R&D, Marketing, and the Success of Next-Generation Products

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This paper studies dynamic competition in markets characterized by the introduction of technologically advanced next-generation products. Firms invest in new product effort in an attempt to attain industry leadership, thus securing high profits and benefiting from advantages relevant for the success of future product generations. The analysis reveals that when the current leader possesses higher research and development (R&D) competence, it tends to invest more in R&D than rivals and to retain its lead position. The leader’s investment exhibits an inverse-U pattern as this advantage increases. In contrast, when the leader enjoys an advantage that originates from the persistence of reputation, it invests less than its followers. Now, followers’ investment exhibits an inverse-U pattern as reputation advantage increases. Depending on the extent of leader reputation, industry structure can either exhibit frequent leadership shifts or prolonged incumbent dominance. The basic framework is extended to allow investments in additional marketing variables (e.g., advertising). Interestingly, the leader takes advantage of strong demand for its current product by focusing more on advertising, whereas the follower expends more on R&D. By shedding light on the implications of industry position for investment incentives and market evolution, the analysis provides valuable insights for formulating marketing strategy in fast-paced, high-tech business environments.

(High-Technology Marketing; New Product Development; Dynamic Capabilities; Technological Competition; Markov-Perfect Equilibrium)

1. Introduction
Fast-changing, technology-intense markets are characterized by the successive introduction of next-generation products. For example, over the past two decades, the PC microprocessor industry has witnessed seven distinct hardware generations, each pushing the technological frontier further by offering faster clock speeds, increased number of instructions per clock cycle, and superior mathematical computing. Likewise, the home video game market has, since the late 1970s, seen six well-defined eras of advanced systems, each delivering higher graphics and sound quality. In both industries, in each generation, several firms introduced their latest product offering at roughly the same time, typically with only one firm securing supranormal profits (see Figure 1).

Although potential profit rewards motivate investment in new product activity, it may not be the case that all firms in a given industry possess the same research and development (R&D) or marketing capabilities needed to master future success. Furthermore, a growing body of empirical work shows that, in many instances, these capability differences are related to the commercial success or
Figure 1  Successive Product Generations

PC Microprocessors:

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<td>Mot. 68000</td>
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Home Video Game Platforms:

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<td>Mattel (Intellivision)</td>
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<td>Nintendo (N64)</td>
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Note. In each period, the major new products introduced and available in volume shipments in each industry are depicted (list is not exhaustive, Mot. = Motorola). Company listed first in boldface made higher than industry average profits with its product (relevant for periods prior to the early 2000s for which data is available), other firms are in alphabetical order. For PC microprocessors, products manufactured under licensed agreements are not included. Data compiled from Burgelman et al. (1996), Malone (1995), Business Week (June 1, 1992, pp. 86–94), AMD 10K reports (1982–1995), Newsweek (January 31, 2000, p. 61), Inforworld (February 28, 2000, p. 27), Electronic Business (March 2000, pp. 38–40), and Schilling (1997).

This paper studies the competitive interaction among firms vying for industry leadership in fast-changing product markets, focusing on the following research questions:

- What governs the incentives of firms to invest in R&D when current leaders enjoy strategic advantages in factors that affect the success of new products? When do such greater capability advantages to leaders encourage follower effort? When do they encourage leader effort?
- How do different capability advantages accruing to successful firms impact market evolution patterns? Are firms that possess higher capability always more likely to succeed in the next generation?
- Given that new product activity involves various elements of the marketing mix, how do leaders and followers allocate resources to R&D when investment in advertising also carries over to affect future success?

To answer these questions, a dynamic model is developed in which firms can undertake costly investments to generate new products in each period. The outcome of these investments is ex ante uncertain, and the products developed compete in the subsequent period in a contestlike fashion. In the basic model setup, the successful firm in each period attains
the lead position, which entails higher than follower per-period profits and bears a strategic capability advantage for next-generation success.

In general, the analysis of competitive dynamics reveals two main forces that affect effort levels and leadership patterns. The first is related to how being a leader impacts a firm’s investment productivity, defined as the marginal change in the probability of winning the next round because of a marginal increase in current investment. The second is related to how the prospect of becoming a leader and attaining the advantage affects future payoffs, either by prolonging the expected future stay in the lead position or by allowing less future investment to achieve the same chances of winning. It is found that, when the advantage is in the form of higher R&D competence (innovative advantage), the investment productivity effect tends to induce greater leader effort and lower follower effort, dominating the future-payoff effect, which in this case gives all firms greater incentive to win because of the longer expected time of retaining the lead position. As the advantage becomes more pronounced, leader investment productivity eventually decreases in the advantage, as even a small effort level results in high success probability, generating an inverse-U pattern of leader investment. When the advantage is not too high, followers invest more than the leader by an amount that exactly compensates them for the leader’s advantage. Hence, in equilibrium, all firms have an equal probability of leading the pack in the next period. Furthermore, for followers, the future-payoff effect dominates, and initially results in greater investment the higher the advantage. When the advantage is large, however, the future-payoff effect does not justify the investment required to match the leader’s ex ante high probability of success, and prolonged incumbent dominance is expected.

The basic framework of the paper is extended to explore investments in additional marketing mix variables (e.g., advertising) that affect current market performance and carry over to influence future new product success. Leader and follower are found to allocate resources between R&D and advertising differently. The leader takes advantage of the strong demand for its current product by focusing more on advertising, whereas the follower spends more on R&D. The implications of increasing either leader or follower advertising carryover on R&D investments are also examined and explained in terms of the forces identified earlier.

The paper is related to several literature streams. In the context of modeling competition for sequences of innovations (Reinganum 1989), prior research has focused on a number of issues different from those examined here. Vickers (1986) investigates how the nature of per-period market competition, as it determines joint profits, affects industry evolution. He finds that, if joint profits are higher when the laggard innovates, the two firms will take turns investing in R&D (and thus winning the next round). Reinganum (1985) concentrates on the standalone incentive each firm has to innovate by modeling R&D competition as a stochastic patent race. She finds that the incumbent, already enjoying the fruits from the previous innovation, has less of an incentive to shorten the time until the next one occurs (known as the replacement effect). Consequently, it invests less than each challenger and has a lower probability of retaining its lead position. As the model to be presented here allows the ability of firms to master new product success to depend on past generation outcomes, our paper contributes to the aforementioned literature by exploring several different ways in which attaining an industry position has an impact on the determinants of future success probability.

