \leq \overline{p}^{**1} \\ 1 & p \in \left[\overline{p}^{**1}, \overline{p}^{*1}\right] \\ 0 & p \geq \overline{p}^{*1} \end{array} . \tag{IA.122}$$

C-4 Case 4: $\kappa \geq 1 - \frac{R\delta}{2+\delta}$

Note that $\kappa \geq 1 - \frac{R\delta}{2+\delta}$ implies the following ordering $\overline{p}^{**3} \leq 1 \leq \overline{p}^{*3} \leq \overline{p}^{**1} \leq \overline{p}^{*1} \leq \overline{p}^{**2} = \overline{p}^{*2}$. Since $p \geq 1$ this implies that $p \geq \overline{p}^{**3}$ in this region. In equilibrium total household expenditure at $t = 1$ as a fraction of wealth is given by (IA.115) and with full commitment is given by (IA.116). In this case, dividing (IA.115) by (IA.116) gives

$$\frac{X_1^*}{X_1^{**}} = \left\{ \begin{array}{cc} 1 + \delta - \frac{\delta}{2}\frac{\overline{p}^{*3}}{p} & p \leq \overline{p}^{*3} \\ \frac{(1+\delta) - \frac{\delta}{2}\frac{\overline{p}^{*3}}{p}}{1+\frac{\delta}{2}\left(1-\frac{\overline{p}^{*3}}{p}\right)} & p \in \left[\overline{p}^{*3}, \overline{p}^{**1}\right] \\ \frac{1+\delta}{1+\frac{\delta}{2}\left(1-\frac{\overline{p}^{*3}}{p}\right)} & p \in \left[\overline{p}^{**1}, \overline{p}^{*1}\right] \\ \frac{1+\delta}{1+\frac{\delta}{2}} & p \geq \overline{p}^{*1} \end{array} \right] . \tag{IA.123}$$

noting the following inequalities:

$$p \in \left[1, \overline{p}^{*3}\right] \Longrightarrow 1 + \delta \geq 1 + \delta - \frac{\delta}{2}\frac{\overline{p}^{*3}}{p} > \frac{1+\delta}{1+\frac{\delta}{2}} \tag{IA.124}$$

$$p \in \left[\overline{p}^{*3}, \overline{p}^{**1}\right] \Longrightarrow \frac{(1+\delta) - \frac{\delta}{2}\frac{\overline{p}^{*3}}{p}}{1+\frac{\delta}{2}\left(1-\frac{\overline{p}^{*3}}{p}\right)} \geq 1 + \frac{\delta}{2} \geq \frac{1+\delta}{1+\frac{\delta}{2}} \tag{IA.125}$$

$$p \in \left[\overline{p}^{**1}, \overline{p}^{*1}\right] \Longrightarrow \frac{1+\delta}{1+\frac{\delta}{2}\left(1-\frac{\overline{p}^{*3}}{p}\right)} > \frac{1+\delta}{1+\frac{\delta}{2}} \tag{IA.126}$$

Analysis of the share of expenditure at $t = 1$ that goes to the durable good is the same as in Case 3 above (both in equilibrium and with commitment).

D *Equilibrium when $p = 1$ and $T \geq 2$*

When $p = 1$ household members will optimally set $C_{i,t} = 0$ each period. What remains is to study the equilibrium choice of durable consumption. I solve the model by backward induction.

D-1 Equilibrium at T

In the final period each household member will spend all available wealth on the durable good. Hence

$$D_{i,T}^{*}=\frac{W_T}{2}+(1-\kappa)D_{i,T-1}. \tag{IA.127}$$

It follows that the value function of member i when entering T is

$$U_{i,T}=\ln\left[\frac{R}{2}\left[\Lambda_{T-1}-D_{i,T-1}-D_{j,T-1}\right]+(1-\kappa)D_{i,T-1}\right]. \tag{IA.128}$$

D-2 Case I: $t\geq k^{*}$

I start by conjecturing that in any subgame perfect equilibrium both members will spend all remaining wealth on the durable good if κ is small enough.

Conjecture 2. *The unique subgame perfect equilibrium from k until T is for both members to set*

$$D_{i,l}^{*}=\frac{W_l}{2}+(1-\kappa)D_{i,l-1} \tag{IA.129}$$

where $l=k,k+1,..,T$ *if*

$$\kappa\leq\overline{\kappa}_k\equiv 1-\frac{R}{2}\left[\frac{\sum_{x=1}^{T-k}\delta^x}{\sum_{x=0}^{T-k}\delta^x}\right]. \tag{IA.130}$$

I now prove Conjecture 2 by induction. Note that when $k=T$ that (IA.130) simply requires $\kappa\leq 1$ which holds by assumption. The analysis of the final period verifies that Conjecture 6 holds when $k=T$.

Next, assume that Conjecture 2 holds for k and I will use this to establish it for $k-1$. By this assumption that value function of member i at k is

$$U_{i,k}=\left(\sum_{x=0}^{T-k}\delta^x\right)\ln\left[\frac{W_k}{2}+(1-\kappa)D_{i,k-1}\right]+\left(\sum_{x=1}^{T-k}x\delta^x\right)\ln(1-\kappa) \tag{IA.131}$$

Using (IA.131) I can write the problem solved by member i in period $k-1$ as

$$\max_{D_{i,k-1}}\ln D_{i,k-1}+\delta\left(\sum_{x=0}^{T-k}\delta^x\right)\ln\left[\Lambda_{k-1}-D_{j,k-1}+\left[2(1-\kappa)R^{-1}-1\right]D_{i,k-1}\right] \tag{IA.132}$$

$$+\delta\left(\sum_{x=0}^{T-k}\delta^x\right)\ln\frac{R}{2}+\delta\left(\sum_{x=1}^{T-k}\delta^x\right)\ln(1-\kappa)$$

subject to

$$D_{i,k-1}\geq(1-\kappa)D_{i,k-2} \tag{IA.133}$$

$$\frac{W_{k-1}}{2}+(1-\kappa)D_{i,k-2}\geq D_{i,k-1} \tag{IA.134}$$

If $\kappa \leq 1-\frac{R}{2}$ the objective (IA.132) is strictly increasing in $D_{i,k-1}$ and hence the solution must be to set $D_{i,k-1}$ as (IA.134) will allow. Thus if $\kappa \leq 1-\frac{R}{2}$ the equilibrium choice of $D_{i,k-1}$ is

$$D^*_{i,k-1} = \frac{W_{k-1}}{2} + (1-\kappa) D_{i,k-2}. \tag{IA.135}$$

Suppose instead that $\kappa > 1-\frac{R}{2}$. The Lagrangian for member i's problem at t with $\Gamma_{D,k-1} \geq 0$ and $\Gamma_{W,k-1} \geq 0$ as the multipliers on the two constraints is

$$\begin{aligned}\max_{D_{i,k-1}} & \ln D_{i,k-1} + \delta \left(\sum_{x=0}^{T-k} \delta^x \right) \ln \left[\Lambda_{k-1} - D_{j,k-1} + \left[2(1-\kappa) R^{-1} - 1 \right] D_{i,k-1} \right] \\ & + \Gamma_{D,k-1} \left[D_{i,k-1} - (1-\kappa) D_{i,k-2} \right] \\ & + \Gamma_{W,k-1} \left[\frac{W_{k-1}}{2} + (1-\kappa) D_{i,k-2} - D_{i,k-1} \right]. \end{aligned} \tag{IA.136}$$

The first order condition with respect to $D_{i,t-1}$ is

$$\frac{1}{D_{i,k-1}} + \frac{\delta \left(\sum_{x=0}^{T-k} \delta^x \right) \left[2(1-\kappa) R^{-1} - 1 \right]}{\Lambda_{k-1} - D_{j,k-1} + \left[2(1-\kappa) R^{-1} - 1 \right] D_{i,k-1}} + \Gamma_{D,k-1} - \Gamma_{W,k-1} = 0. \tag{IA.137}$$

