

Complementary Goods: Creating and Sharing Value

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Taylan Yalcin

Harvard Business School – tyalcin@hbs.edu

Elie Ofek

Harvard Business School – eofek@hbs.edu

Oded Koenigsberg

Columbia Business School – ok2018@columbia.edu

Eyal Biyalogorsky

Arison School of Business – biyalogorsky@yahoo.com

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Abstract

This paper studies the strategic interaction between firms producing strictly complementary products. With strict complements, e.g., video-game consoles and software titles, a consumer derives positive utility only when both products are used together. We show that when the products are developed by separate firms ('non-integrated' development) there exist both value sharing and value creating problems: firms charge higher prices and choose lower quality levels. Moreover, when the firms develop products sequentially the second mover has an advantage: it can choose a lower quality level and secure higher profits than the first mover. However, if the first mover can mandate a royalty or licensing payment from the second firm for permission to produce a compatible product (as often occurs in hardware-software arrangements), the profit advantage flips: the first mover captures a larger share of value created via a high licensing fee. When there is vertically differentiated competition in one of the product markets (e.g., two hardware firms of different quality and a single software firm), we show that with a royalty fee structure there is a possibility of a win-win-win situation where all firms are better off with the low quality firm in the market. This finding is in contrast to the extant literature that suggests that the low quality firm will always be driven out of the market. By shedding light on the incentives to improve quality and the effect of royalty fees, the analysis provides valuable insight for formulating marketing strategy in industries where consumption involves complementarities among multiple goods.

1. Introduction

In a number of prominent markets, consumers need to purchase and use multiple products simultaneously in order to derive positive utility. The goods involved in consumption are therefore highly complementary, and value is derived from their joint consumption. Furthermore, the complex technology and know-how involved in developing each of the products often require specialized organizational skills. Thus, we often see separate firms producing each of the goods, relying on other firms to produce the complement.

There are several noteworthy examples of such an interaction. In the case of computers, one firm typically produces the central processor while another firm produces the operating system, with electric instruments one firm will typically focus on the instrument itself (say the guitar) while other firms focus on the amplifying equipment. In the emerging category of smartphones, typically one firm designs and produces the cell phone and operating platform while others create applications (as is the case with the iPhone and iPhone apps). The video game industry is yet another example that embodies the characteristics of strictly complementary goods: a video game title has no use without the console and a gaming device has no use without a game.¹

Because of the joint consumption characteristic, in many complementary consumption instances there is quality interdependence: the utility derived from one product depends not only on its own quality but also on the complement's quality. A more advanced operating system delivers better performance only if the microprocessor is capable of handling the increase in code complexity.² In the case of video games, since the late 1970s every 6-7 years a new generation of consoles is launched, each delivering higher graphics, sound quality and more innovative game controllers. The user fully benefits from these hardware improvements and novel features only if games are developed that take advantage of the new console's capabilities. Needless to say, improving quality requires costly upfront R&D investments, and this can complicate the quality decision as one producer is reliant on its counterpart's efforts. Therefore, firms producing complementary goods need to consider each others' quality levels when deciding on their quality. The examples above illustrate yet another important aspect of complementary goods: given the need for both products to work together, one of them is usually developed after the other and according to its specifications. For instance, hardware architecture is designed before software.

The fact that consumers need to purchase both goods has strategic implications for the players involved. Specifically, if one views the revenue pie as consisting of the total amount consumers spend on both complementary products, the question arises as to how this pie should be split.

¹ As an exception, the Sony PlayStation 3 can be used as a movie player; but the vast majority of people that buy the device do so to play video games (NPD group report 2009).

² Take the case of 64 bit systems; a microprocessor that wishes to increase processing power by allowing instructions sets that are 64 bits long (instead of 32 bits) can only do so if the operating system is designed to do so.

With video games, for example, total industry revenues in U.S. reached \$21.3 billion³ in 2008, from the sale of consoles, games and accessories—far surpassing movie box office revenues. The issue of sharing the soaring revenues creates tension between the console makers and the game publishers. An important industry practice in this respect is the royalty fee console producers charge game publishers. A similar arrangement is also at play with smartphones and apps that sell separately.

In addition to the strategic interaction between firms that produce complements, there might be competition in one of the markets. More than one firm may produce one of the goods, and this further affects the ability to capture value, and create value by investing in quality. In the case of the PS3 and Xbox 360, a consumer that wants to play a hit game like GTA IV has to purchase the game title and one of the consoles. As a result, PS3 and Xbox 360 not only directly compete with each other but they also need to consider the interaction with the complementary game title, GTA IV.⁴

As the above discussion highlights, the joint consumption but separate production of complementary goods can generate value creating problems in coordinating the qualities of the complements and value sharing problems in setting the prices of each complement. It is not hard to see that there can be an interaction between the two—anticipating the value sharing problems can impact firms' incentives to invest in quality. In this paper we study the strategic interaction among firms producing highly complementary products, focusing on the following research questions:

- How do firms make pricing decisions in light of the joint consumption of their products?
- What governs firms' incentives to invest in improving the quality of complementary products? Should we expect the first mover to enjoy greater profits over its rival?
- How does a royalty structure affect the interaction between firms?
- What impact does competition have on each of the firms? Should we always expect their payoffs to decrease in light of an additional player?

To answer these questions, a stylized model is constructed in which the firms incur costs to develop higher quality products. The end consumer derives utility if and only if she uses both products together. In addition, the marginal utility derived from one product depends on the quality of the complementary product. We investigate several scenarios including a benchmark integrated industry structure in which a single firm produces both complements, a non-integrated industry structure where production is undertaken by separate firms and a competitive case in which one of the complements is produced by two vertically differentiated firms. To capture the

³ Kim, R. (2009, January 16). *Video game announces record revenue*. Retrieved May 30, 2009, from SFGate: <http://www.sfgate.com/cgi-bin/article.cgi?f=/c/a/2009/01/15/BU7L15BAEJ.DTL&type=tech>

⁴ Grand Theft Auto IV only plays on the Playstation 3 and Xbox 360, consoles that are targeted to the hard core gamers.

nature of strict complements, we analyze a sequential model in which one firm needs to develop its product (the “hardware”) earlier than the complementary product (the “software”). In the competitive case, rivalry is in the first mover’s market. Throughout the analysis, because of the sequential nature of the game, the first mover may charge royalty fees to the second mover. As an extension, the first mover’s option to produce the complement in house is also considered.

The analysis reveals that when the products are developed by separate firms not only are there value sharing problems, but the total pie on which the firms argue is smaller due to value creation problems. More specifically, as compared to an integrated firm, separate firms charge higher prices and thus generate less demand. This is due to the fact that prices for complementary products form strategic substitutes, i.e., if one firm tries to cut prices to stimulate demand the other firm has an incentive to increase price.⁵ In addition, quality levels in a non-integrated industry structure are lower. This is because the qualities of complementary products form strategic complements, i.e. if one firm decreases quality to save on development costs the other firm’s best response is to decrease its own quality.

These relationships result in firms’ inability to capture all the value associated with their quality improvements which, in turn, creates an incentive to free-ride on the quality investments of the complement. Consequently, if the firms choose qualities sequentially, the second mover has an advantage in the game and secures a higher profit compared to the first mover by selecting a lower quality. Note that this is different from a second-mover free-riding advantage among firms that produces substitute products, in which case the advantage originates from imitating the technology of the first mover through reverse engineering. In our model, because consumers derive utility from the joint consumption of the goods, the second mover can free-ride on the relatively higher quality of the first mover’s product by investing less in its own quality improvement. However, we find that if the first mover can mandate a royalty payment from the second firm for permission to produce a compatible product, the profit advantage shifts to the first mover, even though the quality gap is exacerbated (i.e., the first mover chooses an even higher quality while the second mover an even lower quality).

When there is vertically differentiated competition in one of the markets, the low quality firm cannot make positive profits without a royalty structure in the industry. On the other hand, if the high quality firm asks a royalty fee from the producer of the complement, an equilibrium in which all the firms are active may exist. Furthermore, in such an equilibrium, all three firms are better off as the presence of a competitor helps alleviate the value creation and value sharing problems: the existence of the low quality product in the competitive market induces the complementor firm to increase quality and decrease price in order to appeal to the lower valuation segment. The firm in the non-competitive sector is better off because of the increased demand while the high quality firm benefits from the complement’s lower price and higher

⁵ Our use of the terms strategic substitutes and complements follows Bulow et al. (1985)

quality. To the best of our knowledge, this is the only model that foresees a win-win-win solution for all three firms producing complementary products where two of them compete directly.

The strategic interaction between firms that produce substitute products has been well researched. Since the game they play is usually zero-sum, the incentives of these firms are naturally conflicting. One could have intuitively suggested that the incentives of firms would be more aligned when they produce strictly complementary products. Our work shows that this is not necessarily the case; producers of strict complements have conflicting incentives in creating and sharing value.

1.1.Related Literature

One of the first analyses of interaction between producers of complementary products is due to Cournot (1838, ch. 9). He models two firms that produce complementary goods (zinc and copper), which in turn are combined to make a composite product (brass). He shows that regardless of differences in marginal costs, both firms share the profits equally. We extend this analysis and look at the quality decisions. Because of asymmetric incentives to invest in quality, the profits are shared unequally in our study.