1 In a continuous and stochastic version, Budd et al. (1993) again find that the technology gap between firms tends to move in the direction of increasing joint per-period profits. In the case of increased dominance by the current leader, the results of both these papers are similar in spirit to those of Gilbert and Newbery (1982). This is known as the efficiency effect. In the context of dynamic technology adoption, Villas-Boas (1992) shows that the joint profit effect is closely related to whether firms’ products are strategic substitutes or complements in the decision variable.

2 In Ghemawat (1991), consumer switching costs for the incumbent’s current product are generation-specific and do not bestow any
In this respect, the paper also bears on recent conceptual work in strategy (e.g., Wernerfelt 1984, Teece et al. 1997) and marketing (Day 1994) that emphasizes firm-specific capabilities and assets as the fundamental determinants of business performance. The proposed analytic model can be viewed as one in which firms repeatedly compete for achieving/sustaining a status that confers greater capability advantages for future success (as well as higher per-period profits). The paper contributes to this literature by exploring the implications, for investment strategies and market structure evolution, of different forms of dynamic, path-dependent capability advantages.

Finally, the paper is related to a growing body of empirical research that suggests a relationship between R&D and marketing capabilities in high-tech industries (Levin et al. 1987, Dutta et al. 1999). Surprisingly, existing analytic models of dynamic technological competition give little attention to non-R&D expenditures. By explicitly allowing investment in both R&D and advertising to be undertaken simultaneously (in §4), the paper highlights the importance of understanding how leader ability to exploit strong current demand through additional elements of the marketing mix impacts the incentives of firms to engage in innovative activity.

The remainder of the paper is organized as follows: The next section develops the basic model set-up. Section 3 analyzes the possibility of different capability advantages accruing to successful firms attaining industry leadership. The model is then extended in §4 to allow expenditure in advertising, as well as R&D. Implications for investment strategies and market structure evolution are highlighted throughout the analysis. The paper ends with concluding remarks.

2. The Basic Model
Consider a market in which in any given period one firm (the leader) is recognized as offering a superior product, whereas all the other firms (followers) offer products perceived to be equally inferior. Let leader and follower per-period profits be denoted $\pi_l$ and $\pi_f$, respectively, and assume that $0 \leq \pi_f < \pi_l < \infty$, expressing the fact that the current leader enjoys some fixed form of extra market power.\(^3\)

In each of infinitely many discrete periods, firms may engage in costly investments in an attempt to improve on the current “state-of-the-art” product and win the next leadership contest. The focus of the analysis will be on stationary equilibria, in which firms’ actions can be expressed as a function of their current state. Let $x_i$ be the choice of R&D investment level of the leader and $x'_j$ be that of each of the $n$ followers with $x_i, x'_j \geq 0$.\(^4\) Capital markets are assumed to be efficient so that firms have access to optimal allocation of resources. The outcome of these investments is stochastic, and the probability that a firm in state $s \in \{l, f\}$ wins the next round, thus attaining leadership, is $P_l$ for the current leader and $P_f$ for followers, with the requirement that $P_l + \sum_{j} P_f = 1$. Given that outcomes are determined through a market contest, let the transition probabilities between states take the form:

$$
P_l = \frac{h_l(x_l)}{h_l(x_l) + \sum_{j=1}^{\infty} h_f(x'_j)},
$$

$$
P_f = \frac{h_f(x'_j)}{h_l(x_l) + \sum_{j=1}^{\infty} h_f(x'_j)},
$$

\(^3\) The prevalence of “winner-take-most” outcomes in technology-based markets (Arthur 1996, Shapiro and Varian 1999), leading to relatively fixed payoffs in each period, can be justified in several ways. For example, with Bertrand competition, the firm with the highest-quality product can price so as to force all other firms to make zero profits. A second possible approach posits the existence of a considerable consumer segment with higher willingness to pay for the product they deem best. Third, if network externalities are present, and consumer expectations track quality improvement, a similar conclusion can be obtained. See also Footnote 17 for examples of real-life market mechanisms resulting in winner-take-most outcomes.

\(^4\) The state of any firm at time $t$, which summarizes the direct impact of past play on the current environment (Fudenberg and Tirole 1998, p. 501) and its current investment level, will be sufficient to completely determine the evolution of all firms in the ensuing period. This implies that optimal investment strategies can be chosen from the class of stationary Markov strategies, and time subscripts can be dropped (see also Ericson and Pakes 1995).
where \( h_l(\cdot) \) reflects the leader’s capability of introducing a successful next-generation product as a function of investment and, similarly, \( h_f(\cdot) \) for followers. These functions satisfy \( h_s(0) \geq 0 \), \( h'_s > 0 \) and \( h''_s \leq 0 \) for each \( s \in \{l, f\} \). (The derivatives are taken with respect to the relevant investment variable (i.e., \( x_l \) for leader and \( x_f \) for each follower)). Hence, all else equal, higher effort yields greater chances of success, with diminishing returns to such effort. If no firm invests, the current leader is assumed to retain its position with probability one.

In every period, each firm will have a state-dependent value function; which gives its expected discounted payoff in the game starting from that state. These functions are denoted \( V_l \), \( V_f \). Assuming firms discount future revenues with a discount factor of \( \delta \in (0, 1) \), the following equations describe the optimization problem firms face in each of the two states:

\[
V_l = \text{Max}_{x_l} \left[ \pi_l - x_l + \delta \left( V_l p_l + V_f \sum_j p^j_f \right) \right], \\
V_f = \text{Max}_{x_f} \left[ \pi_f - x_f + \delta \left( V_l p_f^l + V_f \left( p_f^l + \sum_{j \neq l} p^j_f \right) \right) \right].
\]

(2)

Firms make their decisions on investment levels simultaneously at the beginning of each period and then realize current-period profits. We seek a Markov Perfect Nash Equilibrium (MPNE) of the game. In such an equilibrium, optimal investments and transition probabilities are only functions of payoff-relevant variables, and the profile of Markov strategies yields a Nash equilibrium in every proper subgame (Fudenberg and Tirole 1998). Thus, asymmetries in the ability to master future success based on industry position can be captured (these asymmetries are common knowledge). It should be clear from (2) that, in choosing current investment strategy, each firm takes into account the impact on future investment incentives. Greater investment makes it more probable that success will occur, which would then result in possible relative capability advantages for subsequent new product activity. In an extension discussed in the Conclusion section, an enriched state-space is allowed whereby advantages gradually accumulate/dissipate with repeated success.\(^6\)

Given the above formulation of the game, the following lemma characterizes the equilibrium solution (in subsequent analysis, all proofs have been relegated to the Appendix).