Using (IA.137) along with the standard Kuhn-Tucker conditions gives the following best response function for $D_{i,k-1}$:

$$D^{BR}_{i,k-1} = \left\{ \begin{array}{cc} \frac{W_{k-1}}{2} + (1-\kappa) D_{i,k-2} & \text{if } D_{j,k-1} \leq \rho_{i,k-1} - \rho'_{i,k-1} \\ \frac{\Lambda_{k-1} - D_{j,k-1}}{\left(\sum_{x=0}^{T-(k-1)} \delta^x \right) \left[1 - 2(1-\kappa) R^{-1} \right]} & \text{if } D_{j,k-1} \in \left[\rho_{i,k-1} - \rho'_{i,k-1}, \rho_{i,k-1} \right] \\ (1-\kappa) D_{i,k-2} & \text{if } D_{j,k-1} \geq \rho_{i,k-1} \end{array} \right] \tag{IA.138}$$

where

$$\rho'_{i,k-1} \equiv \left(\sum_{x=0}^{T-(k-1)} \delta^x \right) \left[1 - 2(1-\kappa) R^{-1} \right] \frac{W_{k-1}}{2} \tag{IA.139}$$

$$\rho_{i,k-1} \equiv \Lambda_{k-1} - \left(\sum_{x=0}^{T-(k-1)} \delta^x \right) \left[1 - 2(1-\kappa) R^{-1} \right] (1-\kappa) D_{i,k-2}. \tag{IA.140}$$

Observe from (IA.138) that $D^{BR}_{i,k-1}$ is weakly decreasing in $D_{j,k-1}$. By symmetry, (IA.138) demonstrates that the largest possible value for $D_{j,k-1}$ is $D^{MAX}_{j,k-1} \equiv \frac{W_{k-1}}{2} + (1-\kappa) D_{j,k-2}$. If

$$\rho_{i,k-1} - \left(\sum_{x=0}^{T-k} \delta^x \right) \left[1 - 2(1-\kappa) R^{-1} \right] \frac{W_{k-1}}{2} \geq D^{MAX}_{j,k-1} \tag{IA.141}$$

then (IA.138) implies that i will optimally spend as much on the durable good as possible for any feasible choice of $D_{j,k-1}$. The inequality in (IA.141) can be re-written as

$$\left[\frac{W_{k-1}}{2}+(1-\kappa)D_{i,k-2}\right]\left[1-\left(\sum_{x=0}^{T-(k-1)}\delta^x\right)\left[1-2(1-\kappa)R^{-1}\right]\right]\geq 0 \tag{IA.142}$$

which holds if and only if

$$1\geq\left(\sum_{x=0}^{T-(k-1)}\delta^x\right)\left[1-2(1-\kappa)R^{-1}\right]\Longleftrightarrow\kappa\leq 1-\frac{R}{2}\left[\frac{\sum_{x=1}^{T-(k-1)}\delta^x}{\sum_{x=0}^{T-(k-1)}\delta^x}\right]. \tag{IA.143}$$

Observe that (IA.143) is the counterpart to (IA.130) for the case where $l=k-1$. Notice that $\overline{\kappa}_k$ is strictly increasing in k and so if $\kappa\leq\overline{\kappa}_k$ then it must be that $\kappa<\overline{\kappa}_{k'}$ if $k'>k$. Hence since Conjecture 6 is established for $k=T$ then it must hold for $k=T-1$ and so on by an argument of induction.

If $\kappa\leq\overline{\kappa}_1$ then Conjecture 6 implies directly that the unique subgame perfect equilibrium will have $D^*_{i,1}=\frac{W_1}{2}$ for both members and hence all liquid wealth is spent in the first period. Alternately, if $\kappa>\overline{\kappa}_1$ then $D^*_{i,1}<\frac{W_1}{2}$ and there will be some period after $t=1$ when the household will fully exhaust its liquid wealth (this could be as late as $t=T$). Define

$$k^*=\left\{\begin{array}{ll}\max\left\{\left\lceil T-\frac{\ln\left[\frac{\delta-2(1-\kappa)R^{-1}}{\delta\left[1-2(1-\kappa)R^{-1}\right]}\right]}{\ln\delta}\right\rceil,1\right\} & \text{if } \kappa>1-\frac{R\delta}{2} \\ 1 & \text{if } \kappa\leq 1-\frac{R\delta}{2}\end{array}\right] \tag{IA.144}$$

as the smallest k for which $\kappa\leq\overline{\kappa}_k$. Notice that $k^*\in\{1,2,..,T\}$. I have already fully established the subgame equilibrium for the case where $k^*=1$.

D-3 Case II: $t=k^*-1$

Now consider the case where $k^*>1$. The value function for each member in k^* will be the same as given in (IA.131) and hence I can continue with my analysis of the problem described in (IA.132), (IA.133) and (IA.134). The best response function (IA.138) applies at k^*-1. I conjecture that the equilibrium choice of D_{i,k^*-1} must be on the interior section of (IA.138) so that the best response function is

$$D^{BR}_{i,k^*-1}=\frac{\Lambda_{k^*-1}-D_{j,k^*-1}}{\left(\sum_{x=0}^{T-(k^*-1)}\delta^x\right)\left[1-2(1-\kappa)R^{-1}\right]}. \tag{IA.145}$$

Since (IA.145) is symmetric for both A and B these can be solved simultaneously to give

$$D^*_{i,k^*-1}=\frac{\Lambda_{k^*-1}}{1+\left[1-2(1-\kappa)R^{-1}\right]\left(\sum_{x=0}^{T-(k^*-1)}\delta^x\right)}. \tag{IA.146}$$

Using (IA.146) and the intertemporal budget constraint I have that W_{k^*} will be

$$W_{k^*} = R\Lambda_{k^*-1}\left[\frac{\left[1-2(1-\kappa)R^{-1}\right]\left(\sum_{x=0}^{T-(k^*-1)}\delta^x\right)-1}{1+\left[1-2(1-\kappa)R^{-1}\right]\left(\sum_{x=0}^{T-(k^*-1)}\delta^x\right)}\right] \tag{IA.147}$$

Using (IA.145), (IA.147), and (IA.128) I can write the value function for each member at the start of k^*-1 as

$$U_{i,k^*-1} = \left(\sum_{x=0}^{T-(k^*-1)}\delta^x\right)\ln\Lambda_{k^*-1} + CONST_{k^*-1} \tag{IA.148}$$

where

$$\begin{aligned} CONST_{k^*-1} \equiv & \left(\sum_{x=1}^{T-(k^*-1)}\delta^x\right)\ln\left[\frac{R}{2}\left[1-2(1-\kappa)R^{-1}\right]\left(\sum_{x=0}^{T-(k^*-1)}\delta^x\right)\right] \\ & -\left(\sum_{x=0}^{T-(k^*-1)}\delta^x\right)\ln\left[1+\left[1-2(1-\kappa)R^{-1}\right]\left(\sum_{x=0}^{T-(k^*-1)}\delta^x\right)\right] \\ & +\delta\left(\sum_{x=1}^{T-k^*}x\delta^x\right)\ln(1-\kappa) \end{aligned} \tag{IA.149}$$

D-4 Case III: $t < k^* - 1$

If $k^* \leq 2$ then my analysis so far fully describes the subgame perfect equilibrium. If however $k^* \geq 3$ then I need to characterize the equilibrium for each $t < k^*-1$. For the periods prior to k^*-1 I make the following conjecture.

Conjecture 3. *The value function of each member in periods $t \leq k^*-1$ is of the form*

$$U_{i,t} = \left(\sum_{x=0}^{T-t}\delta^x\right)\ln\Lambda_t + CONST_t \tag{IA.150}$$

I will establish Conjecture 3 by induction. Observe that (IA.148) verifies this conjecture for the case of $t = k^*-1$. I now assume that Conjecture 3 is true for $t = k$ and, assuming this, prove it is true for $t = k-1$. Ignoring the constant in the objective function, the problem solved my member i at $k-1$ is

$$\max_{D_{i,k-1}} \ln D_{i,k-1} + \left(\sum_{x=1}^{T-(k-1)}\delta^x\right)\ln\left[\Lambda_{k-1} - \left(1-(1-\kappa)R^{-1}\right)\left(D_{i,k-1}+D_{j,k-1}\right)\right] \tag{IA.151}$$

subject to

$$D_{i,k-1} \geq (1-\kappa)D_{i,k-2} \tag{IA.152}$$

$$\frac{W_{k-1}}{2} + (1-\kappa)D_{i,k-2} \geq D_{i,k-1} \tag{IA.153}$$

I conjecture that neither (IA.152) and (IA.153) will bind in equilibrium and so the best response of i is described by the standard first order condition:

$$\frac{1}{D_{i,k-1}} - \left(\sum_{x=1}^{T-(k-1)} \delta^x \right) \frac{1-(1-\kappa)R^{-1}}{\Lambda_{k-1} - (1-(1-\kappa)R^{-1})\left(D_{i,k-1} + D_{j,k-1}\right)} = 0 \tag{IA.154}$$

Observe that (IA.154) is symmetric for both members and hence I must have a symmetric equilibrium. The equilibrium level of durable consumption for both members will be

$$D^*_{i,k-1} = \frac{\Lambda_{k-1}}{(1-(1-\kappa)R^{-1})\left(1+\sum_{x=0}^{T-(k-1)} \delta^x\right)}. \tag{IA.155}$$

Using (IA.155) in (IA.151) gives

$$U_{i,k-1} = \left(\sum_{x=0}^{T-(k-1)} \delta^x \right) \ln \Lambda_{k-1} + CONST_{k-1} \tag{IA.156}$$

which establishes Conjecture 3 by an argument of induction. Thus (IA.155) combined with the intertemporal budget constraint fully characterizes the equilibrium durable consumption choice of both members for all $t < k^* - 1$.