Like our model, Economides (1999) looks at quality decisions of complementor firms. He compares an integrated and a non-integrated industry structure, in which the complementary products are produced by a single firm and by two distinct firms, respectively. The basic model in this paper resembles his model. The main difference is in how the qualities of complementary products are aggregated to find the quality of the composite good. In his model, the composite product's quality is equal to the quality of the lesser product. As Economides (1999) notes in his motivation, this approach is more appropriate for a telecom service like long distance calls, which requires the use of a long distance line as well as local lines at the two terminating points. In this case, the sound quality will be equal to the minimum of the qualities of the services used. On the other hand, in our model, the qualities of the complementary products are supermodular. The magnitude of increase in the quality of the composite product from an incremental increase in the quality of one component depends on the absolute quality of the other component. This approach suits many complementary product pairs like video game-console, smartphone-apps, electric guitar-amplifier and operating system-processor better. An increase in, say, the game's quality enhances the gaming experience more when the quality of the console is higher.

Our paper is also related to a stream of literature that looks at "one way complements" (Cheng and Nahm 2007, Chen and Nalebuff 2006). With one way complements, one of the products (A) has value for the consumers by itself, but the other one (B) is useless without the first one. That makes one of the complementary products "essential" and its value can be enhanced by the "non-essential" one. Cheng and Nahm (2007) examine how the ratio of the essential good A's value and the enhanced value of the bundle (AB) affects the pricing game. Our work differs from these

papers in several ways. First, in our model quality is endogenous. Second, both goods have no value without the other in our analysis. By contrast, consider the case when one product has value without the other. In that context, the firm producing it naturally has an incentive to invest in improving quality even without the existence of the other firm. By assuming that the products have no value without each other, we are ruling out this trivial explanation for asymmetric quality investments.

Another stream of literature studies strict complements (Farrell and Katz 2000, Casadesus-Masanell et al. 2007). Both of these papers consider competition in one of the complementing industries but they concentrate on the effects of competition on the ability to capture value and do not fully explore the value creating aspect. Farrell and Katz (2000) build a model where one of the complements (A) is monopolized and the other (B) is supplied by a competitive sector. They find that the monopolist may want to supply its own version of B and destroy incentives to innovate in the competitive B sector. Our paper is similar to theirs because we also look at complementary products and incentives to innovate. In their model, however, the monopolized complement's quality is fixed and so there is no strategic interaction in qualities. By contrast, we look at the incentives to innovate in both markets. Casadesus-Masanell et al. (2007) show that when there are two firms in the A market and a single B firm, the lower quality A firm cannot have positive sales. We confirm this result in the paper, however, we show that adding royalty fees to this specific industry structure can result in all three firms having positive demand at positive prices. Importantly, we characterize conditions for the firms to make higher profits in the presence of competition than in its absence—effectively leading to a win-win-win outcome.

The advantages of first and second movers have been extensively researched. For instance, Lieberman and Montgomery (1988) and Kerin et al. (1992) has excellent reviews of this literature. In these works, the second mover is the producer of a substitute product that has entered the market late. On the other hand, in our model the second mover is the producer of a complementary good and designs its product later than the first mover. Since the products have to work together, the second mover needs to know the specifications of the first mover's architecture in order to develop its own product. Therefore, the source of the second mover (dis)advantages is fundamentally different in our model.

This paper is also loosely related to the bundling literature (McAfee et al. 1989, Venkatesh and Kamakura 2003) as in the integrated case the firm sells the complements as a bundle. But our work is different as the products are perfect complements and hence the value (reservation price) is zero for each complement by itself.

The rest of the paper is organized as follows: Section 2 introduces the basics of the model and analyses the benchmark integrated case. Section 3 examines non-integrated firms and is followed by the introduction of a royalty structure in Section 4. Section 5 investigates the effects of competition. In Section 6, the decision to produce the complementary product in house is

studied. The paper ends with concluding remarks. All proofs have been relegated to the Appendix.

2. The Model

2.1. Products

There are two strictly complementary products, A and B. Consumers derive utility if and only if they use both products together, as the composite good denoted AB. This utility is governed by the quality of the composite good, reflecting the qualities⁶ of each product. We assume that the quality of the composite product is the multiplication of the qualities of each component which captures the notion of strict complementarity. More formally, the quality of the composite product, q , is given by $q = \alpha\beta$, where α is the quality of product A and β is the quality of product B. Note that:

$$\frac{\partial q}{\partial \alpha} = \beta, \quad \frac{\partial q}{\partial \beta} = \alpha. \quad (1)$$

Equation (1) implies that the marginal increase in the quality of the composite good from an incremental increase in the quality of one component is equal to the absolute level of quality of the other component. That is, investing in own product quality is more attractive when the complementing product is of higher quality. For example if the console has a more advanced graphics card, the game developer has a greater incentive to invest in the visual aspects of the game title. In another instance when 64 bit processors started replacing 32 bit processors, Microsoft developed x64 editions of its operating systems that could take advantage of the new processors.

2.2. Consumers

Consumers in the market have the same preference ordering among the (potential) composite products when offered at the same price. All consumers prefer higher quality over lower quality, but they are heterogeneous in their willingness to pay for quality. The marginal valuation of quality, θ , is distributed uniformly on $[0,1]$. The utility a consumer derives from buying product A and product B with quality levels α and β and at prices p_A and p_B respectively, is equal to $U = \theta\alpha\beta - p_A - p_B$. The term $\theta\alpha\beta$ is the maximum willingness to pay for a consumer type θ .

⁶ “Quality” can be thought of simply as an attribute (or a collection of attributes) that the consumers always prefer more of at the same cost. For a console it could be processing speed or graphical capabilities, for a video game it could be the number of levels in the game or simply a rating that represents how good the game is, like the ones given by video gaming magazines.

A consumer buys the products provided her valuation is higher than the total of prices, p_A and p_B . Thus, the indifferent consumer has the taste parameter $\hat{\theta} = \frac{p_A + p_B}{\alpha\beta}$. All consumers of types $\theta \in [\hat{\theta}, 1]$ will purchase. Demand, which is equal for both products is, $1 - \hat{\theta}$.

2.3. Cost Structure

We assume that the main burden of the cost of quality falls on investment in development and design. Variable production costs are assumed constant and normalized to zero. We also assume that the cost function is convex⁷. The specific cost functions for developing a product with quality α and β are $c(\alpha) = \frac{1}{n}k_A\alpha^n$ and $c(\beta) = \frac{1}{n}k_B\beta^n$, respectively, where k_A and k_B are cost parameters. The cost functions are assumed to be sufficiently convex in order to ensure the quasiconcavity of the profit functions. Our results hold for any $n > 2$, and we will use $n = 3$ for mathematical tractability. Note that for $k_A = k_B$ the firms have symmetric costs.

Now that we have laid out the basics of the model, we continue with the integrated firm's solution as a benchmark.

2.4 Benchmark: The Integrated Firm Solution

In this scenario both complementary products A and B are produced by a single profit maximizing firm. The integrated firm decides on the quality levels of the individual products, α and β , and prices. Since consumers need both products together, the integrated firm effectively charges a single price $p_I = p_A + p_B$ for the bundle of AB. The consumers with taste parameters greater than $\hat{\theta}_I = \frac{p_I}{\alpha\beta}$ buy the bundle, and the firm solves the following problem:

$$\max_{p_I, \alpha, \beta} \pi_I = (1 - \hat{\theta}_I)p_I - \frac{1}{3}k_A\alpha^3 - \frac{1}{3}k_B\beta^3. \quad (2)$$

Although it is mathematically equivalent for the integrated firm to decide on the quality levels first and price later or decide everything at once, we solve the problem as if it is a two stage game so that it is easier to compare with the non-integrated firms scenario. In the first stage, the integrated firm decides on qualities and in the second stage price is chosen (Figure 1).

⁷ Given our motivating example of the video gaming industry, it is plausible to assume that R&D costs are crucial. For instance in the latest generation, developing a console required investments in the order of billions of dollars and the budget for a typical video game ranged between \$10 and \$35 million (Ofek 2008).

$$\max_{\beta} \pi_B = \frac{1}{3} \frac{\alpha\beta}{3} - \frac{1}{3} k_B \beta^3. \quad (5)$$

In the first stage firm A decides on quality level, taking into account firm B's optimal reaction $\beta^*(\alpha)$ and the pricing subgame

$$\max_{\alpha} \pi_A = \frac{1}{3} \frac{\alpha\beta^*(\alpha)}{3} - \frac{1}{3} k_A \alpha^3. \quad (6)$$

The optimal qualities and the corresponding prices and profits are stated in Tables 1 and 2, and the next lemma.⁹

Lemma 2 *The non-integrated firms choose lower quality levels compared to an integrated firm.*

Lemma 2 reflects the *value creation* problem between firms that produce complementary products. Firms cannot capture all the benefits of increasing their own product's quality, but they have to pay for all the upfront investment for that increase. Specifically, if firm A increases quality, firm B's incentive to increase quality is not as high because it can increase price to capture some of the value from firm A's quality improvement. More formally, the qualities of complementary products form strategic complements. The firms thus have an incentive to free-ride on each other's quality investments. Free-riding is a serious setback, because firms undersupply quality. In fact, the firms choose such low quality levels, that the total price charged by the non-integrated firms is *less than* the price charged by the benchmark integrated firm. This finding is in sharp contrast with Economides (1999), who finds that the total price asked by the non-integrated firms is higher.