**Lemma 1.** There exists a unique solution for the value functions and investments in (2). Furthermore, the equilibrium investments in an interior solution satisfy:

\[
nh_l(x_f)h'_l(x_l) = [(n-1)h_l(x_f) + h_l(x_l)]h'_l(x_f). \quad (3)
\]

In other words, Lemma 1 states that, in an interior equilibrium, firms’ marginal return on investment will be equal. Note also from (3) that, in the basic model setup, the implicit relationship between \( x_l \) and \( x_f \) does not depend on the per-period profits \( \pi_l \) and \( \pi_f \).

### 3. Advantages for Next-Generation Success

In attaining the lead position, there may arise advantages that considerably affect the leader’s ability to achieve future success. Two types of such state-dependent capability advantages, qualitatively different in nature, are considered. The first, which will be called innovative advantage, is relevant in cases where a

\(^6\)Although cumulative investment outcomes do matter in the model, R&D effort does not stock from period to period. Because of the inherent risk associated with R&D, it is possible that a firm that invests more than a competitor will be unsuccessful. If R&D efforts were then to be treated as a pure “stock” variable, the unsuccessful firm would be at an advantage compared with the successful one. With the outcome of risky R&D investment relevant for the model presented here, it is assumed that the increase in relative capability obtained from success is not linked to the exact amount previously invested (although more investment increases the likelihood of such capability gain). This characterization is consistent with the treatment of R&D in economic growth and trade theory (e.g., Grossman and Helpman 1991, p. 564) and the study of dynamic technological progress (e.g., Teece et al. 1997).
firm’s past success renders its current investment relatively more effective in developing future products. Such path dependence of R&D competence has been demonstrated, for example, in chemicals and drugs (Grabowski 1968, Dasgupta 1986) and semiconductor logic design (Teece 1998, p. 61).

The second type of advantage, which will be called reputation advantage, captures an increase in the probability of achieving future success independent of current R&D effort. This type of advantage may be relevant when factors, such as brand image or established supply chain relations, are part of the determinants of next-generation success and are enjoyed by the firm currently in the lead position. Examples of industry settings in which such advantages have empirically been shown to be present include extension of brand name recognition across product generations in the personal computer market (Bresnahan et al. 1997) and incumbent preferential access to distribution channels in the diagnostic imaging industry (Mitchell 1989). In what follows, implications of these two capability advantages are examined in turn.

3.1. Innovative Advantage

Consistent with the previous discussion, this type of advantage can be captured by imposing the following requirement on the capability functions: \( h_i(x) > h_i^*(x), \forall x \geq 0 \). For simplicity, a linear specification for the \( h_i \) functions is assumed (the results do not change qualitatively when these functions are concave) with \( h_i(x_i) = k_i x_i, h_i(x_i') = k_i x_i', \) and \( k = k_i/k_j > 1 \). The MPNE solution in the presence of innovative advantage is now provided.

Proposition 1. Let \( k \) be the extent of leader innovative advantage. In equilibrium, next-generation success probabilities satisfy \( P_i^* > P_j^* \), with an expected amount of time a firm will spend in the lead position \( k + 1/n \) periods. Investment strategies satisfy \( x_i^* \geq x_j^* \).

What are the implications of Proposition 1 for the expected pattern of industry leadership? As long as there is innovative advantage \((k > 1)\), a firm that secures the lead position is expected to stay in that position for a longer than random amount of time before it is overthrown. With respect to investment strategies, armed with innovative advantage, the leader tends to invest more than any of its rivals (strictly whenever \( n > 1 \)), even though equal investment still would have resulted in a higher probability of success. This occurs because innovative advantage makes the leader’s investment productivity (i.e., the rate of change of winning probability because of a marginal increase in current R&D investment \((\partial P_i/\partial x_i)\)) greater than that of each follower (when evaluated at the same investment levels). Coupled with diminishing marginal return to investment, and consistent with Lemma 1, the leader has an incentive to invest more than each follower.

“Success breeding success” as a result of more effective R&D capability has in fact been observed empirically. In a study of the pharmaceutical industry (1975–1988), Henderson and Cockburn (1994a, b) find that research productivity increased with past success, and that successful firms tended to invest more and sustain their lead position over time. A similar pattern has also been theoretically produced in other contexts (e.g., Rogerson 1982, Barro and Sala-i-Martin 1995).

We now turn to examine how the degree of innovative advantage affects investments depending on a firm’s industry position:

Result 1. The investment level of the leader is initially increasing and subsequently decreasing in innovative advantage, whereas for \( n > 2 \), the investment level of followers is decreasing in this advantage.

Thus, leader effort has an inverse-U shape as a function of innovative advantage, whereas followers tend to monotonically exert less effort as \( k \) increases. To understand this comparative static result, first
note that an increase in innovative advantage has a *future-payoff effect* (i.e., it impacts the difference between winning and losing continuation value functions \((V_i - V_j)\)), and the effect is positive, as on expectation the higher \(k\), the more periods the winning firm expects to stay in the lead. When innovative advantage is small, leader investment productivity is increasing in \(k\); coupled with the future-payoff effect, the leader is prompted to invest more. On the other hand, follower investment productivity is decreasing in both innovative advantage and leader effort (strategic substitutes with respect to R&D effort). Under the condition in Result 1, these forces dominate the future-payoff effect, causing each follower to invest less. When innovative advantage becomes large, leader equilibrium investment productivity begins decreasing in \(k\) (as \(P_i \to 1\), it becomes less sensitive to changes in \(kx_j\)) and, in addition, the leader now has an incentive to match changes in follower effort (strategic complements). With followers still reducing effort as \(k\) increases, both factors reinforce each other and at some point dominate the future-payoff effect, causing the leader to decrease R&D investment.\(^9\) Although the region of increasing leader investment is sensitive to the discount factor through the future-payoff effect, in both Proposition 1 and Result 1, investment patterns are largely determined by the impact of innovative advantage on current investment productivity and would also be present in a static model.