E *Full Commitment Allocation when $p = 1$ and $T \geq 2$*

The full commitment solution for the household is identical to dividing W_0 between each member in proportion to their bargain power (ηW_0 to A and $(1-\eta)W_0$ to B) and then allowing them to decide how to spend that wealth on durable consumption over their lifetime. Therefore I characterize the full commitment allocation by finding the consumption allocation chosen by a single time-consistent representative agent with the same preferences as the individual household members.

In the final period the representative agent will optimally spend all remaining wealth on the durable good. Hence

$$D^{r*}_T = \Lambda_T. \tag{IA.157}$$

It follows that the value function of the representative agent when entering T is

$$U_{r,T} = \ln\left[\Lambda_{T-1} - \left(1-(1-\kappa)R^{-1}\right) D^r_{T-1}\right] + \ln R. \tag{IA.158}$$

Conjecture 4. *The representative agent will optimally spend all remaining wealth on the durable good ($D^{r*}_t = \Lambda_t$) in periods $t \geq k^r$ for some $k^r \in \{1,2,..,T\}$.*

Observe that Conjecture 4 is verified for $k^r = T$. I now assume it to hold for some $k \geq k^r$ and prove it by induction by studying $k-1$. The representative agent will solve the following problem at

$k-1$

$$\max_{D^r_{k-1}} \ln D^r_{k-1} + \sum_{x=1}^{T-(k-1)} \delta^x \ln\left[\Lambda_{k-1} - \left(1-(1-\kappa)R^{-1}\right) D^r_{k-1}\right] \tag{IA.159}$$

subject to

$$D^r_{k-1} \leq \Lambda_{k-1} \tag{IA.160}$$

$$D^r_{k-1} \geq (1-\kappa) D^r_{k-2} \tag{IA.161}$$

The Lagrangian for this problem is

$$\max_{D^r_{k-1}} \tag{IA.162}$$

where $\lambda_\Lambda \geq 0$ and $\lambda_D \geq 0$ are the Lagrange multipliers on the constraints (IA.160) and (IA.161). The first order condition with respect to D^r_{k-1} is

$$\frac{1}{D^r_{k-1}} - \frac{\sum_{x=1}^{T-(k-1)} \delta^x \left(1-(1-\kappa)R^{-1}\right)}{\Lambda_{k-1} - \left(1-(1-\kappa)R^{-1}\right) D^r_{k-1}} - \lambda_\Lambda + \lambda_D = 0 \tag{IA.163}$$

I have an interior solution at $k-1$ if neither (IA.160) or (IA.161) which implies $\lambda_\Lambda = \lambda_D = 0$. For now I will ignore (IA.161) and verify that it is satisfied at the proposed solution so that $\lambda_D = 0$. At an interior solution (IA.163) implies

$$D^r_{k-1} = \frac{\Lambda_{k-1}}{\left(1-(1-\kappa)R^{-1}\right)\left[1+\sum_{x=1}^{T-(k-1)} \delta^x\right]}. \tag{IA.164}$$

For (IA.160) to be satisfied at (IA.164) requires $\kappa \geq \overline{\kappa}^r_{k-1}$ where

$$\overline{\kappa}^r_t \equiv 1 - \frac{R\sum_{x=1}^{T-t} \delta^x}{1+\sum_{x=1}^{T-t} \delta^x} \tag{IA.165}$$

If $\kappa < \overline{\kappa}^r_{k-1}$ then $D^{r*}_{k-1} = \Lambda_{k-1}$. Notice that $\overline{\kappa}^r_t$ is strictly increasing in t and hence if $\kappa < \overline{\kappa}^r_{k-1}$ then $\kappa < \overline{\kappa}^r_t\ \forall t \geq k-1$. This proves immediately that if $D^{r*}_{k-1} = \Lambda_{k-1}$ then $D^{r*}_t = \Lambda_t\ \forall t \geq k-1$ which along with (IA.157) establishes Conjecture 4 by an argument of induction.

If $\kappa < \overline{\kappa}^r_{k-1}$ which implies $D^{r*}_{k-1} = \Lambda_{k-1}$ then the value function of the representative agent at the start of $k-1$ is

$$U_{r,k-1} = \left(1+\sum_{x=1}^{T-(k-1)} \delta^x\right) \ln \Lambda_{k-1} + \sum_{x=1}^{T-(k-1)} x \ln(1-\kappa). \tag{IA.166}$$

Alternately if $\kappa \geq \overline{\kappa}^r_{k-1}$ then (IA.164) holds. In this case

$$\Lambda_k = R\Lambda_{k-1}\left[\frac{\sum_{x=1}^{T-(k-1)} \delta^x}{1+\sum_{x=1}^{T-(k-1)} \delta^x}\right] \tag{IA.167}$$

and thus the value function of the representative agent at the start of $k-1$ is

$$U_{r,k-1} = \left(1 + \sum_{x=1}^{T-(k-1)} \delta^x\right) \ln \Lambda_{k-1} + CONSTANT. \tag{IA.168}$$

Since both (IA.166) and (IA.168) have the same form as the continuation value function used in (IA.159) then the analysis of $k-1$ applies to all periods $t < k-1$. To summarize the full equilibrium depending on the magnitude of κ. If $\kappa < \overline{\kappa}_1^r$ then $D_1^{r*} = RW_0$ and $D_t^{r*} = (1-\kappa) D_{t-1}^{r*}$ $\forall t \geq 2$. Note that (IA.161) is satisfied in this case. If $\kappa \geq \overline{\kappa}_1^r$ then

$$D_t^{r*} = \frac{\Lambda_t}{(1-(1-\kappa)R^{-1})\left[1+\sum_{x=1}^{T-t}\delta^x\right]} \tag{IA.169}$$

for each t where $\kappa \geq \overline{\kappa}_1^r$. Let k^{r*} denote the first period in which $\kappa < \overline{\kappa}_t^r$. Thus (IA.169) will apply for all $t < k^{r*}$, $D_{k^{r*}}^{r*} = \Lambda_{k^{r*}}$ and $D_t^{r*} = (1-\kappa) D_{t-1}^{r*}$ $\forall t \geq k^{r*}+1$. Using (IA.165) I can solve for k^{r*} by finding the smallest t for which $\kappa < \overline{\kappa}_t^r$ subject to the constraint that $k^{r*} \in \{1,2,..,T\}$. This gives

$$k^{r*} = \max\left\{\left\lceil T - \frac{\ln\left[1 - \frac{(1-\kappa)(1-\delta)}{(\kappa+R-1)\delta}\right]}{\ln \delta}\right\rceil, 1\right\}. \tag{IA.170}$$

Finally I need to check that (IA.161) is satisfied when $t < k^{r*}$. The evolution of durable consumption determined by (IA.169) and the intertemporal budget constraint is

$$\frac{D_t^{r*}}{D_{t-1}^{r*}} = \frac{R\sum_{x=1}^{T-(t-1)}\delta^x}{1+\sum_{x=1}^{T-t}\delta^x}. \tag{IA.171}$$

Thus to satisfy (IA.161) requires $\kappa \geq \widetilde{\kappa}_t^r$ where

$$\widetilde{\kappa}_t^r \equiv 1 - \frac{R\sum_{x=1}^{T-(t-1)}\delta^x}{1+\sum_{x=1}^{T-t}\delta^x}. \tag{IA.172}$$

Observe that $\widetilde{\kappa}_t^r < \overline{\kappa}_t^r$ and since by construction $\kappa \geq \overline{\kappa}_t^r$ when $t < k^{r*}$ it must be that (IA.161) is satisfied.