In the non-integrated case, the firms develop products with less than efficient qualities, as their marginal revenue from increasing quality is only a fraction of the marginal revenue of the industry from the same increase. Both of the firms would be better off had they mutually invested more in improving quality, closer to the investment of the integrated firm. However, neither of them wants to increase its investment unilaterally. Note also that the firms' optimal quality levels are not symmetric even when the development costs are symmetric (See Table 2).

Proposition 1 *In the non-integrated case, the second mover firm shirks more on quality and makes a higher profit than the first mover.*

Although both firms shirk in quality choices (per Lemma 2), they do not end up with the same quality levels even when they have the same cost parameters: $k_A = k_B$. Specifically, the second mover has an advantage when selecting quality, and it does not have to select a level as high as the first mover. This translates to an advantage in profits; the firms charge the same price and

⁹ We have also solved this game with simultaneous product development. In this case optimal qualities are even lower than in the sequential game. Furthermore, both firms choose the same quality level provided they have the same cost parameter k .

face the same demand, while firm B saves on R&D investments. The cause of the second mover advantage in the model is different than the extant late mover advantages, as the firms produce complementary products instead of substitutes in our model.¹⁰

Another way to think about the result in Proposition 1 is to consider a case in which both firms develop products simultaneously. The firms' optimal quality levels would be very low in this hypothetical case, since free-riding is extremely severe. If the firms could commit to quality choices, they would both prefer higher qualities. In the sequential move model, the first firm can credibly commit to a relatively higher level of quality compared to the simultaneous model, by developing its product first. In turn, firm B also selects higher quality than it would in a simultaneous product development model; because with the superior quality of product A, firm B's marginal return from innovation investment increases. In the end both firms come up with higher quality levels than they would in a simultaneous case. Nevertheless, firm B's best reply to firm A's chosen quality is lower than firm A's quality. This creates the second mover advantage and firm B earns greater profits as it invests less in R&D.

Compared to an integrated firm that produces both goods A and B, the non-integrated firms suffer from the value sharing problem at the pricing stage (Lemma 1) and the value creation problem at the product development stages (Lemma 2). A relevant question at this point is how severe each of these problems is. In order to assess this we define $\pi_i(\alpha^*, \beta^*)$ as the profit of an integrated firm that produces goods with the non-integrated firms' equilibrium quality levels; and $\pi_i(\alpha_i^*, \beta_i^*)$ as the profit of firm i when both qualities are at the optimal level for the integrated firm.

Corollary 1 *The value creation problem is more severe than the value sharing problem between the firms that produce strictly complementary products. Specifically, we have:*

$$\pi_i > \pi_1(\alpha_1^*, \beta_1^*) + \pi_2(\alpha_1^*, \beta_1^*) > \pi_i(\alpha^*, \beta^*) > \pi_1 + \pi_2$$

If we normalize π_i to 1, we get the inequality with the numerical values:

$$\pi_i > \pi_1(\alpha_1^*, \beta_1^*) + \pi_2(\alpha_1^*, \beta_1^*) > \pi_i(\alpha^*, \beta^*) > \pi_1 + \pi_2$$

$$(1) \qquad (0.67) \qquad (0.56) \qquad (0.46)$$

The qualities of the non-integrated firms' products α^* and β^* , are so low that even without the value sharing problem profit is nearly halved to 0.56. On the other hand the total profit of the

¹⁰ Note that this difference in the cause of the second mover advantage is qualitative, as the relationship between the firms is fundamentally different (ie. substitutes vs. complements). However, on a technical level our model is in line with Gal-Or's (1985) finding that "the player that moves first earns lower profits than the player that moves second if the reaction functions of the players are upwards sloping—the actions are strategic complements". (Italic part added.)

non-integrated firms that produce the efficient quality levels α_1^* and β_1^* , is 0.67 of the integrated firm. This comparison highlights the significance of the value creation problem in the production of strict complements.

4. Non-integrated Firms with Royalty Fees

Clearly firm A, the first mover, is at a disadvantage in the non-integrated industry structure. Firm A may try to relieve these problems with some contractual agreements. For instance, a common practice in the video gaming industry is for console makers to charge a royalty fee per game copy to the game publishers, in return for permission to develop games compatible with the console. Smartphones and apps also have a similar royalty structure. In order to incorporate this, we modify the timeline of the original model such that in stage 1 firm A also decides the royalty fee r , which is defined as a percentage of p_B^R , to be asked from firm B. With the royalty fees, firm A can capture part of firm B's revenues.

The new profit functions of the firms are as follows:

$$\pi_A^R = (1 - \hat{\theta}_R)(p_A^R + rp_B^R) - \frac{1}{3}k_A\alpha_R^3, \quad (7)$$

$$\pi_B^R = (1 - \hat{\theta}_R)(1 - r)p_B^R - \frac{1}{3}k_B\beta_B^3. \quad (8)$$

Where $\hat{\theta}_R$ is the marginal consumer who buys the products in the royalty model, $r \in [0,1]$ is the royalty percentage, and the superscript/subscript R on prices, profits and qualities indicate the corresponding values in a royalty structure.

The model is solved by backwards induction as usual. The results with the optimal royalty fee are given in Tables 1 and 2.

Proposition 2.a *Relative to the non-integrated case without the royalty structure:*

- *Firm A provides higher quality, firm B provides lower quality and the quality of the composite product is lower.*
- *Firm A charges a lower price and firm B charges a higher price.*

Because of the royalty payment by firm B, firm A has a higher stake in industry revenues and is willing to invest more in improving quality. On the other hand, because it has to remit a royalty for each unit it sells, firm B has less incentive to invest in quality. The net effect of the changes in qualities in the presence of royalty fees is negative for the composite product. The prices also change drastically. Without the royalty fees, the firms set the same price. However, since firm A

has a larger share of the pie with the royalty fees, it sells a higher quality product for a lower price to enhance industry sales. This is again reversed for firm B, who sells a lower quality product for a higher price. The decrease in total price is sharper compared to quality reduction in the composite good, hence the market coverage increases to $5/12$.

Proposition 2.b *In the presence of royalty fees, firm A makes more profit compared to firm B.*

Basically, royalty fees act as a device that transfers value from the second mover to the first mover. It helps firm A get closer to the efficient level of quality, but causes firm B to get further away from the efficient level. Notice that the industry profits also increase (though very slightly) under a royalty structure. This is because of firm A's increased stake in total industry revenues. In order to boost industry sales, firm A cuts prices and improves quality so much that even with the amplified free-riding by firm B, total demand and industry profits increase. In short, the royalty structure slightly relieves the value sharing problem but exacerbates the value creation problem by spreading the gap between α and β .

5. Competition

Recall from our previous analysis that when a single firm participates in the development and sale of each of the complementary products, firms undersupply quality relative to the integrated case. Introducing royalty transfers improved the situation for the first mover, firm A, but made things worse for firm B. Moreover, the quality gap between the firms was actually exacerbated by royalties—firm A increased its quality while firm B further decreased its quality; thus total industry profits were not much affected. We now seek to understand how competition in the A market impacts our findings so far. One might think that such competition can benefit only the B firm because it will reduce the power the A firm has. We will show that competition in the A market actually results in more intricate effects that alleviate the quality gap problem and can result in all firms being better off with rivalry in one market.

Let there be two firms producing good A and a single firm producing good B. The firms in the A market are differentiated vertically by their qualities. We will first establish that with a simple pricing structure (no royalty payments) we cannot have an equilibrium with two distinct A firms of different quality present in the market. Then we will incorporate royalty fees and show that this result reverses: all three firms can make positive profits in equilibrium. We conclude by characterizing conditions for all three firms to be better off under competition compared to an equilibrium with only one A firm.

Consider a multistage game between 2 firms producing good A and 1 firm producing good B. Analyzing the last stage (pricing) is sufficient to show that two A firms of different qualities cannot be in the market (i.e., sell positive quantities) at the same time. In order to find the pricing equilibrium, we take qualities as given. Let α_H and α_L denote the levels of high and low quality

products in the A market, respectively ($\alpha_H > \alpha_L$). Accordingly, denote the two A firms by A_H and A_L , respectively. The quality of firm B's product is denoted β as before. The marginal costs of all the products are zero.