The following result examines how investment levels depend on the number of firms contesting the current leader.

**Result 2.** The investment level of each follower is monotonically decreasing in the number of competing firms. The investment of the leader can be monotonically decreasing, initially increasing and subsequently decreasing, or monotonically increasing in the number of competitors, depending on the values of \(k\) and \(\delta\).

\(^9\) Technically, the dependence of investment productivity on any given variable \(\mu\) is \(\partial P_i / \partial \mu\) and that of the future-payoff effect \(\partial (\Delta V) / \partial \mu\), where \((\Delta V)\) is the difference between next-period winning and losing continuation value functions.

\(^*\) We thank the area editor and an anonymous reviewer for asking us to clarify these intuitions.

The intuition behind this result relates to the fact that each additional competitor imposes a negative future-payoff effect on all firms, because as \(n\) increases a firm expects to stay in the lead position for fewer periods (see Proposition 1). Followers, who are also disadvantaged in terms of investment productivity, unambiguously reduce effort with more contenders. For the leader, however, there is an interplay between the future-payoff effect and investment productivity that can result, depending on the values of \(k\) and \(\delta\) (see Appendix for a formal characterization) in either decreasing or increasing investment as more followers vie for leadership (see Figure 2).

### 3.2. Reputation Advantage

In the previous section, asymmetries in the relative effectiveness of resources committed to new product development were explored. In this section, the analysis focuses on the case in which the advantage to leadership is independent of resources allocated specifically to the R&D process. Rather, it may result from ability to influence the perceived benefits of the next-generation product, thus augmenting the chances of winning.\(^10\) To capture such an advantage to leadership, assume that \(h_i(x) = h_i(x + m), \forall x \geq 0\), where the parameter \(m \geq 0\) reflects an additional non-R&D success factor. Using the linear specification, the following proposition gives the MPNE outcome in this case.

\(^{10}\) For example, as new technology often embodies considerable uncertainty for customers, a firm will more easily be able to convince retailers and consumers to adopt its next-generation product if its current product is a success (Chandy and Tellis 2000). Work on brand extensions in marketing (e.g., Aaker and Keller 1990, Park and Srinivasan 1994) has also shown that consumers’ higher-quality perceptions toward the original brand are associated with more-favorable attitudes towards an extension when there is some fit between the two. One can expect the type of extension modeled here, a next-generation improved version of the product, to in fact exhibit a close fit in the eyes of customers. Thus, positive evaluations are likely to transfer from the current product to the next-generation one. Although again, such mechanisms are not micromodeled, §4 does allow the increase in probability to be endogenously related to the choice of a strategic variable (other than R&D) indirectly linked to higher demand for the leader’s current product.
**Figure 2** R&D Investments as a Function of the Number of Followers, the Extent of Innovative Advantage, and the Discount Factor

Notes. $x_l$, $x_f$ = leader and follower investments, respectively; $n$ = number of followers; $k$ = innovative advantage; $(\pi_l - \pi_f) = 1$.

**Proposition 2.** Let $h_i(x_l) = x_l + m$ and $h_f(x_f^i) = x_f^i$. Then, depending on the relevant region for $m$, the equilibrium next-generation success probabilities and investments satisfy:

$$m < m^* = \frac{\delta n (\pi_l - \pi_f)}{(n+1)^2 - \delta n},$$

$$P_l^* = P_f^*, \quad x_f^* - m = x_l^* > 0.$$ \hspace{1cm} (4)

$$m^* < m < m^{**} = \frac{\delta (\pi_l - \pi_f)}{1 - \delta},$$

$$P_l^*> P_f^*> 0, \quad m > x_f^* > x_l^* = 0.$$ \hspace{1cm} (4)

$$m^{**} < m, \quad P_l^* = 1 > P_f^* = 0, \quad x_l^* = x_f^* = 0.$$ \hspace{1cm} (4)

What are the implications of Proposition 2 for market structure evolution? When reputation advantage is small $(0 < m < m^*)$, all firms have equal chances of leading the pack in the next period (i.e., the identity of the leader is expected to exhibit relatively frequent shifts). This occurs because reputation advantage replaces part of the leader’s need to invest, reducing its investment productivity, compared with that of followers (when evaluated at equal investment levels). In equilibrium, based on Lemma 1, this results in each follower investing more than the leader. While Vickers (1986) identified conditions for an alternating pattern of leadership, there are two fundamental differences between his result and the one presented here. First, in his duopoly model, firms take turns investing in R&D, and thus leading. In contrast, in the first region of (4), the identity of the next-period leader is always random (on expectation, each firm wins with equal probability), and all firms positively invest. The second difference comes from the reason for expecting an alternating pattern. In Vickers (1986), the result is driven by the form of product market competition (Cournot versus Bertrand), which affects joint per-period profits. Here, it originates from factors that impact the process by which the leader is determined and that affect investment productivity $(\delta P_l/\partial x_l)$. 

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When reputation plays a more significant role \(m^* < m < m^{**}\), leader chances of winning the next round are higher than that of each follower. Although such a pattern of market evolution was observed in the previous subsection as well, there is a substantive difference between the two cases. With innovative advantage, the leader tends to invest more than followers and wins with higher probability because its next-generation product embodies greater R&D. However, when reputation advantage is high enough, each follower invests strictly more in product development, and yet the leader is more likely to win. In the extreme event that \(m^{**} < m\), no firm wishes to challenge the current leader who enjoys uncontested dominance.

The pattern of investment strategies just described is reinforced by the following result.

