Characterizing Myopic Intertemporal Demand

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Abstract

In the standard certainty multiperiod demand problem it is well-known that if a consumer's preferences are log additive (or equivalently Cobb-Douglas), demand in each period is myopic in the sense of being independent of future prices. As a result, less stringent informational requirements in terms of price expectations are imposed on the consumer. Given the general aversion of Fisher (1930), Hicks (1965) and Lucas (1978), among others, to requiring preferences to be additively separable, it is natural to ask whether myopia can hold for non-additive forms of utility. In a multigood, multiperiod setting, we first show that neither additive separability nor logarithmic period utility is required for myopia and then characterize the form of utility which generates myopic demand. As an application, we derive simple restrictions on equilibrium interest rates which are necessary and sufficient for utility to take the myopic form. The resulting conditions for myopic utility are arguably less restrictive than those implied by preferences which are additively separable.

KEYWORDS. Myopic demand, additive separability, utility, equilibrium interest rates

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1 Introduction

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Consider a multigood, multiperiod setting in which a well-behaved utility function is maximized subject to a standard intertemporal budget constraint. In order to derive an optimal consumption plan, the consumer needs to know all current and future prices. It is natural to ask when these informational requirements can be reduced. Kurz (1987) defines an optimal consumption plan in a given time period to be myopic if and only if that period's demand function depends only on exogenous variables in current and past periods and not in future periods.[1]

It is well known that a sufficient condition for myopia to hold in each time period is that the representation of preferences is log additive (or equivalently Cobb-Douglas).[2] As a result, two questions naturally arise which to our knowledge have not been addressed. First given the general reluctance, following for example Fisher (1930), Hicks (1965) and Lucas (1978), to require preferences to be additively separable, is it possible for consumption plans to be myopic for non-additive forms of utility? Second, is it possible to exhibit less extreme forms of myopia where, for instance, demand in period $t \in \{1, 2, ..., T\}$ is a function of prices in periods 1 to $t+1$ but not of prices in subsequent periods? We characterize in a general multigood, multiperiod setting the restrictions on preferences that are necessary and sufficient for demand to be myopic in (i) the current period, (ii) all periods and (iii) a subset of periods. In general, there is no requirement for these myopic preferences to be additively separable over time or homothetic as in the log additive case. Examples of these more general forms are provided.

As an application, we consider the implications of preferences being myopic for equilibrium interest rates. In a standard representative agent equilibrium setting, the assumption that preferences are myopic is seen to imply and be implied by a very simple restriction on equilibrium interest rates. This restriction is quite different from the restrictions implied by preferences being additively separable or homothetic. Interestingly, some may view the implications of additive separability as being more restrictive than those corresponding to myopia – thereby providing an equilibrium justification for the preference based reservations associated with

[1] It should be noted that the notion of a myopic plan as defined by Kurz and employed throughout this paper differs from its use in the changing tastes literature. In Strotz (1956), for example, a myopic plan is used interchangeably with a naive plan where a consumer bases her plan for current and future consumption on current multiperiod preferences ignoring the fact that her preferences in the next period for the then remaining consumption vector may differ.

[2] In a multiperiod uncertainty setting, if intertemporal preferences are represented by log additive Expected Utility, then the consumer's multiperiod investment plan is myopic (see, for example, Rubinstein 1974). In the finance literature, an alternative notion of myopic investment plans is also considered where investors are assumed to maximize Expected Utility of terminal wealth. See, for example, Mossin (1968) and Hakansson (1971). In this paper however, we focus only on the certainty case.

additive separability cited above.

In the next Section, we derive the necessary and sufficient conditions for demand to be myopic in one and multiple periods. Section 3 investigates the implications of myopia for equilibrium interest rates.