We define $\hat{\theta}_i$ to be the lowest type consumer who gets non-negative utility from purchasing $\alpha_i\beta$ for $i=\{H, L\}$ and $\tilde{\theta}$ to be the consumer indifferent between the high quality pair $A_H B$ and the low quality pair $A_L B$. We have $\hat{\theta}_H = \frac{P_{AH} + P_B}{\alpha_H \beta}$, $\hat{\theta}_L = \frac{P_{AL} + P_B}{\alpha_L \beta}$, and $\tilde{\theta} = \frac{P_{AH} - P_{AL}}{(\alpha_H - \alpha_L)\beta}$, where p_{A_i} is the price of the product with quality i , for $i=\{H, L\}$. D_B denotes the demand for good B and D_{A_i} denotes the demand for product A_i . Figure 3 depicts the consumer space and the demand structure when $\tilde{\theta} \geq \hat{\theta}_H \geq \hat{\theta}_L$ is satisfied. Notice that unless this condition is satisfied all three firms cannot have positive market shares simultaneously. This condition boils down to:

$$p_{AH}\alpha_L - p_{AL}\alpha_H \geq p_B(\alpha_H - \alpha_L). \quad (9)$$

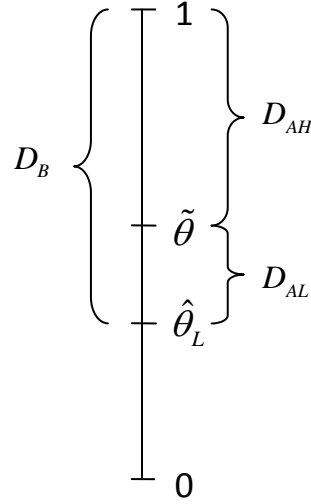


Figure 3

The corresponding revenue (sales) functions S_{AH} , S_{AL} and S_B , for the high quality A firm, the low quality A firm and the B firm respectively, are listed below:

$$S_{AH} = (1 - \tilde{\theta})p_{AH}, \quad S_{AL} = (\tilde{\theta} - \hat{\theta}_L)p_{AL} \quad \text{and} \quad S_B = (1 - \hat{\theta}_L)p_B. \quad (10)$$

The equilibrium in the pricing subgame is:

$$p_{AH}^* = \frac{(\alpha_H - \alpha_L)\beta}{2}, \quad p_{AL}^* = 0, \quad p_B^* = \frac{\alpha_L\beta}{2}. \quad (11)$$

Note that the low quality firm's best response price and demand are equal to zero. Thus, if the low quality firm enters the market it will not make any revenues. But in order to have a product of quality α_L to offer in the last stage pricing game, the low quality firm needs to invest in development. Consequently, the low quality firm does not have an incentive to develop the product in the first stage. Furthermore, when the low quality firm is not in the market, the high quality firm and firm B interact as in the non-integrated firms case developed in Section 4. This result is stated in the following lemma:

Lemma 3 *With a simple price structure (no royalties), there does not exist an equilibrium where the low quality firm enters the market.*

The pricing equilibrium in (11) is remarkably different from that of competition between vertically differentiated duopolists without a complementary product (ala Shaked and Sutton, 1982) where the lower quality product's price is not driven to zero. In essence, the existence of product B creates a downward spiral in prices which results in the low quality firm to be driven out. Because of the joint consumption characteristic, consumers need to always purchase good B and hence firm B has an advantage in extracting value over the A firms. Since there are two competing firms in the A market and a single player in B that can extract value, the low quality product's price is driven to marginal cost. Consider a situation where the low quality firm has positive demand, in this case decreasing own price expands the high quality firm's demand quickly as stealing demand from the low quality firm is easier than creating demand from the uninterested customers with very low valuations. However at the point where the low quality firm has no demand (but just on the margin) the returns from cutting prices drop sharply, supporting the pricing equilibrium in (11). Firm B expects this result and thus charges p_B^* in order not to leave any value on the table to the low quality firm.

This result is in line with Casadesus-Masanell et al. (2007) who find that an equilibrium with two firms in the processor market making positive profits and one operating system firm does not exist. However, in reality we do see complementary products of different qualities in the marketplace. To explain this, Casadesus-Masanell et al. (2007) introduce negative marginal costs in the form of future profits from the installed base and give the interaction between Intel, AMD and Microsoft as an example. We offer a different explanation utilizing royalty fees, which is a common practice in a number of industries (video games, smartphone and apps, etc.).

In order to incorporate royalty fees and ensure mathematical tractability, we make the following simplifying assumptions:¹¹

- α_H is exogenous and normalized to 1.
- Only the high quality A firm charges a royalty fee. Firm B pays a royalty fee (r percent of p_B) to firm A_H for a permission to sell a compatible product.

The firms play a 4-stage game depicted in Figure 4. In the first stage the low quality firm decides on $\alpha_L \in [0,1)$ and the consequent cost of product development is denoted $C(\alpha_L)$. Choosing $\alpha_L = 0$ means that the firm voluntarily does not enter the market and hence $C(\alpha_L) = 0$. On the other hand, when $\alpha_L \in (0,1)$ product development cost is strictly positive and increasing in α_L . In the second stage, the high quality firm chooses the royalty percentage, $r \in [0,1]$ and in the third stage firm B chooses quality, β . The firms choose prices in the last stage.

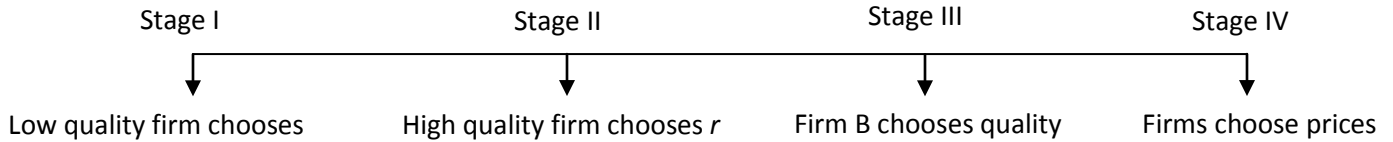


Figure 4

When solving the game we need to take into account two possible discontinuities in payoffs. First, if the low quality firm decides not to enter the market in the first stage, the remaining firms interact as in Section 4. In this case the low quality firm makes no profits. The second discontinuity is more subtle: If A_L enters the market, the high quality firm's choice of r in the second stage results in discontinuous payoffs. In particular, choosing a high r softens the competition between the A firms and enables A_L to make positive revenues, while choosing a low r results in zero revenues for the low quality firm. In the following analysis we will call these two possible actions by the high quality firm as “accommodate” and “fight”, respectively.

¹¹ A product of quality level “1” can be considered the “state of the art” such that it is not profitable for a competitor to mass produce a higher quality product. Thus the second A firm automatically becomes “the low quality firm”. What is relevant for our subsequent analysis is the fraction of the high-quality firm's quality level that the rival A_L firm chooses; thus our normalization is without loss of generality. Extending the analysis to allow the A_H firm to select its quality would not change the nature of the results we present. The cost of developing a product of quality level 1, $C(1)$, is a fixed cost and is not crucial to the model. As long as the profits of the high quality firm is higher than this fixed cost all the results hold. An equilibrium with all three firms making positive revenues is also possible with both A firms asking royalty fees. However this complicates the analysis substantially and our simulations show that the results are qualitatively similar.

When the low quality firm enters the market the profit functions become:

$$\begin{aligned}
\pi_{AH} &= (1 - \tilde{\theta})(p_{AH} + rp_B) - C(1), \\
\pi_{AL} &= (\tilde{\theta} - \hat{\theta}_L)p_{AL} - C(\alpha_L), \\
\pi_B &= (1 - \tilde{\theta})(1 - r)p_B + (\tilde{\theta} - \hat{\theta}_L)p_B - \frac{1}{3}k_B\beta^3.
\end{aligned} \tag{12}$$

We will first analyze the accommodation case, i.e. when the royalty fee is such that the low quality firm can make positive profits. Taking derivatives with respect to prices and solving gives the best response prices in the last stage:

$$\begin{aligned}
p_{AH}^{acc*} &= \frac{\beta(1 - \alpha_L)(3 - \alpha_L(3 + 2r(1 - r)))}{6 - \alpha_L(6 + r(2(1 - r) + \alpha_L(r - 2)))}, \\
p_{AL}^{acc*} &= \frac{\alpha_L\beta(1 - \alpha_L)(1 + \alpha_L(r - 2))r}{6 - \alpha_L(6 + r(2(1 - r) + \alpha_L(r - 2)))}, \\
p_B^{acc*} &= \frac{\alpha_L\beta(1 - \alpha_L)(3 - 2r)}{6 - \alpha_L(6 + r(2(1 - r) + \alpha_L(r - 2)))}.
\end{aligned} \tag{13}$$

All the prices and corresponding demands are positive as long as $r > 0$ and $r > 2 - (1/\alpha_L)$ is satisfied, which is nothing but (9). We can combine these two conditions that are required for accommodation into one for easier reference:

$$r^2 > r[2 - (1/\alpha_L)]. \tag{14}$$

More specifically, p_B^{acc*} is always positive, $p_{AH}^{acc*} > p_{AL}^{acc*}$ and p_{AL}^{acc*} is positive as long as (14) holds. Examining the expression for p_{AL}^{acc*} in (13), we see that (in accordance with Lemma 1) if $r = 0 \Rightarrow p_{AL}^{acc*} = 0$.¹² Thus, the low quality firm can sell its product at a positive price only if r is high enough, such that (14) holds. The reason for the reversal of the finding in Lemma 1 has to do with the effect of the royalty percentage on firm B's best response price. When the high quality firm asks royalties, the B firm needs to pay r percent of every product it sells that is compatible with the high quality product. Intuitively, as r increases firm B becomes more willing to sell some of its product paired with the low quality product and avoid paying royalties to the high quality firm. But in order to do that firm B needs to decrease its price, because the low quality pair $A_L B$ delivers less value to the customers. In other words, a choice of high r (that satisfies 14) accommodates the low quality firm because it decreases the price of the low quality

¹² In fact, when $r = 0$ all the best response prices given in (13) are equal to the ones in (11).

pair (by decreasing the price of good B) to a level that stimulates demand for A_L and consequently the low quality firm makes positive revenues.