III Proofs for Section IV (Separate Accounts)

Many of the derivations for Section IV are contianed within the paper and hence not repeated here. Here I present derivations to support the analysis of decisions at $t = 1$ for the case where $\sigma = 0$.

A Best Response Functions

When there is no wealth shock at all the objective function of each member is a piecewise continuous function. The objective of member i at $t = 1$ is

$$U(C_{i,1}) = \left\{ \begin{array}{ll} U^{(1)}(C_{i,1}) & \text{if } C_{i,1} \leq C''_{i,1} \\ U^{(2)}(C_{i,1}) & \text{if } C_{i,1} \geq C''_{i,1} \end{array} \right] \tag{IA.173}$$

where

$$U^{(1)}(C_{i,1}) = \left\{ \begin{array}{ll} U^{(1.1)}(C_{i,1}) & \text{if } C_{i,1} \leq C'_{i,1} \\ U^{(1.2)}(C_{i,1}) & \text{if } C_{i,1} \geq C'_{i,1} \end{array} \right] \tag{IA.174}$$

$$\begin{aligned} U^{(1.1)}(C_{i,1}) &\equiv \frac{1+\Delta}{2}\ln C_{i,1} + \frac{1-\Delta}{2}\ln C_{j,1} \\ &\quad + \delta\left[\ln\left(\widetilde{W}_{i,1} + \widetilde{W}_{j,1} - C_{i,1} - C_{j,1}\right) + \ln R + v_2^{High}\right] \end{aligned} \tag{IA.175}$$

$$\begin{aligned} U^{(1.2)}(C_{i,1}) &\equiv \frac{1+\Delta}{2}\ln C_{i,1} + \frac{1-\Delta}{2}\ln C_{j,1} \\ &\quad + \delta\left[\frac{1+\Delta}{2}\ln\left(\widetilde{W}_{i,1} - C_{i,1}\right) + \frac{1-\Delta}{2}\ln\left(\widetilde{W}_{j,1} - C_{j,1}\right) + \ln R\right] \end{aligned} \tag{IA.176}$$

$$\begin{aligned} U^{(2)}(C_{i,1}) &\equiv \frac{1+\Delta}{2}\ln C_{i,1} + \frac{1-\Delta}{2}\ln C_{j,1} \\ &\quad + \delta\left[\ln\left(\widetilde{W}_{i,1} + \widetilde{W}_{j,1} - C_{i,1} - C_{j,1}\right) + \ln R + v_2^{Low}\right] \end{aligned} \tag{IA.177}$$

$$C'_{i,1} \equiv \widetilde{W}_{i,1} - \frac{1+\Delta}{1-\Delta}\left(\widetilde{W}_{j,1} - C_{j,1}\right) \tag{IA.178}$$

$$C''_{i,1} \equiv \widetilde{W}_{i,1} - \frac{1-\Delta}{1+\Delta}\left(\widetilde{W}_{j,1} - C_{j,1}\right) \tag{IA.179}$$

$$v_2^{High} \equiv \frac{1+\Delta}{2}\ln\frac{1+\Delta}{2} + \frac{1-\Delta}{2}\ln\frac{1-\Delta}{2} \tag{IA.180}$$

$$v_2^{Low} \equiv \frac{1+\Delta}{2}\ln\frac{1-\Delta}{2} + \frac{1-\Delta}{2}\ln\frac{1+\Delta}{2}. \tag{IA.181}$$

Note that $C'_{i,1} < C''_{i,1}$. The problem of member i at $t = 1$ is to take $C_{j,1}$ as given and maximize (IA.173). I ignore the non-negativity constraint on $C_{i,1}$ since it is easy to verify that these will be slack at any solution because $U(C_{i,1})$ will go to negative infinity as $C_{i,1}$ approaches zero. This implies that if $C'_{i,1} \leq 0$ then the optimal choice of $C_{i,1}$ must be greater than $C'_{i,1}$. The same applies to the case where $C''_{i,1} \leq 0$. By the same argument, if j sets $C_{j,1} \geq \widetilde{W}_{j,1}$ that i's best response must be to choose $C_{i,1} < C'_{i,1}$ anticipating the need to transfer to j at $t = 2$.

$U(C_{i,1})$ is a continuous function. Maximizing $U(C_{i,1})$ is complicated by the fact that it is not strictly concave and has a discontinuous derivative at $C''_{i,1}$. Specifically, $U^{(1)}(C_{i,1})$, $U^{(2)}(C_{i,1})$, $U^{(1.1)}(C_{i,1})$, and $U^{(1.2)}(C_{i,1})$ are all strictly concave functions. $U(C_{i,1})$ has a continuous derivative around $C'_{i,1}$. Specifically:

$$\left.\frac{\partial U^{(1.1)}(C_{i,1})}{\partial C_{i,1}}\right|_{C'_{i,1}} = \left.\frac{\partial U^{(1.2)}(C_{i,1})}{\partial C_{i,1}}\right|_{C'_{i,1}} = \frac{\frac{1+\Delta}{1-\Delta}(1+\delta)\left(\widetilde{W}_{j,1}-C_{j,1}\right)-\delta\widetilde{W}_{i,1}}{C'_{i,1}\frac{2}{1-\Delta}\left(\widetilde{W}_{j,1}-C_{j,1}\right)}. \tag{IA.182}$$

which indicates that $U^{(1)}(C_{i,1})$ is a strictly concave function with a continuous first derivative. If

$$\left.\frac{\partial U^{(1.1)}(C_{i,1})}{\partial C_{i,1}}\right|_{C'_{i,1}} \leq 0 \Longleftrightarrow C_{j,1} \geq C^{(2)}_{j,1} \equiv \widetilde{W}_{j,1} - \left(\frac{\delta}{1-\delta}\right)\left(\frac{1-\Delta}{1+\Delta}\right)\widetilde{W}_{i,1} \tag{IA.183}$$

then $U^{(1)}(C_{i,1})$ must be maximized at some $C_{i,1} \leq C'_{i,1}$. Conversely if $C_{j,1} \leq C^{(2)}_{j,1}$ then $U(C_{i,1})$ must be maximized at some $C_{i,1} > C'_{i,1}$.

Next consider the way the slope of $U(C_{i,1})$ behaves around $C''_{i,1}$:

$$\left.\frac{\partial U^{(1.2)}(C_{i,1})}{\partial C_{i,1}}\right|_{C''_{i,1}} = \frac{\frac{1+\Delta}{2}}{C''_{i,1}} - \frac{\delta\left(\frac{1+\Delta}{2}\right)^2}{\left(\frac{1-\Delta}{2}\right)\left(\widetilde{W}_{j,1}-C_{j,1}\right)} \text{ and} \tag{IA.184}$$

$$\left.\frac{\partial U^{(2)}(C_{i,1})}{\partial C_{i,1}}\right|_{C''_{i,1}} = \frac{\frac{1+\Delta}{2}}{C''_{i,1}} - \frac{\delta\left(\frac{1+\Delta}{2}\right)}{\left(\widetilde{W}_{j,1}-C_{j,1}\right)}. \tag{IA.185}$$