2 Myopia: Necessary and Sufficient Conditions

Assume a T period, multigood consumption setting. In each period, one or more goods are consumed. The quantity (purchase) and price of good i in period t are denoted by c_{ti} and p_{ti}, respectively.[3] The corresponding consumption and price vectors in period t are denoted by $\mathbf{c}_t$ and $\mathbf{p}_t$. Assume a well-behaved T period utility function $U(\mathbf{c}_1, ..\mathbf{c}_T)$ which is maximized subject to the budget constraint

$$\sum_{t=1}^{T} \mathbf{p}_t \cdot \mathbf{c}_t = I, \tag{1}$$

where I is period one income or wealth. The price and consumption vectors are elements of the positive orthant. Define the set $\mathcal{U}$ to be the collection of real-valued functions defined on (a subset of) the positive orthant of a Euclidean space, which are C^2, strictly increasing in each of their arguments and strictly quasiconcave. Throughout this paper, it will be assumed that the T period utility function $U(\mathbf{c}_1, ..\mathbf{c}_T) \in \mathcal{U}$. (As can be easily verified, these assumptions are satisfied by each of the utility functions employed in the examples below.) Unless otherwise stated, we will always assume that U is defined on the whole positive orthant. It should be stressed that our setting is static even though the consumer confronts a multiperiod decision problem, since we only consider her optimal consumption plan as set at the beginning of the initial time period $t = 1$.

Following Kurz (1987), myopic demand is defined as follows.

Definition 1 *Optimal demand in period t, $\mathbf{c}_t$, is said to be myopic if and only if it depends on past and current prices $\{\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_t\}$ but not on future prices $\{\mathbf{p}_{t+1}, ..., \mathbf{p}_T\}$.*

Remark 1 *Definition 1 implies that $\mathbf{c}_t$ is myopic if and only if for any $k \in \{1, 2, ..., T-t\}$*

$$\frac{\partial \mathbf{c}_t}{\partial p_{t+k,i}} = \mathbf{0} \quad \forall i, \tag{2}$$

where i is the index of good i in period $t+k$. Using the classic Slutsky demand equation (see, for example, Mas-Colell, Whinston and Green 1995, p. 71), eqn. (2)

[3] If there is only one good in each period, the subscript i will be ignored.

is equivalent to

$$\left(\frac{\partial \mathbf{c}_t}{\partial p_{t+k,i}}\right)_{U=const} = c_{t+k,i}\frac{\partial \mathbf{c}_t}{\partial I}, \tag{3}$$

where the left- and right-hand sides correspond respectively to the substitution effect and minus the income effect. Thus $\mathbf{c}_t$ being myopic is equivalent to the income and substitution effects exactly offsetting each other. As we will see, eqn. (3) can be viewed as a more general characterization of myopic behavior. We return to this characterization in the equilibrium analysis in the next Section.

The representation of preferences generally associated with myopic demand is log additive (or equivalently Cobb-Douglas) utility. However, since these preferences are (ordinally) additively separable and homothetic, it is natural to wonder whether these properties are necessary to generate myopic demand.[4] The fact that homotheticity is not required is easily demonstrated by the following non-homothetic utility which generates myopic demands

$$U(c_1, c_2, c_3) = -\exp(-c_1) + \ln c_2 + \ln c_3. \tag{4}$$

The question of whether additive separability is required is more involved since none of the widely used non-additively separable utility functions, of which are we aware, results in myopic consumption plans. We return to this issue below.

In this paper we seek to fully characterize the class of preferences which imply and are implied by myopic demand in a multigood, multiperiod setting. We begin by establishing the necessary and sufficient condition for the period one consumption vector to be myopic.[5] This condition is then applied recursively in Result 1 to characterize the form of utility associated with the consumption vector being myopic in several (or all) periods.