If the high quality firm decreases r , the low quality firm needs to compensate by decreasing its own price, more formally, $\partial p_{AL}^{acc^*} / \partial r > 0$. However at a certain threshold—given by $r = \max\{0, 2 - (1/\alpha_L)\}$ —we get $p_{AL}^* = 0$, and the low quality firm cannot compensate by decreasing price anymore and makes zero revenues. In other words, the high quality firm may fight by choosing an $r > 0$ that does not satisfy (14). In this case the best response prices become:

$$\begin{aligned}
 p_{AH}^{fight^*} &= \frac{\beta(1-\alpha_L)(2-\alpha_L)(2-r)(1+r)}{4-4\alpha_L+r^2\alpha_L}, \\
 p_{AL}^{fight^*} &= 0, \\
 p_B^{fight^*} &= \frac{\alpha_L\beta(1-\alpha_L)(2-r)}{4-4\alpha_L+r^2\alpha_L}.
 \end{aligned} \tag{15}$$

The rest of the model is solved by backward induction. The equilibrium choice of r (and hence the high quality firm's choice of whether to accommodate or fight) depends on the payoffs from these actions, which are, in turn, functions of α_L . Taking this into account, the low quality firm will never choose an α_L that will induce the high quality firm to fight as this results in a loss for the low quality firm (it incurs the development cost $C(\alpha_L)$ but does not make any product market revenues). However, as presented in the next proposition there exists an accommodating equilibrium if $C(\alpha_L)$ is low.

Proposition 3. *If the cost for developing the low quality product is less than 7×10^{-6} , there exists an accommodating equilibrium with three firms in the market.*

This result shows that for $\alpha_L \in (1/3, 0.463)$ the high quality firm's optimal action is to accommodate the low quality firm by setting a royalty rate that meets (14). Next, we compare the equilibrium profits of the three-firm equilibrium with an equilibrium with only the high quality A firm and the B firm. The results are given in the next proposition:

Proposition 4. *When the low quality firm is in the market along with the high quality firm:*

- All firms are better off.
- Firm A_H chooses a higher price; Firm B chooses a higher quality and a lower price.

compared to a case when it is not

It might be expected that firm B is better off when there is competition in the A market. One could intuit that competition in the A market will lead to a decrease in prices for the A products and consequently leave a larger share of the pie for firm B, which can increase its price. However this is not the case, with the introduction of the low quality firm in the market, p_{AH}^* actually goes up and p_B^* goes down. This is because of the decrease in the optimal r . When there are two firms in the A market, the high quality firm does not ask a royalty fee as high as it would in the case when it is a monopoly. The lower r in the three firm case means that the high quality firm will get a lower share of firm B's revenues. As a result the high quality firm has a lesser stake in industry sales and increases price. On the other hand, since firm B can keep a higher portion of its revenues, demand generation becomes more important and the equilibrium price of B decreases while the equilibrium quality of B increases. The net effect is that firm B is better off (the lower royalty and greater demand more than compensates for the decrease in p_B^* and the additional cost of higher quality). The low quality firm is, of course, better off as it makes positive profits in the three firm equilibrium.

A more interesting result is that the high quality A firm is better off when the low quality firm is active. In fact, firm A_H accommodates firm A_L by choosing a high r , even if it has the option to drive the price of the low quality product to zero and crowd it out of the market. To understand this finding, one has to examine the implications of firm B's quality level choice in light of competition in the A market. When there is a lower quality firm active in the A market, firm B wants to sell some of its product to be paired with A_L instead of paying a high royalty fee to the high quality firm. However, in order to do so firm B cannot shirk on its quality as much as it does when there is only the high quality firm— because then the quality of $A_L B$ would be too low to generate any substantial demand. In addition, the buyers of $A_L B$ are the consumers with relatively lower valuation. To appeal to this segment, firm B also needs to decrease its price. These two effects benefit the high quality A firm to a great extent as it can increase its price: both because of firm B's higher contribution to the end product's quality and its lower price. Thus, from firm A_H 's standpoint, having the lower quality competitor is beneficial because it helps mitigate both the value sharing and value creating problems with firm B.

6. The option of producing the complementary good in-house

One simple solution to the difficulties arising from separate development of the complements would be producing the other good in-house. This option is superior if one of the firms has the technical skills to produce the other good efficiently. For example a firm that has the technology to design and produce a left shoe also has the technology to design and produce a right shoe. Therefore, we never see separate firms manufacturing only one of these complements. On the other hand it is hard to say the same for another complement pair like a processor and an

operating system. Firms can typically achieve high levels of competence at developing and producing one of the goods, but if they also try to master developing and producing the other good they will not be as efficient.¹³ For instance, if we assume firm A tries to develop product B in house, it will have a cost parameter $\tilde{k}_B > k_B$, which determines the amount of spending for a given quality improvement. Firm A has an incentive to develop good B by itself, if its profit from doing so, $\tilde{\pi}_A$, is higher than the profit from only producing good A.

$$\tilde{\pi}_A > \pi_A, \text{ or}$$

$$\frac{1}{2^6 3 k_A \tilde{k}_B} > \frac{1}{2^2 3^5 k_A k_B} \Rightarrow \tilde{k}_B < 5.06 k_B.$$

Thus, it is not profitable for firm A to develop product B by itself, unless its cost parameter for product B is less than 5.06 times the cost parameter of firm B, which specializes in good B. We next investigate firm A's willingness to produce good B in house when it charges royalty fees.

Conducting a similar analysis when the A firm can impose a royalty fee, shows that firm A prefers developing product B itself when its cost parameter for developing good B satisfies

$$\tilde{k}_B < 3.32 k_B.$$

Thus, compared to the original model, firm A is much less willing to produce good B when it can charge a royalty fee. Said differently, when royalty payments can be imposed, we should expect to see much more specialization in an industry whereby separate firms develop and produce each of the complementary products. For example, consider an industry in which firm A has a cost parameter for developing product B that is 4 times as big as firm B's cost for developing product B ($\tilde{k}_B = 4k_B$). When there are no royalties, firm A wants to develop product B in house and be an integrated firm. By contrast, if it can charge royalty fees, firm A is better off letting firm B, who is more efficient, develop good B and then extract surplus from firm B through the royalty payments.

7. Conclusion

In this paper, we analyzed the strategic interaction between complementor firms that need to make decisions on pricing and quality. Our study has yielded important managerial insights.

We have shown that an integrated firm is much more effective compared to non-integrated firms in producing complementary products, as it can internalize all the gains from its actions. In other

¹³ As an example, in the video game market only Nintendo's game publishing subsidiary is a main source of hit games for the company's console. It is important to note that Nintendo develops games for its consoles since 1970s.

words an integrated firm is immune to the value sharing and value creating problems. In the non-integrated case both firms price selfishly and have an incentive to free-ride on each other's investments in quality. Moreover, the second mover is at an advantage because it decides after the first firm commits to a quality level and chooses an even lower quality.

The first mover may extract surplus from the second mover via a royalty structure, which can reverse the first mover's disadvantage in profits. However, royalty fees change the stakes of the firms in the industry and make the second firm's shirking problem even more serious; the quality gap problem gets exacerbated. This may result in lower quality for the composite product and less value to consumers. Nevertheless, the actions of firm A, who has a larger stake in the industry profit results in higher total profits and a higher coverage of the market.

In the presence of competition between vertically differentiated firms in developing and producing one of the complements, utilizing royalty fees can result in an industry outcome where all firms are better off compared to a case in which only one A firm and one B firm is in the market. This is because existence of the low quality firm alleviates the value sharing and value creating problems. When faced with competition, the high quality firm lowers the royalty fee and this benefits firm B. Moreover, when the low quality firm is in the market firm B increases its quality to compensate for A_L 's low quality and decreases price to increase demand from people with low willingness to pay. Both of these actions boost firm A_H 's profits. Needless to say a three firm equilibrium is better for the low quality firm as it makes positive profits. Therefore having a low quality competitor in the A market makes all firms better off.

Our analysis is based on a stylized model in order to address the issues of quality and pricing decisions regarding firms that produce complementary products. The focus has been on the firm(s) producing one of the complements whose quality decision needs to be resolved earlier. In this context there are several issues related to the decisions of the firm producing the other complement which is not considered in this paper. The most obvious one is competition in the B market. Firm B may also want to consider making its product exclusive with one of the A products. So for firm B, compatibility is a strategic decision which possibly requires a complicated contract with side payments. These important issues are deferred to future research.