$\left.\frac{\partial U^{(1.2)}(C_{i,1})}{\partial C_{i,1}}\right|_{C''_{i,1}} > \left.\frac{\partial U^{(2)}(C_{i,1})}{\partial C_{i,1}}\right|_{C''_{i,1}}$ if and only if $\widetilde{W}_{j,1}-C_{j,1} > 0$ which is necessary for $U^{(1.2)}(C_{i,1})$ and $U^{(2)}(C_{i,1})$ to be relevant to the analysis. Note that since $U(C_{i,1})$ is strictly convex around $C''_{i,1}$ that this ensures that $C_{i,1} = C''_{i,1}$ can never be an optimal choice. If

$$\left.\frac{\partial U^{(1.2)}(C_{i,1})}{\partial C_{i,1}}\right|_{C''_{i,1}} \geq 0 \Longleftrightarrow C_{j,1} \leq C^{(1)}_{j,1} \equiv \widetilde{W}_{j,1} - \left(\frac{\delta}{1-\delta}\right)\left(\frac{1+\Delta}{1-\Delta}\right)\widetilde{W}_{i,1} \tag{IA.186}$$

then the unique maximum of $U(C_{i,1})$ must be at some $C_{i,1} > C''_{i,1}$ on $U^{(2)}(C_{i,1})$. Conversely if

$$\left.\frac{\partial U^{(2)}(C_{i,1})}{\partial C_{i,1}}\right|_{C''_{i,1}} \leq 0 \Longleftrightarrow C_{j,1} \geq C^{(3)}_{j,1} \equiv \widetilde{W}_{j,1} - \left(\frac{(1+\Delta)\delta}{1+\Delta+\delta(1-\Delta)}\right)\widetilde{W}_{i,1} \tag{IA.187}$$

then the unique local maximum of $U(C_{i,1})$ must be at some $C_{i,1} < C''_{i,1}$. Note that $C^{(3)}_{j,1} > C^{(1)}_{j,1}$ and $C^{(2)}_{j,1} > C^{(1)}_{j,1}$. When $C_{j,1} \in \left[C^{(1)}_{j,1}, C^{(3)}_{j,1}\right]$ then $U(C_{i,1})$ has a local maximum both above and below $C''_{i,1}$ and the optimal solution will be found by comparing the value of $U(C_{i,1})$ at each. The optimal choice of $C_{i,1}$ will jump at the value of $C_{j,1}$ where i is indifferent between the local optimum on

$U^{(1)}(C_{i,1})$ and $U^{(2)}(C_{i,1})$. Observe that

$$U^{(1.1)}(C_{i,1}) - U^{(2)}(C_{i,1}) = \delta\left[\frac{1+\Delta}{2}\ln\left(\frac{1+\Delta}{1-\Delta}\right) + \frac{1-\Delta}{2}\ln\left(\frac{1-\Delta}{1+\Delta}\right)\right] > 0 \tag{IA.188}$$

and hence the indifference point $C_{j,1}^{Jump}$ between the local maximum above and below $C''_{i,1}$ must occur between $U^{(1.2)}(C_{i,1})$ and $U^{(2)}(C_{i,1})$ and hence it must be that $C_{j,1}^{Jump} \in \left(C_{j,1}^{(1)}, \min\left\{C_{j,1}^{(2)}, C_{j,1}^{(3)}\right\}\right)$

Next, the first order conditions for $U^{(1.1)}$, $U^{(1.2)}$, and $U^{(2)}$ characterize the potential local optimum on each segment of $U(C_{i,1})$. These are:

$$\frac{\partial U^{(1.1)}(C_{i,1})}{\partial C_{i,1}} = 0 \Longleftrightarrow C_{i,1} = C_{i,1}^{BR(1.1)} \equiv \frac{1+\Delta}{1+\Delta+2\delta}\left(\widetilde{W}_{i,1} + \widetilde{W}_{j,1} - C_{j,1}\right) \tag{IA.189}$$

$$\frac{\partial U^{(1.2)}(C_{i,1})}{\partial C_{i,1}} = 0 \Longleftrightarrow C_{i,1} = C_{i,1}^{BR(1.2)} \equiv \frac{1}{1+\delta}\widetilde{W}_{i,1} \tag{IA.190}$$

$$\frac{\partial U^{(2)}(C_{i,1})}{\partial C_{i,1}} = 0 \Longleftrightarrow C_{i,1} = C_{i,1}^{BR(2)} \equiv \frac{1+\Delta}{1+\Delta+2\delta}\left(\widetilde{W}_{i,1} + \widetilde{W}_{j,1} - C_{j,1}\right) \tag{IA.191}$$

The final thing required to characterize i's best response to any $C_{j,1}$ is to find $C_{j,1}^{Jump}$: the point at which i chooses to jump from $C_{i,1}^{BR(1.2)}$ to $C_{i,1}^{BR(2)}$. This is characterized by the indifference condition $U^{(1.2)}\left(C_{i,1}^{BR(1.2)}\right) = U^{(2)}\left(C_{i,1}^{BR(2)}\right)$ which implicitly defines $C_{j,1}^{Jump}$ as the solution to the following condition:

$$\left(W_{j,1} - C_{j,1}^{Jump}\right)^{\delta\left(1-\frac{1+\Delta}{2}\right)} = \Theta\left(\widetilde{W}_{i,1} + \widetilde{W}_{j,1} - C_{j,1}^{Jump}\right)^{\left(\frac{1+\Delta}{2}+\delta\right)} \tag{IA.192}$$

$$\text{where } \Theta \equiv e^{\frac{1+\Delta}{2}\ln\left(\frac{\frac{1+\Delta}{2}}{\frac{1+\Delta}{2}+\delta}\right)+\delta\left[\ln\left(\frac{\delta}{\frac{1+\Delta}{2}+\delta}\right)+v_2^{Low}\right]-(1+\delta)\left(\frac{1+\Delta}{2}\right)\ln\left(\frac{\widetilde{W}_{i,1}}{1+\delta}\right)-\delta\frac{1+\Delta}{2}\ln\delta} \tag{IA.193}$$

No explicit analytical solution is possible.

Combining these results gives the best response function of i at $t = 1$:

$$C_{i,1}^{BR} = \left\{\begin{array}{cc} C_{i,1}^{BR(2)} & \text{if } C_{j,1} \leq C_{j,1}^{Jump} \\ C_{i,1}^{BR(1.2)} & \text{if } C_{j,1} \in \left[C_{j,1}^{Jump}, C_{j,1}^{(2)}\right] \\ C_{i,1}^{BR(1.1)} & \text{if } C_{j,1} \geq C_{j,1}^{(2)} \end{array}\right] \tag{IA.194}$$

B Equilibrium consumption choices

First I prove that an equilibrium in pure strategies always exists for any set of parameters. Two such equilibria are possible: (1) an equilibrium in which both agents anticipate that no transfers will occur at $t = 2$ and (2) an equilibrium in which agents anticipate that one will transfer wealth to the other at $t = 2$.

Note first that $C_{i,1}^{BR}$ is weakly downward sloping in $C_{j,1}$ and has a value defined for each possible

$C_{j,1} \leq \widetilde{W}_{i,1} + \widetilde{W}_{j,1}$. It is single valued at each $C_{j,1}$ apart from $C_{j,1}^{Jump}$ where it takes on two values. Next observe that the slope of $C_{i,1}^{BR}$ is always greater than -1:

$$\frac{\partial C_{i,1}^{BR}}{\partial C_{j,1}} = \begin{cases} \frac{-\frac{1+\Delta}{2}}{\frac{1+\Delta}{2}+\delta} & \text{if } C_{j,1} \notin \left[C_{j,1}^{Jump}, C_{j,1}^{(2)}\right] \\ 0 & \text{if } C_{j,1} \in \left[C_{j,1}^{Jump}, C_{j,1}^{(2)}\right] \end{cases} \tag{IA.195}$$

Also note that

$$C_{i,1}^{BR}\left(C_{j,1}=0\right) \leq \frac{\frac{1+\Delta}{2}}{\frac{1+\Delta}{2}+\delta}\left(\widetilde{W}_{i,1} + \widetilde{W}_{j,1} - C_{j,1}\right) \tag{IA.196}$$

$$C_{i,1}^{BR}\left(C_{j,1}\right) = 0 \text{ if and only if } C_{j,1} = \widetilde{W}_{i,1} + \widetilde{W}_{j,1}. \tag{IA.197}$$

This implies that there must be at least one point where the best response function of member i intersects the best response function of j. The only complication occurs when j's best response function passes through the jump in i's best response function (or vice versa). In this case two pure strategy equilibrium exist - one which anticipates a transfer and another which does not. In addition there also exists a third mixed strategy equilibrium in which j consumes $C_{j,1}^{Jump}$ and i randomizes between her two best responses to this choice. To simplify the analysis I focus on pure strategy equilibrium and assume that when two exist the household always coordinates on the one in which no future transfers are made since this is dynamically efficient. Numerical simulations indicate that generally members disagree over which of the two equilibrium they prefer (the member who anticipates receiving a transfer in the equilibrium prefers the transfer inducing equilibrium). The analysis of transfer decisions which follows is not qualitatively affected by which pure strategy equilibrium is selected as long as each member correctly anticipates which equilibrium the household will coordinate on.