Proposition 1 *Assume that in the first period, there are m goods, where the quantities are denoted by $c_1, c_2, ..., c_m$. In periods 2 to T there are n goods, where the quantities are denoted by $c_{m+1}, c_{m+2}, ..., c_{m+n}$ and the distribution of goods across periods is arbitrary. The utility function $U(c_1, ..., c_{m+n})$ is maximized subject to*

$$\sum_{i=1}^{m+n} p_i c_i = I. \tag{5}$$

[4]Homothetic preferences are characterized as being representable by a homogeneous function. Moreover the preferences give rise to linear Engel curves. See, for example, Chipman (1974).

[5]For the following Proposition, since the distribution of goods in periods doesn't matter, we use c_i instead of c_{ti} in order to simplify the notation in the proof.

The optimal period one consumption vector $(c_1, ..., c_m)$ *is myopic if and only if* $U(c_1, ..., c_{m+n})$ *takes the form*[6]

$$U(c_1, ..., c_{m+n}) = f(g(c_1, ..., c_m)\, c_{m+1}, g(c_1, ..., c_m)\, c_{m+2}, ..., g(c_1, ..., c_m)\, c_{m+n}), \tag{6}$$

where $f, g \in \mathcal{U}$ *and* $g > 0$.[7]

Proof. Although a T period setting is assumed in Proposition 1, since we are only interested in when the period one consumption vector is myopic, we can combine all of the future periods into one long period and the problem effectively becomes a two period problem. Therefore, to prove Proposition 1, we need only verify the following Lemma. ■

Lemma 1 *For the two period case, assume that goods* $1, ..., m$ *are consumed in period one and* $m+1, ..., m+n$ *in period two. The utility function* $U(c_1, ..., c_{m+n})$ *is maximized subject to the budget constraint*

$$\sum_{i=1}^{m+n} p_i c_i = I. \tag{7}$$

The optimal period one consumption vector $(c_1, ..., c_m)$ *is myopic if and only if* $U(c_1, ..., c_{m+n})$ *takes the form*

$$U(c_1, ..., c_{m+n}) = f(g(c_1, ..., c_m)\, c_{m+1}, g(c_1, ..., c_m)\, c_{m+2}, ..., g(c_1, ..., c_m)\, c_{m+n}), \tag{8}$$

where $f, g \in \mathcal{U}$ *and* $g > 0$.

Proof. First prove sufficiency. Introduce the following notation

$$f_i = \frac{\partial f}{\partial (g(c_1, ..., c_m)\, c_{m+i})} \quad \text{and} \quad g_j = \frac{\partial g(c_1, ..., c_m)}{\partial c_j}, \tag{9}$$

where $i \in \{1, 2, ..., n\}$ and $j \in \{1, 2, ..., m\}$. The first order conditions are

$$\frac{f_i g(c_1, ..., c_m)}{f_j g(c_1, ..., c_m)} = \frac{f_i}{f_j} = \frac{p_{m+i}}{p_{m+j}} \quad i, j \in \{1, 2, ..., n\}, \tag{10}$$

[6] Given the interest in utility functions with translated origins such as members of the Modified Bergson family and habit formation models (see, for example, Pollak 1970), it is natural to ask whether any of these utilities can exhibit myopic demand. It is clear from the general form (6) that this is not possible where the origins for $c_{m+1}, ..., c_{m+n}$ are translated.

[7] In order to simplify the statement of this result, we follow the convention throughout the paper of assuming without loss of generality that $g > 0$. However it should be noted that for the proof of sufficiency, $f, g \in \mathcal{U}$ and $g > 0$ cannot guarantee that $U \in \mathcal{U}$, which is always assumed in this paper, since the strict quasiconcavity of f and g cannot ensure the strict quasiconcavity of U. For necessity if $U \in \mathcal{U}$ and $g > 0$, we prove that this implies $f, g \in \mathcal{U}$. It should be emphasized that when $g < 0$, one can always reverse the sign of g and the signs of the arguments in f such that the form of U remains the same and $f, g \in \mathcal{U}$ and $g > 0$.