	Integrated	Non-integrated	Royalty
Quality	$\alpha_I = \frac{1}{2^2 k_A^{2/3} k_B^{1/3}}$ $\beta_I = \frac{1}{2^2 k_A^{1/3} k_B^{2/3}}$ $q_I = \frac{1}{2^4 k_A k_B}$	$\alpha = \frac{1}{2^{2/3} 3^{4/3} k_A^{2/3} k_B^{1/3}}$ $\beta = \frac{1}{2^{1/3} 3^{5/3} k_A^{1/3} k_B^{2/3}}$ $q = \frac{1}{3^3 2 k_A k_B}$	$\alpha_R = \frac{5^{5/3}}{2^{13/3} 3^{4/3} k_A^{2/3} k_B^{1/3}}$ $\beta_R = \frac{5^{4/3}}{2^{11/3} 3^{5/3} k_A^{1/3} k_B^{2/3}}$ $q_R = \frac{5^3}{2^8 3^3 k_A k_B}$
Price	$p_I = \frac{1}{2^5 k_A k_B}$	$p_A = p_B = \frac{1}{3^4 2 k_A k_B}$	$p_A^R = \frac{5^3}{2^9 3^4 k_A k_B}$ $p_B^R = \frac{5^4}{2^{10} 3^4 k_A k_B}$
Market coverage	$1 - \hat{\theta}_I = \frac{1}{2}$	$1 - \hat{\theta} = \frac{1}{3}$	$1 - \hat{\theta}_R = \frac{5}{12}$
Profit	$\pi_I = \frac{1}{2^6 3 k_A k_B}$	$\pi_A = \frac{1}{2^2 3^5 k_A k_B}$ $\pi_B = \frac{1}{3^6 k_A k_B}$ $\pi_A + \pi_B = \frac{7}{2^2 3^6 k_A k_B}$	$\pi_A^R = \frac{5^5}{2^{13} 3^5 k_A k_B}$ $\pi_B^R = \frac{5^4}{2^{10} 3^6 k_A k_B}$ $\pi_A^R + \pi_B^R = \frac{5^4 23}{2^{13} 3^6 k_A k_B}$

Table 1

	Integrated	Non-integrated	Royalty
Quality	$\alpha_I = 2.5$ $\beta_I = 2.5$ $q_I = 6.25$	$\alpha = 1.46$ $\beta = 1.27$ $q = 1.85$	$\alpha_R = 1.68$ $\beta_R = 1.08$ $q_R = 1.81$
Price	$p_I = 3.13$	$p_A = p_B = 0.62$	$p_A^R = 0.30$ $p_B^R = 0.75$
Market coverage	$1 - \hat{\theta}_I = 50\%$	$1 - \hat{\theta} = 33\%$	$1 - \hat{\theta}_R = 42\%$
Profit	$\pi_I = 0.52$	$\pi_A = 0.103$ $\pi_B = 0.137$ $\pi_A + \pi_B = 0.240$	$\pi_A^R = 0.157$ $\pi_B^R = 0.084$ $\pi_A^R + \pi_B^R = 0.241$

Table 2

The numerical values have been calculated assuming $k_A = k_B = 0.1$

Appendix

Proof of Lemma 1. We will first solve the last stage of the integrated case. Differentiating (2) with respect to p_I , setting the derivative to zero and solving for p_I^* yields $p_I^* = \frac{\alpha\beta}{2}$. In the last stage of the non-integrated case, firm A's profit function is $\pi_A = (1 - \frac{p_A + p_B}{\alpha\beta})p_A$. Differentiating with respect to p_A , setting the derivative to zero and solving for p_A^* yields $p_A^* = \frac{\alpha\beta - p_B^*}{2}$, by symmetry $p_B^* = \frac{\alpha\beta - p_A^*}{2}$. Solving these together we get $p_A^* = p_B^* = \frac{\alpha\beta}{3}$. Both solutions are maxima as the second order derivatives are negative. Substituting the equilibrium prices gives the marginal consumer's valuation in each case as $\hat{\theta}_I = 1/2$ and $\hat{\theta} = 2/3$.

Proof of Lemma 2 and Proposition 1. Again, we will first solve the integrated case. When we substitute the equilibrium price from the last stage, $p_I^* = \frac{\alpha\beta}{2}$, the integrated firm's profit function becomes $\pi_I = \frac{\alpha\beta}{4} - \frac{1}{3}k_A\alpha^3 - \frac{1}{3}k_B\beta^3$. Setting the first order conditions for α and β to zero and solving simultaneously yields $\alpha_I^* = \frac{1}{2^2 k_A^{2/3} k_B^{1/3}}$ and $\beta_I^* = \frac{1}{2^2 k_A^{1/3} k_B^{2/3}}$. Since the second order conditions are both negative, the solution is the maximum. In order to solve for optimal quality decisions in the non-integrated case we first need to analyze the second stage of the game and find firm B's best response function. Substituting the equilibrium prices from the third stage, $p_A^* = p_B^* = \frac{\alpha\beta}{3}$ we get firm B's profit function, (5). Differentiating (5) with respect to β and setting derivative to zero we get the best response function $\beta^*(\alpha) = \frac{\alpha^{1/2}}{3k_B^{1/2}}$. Consequently firm A's profit function in the first stage is updated to (6) and the optimal quality level of firm A is $\alpha^* = \frac{1}{2^{2/3} 3^{4/3} k_A^{2/3} k_B^{1/3}}$. Using the best response function we also find firm B's optimal quality $\beta^* = \frac{1}{2^{1/3} 3^{5/3} k_A^{1/3} k_B^{2/3}}$. The second order conditions in both stages are again negative.

Proof of Proposition 2. As always we will go backwards starting from the last stage. Differentiating (7) with respect to p_A^R , setting the derivative to zero and solving for p_A^{R*} yields

$p_A^{R*} = \frac{\alpha_R \beta_R - (1-r)p_B^R}{2}$. Differentiating (8) with respect to p_B^R , setting the derivative to zero and

solving for p_B^{R*} yields $p_B^{R*} = \frac{\alpha_R \beta_R - p_A^R}{2}$. Solving these together we get $p_A^{R*} = \frac{(1-r)\alpha_R \beta_R}{3-r}$ and

$p_B^{R*} = \frac{\alpha_R \beta_R}{3-r}$. In the second stage, firm B's profit function is updated to

$\pi_B^R = \frac{(1-r)\alpha_R \beta_R}{(3-r)^2} - \frac{1}{3}k_B \beta_R^3$ by substituting the equilibrium prices. Firm B's profit maximizing

best response is $\beta_R^*(\alpha_R^*) = \frac{(1-r)^{1/2} \alpha_R^{1/2}}{(3-r)k_B^{1/2}}$. In the first stage firm A maximizes

$\pi_A^R = \frac{(1-r)^{1/2} \alpha_R^{3/2}}{(3-r)^3} - \frac{1}{3}k_A \alpha_R^3$ with respect to α_R and r . We will first tackle the quality choice by

differentiating and setting the first order condition to zero which yields $\alpha_R^* = \frac{3^{2/3}(1-r)^{1/3}}{2^{2/3}(3-r)^2 k_A^{2/3} k_B^{1/3}}$.

Substituting it into firm B's best response function for quality we get $\beta_R^* = \frac{3^{1/3}(1-r)^{2/3}}{2^{1/3}(3-r)^2 k_A^{1/3} k_B^{2/3}}$.

Taking these optimal qualities into account we have $\pi_A^R = \frac{3(1-r)}{2^2(3-r)^6 k_A k_B}$ and this is maximized

at $r^* = \frac{3}{5}$. All of the second order conditions are met. Substituting the equilibrium prices gives

the marginal consumer's valuation $\hat{\theta}_R = 5/12$.

Proof of Lemma 3. Differentiating the three revenue functions in (10) with respect to p_{AH} , p_{AL} and p_B respectively and solving the first order conditions together we get the best response

prices: $p_{AH}^* = \frac{(\alpha_H - \alpha_L)\beta}{2}$, $p_{AL}^* = 0$ and $p_B^* = \frac{\alpha_L \beta}{2}$. The second order conditions are negative.

Proof of Propositions 3 & 4. Differentiating the three profit functions in (12), with respect to p_{AH} , p_{AL} and p_B respectively and solving the first order conditions together we get the best response prices in (13). In order to ensure that all firms make positive revenues, we need to check the signs of prices and positive demand condition (9). We will first check the signs of the prices. All of the prices share the same denominator: $6 - \alpha_L(6 + r(2(1-r) + \alpha_L(r-2)))$.

Expanding, rearranging and adding and subtracting $4r - 4\alpha_L r$ we get

$-\alpha_L^2(r^2 - 2r) + 2\alpha_L(r^2 - 2r) - 2\alpha_L r + 4r + 6 - 6\alpha_L - 4r + 4\alpha_L r$. The first four terms can be factorized as $r(2 - \alpha_L)(\alpha_L r + 2 - 2\alpha_L)$. Since r and α_L are both in $(0,1)$ this expression is positive. The last four terms can be factorized as $2(\alpha_L - 1)(2r - 3)$. This expression is also positive since both parentheses are negative. Since the denominators are positive the prices have the same sign as their numerators. A simple inspection reveals that p_B^{acc*} is always positive, p_{AL}^{acc*} has the same sign with $1 + \alpha_L(r - 2)$; p_{AH}^{acc*} and has the same sign with $3 - \alpha_L(3 + 2r(1 - r))$. If we rearrange the second expression and add/subtract we get $2[1 + \alpha_L(r - 2)] + (1 - \alpha_L r^2) + \alpha_L(1 - r^2)$. The terms in the parentheses are positive and the term in brackets is the same as the first expression. So we conclude that as long as $1 + \alpha_L(r - 2) > 0$ (14) is met all the prices are positive. In order to check the demand we substitute the best response prices in (13). Straightforward algebra shows that this also boils down to (14).

We will first assume that (14) holds (accommodate mode) and check if the high-quality firm's choice of r in the second stage satisfies this condition. Then, we will analyze the model assuming the high-quality firm chooses a royalty that corresponds to the fight mode. We will establish the profits in each case and determine which action will be chosen in equilibrium.

Assuming (14) holds we go to the third stage to analyze accommodation case. Substituting p_B^{acc*} in π_B^{III} , differentiating with respect to β , setting the first order condition to zero and solving gives $\beta_{acc}^* = \pm \sqrt{\frac{\alpha_L(1 - \alpha_L)^2(3 - 2r)^2}{k_B(6 - \alpha_L(6 + r(2(1 - r) + \alpha_L(r - 2))))^2}}$. Since quality cannot be negative we select the positive root.