\textbf{Result 3.} \textit{The investment level of each follower is initially increasing and subsequently decreasing in reputation advantage, whereas the investment of the leader is non-increasing in this advantage.}

This pattern of investments sharply contrasts with Result 1, because now follower effort exhibits an inverse-U relationship with increased leader advantage. This occurs because for all firms the future-payoff effect is increasing in the extent of reputation (as the winning firm expects to invest less in the lead position to achieve the same chances of next-generation success). At the same time, for the leader investment productivity is sharply decreasing in reputation advantage, with this latter effect dominating. Followers’ investment productivity is much less sensitive to changes in this advantage, and thus for them, initially as \(m\) increases the future-payoff effect dominates (the fact that leader-follower investments are strategic substitutes reinforces this tendency). However, because the future benefits arising from reputation advantage are bounded, eventually the countervailing effect of lower R&D investment productivity takes over, and follower effort decreases in \(m\). It is noteworthy that \(m^*\) and \(m^{**}\) (see (4)) are positively related to the discount factor \(\delta\). Thus, the more the future matters (or the smaller the time lag between successive periods), the greater the impact of the future-payoff effect; consequently, the greater the region in which follower effort is increasing in reputation advantage and in which all firms have equal next-generation success probabilities.\(^{11}\)

\section{4. Balancing R&D with Advertising}

Investments in previous sections had an impact on a firm’s future success, but did not influence current market performance. In this respect, they capture an R&D effort for which there is a considerable time lag between resources committed and their potential outcome. On the other hand, there are expenditures that firms undertake to boost current-period sales that may also carry over to influence future success. For example, by spending on advertising a firm creates better awareness for its currently offered product, thereby affecting immediate profits. Typically though, advertising may also have a long-term effect of generating strong brand value transferable to the next-generation product. Other marketing activities, such as building supply chain relations, have a similar dual effect. Because in many industries resources are allocated to R&D as well as such marketing variables, the basic framework is modified to simultaneously incorporate them. The model setup will remain similar to that in §2, except that now, because one type of expenditure also affects current period sales, product market competition will be modeled explicitly. To highlight the strategic implications, it suffices to examine the duopoly case. Advertising is used as a working example of a marketing expenditure with current and future impact.\(^{12}\)

Consider two firms engaged in a multiperiod competition for leadership. In each period, after the

\(^{11}\) In a static, single-innovation model, follower effort would not depend on reputation advantage as modeled here. This is because unless there is a continuation of the game (beyond one period), a follower would not benefit from the advantage that influences the success of a next-generation product (i.e., there is no future-payoff effect to the advantage in the single-innovation case).

\(^{12}\) Villas-Boas (1993) has looked at dynamic competition between firms where advertising was the sole strategic variable. The analysis here will focus on how advertising may substitute/complement investment in product development.
identity of the leader has been established, assume per-period profits are determined as follows:

\[
\pi_l = \left[ w_l - \frac{p_l}{1 + a_l} + \frac{p_f}{1 + a_f} \right] p_l - \frac{1}{2} ba_l^2,
\]

\[
\pi_f = \left[ w_f - \frac{p_f}{1 + a_f} + \frac{p_l}{1 + a_l} \right] p_f - \frac{1}{2} ba_f^2,
\]

where \( p_l, p_f \) and \( a_l, a_f \) are prices and advertising levels of leader and follower, respectively. Terms in square brackets in Equation (5) reflect demand schedules given pricing and advertising. The effect of advertising in the current period, which comes at cost \( (ba_l^2)/2 \), is to decrease elasticity of demand for one’s own product while increasing that of the competitor. As the leader’s product is perceived better, all else being equal, the basic demand it generates is assumed to be higher (i.e., \( w_l > w_f \geq 1 \)), with total demand constant at \( w_l + w_f \). For simplicity, production costs are taken to be zero (positive production costs do not affect the results).

As argued, advertising also has a long-term effect and serves as a determinant of possible future success. Once again, let \( x_l \) and \( x_f \) be leader and follower investments in R&D, respectively. Using the linear formulation as before on how current investments, both in advertising and R&D, stochastically affect future outcomes, let \( h_l = x_l + \lambda_l a_l \), \( h_f = x_f + \lambda_f a_f \), where \( \lambda_s \) measures the strength of advertising carryover for each of the two firms. The transition probabilities become:

\[
P_l = \frac{(x_l + \lambda_l a_l)}{(x_l + \lambda_l a_l) + (x_f + \lambda_f a_f)},
\]

\[
P_f = \frac{(x_f + \lambda_f a_f)}{(x_l + \lambda_l a_l) + (x_f + \lambda_f a_f)}.
\]

In each period, firms first make decisions on advertising and R&D allocations, which then become common knowledge, and decide on pricing strategies in a second stage (decisions in each stage are simultaneous). The MPNE solution when both firms have a positive incentive to invest in R&D (\( \lambda_s \) small) is now characterized.

**Proposition 3.** Assume \( \lambda_l \geq \lambda_f \geq 0 \). Then, in equilibrium, prices in each period satisfy \( p_l > p_f \), advertising and R&D strategies satisfy \( a_l > a_f \), \( x_l > x_f \), respectively. Furthermore, firms have equal next-generation success probabilities as \( P_l = P_f \).

The proposition indicates a contrast in the investment strategies of the two firms. The intuition is related to the fact that advertising affects both current profits and the probability of winning the next round. The return to advertising in the current period (\( \partial \pi_l / \partial a_l \)) is higher for the leader, as it can leverage its existing stronger appeal to consumers (reflected by \( w_l > w_f \)) to increase profits. Thus, even if \( \lambda_l = \lambda_f \), the leader has greater incentive than the follower to invest in advertising (this tendency becomes stronger if \( \lambda_l > \lambda_f \), see (A16) in the Appendix). The follower, on the other hand, prefers to forgo harsh competition in this period, where basic demand for its product is lower, in an attempt to be in a better position in the future by improving its product through R&D. As a result, both have the same probability of winning in the next period. The leader wins in large part because of advertising effort, whereas the follower wins in large part from investment in R&D.14 The empirical finding that marketing capability is most effective for firms that have first built a strong technology base (Dutta et al. 1999) is consistent with the implications of this proposition.

It is clear from the equilibrium outcome (see (A16) in the Appendix) that as a firm’s advertising carryover parameter increases, it invests more in advertising. The following result describes how advertising carryover affects R&D investments.

**Result 4.** In equilibrium, an increase in leader advertising carryover, \( \lambda_f \), induces less R&D effort by the leader

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13 This formulation is consistent with empirical evidence in the finance literature, which shows that R&D and advertising are both forms of investment in intangible assets with predictably positive effects on future cash flow (e.g., Chauvin and Hirschey 1993).