Using (IA.194) I can solve for the equilibrium consumption choices in each of the two possible equilibrium. In the equilibrium in which no transfer is anticipated at at $t = 2$ the equilibrium consumption choice of member i at $t = 1$ is

$$C_{i,1}^{*,NoTransfer} = \frac{1}{1+\delta}\widetilde{W}_{i,1} \tag{IA.198}$$

Conversely, in the equilibrium in which a transfer from one agent to the other is anticipated at $t = 2$ the equilibrium consumption choice of member i at $t = 1$ is

$$C_{i,1}^{*,Transfer} = \left(\frac{\frac{1+\Delta}{2}}{1+\Delta+\delta}\right)\left(\widetilde{W}_{i,1} + \widetilde{W}_{j,1}\right) \tag{IA.199}$$

Notice that equilibrium consumption choices in the transfer equilibrium are determined only by the household's combined wealth and are therefore unaffected by any transfers that may occur between household members at the start of $t = 1$. In order for a transfer equilibrium to exist it must be that either i anticipates making a transfer to j at $t=2$ which requires

$$C_{i,1}^{*,Transfer} \leq C'_{i,1}\Big|_{C_{j,1}^{*,Transfer}} \Leftrightarrow \frac{\widetilde{W}_{i,1}}{\widetilde{W}_{j,1}} \geq \left(\frac{1+\Delta}{1-\Delta}\right)\left(\frac{1+\delta}{\delta}\right) \tag{IA.200}$$

or that j anticipates making a transfer to i at $t=2$ which requires

$$C_{j,1}^{*,Transfer} \leq C'_{j,1}\Big|_{C_{i,1}^{*,Transfer}} \Leftrightarrow \frac{\widetilde{W}_{i,1}}{\widetilde{W}_{j,1}} \leq \left(\frac{1-\Delta}{1+\Delta}\right)\left(\frac{\delta}{1+\delta}\right). \tag{IA.201}$$

It follows immediately that when

$$\frac{\widetilde{W}_{i,1}}{\widetilde{W}_{j,1}} \in \left(\left(\frac{1-\Delta}{1+\Delta}\right)\left(\frac{\delta}{1+\delta}\right), \left(\frac{1+\Delta}{1-\Delta}\right)\left(\frac{1+\delta}{\delta}\right)\right) \tag{IA.202}$$

which is a strictly non-empty set, that an equilibrium in which one member anticipates transferring wealth the other at $t=2$ does not exist and hence the unique equilibrium must be for members to choose $C_{i,1}^{*,NoTransfer}$ at $t=1$ and for no transfer to occur at $t=2$. Define $w_{i,1} \equiv \frac{\widetilde{W}_{i,1}}{\widetilde{W}_{j,1}}$. It follows that there exists two cutoffs

$$\begin{aligned} w'_{i,1} &\leq \left(\frac{1-\Delta}{1+\Delta}\right)\left(\frac{\delta}{1+\delta}\right) \text{ and} \\ w''_{i,1} &\geq \left(\frac{1+\Delta}{1-\Delta}\right)\left(\frac{1+\delta}{\delta}\right) \end{aligned} \tag{IA.203}$$

such that equilibrium consumption choices at $t=1$ are

$$C_{i,1}^{*} = \left\{ \begin{array}{cc} C_{i,1}^{*,Transfer} & \text{if } w_{i,1} \leq w'_{i,1} \\ C_{i,1}^{*,NoTransfer} & \text{if } w_{i,1} \in \left[w'_{i,1}, w''_{i,1}\right] \\ C_{i,1}^{*,Transfer} & \text{if } w_{i,1} \geq w''_{i,1} \end{array} \right] \tag{IA.204}$$

The resulting expected discounted utility of each member as a function of $\widetilde{W}_{i,1}$ and $\widetilde{W}_{j,1}$ is

$$V_{i,1}=\left\{\begin{array}{lc} V_i^{(1)}=(1+\delta)\ln\left(\widetilde{W}_{i,1}+\widetilde{W}_{j,1}\right)+v_i^{(1)} & \text{if } w_{i,1}\leq w_{i,1}^{'} \\ V_i^{(2)}=(1+\delta)\left[\frac{1+\Delta}{2}\ln\widetilde{W}_{i,1}+\left(\frac{1-\Delta}{2}\right)\ln\widetilde{W}_{j,1}\right]+v_i^{(2)} & \text{if } w_{i,1}\in\left[w_{i,1}^{'},w_{i,1}^{''}\right] \\ V_i^{(3)}=(1+\delta)\ln\left(\widetilde{W}_{i,1}+\widetilde{W}_{j,1}\right)+v_i^{(3)} & \text{if } w_{i,1}\geq w_{i,1}^{''}\end{array}\right] \tag{IA.205}$$

where

$$\begin{aligned} v_i^{(1)} &\equiv (1+\delta)\left[\frac{1+\Delta}{2}\ln\frac{1+\Delta}{2}-\ln(1+\Delta+\delta)\right] \\ &+\left(\frac{1-\Delta}{2}\right)\ln\left(\frac{1+\Delta}{2}\right)+\delta\left[\ln R+\ln\delta+\left(\frac{1-\Delta}{2}\right)\ln\left(\frac{1-\Delta}{2}\right)\right] \\ v_i^{(2)} &\equiv \delta\left[\ln\delta+\ln R\right]-(1+\delta)\ln(1+\delta) \\ v_i^{(3)} &\equiv (1+\delta)\left[\frac{1-\Delta}{2}\ln\left(\frac{1+\Delta}{2}\right)-\ln(1+\Delta+\delta)\right] \\ &+\frac{1+\Delta}{2}\ln\left(\frac{1+\Delta}{2}\right)+\delta\left[\ln R+\ln\delta+\frac{1+\Delta}{2}\ln\left(\frac{1-\Delta}{2}\right)\right] \end{aligned} \tag{IA.206}$$

C Equilibrium transfer choices

I now characterize the equilibrium set of non-negative voluntary transfers $\left\{\Psi_{A,1},\Psi_{B,1}\right\}$ made between member at the start of $t=1$. The wealth of member i at $t=1$ is determined by their her initial endowment net of any transfers made to or received from the other household member: $W_{i,1}=RW_{i,0}$.

Without loss of generality I focus on equilibria in which there are no redundant transfers so that at least one of $\Psi_{A,1},\Psi_{B,1}$ are zero in equilibrium. As a result I characterize i's best response to $\Psi_{j,1}=0$. Observe that $V_i^{(1)}$ and $V_i^{(3)}$ are invariant to transfers. If $w_{i,1}\leq w_{i,1}^{'}$ then i can only further lower $w_{i,1}$ by making a transfer to j and hence her optimal response is to transfer nothing.

By construction $V_i^{(1)}=V_i^{(2)}$ at $w_{i,1}=w_{i,1}^{'}$ because i's best response here is characterized by her indifference around $C_{j,1}^{Jump}$. It must also be that $V_i^{(2)}>V_i^{(3)}$ at $w_{i,1}=w_{i,1}^{''}$ because $\left\{C_{A,1}^{*,NoTransfer},\right.$ $\left.C_{B,1}^{*,NoTransfer}\right\}$ implements a Pareto efficient equilibrium and $\left\{C_{A,1}^{*,Transfer},C_{B,1}^{*,Transfer}\right\}$ is identical to the dynamically inefficient equilibrium achieved with a single joint account. Since j is indifferent around this point then it must be that i captures the full pareto improvement and is strictly better off. It follows that if $\frac{W_{i,1}}{W_{j,1}}>w_{i,1}^{''}$ then i will optimally transfer enough wealth to j at $t=1$ to ensure $w_{i,1}\leq w_{i,1}^{''}$.