In the second stage substituting the best response prices and quality in π_{AH}^{III} , and differentiating with respect to r gives the first order condition:

$$\frac{6\alpha_L(2 - \alpha_L)(1 - \alpha_L)^3(3 - 3\alpha_L + \alpha_L r)(3 - 2r)[-3 + \alpha_L(12 - 14r + 5r^2 - 9\alpha_L + \alpha_L r(14 - 7r + r^2))]}{\sqrt{\alpha_L(1 - \alpha_L)^2(3 - 2r)^2(6 - \alpha_L(6 + r(2(1 - r) + \alpha_L(r - 2))))^3} \sqrt{k_B(6 - \alpha_L(6 + r(2(1 - r) + \alpha_L(r - 2))))^2}} = 0.$$

An inspection shows that the only part that can be equal to zero in this expression is:

$-3 + \alpha_L(12 - 14r + 5r^2 - 9\alpha_L + \alpha_L r(14 - 7r + r^2))$. Expanding and rearranging this term we get $\alpha_L^2 r^3 - \alpha_L(5 - 7\alpha_L)r^2 - 14\alpha_L(1 - \alpha_L)r + 3(3\alpha_L - 1)(1 - \alpha_L)$. This cubic expression has three roots:

$$r_1 = \frac{\alpha_L(7\alpha_L - 5)}{3\alpha_L^2} + \frac{(1 - \sqrt{3}i)(-25\alpha_L^2 + 28\alpha_L^3 - 7\alpha_L^4)}{2^{2/3}3\alpha_L^2(-250\alpha_L^3 + 501\alpha_L^4 - 282\alpha_L^5 + 47\alpha_L^6 + 3\sqrt{3}\sqrt{-1500\alpha_L^7 + 3863\alpha_L^8 - 3728\alpha_L^9 + 1706\alpha_L^{10} - 372\alpha_L^{11} + 31\alpha_L^{12}})^{1/3}}$$

$$\frac{(1 + \sqrt{3}i)(-250\alpha_L^3 + 501\alpha_L^4 - 282\alpha_L^5 + 47\alpha_L^6 + 3\sqrt{3}\sqrt{-1500\alpha_L^7 + 3863\alpha_L^8 - 3728\alpha_L^9 + 1706\alpha_L^{10} - 372\alpha_L^{11} + 31\alpha_L^{12}})^{1/3}}{2^{4/3}3\alpha_L^2},$$

$$r_2 = \frac{\alpha_L(7\alpha_L - 5)}{3\alpha_L^2} + \frac{(1 + \sqrt{3}i)(-25\alpha_L^2 + 28\alpha_L^3 - 7\alpha_L^4)}{2^{2/3}3\alpha_L^2(-250\alpha_L^3 + 501\alpha_L^4 - 282\alpha_L^5 + 47\alpha_L^6 + 3\sqrt{3}\sqrt{-1500\alpha_L^7 + 3863\alpha_L^8 - 3728\alpha_L^9 + 1706\alpha_L^{10} - 372\alpha_L^{11} + 31\alpha_L^{12}})^{1/3}}$$

$$\frac{(1 - \sqrt{3}i)(-250\alpha_L^3 + 501\alpha_L^4 - 282\alpha_L^5 + 47\alpha_L^6 + 3\sqrt{3}\sqrt{-1500\alpha_L^7 + 3863\alpha_L^8 - 3728\alpha_L^9 + 1706\alpha_L^{10} - 372\alpha_L^{11} + 31\alpha_L^{12}})^{1/3}}{2^{4/3}3\alpha_L^2},$$

$$r_3 = \frac{\alpha_L(7\alpha_L - 5)}{3\alpha_L^2} - \frac{2^{1/3}(-25\alpha_L^2 + 28\alpha_L^3 - 7\alpha_L^4)}{3\alpha_L^2(-250\alpha_L^3 + 501\alpha_L^4 - 282\alpha_L^5 + 47\alpha_L^6 + 3\sqrt{3}\sqrt{-1500\alpha_L^7 + 3863\alpha_L^8 - 3728\alpha_L^9 + 1706\alpha_L^{10} - 372\alpha_L^{11} + 31\alpha_L^{12}})^{1/3}}$$

$$+ \frac{(-250\alpha_L^3 + 501\alpha_L^4 - 282\alpha_L^5 + 47\alpha_L^6 + 3\sqrt{3}\sqrt{-1500\alpha_L^7 + 3863\alpha_L^8 - 3728\alpha_L^9 + 1706\alpha_L^{10} - 372\alpha_L^{11} + 31\alpha_L^{12}})^{1/3}}{2^{1/3}3\alpha_L^2}.$$

Now, we will check if any of these roots fall in the interval (0,1) for $0 < \alpha_L < 1$. First we need to find the cubic root of the expression that is common in second term denominator and third term numerator in all three roots:

$$(-250\alpha_L^3 + 501\alpha_L^4 - 282\alpha_L^5 + 47\alpha_L^6 + 3\sqrt{3}\sqrt{-1500\alpha_L^7 + 3863\alpha_L^8 - 3728\alpha_L^9 + 1706\alpha_L^{10} - 372\alpha_L^{11} + 31\alpha_L^{12}})^{1/3}.$$

The term under the root is negative for all $\alpha_L \in (0,1)$. We name this expression $z = x + yi$ where

$$x = -250\alpha_L^3 + 501\alpha_L^4 - 282\alpha_L^5 + 47\alpha_L^6 \text{ and}$$

$$y = 3\sqrt{3}\sqrt{1500\alpha_L^7 - 3863\alpha_L^8 + 3728\alpha_L^9 - 1706\alpha_L^{10} + 372\alpha_L^{11} - 31\alpha_L^{12}}. \text{ We define } \rho = \sqrt{x^2 + y^2} \text{ and}$$

$$\eta = \arctan(y/x) \text{ in order to write } z \text{ in polar form: } z = \rho(\cos \eta + i \sin \eta). \text{ The cubic root of } z \text{ is}$$

$$\text{given by: } z^{1/n} = \rho^{1/n} \left\{ \cos\left(\frac{\eta + 2k\pi}{n}\right) + i \sin\left(\frac{\eta + 2k\pi}{n}\right) \right\} \quad k = 0, 1, \dots, n-1. \text{ The actual calculation}$$

$$\text{gives: } z^{1/3} = 2^{1/3} \alpha_L \sqrt{7\alpha_L^2 - 28\alpha_L + 25} \left\{ \cos\left(\frac{\eta}{3}\right) + i \sin\left(\frac{\eta}{3}\right) \right\}. \text{ Using Euler's formula we can transform}$$

$$\text{this into } z^{1/3} = 2^{1/3} \alpha_L \sqrt{7\alpha_L^2 - 28\alpha_L + 25} e^{i\eta/3}. \text{ When we substitute this into the first root and}$$

$$\text{simplify a little we get: } r_1 = \frac{7\alpha_L - 5}{3\alpha_L} - \frac{\sqrt{7\alpha_L^2 - 28\alpha_L + 25}}{6\alpha_L} \left[\frac{1 - \sqrt{3}i}{e^{i\eta/3}} + (1 + \sqrt{3}i)e^{i\eta/3} \right]. \text{ The term in the}$$

brackets can be rearranged as:

$$(1 - \sqrt{3}i)e^{-i\phi} + (1 + \sqrt{3}i)e^{i\phi} \quad \text{where } \phi = \eta/3$$

$$(1 - \sqrt{3}i)(\cos \phi - i \sin \phi) + (1 + \sqrt{3}i)(\cos \phi + i \sin \phi)$$

$$\cos \phi - \sqrt{3}i \cos \phi - i \sin \phi - \sqrt{3} \sin \phi + \cos \phi + i \sin \phi + \sqrt{3}i \cos \phi - \sqrt{3} \sin \phi$$

$$2(\cos \phi - \sqrt{3} \sin \phi) \text{ and } r_1 \text{ becomes: } r_1 = \frac{7\alpha_L - 5}{3\alpha_L} - \frac{\sqrt{7\alpha_L^2 - 28\alpha_L + 25}}{3\alpha_L} (\cos \phi - \sqrt{3} \sin \phi).$$

The values taken by r_1 for $0 < \alpha_L < 1$ is given in Figure A1. For $1/3 < \alpha_L < 1$, $r_1 \in (0,1)$. A similar calculation for the other roots shows that both of them are not in $(0,1)$ for any $\alpha_L \in (0,1)$ so we can eliminate them. Therefore $r_{acc}^* = r_1 \cdot r_{acc}^*$ satisfies (14) for $\alpha_L \in (1/3, 0.586)$ enabling all three firms to make positive revenues.