14 If the leader also enjoys relative innovative advantage \( k \), then higher R&D investment productivity tends to induce greater leader R&D effort (for the reasons explained in §3.1). If \( k \) is very large and \( w \) close to \( w_f \) (so that the leader does not have much to gain in the current period from advertising), then the leader will actually spend more than the follower on R&D and less than the follower on advertising.
5. Conclusion

This paper has examined dynamic competition in markets where new product activity is the focus of business strategy and where a firm’s industry position considerably affects its ability to achieve future success with a next-generation product. In particular, the possibility of technology-related and market-related capability advantages accruing to successful firms has been explored. The analysis reveals that to understand the impact of such advantages, one has to consider two primary effects: first, how the advantage influences investment productivity and, second, how the advantage acts to influence the future payoff to success (versus failure). The net impact of these effects, which can differ across firms based on past success, ultimately determines investment incentives and market evolution patterns.

When success enhances R&D competence, in general, the dominant effect is higher leader investment productivity. Thus, exactly because it enjoys this advantage, the firm in the lead position tends to invest more than rivals, consequently sustaining its position with higher probability. This kind of advantage also drives the leader to initially invest more as its R&D competence increases. In contrast, when success in one period increases chances of future success through reputation-like factors, the firm in the lead position finds its investment productivity to be lower than that of followers (because reputation replaces part of the leader’s need to invest) and expends less effort the greater the advantage. Success, on the other hand, foresees the benefits of gaining this type of advantage (less future effort for achieving equal chances of success) as the dominant factor, and initially increase their R&D effort as the advantage becomes more pronounced. As long as reputation advantage is moderate, all firms are equally likely to attain leadership in the next period. When reputation advantage is high, the future-payoff effect does not offset the large investment needed to match leader probability of winning, and prolonged incumbent dominance is expected.

Another interpretation of this outcome is related to changes in the fraction R&D represents in the (R&D + advertising) budget. The model (Proposition 3) predicts a negative correlation between changes in the relative weight given to R&D by two competitors in a particular product market. This implication is borne out in the microprocessor industry between the years 1982–1997 for two major competitors, Intel and AMD. Using data from the Compustat database, the correlation between annual changes in the percent each firm allocated to R&D, as a fraction of the (R&D + advertising) budget, is found to be −0.353 (smaller than 0 at the 10% significance level; one-tail test). The hypothesis is also supported in this case by running a more rigorous simultaneous equations specification (see Ofek 2000, pp. 32–36).
The above intuitions were also examined in a more elaborate state-space structure in which relative capability and per-period profits gradually evolve. In this model setup, an additional intermediate state exists, into which the market transitions when a follower is successful, where firms possess equal capabilities and per-period profits. Behavior in the leader-follower states is found to be robust to the inclusion of the intermediate market structure, after controlling for the difference in joint profits that now depend on whether leader or follower succeed. In the equal state, investment is shown to always be higher than in the other two states and, interestingly, to increase in both types of leader advantage given that only the future-payoff effect is relevant (details of the analysis are available from the first author on request).

The paper has also explored the implications of allowing firms to invest in a second marketing variable that affects both current profits and carries over to enhance the prospects of next-generation success. The leader was shown to take advantage of higher basic demand for the current product by investing more in the non-R&D variable. The follower, on the other hand, allocates more resources to R&D. Greater leader marketing carryover has a positive future-payoff effect, thereby inducing greater follower R&D effort, but at the same time reduces the investment productivity of R&D relative to the marketing variable for the leader.

Although in reality capability advantages accruing to successful firms in the stylized form modeled here are not the only factors affecting firm behavior in high-tech markets, there is evidence of the role they play in shaping competition in such environments. For example, in the PC microprocessor industry, the higher efficacy of Intel’s proprietary design tools, which built on ongoing successful product and process development, was often cited in the mid-1990s as a strategic advantage over competitors (Burgelman et al. 1996). The fact that Intel intensified R&D effort in the face of competition (Economist 1993) and the repeated pattern of its success (Figure 1) is consistent with the analysis presented here. In the home video game market, the success of a firm’s current platform bears several advantages for its next-generation product that are independent from the degree of technological advance of the new system, such as an “installed base” of game titles (through backward compatibility), and strong brand association and loyalty that are typically created (Deshpandé 2001). The fact that in several of the six generations of home video game platforms rivals introduced systems technically superior to those introduced by incumbents (Schilling 1997, Deshpandé 2001) and that relatively frequent leadership shifts have occurred (Figure 1), are noteworthy.16

There are several limitations to this study. The assumption that market outcomes in each period are determined in a contestlike manner (with fixed payoffs) potentially abstracts from reality, as the extent by which a firm wins in each period is not part of the model.17 In addition, capital markets have been assumed to be efficient so as not to impose constraints on optimal investment strategies. Although this assumption is reasonable in many cases, especially with the abundance of venture capital funding, it is possible that cash flow may impose temporary restrictions in some cases.

16With moderate reputation advantage, the model still assigns some likelihood that the same firm will be in the lead for two consecutive periods (as, e.g., is likely to be the case with the Sony PS2). In many industry settings, a successful firm may actually enjoy both types of advantages presented here. For example, clearly Intel also enjoys some brand recognition (in particular, subsequent to the “Intel Inside” campaign, Burgelman et al. 1996). Ofek (2000, pp. 24–25) shows that when both advantages are present and reputation advantage is not too big, innovative advantage mitigates the leader’s tendency to “rest on its laurels” because of reputation, and leadership shifts are affected by innovative advantage alone.

17Real-life examples of contestlike competitive interaction, where typically one firm achieves supernormal profits in a given generation, include both explicit and implicit market mechanisms. Explicit mechanisms include trade show product contests (such as “The Best of COMDEX”) and awards by professional magazines (such as PC Magazine’s annual top picks). Implicit mechanisms include supply contracts awarded to one contender, and network externalities (Shapiro and Varian 1999) that typically result in market coordination on a particular firm’s platform and in “winner-take-all” outcomes.
The model developed here has several implications for empirical work. First, it reinforces the recent call for quantifying the productivity and efficiency of R&D effort (consistently rated among the biggest issues facing technology leaders, IRI 2000; see also Hauser 1998), and the recent attempts by market research companies to develop standardized instruments quantifying the extent of reputation firms enjoy (like the scale developed by Harris Interactive, Wall Street Journal, 1999). The model also bears on work examining competitive responsiveness in high-tech markets by suggesting how a firm’s market position, as it affects the productivity of investment and confers advantages for future success, can impact investment tendencies. This may be particularly relevant when using time-series data and when firms can react on more than one strategic investment variable.18

Acknowledgments
This paper is based on research conducted by the first author as part of his dissertation thesis at Stanford University (Ofek 2000). The first author thanks Peter Reiss, Paul Romer, Steven Tadelis, J. Miguel Villas-Boas, Robert Wilson, and Muhamet Yildiz for their valuable suggestions. Very special thanks to V. “Seenu” Srinivasan and J. Miguel Villas-Boas, Robert Wilson, and Muhamet Yildiz for their part of his dissertation thesis at Stanford University (Ofek 2000).