Next observe that

$$\frac{\partial V_i^{(2)}}{\partial\Psi_{i,1}}=(1+\delta)\left[\frac{-\left(\frac{1+\Delta}{2}\right)}{W_{i,1}-\Psi_{i,1}}+\frac{\frac{1-\Delta}{2}}{W_{j,1}+\Psi_{i,1}}\right] \tag{IA.207}$$

and hence the value of $\Psi_{i,1}$ that maximizes $V_i^{(2)}$ is

$$\Psi_{i,1} = \max\left\{0, \left(\frac{1-\Delta}{2}\right) W_{i,1} - \left(\frac{1+\Delta}{2}\right) W_{j,1}\right\} \tag{IA.208}$$

which is strictly positive if and only if

$$\frac{W_{i,0}}{W_{j,0}} > \frac{1+\Delta}{1-\Delta}. \tag{IA.209}$$

Collecting these results together gives the equilibrium set of transfers at $t = 1$:

$$\left\{\Psi^*_{A,1}, \Psi^*_{B,1}\right\} = \left\{ \begin{array}{cc} \left\{\left(\frac{1-\Delta}{2}\right) W_{A,1} - \left(\frac{1+\Delta}{2}\right) W_{B,1}, 0\right\} & \text{if } \frac{W_{A,1}}{W_{B,1}} \geq \frac{1+\Delta}{1-\Delta} \\ \{0,0\} & \text{if } \frac{W_{A,1}}{W_{B,1}} \in \left(\frac{1-\Delta}{1+\Delta}, \frac{1+\Delta}{1-\Delta}\right) \\ \left\{0, \left(\frac{1-\Delta}{2}\right) W_{B,1} - \left(\frac{1+\Delta}{2}\right) W_{A,1}\right\} & \text{if } \frac{W_{A,1}}{W_{B,1}} \leq \frac{1-\Delta}{1+\Delta} \end{array} \right]. \tag{IA.210}$$

The resulting equilibrium level of wealth net of transfers for both members in $t = 1$ is

$$\left\{\widetilde{W}^*_{A,1}, \widetilde{W}^*_{B,1}\right\} = \left\{ \begin{array}{cc} \left\{\frac{1+\Delta}{2}\left(W_{A,1} + W_{B,1}\right), \left(\frac{1-\Delta}{2}\right)\left(W_{A,1} + W_{B,1}\right)\right\} & \text{if } \frac{W_{A,1}}{W_{B,1}} \geq \frac{1+\Delta}{1-\Delta} \\ \left\{W_{A,1}, W_{B,1}\right\} & \text{if } \frac{W_{A,1}}{W_{B,1}} \in \left(\frac{1-\Delta}{1+\Delta}, \frac{1+\Delta}{1-\Delta}\right) \\ \left\{\left(\frac{1-\Delta}{2}\right)\left(W_{A,1} + W_{B,1}\right), \frac{1+\Delta}{2}\left(W_{A,1} + W_{B,1}\right)\right\} & \text{if } \frac{W_{A,1}}{W_{B,1}} \leq \frac{1-\Delta}{1+\Delta} \end{array} \right] \tag{IA.211}$$

which ensures that equilibrium consumption choices at $t = 1$ are $C^{*,NoTransfer}_{i,1}$ for each member and rules out the possibility of additional transfers at $t = 2$. This establishes Proposition 4.

IV Consecutive Consumption Choices

In the base model presented in Section I I assumed that household members simultaneously made consumption decisions each period. This assumption creates the theoretical possibility that household members could in any period attempt to spend more than the total amount of all household wealth. To avoid specifying arbitrary tie breaking rules to deal with such a scenario I assumed in (6) that each member is able to spend no more than half of the household's wealth in any period. The purpose of this section is to show that this arbitrary assumption is not important for the results of that model. I do this by assuming that household members make consumption decisions consecutively within any period. This allows (6) to be replaced by a standard budget constraint whereby each member can spend up to the full amount of remaining household wealth each time they consume. I show that when N is large the simultaneous move equilibrium studied above is the limiting case of the unique equilibria reached in the consecutive move setup.

A Consecutive Move Setup

Assume that the preferences of the household members is unchanged from the setup in Section I. For brevity I focus on the case where household members consume only private consumption

($\mu = 1$) although the conclusions carry over the case where both public and private consumption goods are valued. The timing of decisions and the budget constraint facing each member is now as follows. The household starts the period with wealth of W_t. Without loss of generality, assume that member A is able to decide her own level of consumption first subject to

$$C_{A,t} \leq W_t. \tag{IA.212}$$

Thus A is free to spend up to all of the household's remaining wealth. After this decision is made, the interim level of household wealth is

$$\widetilde{W}_t = W_t - C_{A,t}. \tag{IA.213}$$

Member B learns how much wealth the household has remaining and chooses her own consumption level subject to

$$C_{B,t} \leq \widetilde{W}_t. \tag{IA.214}$$

Thus B is able to spend up to the full amount of remaining household wealth. From one period to the next wealth evolves in the same way as before as specified in (4). As before consumption choices are chosen non-cooperatively and are found as subgame perfect best responses at each point in time.

B Non-Cooperative Equilibrium Consumption Choices

The consecutive move version of the model is solved in the Appendix. The unique equilibrium consumption choice of member's A and B as a function of W_t are

$$\begin{aligned} C_{A,t}^{*Con} &= \frac{\frac{1+\Delta}{2}}{1+\sum_{x=1}^{NT-t}\delta^{\frac{x}{N}}} W_t \text{ and} \\ C_{B,t}^{*Con} &= \left(\frac{\frac{1+\Delta}{2}}{\frac{1+\Delta}{2}+\sum_{x=1}^{NT-t}\delta^{\frac{x}{N}}}\right)\left(\frac{1+\sum_{x=1}^{NT-t}\delta^{\frac{x}{N}}-\left(\frac{1+\Delta}{2}\right)}{1+\sum_{x=1}^{NT-t}\delta^{\frac{x}{N}}}\right) W_t. \end{aligned} \tag{IA.215}$$

The unique equilibrium level of total consumption in any period is

$$X_t^{*Con} = \left(\frac{1}{1+\sum_{x=1}^{NT-t}\delta^{\frac{x}{N}}}\right)\left(\frac{(1+\Delta)\sum_{x=1}^{NT-t}\delta^{\frac{x}{N}}+\frac{1+\Delta}{2}}{\frac{1+\Delta}{2}+\sum_{x=1}^{NT-t}\delta^{\frac{x}{N}}}\right) W_t. \tag{IA.216}$$

The equilibrium consumption choices are slightly complicated because of the Stackelberg leader and follower dynamics within each period. This encourages A to consume slightly more to strategically lower the amount of consumption from B. Apart from this within period strategic consumption motive the forces governing both consumption decisions are identical to before. As the length of each period becomes arbitrarily small (i.e. N gets large) then the magnitude of these within period strategic incentives will diminish as well. This is established formally in the following Proposition which is proved below.

Proposition 6. *: As $N \to \infty$ the equilibrium consumption choices of the consecutive move game become arbitrarily close to the simultaneous move equilibrium as defined in (18) and (19). For-*

mally,

$$\lim_{N\to\infty}\frac{C_{i,t}^{*Con}}{C_{i,t}^{*}}=1 \text{ and } \lim_{N\to\infty}\frac{X_{t}^{*Con}}{X_{t}^{*}}=1. \tag{IA.217}$$

Proposition 6 establishes that the equilibrium studied in the simultaneous move model is not a by-product of the arbitrary expenditure limits assumed in (6).

C Proof of Proposition 6

C-1 Equilibrium at $t = NT$

In the final period member B will optimally consume all remaining wealth:

$$C_{B,NT}^{*Con}=\widetilde{W}_{NT}. \tag{IA.218}$$

Anticipating (IA.218), member A will choose $C_{A,NT}$ to solve

$$\max_{C_{A,NT}}\left(\frac{1+\Delta}{2}\right)\ln C_{A,NT}+\left(\frac{1-\Delta}{2}\right)\ln\left(W_{NT}-C_{A,NT}\right) \tag{IA.219}$$

$$\text{subject to } C_{A,NT}\geq 0 \text{ and} \tag{IA.220}$$

$$W_{NT}-C_{A,NT}\geq 0. \tag{IA.221}$$

Ignoring (IA.220) and (IA.221) since they will not bind at the optimal choice, A's consumption choice is characterized by the first order condition

$$\frac{\frac{1+\Delta}{2}}{C_{A,NT}^{*}}-\frac{\frac{1-\Delta}{2}}{W_{NT}-C_{A,NT}^{*}}=0. \tag{IA.222}$$

Rearranging (IA.222) and combing with (IA.218) gives the equilibrium consumption levels for A and B in $t = NT$:

$$C_{A,NT}^{*}=\left(\frac{1+\Delta}{2}\right)W_{NT} \text{ and} \tag{IA.223}$$

$$C_{B,NT}^{*}=\left(\frac{1-\Delta}{2}\right)W_{NT}. \tag{IA.224}$$

And total equilibrium consumption in $t = NT$ is simply

$$X_{NT}^{*}=W_{NT}. \tag{IA.225}$$

C-2 Solve for the Subgame Perfect Consumption path by Induction

I conjecture the following form for the subgame perfect household allocation.