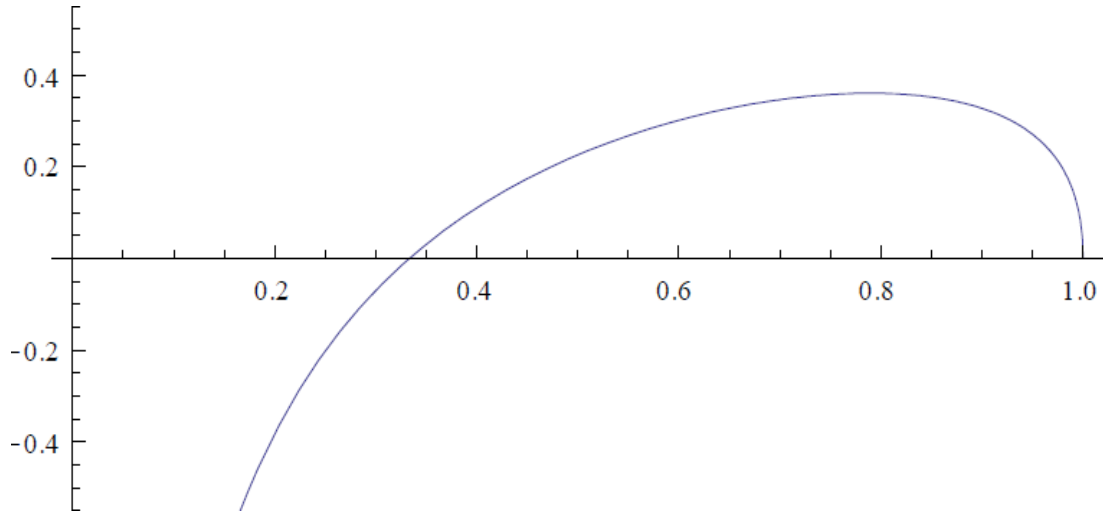


Figure A1

The analysis of the fight case is similar. Differentiating the three profit functions in (12), with respect to p_{AH} , p_{AL} and p_B respectively and solving the first order conditions together we get the best response prices in (15) when (14) does not hold. Substituting p_B^{**} in π_B^{III} , differentiating with respect to β , setting the first order condition to zero and solving gives

$$\beta_{fight}^* = \pm \sqrt{\frac{\alpha_L(1-\alpha_L)^2(2-r)^2}{k_B(4-4\alpha_L+r^2\alpha_L)^2}}. \text{ Since quality cannot be negative we select the positive root.}$$

In the second stage substituting the best response prices and quality in π_{AH}^{III} , and differentiating with respect to r gives:

$$\frac{(1-\alpha_L)(2-2\alpha_L+r\alpha_L)(3\alpha_L^2(r-2)^3+2\alpha_L(2-r)(8-5r)-8)}{(2-r)(4-4\alpha_L+r^2\alpha_L)^3} \sqrt{\frac{\alpha_L(1-\alpha_L)^2(2-r)^2}{k_B(4-4\alpha_L+r^2\alpha_L)^2}}. \text{ An inspection}$$

shows that the only part that can be equal to zero in this expression is:

$3\alpha_L^2(r-2)^3 + 2\alpha_L(2-r)(8-5r) - 8$. Expanding and rearranging this term we get

$3\alpha_L^2r^3 + 2\alpha_L(5-9\alpha_L)r^2 - 36\alpha_L(1-\alpha_L)r + 8(3\alpha_L-1)(1-\alpha_L)$. This cubic expression has three roots:

$$r_4 = \frac{2(9\alpha_L-5)}{9\alpha_L} + \frac{(1-\sqrt{3}i)(9\alpha_L-25)}{9(-125\alpha_L^3+189\alpha_L^4+9\sqrt{3}\sqrt{-125\alpha_L^7+122\alpha_L^8+3\alpha_L^9})^{1/3}} - \frac{(1+\sqrt{3}i)(-125\alpha_L^3+189\alpha_L^4+9\sqrt{3}\sqrt{-125\alpha_L^7+122\alpha_L^8+3\alpha_L^9})^{1/3}}{9\alpha_L^2},$$

$$r_5 = \frac{2(9\alpha_L-5)}{9\alpha_L} + \frac{(1+\sqrt{3}i)(9\alpha_L-25)}{9(-125\alpha_L^3+189\alpha_L^4+9\sqrt{3}\sqrt{-125\alpha_L^7+122\alpha_L^8+3\alpha_L^9})^{1/3}} - \frac{(1-\sqrt{3}i)(-125\alpha_L^3+189\alpha_L^4+9\sqrt{3}\sqrt{-125\alpha_L^7+122\alpha_L^8+3\alpha_L^9})^{1/3}}{9\alpha_L^2},$$

$$r_6 = \frac{2(9\alpha_L-5)}{9\alpha_L} + \frac{2(9\alpha_L-25)}{9(-125\alpha_L^3+189\alpha_L^4+9\sqrt{3}\sqrt{-125\alpha_L^7+122\alpha_L^8+3\alpha_L^9})^{1/3}} - \frac{2(-125\alpha_L^3+189\alpha_L^4+9\sqrt{3}\sqrt{-125\alpha_L^7+122\alpha_L^8+3\alpha_L^9})^{1/3}}{9\alpha_L^2},$$

Now, we will check if any of these roots fall in the interval (0,1) for $0 < \alpha_L < 1$. First we need to find the cubic root of the expression that is common in second term denominator and third term numerator in all three roots: $(-125\alpha_L^3+189\alpha_L^4+9\sqrt{3}\sqrt{-125\alpha_L^7+122\alpha_L^8+3\alpha_L^9})^{1/3}$. The term under the root is negative for all $\alpha_L \in (0,1)$. We name this expression $z = t + wi$ where $t = -125\alpha_L^3+189\alpha_L^4$ and $w = 9\sqrt{3}\sqrt{-125\alpha_L^7+122\alpha_L^8+3\alpha_L^9}$. We define $\rho = \sqrt{t^2 + w^2}$ and $\eta = \arctan(w/t)$ in order to write z in polar form: $z = \rho(\cos \eta + i \sin \eta)$. The cubic root of z is given by:

$$z^{1/n} = \rho^{1/n} \left\{ \cos\left(\frac{\eta + 2k\pi}{n}\right) + i \sin\left(\frac{\eta + 2k\pi}{n}\right) \right\} \quad k = 0, 1, \dots, n-1. \text{ The actual calculation gives:}$$

$$z^{1/3} = \alpha_L \sqrt{25-9\alpha_L} \left\{ \cos\left(\frac{\eta}{3}\right) + i \sin\left(\frac{\eta}{3}\right) \right\}. \text{ Using Euler's formula we can transform this into}$$

$$z^{1/3} = \alpha_L \sqrt{25-9\alpha_L} e^{i\eta/3}. \text{ When we substitute this into the first root and simplify a little:}$$

$$r_1 = \frac{2(9\alpha_L-5)}{9\alpha_L} - \frac{\sqrt{25-9\alpha_L}}{9\alpha_L} \left[\frac{1-\sqrt{3}i}{e^{i\eta/3}} + (1+\sqrt{3}i)e^{i\eta/3} \right]. \text{ From the analysis of accommodation case}$$

we know that the term in the brackets is equal to $2(\cos \psi - \sqrt{3} \sin \psi)$ where $\psi = \eta/3$ and r_4

$$\text{becomes: } r_4 = \frac{2(9\alpha_L-5)}{9\alpha_L} - \frac{2\sqrt{25-9\alpha_L}}{9\alpha_L} (\cos \psi - \sqrt{3} \sin \psi). \text{ For } 1/3 < \alpha_L < 1, r_1 \in (0,1). \text{ A similar}$$

calculation for the other roots shows that both of them are not in (0,1) for any $\alpha_L \in (0,1)$ so we can eliminate them. For $\alpha_L \in (0,1/3]$ r_4 is negative, so the optimal r for fighting is 0 for this range. Therefore, $r_{fight}^* = \{0 \text{ for } \alpha_L \in [0,1/3] \text{ and } r_4 \text{ for } \alpha_L \in (0.583,1)\}$.

r_{fight}^* does not satisfy (14) for $\alpha_L \in [0,1/3] \cup (0.583,1)$ resulting in zero revenues for the low quality firm and positive revenues for the other firms. Note that the low quality firm's choice of α_L largely determines the high quality firm's reaction in terms of accommodating or fighting as

the regions only overlap for $\alpha_L \in (0.583, 0.586)$. Comparing the high quality firm's profits in the accommodate and fight cases we establish the optimal r as a function of α_L :

$$r^*(\alpha_L) = \{ r_F^* \text{ for } \alpha_L \in [0, 1/3] \cup [0.584, 1) \text{ and } r_A^* \text{ for } \alpha_L \in (1/3, 0.584) \}$$

In the first stage the low quality firm can either choose $\alpha_L = 0$ and stay out of the market or chose $\alpha_L \in (0, 1)$ and enter. It is easy to see that if the low quality firm decides to enter, it will never choose $\alpha_L \in [0, 1/3] \cup [0.584, 1)$ which makes the high quality firm fight. On the other hand if A_L chooses $\alpha_L \in (1/3, 0.584)$ the high quality firm chooses r_{acc}^* , firm B chooses β_{acc}^* and the best response prices are as in (13) resulting in $S_{AL}(\alpha_L)$ which is positive. The only thing left to check for the existence of an accommodating equilibrium is whether

$\pi_{AL}^{III}(\alpha_L) = S_{AL}(\alpha_L) - C(\alpha_L) > 0$. If the cost of developing a product is less than the revenues of the low quality firm, the low quality firm will enter the market. In the accommodating case, the revenues of the low quality firm is quasiconcave and it reaches its maximum value at $\alpha_L = 0.463$ where the revenues are 7×10^{-6} . If $C(\alpha_L)$ is less than 7×10^{-6} , then there exists an accommodating equilibrium.

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