Appendix

Proof of Lemma 1. First, the properties of the interior solution are established, and afterwards uniqueness is addressed.

Denoting \( \theta = \sum h'_i, \theta_i = \sum h'_i, \) and \( \Delta V = V_f - V_i, \) the following set of equations must hold simultaneously at an interior solution:

\[
\begin{align*}
\delta h_i(x_0) \Delta V &= [h_i(x_0) + \theta_0]^2, \\
\delta(\theta_i + h_i(x_0)) h'_i(x_0) \Delta V &= [h_i(x_0) + \theta_0]^2.
\end{align*}
\]

These equations are derived from leader and follower \( i \)'s first-order conditions, respectively. Writing out similar expressions for all other followers leads to \( x_j' = x_j, \) \( \forall j. \) One can then write (A1) as:

\[
\begin{align*}
\delta n h_i(x_j) h'_i(x_j) \Delta V &= [h_i(x_j) + n h_i(x_j)]^2 \\
&= \delta((n-1)h_i(x_j) + h_i(x_j)) h'_i(x_j) \Delta V. \quad (A2)
\end{align*}
\]

This immediately leads to the relationship in (3). Note that:

\[
\begin{align*}
\delta \theta \Delta V &\left[ (h_i(x_0) + \theta) h'_i(x_0) - 2(h_i(x_0))^2 \right], \\
\delta(\theta_i + h_i(x_0)) \Delta V &\left[ (h_i(x_0) + \theta) h'_i(x_0) - 2(h_i(x_0))^2 \right],
\end{align*}
\]

are both negative under the assumptions in \( \S 2 \) for the \( h_i. \) functions. Hence the second-order conditions are satisfied. For uniqueness, note that the system in (2) can be written as:

\[
T(V(s)) = \max\left[ \pi_i - x_i + \delta (V_f + V_i(1 - P_i)) \right].
\]

Under the assumptions in \( \S 2 \) for \( \pi_i, P_i, \) and \( h_i. \) the operator \( T(T: \mathbb{R}^2 \to \mathbb{R}^2) \) with modulus \( \delta \) satisfies Blackwell’s conditions for a contraction mapping (Stokey and Lucas 1989, pp. 39–65). Thus, we are guaranteed \( T \) has exactly one fixed point. □

Proof of Proposition 1. Subtracting the set of equations in (2), \( (V_f - V_i) \) can be expressed in terms of investment levels and the parameters of the model. From the results of Lemma 1, the equilibrium relationship between \( x_i \) and \( x_j \) is derived. Using this relationship, and substituting the expression for \( (V_f - V_i) \) in (A2), we get:

\[
\begin{align*}
x_j' &= x_j' \frac{nk - (n-1)}{k}, \\
x_j' &= \frac{\delta nk(\pi_i - \pi_j)}{(nk+1)^2 - \delta nk(k-1)^2 - 2\delta nk(k-1)}.
\end{align*}
\]

Plugging the investment levels into (1), the transition probabilities can be explicitly expressed:

\[
\begin{align*}
P_i' &= \frac{nk - (n-1)}{nk+1}, \\
P_j' &= \frac{1}{nk+1}. \quad \Box
\end{align*}
\]

Proof of Result 1. From the derivatives of the equilibrium investments in (A5) with respect to \( k, \) we get:

\[
\begin{align*}
\text{sign} (\frac{\partial x_i'}{\partial k}) &= \text{sign} (\delta nk^2(k-1) + 2\delta (nk-n+1) \\
&+ (nk+1)(2n-nk-n-1)].
\end{align*}
\]

This is a concave quadratic equation in \( k, \) which is positive for \( k \to 1 \) and becomes negative for values of \( k > 1/(n+1) + \sqrt{(n^2+\delta)/(1-\delta)}. \) For followers:

\[
\begin{align*}
\text{sign} (\frac{\partial x_i'}{\partial k}) &= \text{sign} [1 - n^2k^2(1-\delta) - \delta n(n-2)],
\end{align*}
\]

which is always negative for \( n > 2. \, \Box \)

18 Thus, for example, one could test whether greater past R&D success (“Technological Output,” Dutta et al. 1999, provides a good way to operationalize this) decreases the relative intensity of R&D expenditure by rivals, and whether greater possession of market-related skills/assets (see Mitchell 1989 for possible ways to operationalize such assets) results in lower R&D effort as the model predicts. Footnote 15 describes empirical implications of the model findings for competitive reactions when simultaneously examining R&D and additional marketing expenditures.

19 When \( n = 1 \) (duopoly case), \( x_i' = x_j' \) and both firms will exhibit the inverse-U pattern of investment. When \( n = 2, \) followers’ investment will either be decreasing or inverse-U in \( k, \) depending on \( \delta. \)
Proof of Result 2. Differentiating follower equilibrium R&D investment with respect to \( n \), we get:

\[
\text{sign} [\alpha x^*_i / \partial n] = \text{sign} [-n^2(k^*(1-\delta) + (2k-1)\delta)+1].
\] (A9)

The expression on the right-hand side of (A9) is negative for all \( \delta < 1 \). As for the leader:

\[
\text{sign} [\alpha x^*_i / \partial n] = \text{sign} [n^2(k(1-\delta)(k-2) + 2n(k-1)+1)].
\] (A10)

Letting \( k \) approach 1, we find that \( \lim_{k \to 1} \text{sign} [\alpha x^*_i / \partial n] = -n^2 + 1 \), which is negative. Define \( \tau = k(k-2) \), if \( \delta < \tau/(\tau + 1) \) (or \( k > k^* = 1 + \sqrt{1/(1-\delta)} \)), the right-hand side of (A10) is positive for all \( n > \hat{n} \). The above also imply that there exists a \( k^* \), such that for \( 1 < k < k^* \), the investment level of the leader is initially increasing and subsequently decreasing as a function of \( n \). \( \square \)

Proof of Proposition 2.