Conjecture 5. *The subgame perfect equilibrium household allocation from t until NT is proportional to W_t. That is, for any period $t\in\{1,...,NT\}$ the subgame perfect equilibrium levels of $C_{A,t}^{*}$*

and $C^*_{B,t}$ can be written as $C^*_{t+x} = g_{i,t+x} W_t$ for $x \in \{0,1,...,NT-t\}$ where $g_{i,t+x}$ are strictly positive constants independent of W_t.

I establish Conjecture 5 by induction. The problem that member B solves in any period t taking $\widetilde{W}_t$ as given is:

$$\max_{C_{B,t}} \left(\frac{1+\Delta}{2}\right) \ln C_{B,t} + \sum_{x=1}^{TN-t} \delta^{\frac{x}{N}} \left[\left(\frac{1-\Delta}{2}\right) \ln C^{Con*}_{A,t+x} + \left(\frac{1+\Delta}{2}\right) \ln C^{Con*}_{B,t+x}\right] \tag{IA.226}$$

subject to

$$W_{t+1} = R^{\frac{1}{N}} \left(\widetilde{W}_t - C_{B,t}\right), \tag{IA.227}$$

$$C_{B,t} \geq 0, \text{ and} \tag{IA.228}$$

$$\widetilde{W}_t - C_{B,t} \geq 0. \tag{IA.229}$$

Conjecture 5 implies that

$$\sum_{x=1}^{TN-t} \delta^{\frac{x}{N}} \left[\left(\frac{1-\Delta}{2}\right) \ln C^{Con*}_{A,t+x} + \left(\frac{1+\Delta}{2}\right) \ln C^{Con*}_{B,t+x}\right] = \Sigma_t \ln W_{t+1} + k_{i,t} \tag{IA.230}$$

where

$$\Sigma_t \equiv \sum_{x=1}^{TN-t} \delta^{\frac{x}{N}} \tag{IA.231}$$

and $k_{i,t}$ is a constant. In equilibrium (IA.228) and (IA.229) will not bind and hence I ignore those constraints and verify this later. Using (IA.230) in (IA.226) and substituting in the intertemporal budget constraint (IA.227) allows me to simplify B's problem to:

$$\max_{C_{B,t}} \left(\frac{1+\Delta}{2}\right) \ln C_{B,t} + \Sigma_t \ln\left(\widetilde{W}_t - C_{B,t}\right). \tag{IA.232}$$

The first order condition is

$$\frac{\left(\frac{1+\Delta}{2}\right)}{C_{B,t}} - \frac{\Sigma_t}{\widetilde{W}_t - C_{B,t}} = 0 \tag{IA.233}$$

which gives B's best response for any given level of $\widetilde{W}_t$:

$$\widetilde{C}^*_{B,t} = \frac{\frac{1+\Delta}{2}}{\frac{1+\Delta}{2} + \Sigma_t} \widetilde{W}_t. \tag{IA.234}$$

Note that (IA.234) verifies that (IA.228) and (IA.229) are satisfied in equilibrium.

Member A will anticipate (IA.234) and choose $C_{A,t}$ to solve

$$\max_{C_{A,t}} \left(\frac{1+\Delta}{2}\right) \ln C_{A,t} + \left(\frac{1-\Delta}{2}\right) \ln \widetilde{C}^{*}_{B,t} + \sum_{x=1}^{TN-t} \delta^{\frac{x}{N}} \left[\left(\frac{1+\Delta}{2}\right) \ln C^{Con*}_{A,t+x} + \left(\frac{1-\Delta}{2}\right) \ln C^{Con*}_{B,t+x} \right] \tag{IA.235}$$

subject to (IA.234),

$$W_{t+1} = R^{\frac{1}{N}} \left(W_t - C_{A,t} - \widetilde{C}^{*}_{B,t} \right), \tag{IA.236}$$

$$C_{A,t} \geq 0, \text{ and} \tag{IA.237}$$

$$W_t - C_{A,t} \geq 0. \tag{IA.238}$$

I ignore (IA.237) and (IA.238) and verify that they are satisfied at the end. Using the analog of (IA.228) for A and substituting (IA.236) and (IA.234) into (IA.235) I rewrite A's problem (ignoring constants) as:

$$\max_{C_{A,t}} \left(\frac{1+\Delta}{2}\right) \ln C_{A,t} + \left(\left(\frac{1-\Delta}{2}\right) + \Sigma_t \right) \ln \left(W_t - C_{A,t} \right). \tag{IA.239}$$

The first order condition is

$$\frac{\left(\frac{1+\Delta}{2}\right)}{C^{Con*}_{A,t}} - \frac{\left(\frac{1-\Delta}{2}\right) + \Sigma_t}{W_t - C^{Con*}_{A,t}} = 0. \tag{IA.240}$$

Which gives A's optimal consumption choice as

$$C^{Con*}_{A,t} = \frac{\frac{1+\Delta}{2}}{1+\Sigma_t} W_t. \tag{IA.241}$$

Note that (IA.241) demonstrates that (IA.237) and (IA.238) are satisfied as conjectured. Substituting (IA.241) into (IA.234) gives B's equilibrium consumption choice as a function of W_t:

$$C^{Con*}_{B,t} = \frac{\frac{1+\Delta}{2}}{\frac{1+\Delta}{2} + \Sigma_t} \left(\frac{1 + \Sigma_t - \frac{1+\Delta}{2}}{1+\Sigma_t} \right) W_t. \tag{IA.242}$$

Adding (IA.241) and (IA.242) gives the equilibrium level of total consumption in period t:

$$X^{Con*}_t = \left(\frac{1}{1 + \sum_{x=1}^{TN-t} \delta^{\frac{x}{N}}} \right) \left(\frac{\frac{1+\Delta}{2} + (1+\Delta) \sum_{x=1}^{TN-t} \delta^{\frac{x}{N}}}{\frac{1+\Delta}{2} + \sum_{x=1}^{TN-t} \delta^{\frac{x}{N}}} \right) W_t. \tag{IA.243}$$

Note finally that Conjecture 3 was verified about for the case of $t = NT$. Moreover (IA.241) and (IA.242) demonstrate that it is true for $t = NT - 1$ and so on by iteration. This establishes Conjecture 5 by induction.

C-3 Comparison of Consecutive and Simultaneous Move Equilibria

Comparing (19) to (IA.243) gives

$$\frac{X_t^*}{X_t^{Con*}} = \frac{2\Sigma_t^2 + (3+\Delta)\Sigma_t + 1 + \Delta}{2\Sigma_t^2 + (3+2\Delta)\Sigma_t + 1 + \Delta}. \tag{IA.244}$$

Taking the limit of this ratio as $N \to \infty$ requires finding $\lim_{N\to\infty} \frac{X_t^*}{X_t^{Con*}}$. Since both the numerator and denominator tend to infinity I can apply L'Hopital's rule to get

$$\lim_{N\to\infty} \frac{X_t^*}{X_t^{Con*}} = \lim_{N\to\infty} \frac{4\Sigma_t + 3 + \Delta}{4\Sigma_t + 3 + 2\Delta}. \tag{IA.245}$$

Again both the numerator and denominator tend to infinity so I can re-apply L'Hopital's rule to get

$$\begin{aligned} \lim_{N\to\infty} \frac{X_t^*}{X_t^{Con*}} &= \lim_{N\to\infty} \frac{4\Sigma_t + 3 + \Delta}{4\Sigma_t + 3 + 2\Delta} \\ &= \lim_{N\to\infty} \frac{4\frac{\partial \Sigma_t}{\partial N}}{4\frac{\partial \Sigma_t}{\partial N}} \\ &= 1. \end{aligned} \tag{IA.246}$$

Comparing the ratio of consumption choices of A and B within any period gives

$$\frac{C_{A,t}^{Con*}}{C_{B,t}^{Con*}} = \frac{\frac{1+\Delta}{2} + \Sigma_t}{\frac{1-\Delta}{2} + \Sigma_t}. \tag{IA.247}$$

Taking the limit of this ratio as $N \to \infty$ by applying L'Hopital's rule gives

$$\lim_{N\to\infty} \frac{C_{A,t}^{Con*}}{C_{B,t}^{Con*}} = \frac{\frac{\partial \Sigma_t}{\partial N}}{\frac{\partial \Sigma_t}{\partial N}} = 1 \tag{IA.248}$$

which is the same as (*18*). The combination of (IA.246) and (IA.248) establishes Proposition 6.