Case 1. \( m < m^* = \delta(n(m - \pi_i - \pi_f))/(n + 1)^2 - \delta n \).

When \( m < m^* \), we look for an interior solution because if \( x_i = 0 \), all followers can increase their value function by selecting \( x_i = 0 \). However, given this choice by followers, \( V_f \) can be increased by choosing \( x_i > 0 \). The same holds if any follower chooses zero investment. Then, applying the result of Lemma 1 yields \( x_i^* = m = x_i \), which immediately establishes \( P_f^* = P_f^\prime = 1/(n+1) \). Using the set of equations in (2) and (A2), it is then possible to get an explicit solution:

\[
x_i^* = \frac{\delta n(m + \pi_i - \pi_f)}{(n + 1)^2},
\] (A11)

Case 2. \( m^* < m < m^{**} = \delta(n - \pi_i)/(1 - \delta) \).

When assuming \( x_i = 0 \) and solving the system in (2), one gets:

\[
x_i^* = -\beta + \sqrt{\beta^2 - 4\alpha \gamma}/2\alpha,
\] (A12)

where \( \beta = [mn(2-\delta) - \delta(n - \pi_i)/(n+1)] \), \( \alpha = [n^2 + \delta] \), and \( \gamma = [m(n - \delta(n+1) - \pi_i)] \). Using the first-order condition for a representative follower when \( m^* < m \), the first-order condition for the leader at \( x_i = 0 \) is \( (x_i^* - m)/(m+1) - x_i^* \). From (A12) (with tedious algebra), \( x_i^* < m \), which establishes that the optimal response for the leader is in fact \( x_i^* = 0 \) and that \( P_f^* > P_f^\prime \).

Case 3. \( m^{**} < m \).

Consider follower \( i \) and assume the leader and each of the remaining followers choose not to invest. Then, \( V_f - V_l \leq (\pi_i - \pi_f + x_i^*)/(x_i^* + m)(x_i^* - n - x_i^*)/(x_i^* + m - \delta(m - x_i^*)) \) and follower \( i \)’s first-order condition is \( -1 + \delta(\pi_i - \pi_f + x_i^*)/(x_i^* + m)^2 - \delta(m - x_i^*)/(x_i^* + m(x_i^* + m)) \), which is decreasing in \( x_i^* \) for \( x_i^* \geq 0 \) and negative at \( x_i^* = 0 \) for \( m^* < m \). Thus, the optimal investment for follower \( i \) is zero (and by symmetry for all other followers). The leader obviously does not invest if all followers are not investing. We then get \( P_f^* = 1, V_f = \pi_i/(1-\delta) \) and \( V_l = \pi_f/(1-\delta) \).

Proof of Proposition 3. The proof begins by solving for equilibrium per-period prices, and given these prices, the MPNE advertising and R&D levels are established.

Differeniatating the per-period profit functions in (5) with respect to prices, and keeping advertising levels constant, yields the following two first-order conditions: \( p_i = 0.5(1 + a_f)(w_i + p_i)/(1 + a_f) \) and \( p_f = 0.5(1 + a_f)(w_f + p_f)/(1 + a_f) \). Solving these two equations simultaneously yields:

\[
p_i^\prime = \frac{1 + a_f}{3} (2w_i + w_f), \quad p_f^\prime = \frac{1 + a_f}{3} (2w_f + w_f). \] (A13)

Using these prices, and defining for convenience \( \phi_i = \frac{1}{2} (2w_i + w_f)^2 \) and \( \phi_f = \frac{1}{2} (2w_f + w_f)^2 \), the optimization problem firms face in each of the two states can be written as:

\[
V_i = \max_{m,n} \left[ (1 + a_f)\phi_i - \frac{1}{2} m^2 \right] \delta P_i V_i + \delta P_l V_l, \quad V_f = \max_{m,n} \left[ (1 + a_f)\phi_f - \frac{1}{2} m^2 \right] \delta P_f V_f + \delta P_l V_l, \] (A14)

where \( P_i \) are as defined in (6). Differeniatating the first expression in (A14) with respect to \( x_i \) and the second with respect to \( x_f \) leads to the following condition that must hold in equilibrium:

\[
x_i + \lambda_i a_i = x_f + \lambda_f a_f. \] (A15)

Differeniatating the corresponding expressions in (A14) with respect to advertising levels (and combining the above condition) we get:

\[
\phi_i - a_i b + \lambda_i = 0, \quad \phi_f - a_f b + \lambda_f = 0, \quad \text{or} \quad a_i = \frac{\phi_i + \lambda_i}{b}, \quad a_f = \frac{\phi_f + \lambda_f}{b}. \] (A16)

Combining (A16) with (A15) and subtracting the two equations in (A14), yields:

\[
x_i^* = \delta \left[ \phi_i - \phi_f + \frac{\phi_i + \lambda_i - (\phi_f + \lambda_f)^2}{2b} \right] - \lambda_i (\phi_i + \lambda_i),
\]

\[
x_f^* = \frac{\phi_i + \lambda_i}{b} + \lambda_i (\phi_i + \lambda_i). \] (A17)

To ensure positive R&D investments, we need to consider \( \lambda_i \) small. From the expression for \( x_f^* \), the condition is \( \lambda_i (\phi_i + \lambda_i) \). Denote the expressions in squared brackets of (A14) \( l(\cdot) \) and \( f(\cdot) \), respectively. Taking second-order derivatives, one finds that \( l_{x_1 x_1 x_1} \) and \( f_{x_1 x_1 x_1} \) are negative and \( l_{x_1 x_1} f_{x_1 x_1} - f_{x_1 x_1} l_{x_1 x_1} \) are positive; hence, the Hessian matrix is negative definite, and we are guaranteed concavity. \( \square \)

Proof of Result 4. This result is obtained in a straightforward manner by differentiating the equilibrium investments given in (A17) with respect to \( \lambda_i \) and \( \lambda_f \). \( \square \)
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*This paper was received July 25, 2000, and was with the authors 16 months for 3 revisions; processed by Sridhar Moorthy.*