$$\frac{g_i \sum_{k=1}^{n} f_k c_{m+k}}{g_j \sum_{k=1}^{n} f_k c_{m+k}} = \frac{g_i}{g_j} = \frac{p_i}{p_j} \qquad i, j \in \{1, 2, ..., m\}, \tag{11}$$

and

$$\frac{g_i \sum_{k=1}^{n} f_k c_{m+k}}{f_j g(c_1, ..., c_m)} = \frac{p_i}{p_{m+j}} \quad i \in \{1, 2, ..., m\}, j \in \{1, 2, ..., n\}. \tag{12}$$

Combining (10) and (12), we have

$$\sum_{k=1}^{n} f_k p_{m+j} c_{m+k} = f_j \sum_{k=1}^{n} p_{m+k} c_{m+k} = \frac{p_i f_j g(c_1, ..., c_m)}{g_i}, \tag{13}$$

which is equivalent to

$$\sum_{k=1}^{n} p_{m+k} c_{m+k} = \frac{p_i g(c_1, ..., c_m)}{g_i}. \tag{14}$$

Substitution of the above equation into the budget constraint, yields

$$\sum_{i=1}^{m} p_i c_i + \frac{p_j g(c_1, ..., c_m)}{g_j} = I \tag{15}$$

for $\forall j \in \{1, 2, ..., m\}$. Choosing for example $j = 1$ in (15) and $i = 1$ in (11), we get the following system of m equations in the m variables $c_1, ..., c_m$:

$$\begin{aligned} \sum_{i=1}^{m} p_i c_i + \frac{p_1 g(c_1, ..., c_m)}{g_1} - I &= 0, \\ p_1 g_2 - p_2 g_1 &= 0, \\ &\vdots \\ p_1 g_m - p_m g_1 &= 0. \end{aligned} \tag{16}$$

This system is functionally independent. In fact, the Jacobian matrix of derivatives with respect to the variables $c_1, ..., c_m$ is equal to

$$\begin{pmatrix} 2p_1 & ... & 2p_m \\ p_1 g_{12} - p_2 g_{11} & ... & p_1 g_{2m} - p_2 g_{1m} \\ \vdots & \vdots & \vdots \\ p_1 g_{1m} - p_m g_{11} & ... & p_1 g_{mm} - p_m g_{1m} \end{pmatrix}. \tag{17}$$

An easy computation shows that the determinant of (17) is just the bordered Hessian determinant of g. Since g is strictly quasiconcave, by Theorem VI of Bernstein and Toupin (1962) this determinant does not vanish in a dense open subset of every indifference surface of g, so that the system (17) determines $c_1, ..., c_m$. Noticing that both (11) and (15) are independent of $p_{m+1}, ..., p_{m+n}$, the optimal period one consumption vector $(c_1, ..., c_m)$ is myopic.

Next prove necessity. Introduce the following notation

$$U_i = \frac{\partial U}{\partial c_i} \quad \text{and} \quad U_{ij} = \frac{\partial^2 U}{\partial c_i \partial c_j} \qquad i, j \in \{1, 2, ..., m+n\}. \tag{18}$$

The first order conditions give

$$p_i U_1 - p_1 U_i = 0 \quad (i = 2, 3, ..., m+n). \tag{19}$$

Since the optimal $(c_1, ..., c_m)$ depends only on $(p_1, ..., p_m)$, differentiating both sides of the i^{th} first order condition with respect to p_k ($k \in \{m+1, m+2, ..., m+n\}$) we have

$$\sum_{j=m+1}^{m+n} (p_1 U_{ij} - p_i U_{1j}) \frac{\partial c_j}{\partial p_k} = \delta_{ik} U_1, \tag{20}$$

where δ_{ik} is the Kronecker δ. Differentiation of the left hand side of the budget constraint with respect to p_k, yields

$$\sum_{j=m+1}^{m+n} p_j \frac{\partial c_j}{\partial p_k} = -c_k. \tag{21}$$

We have a system of $n+1$ linear equations with n unknowns $\frac{\partial c_j}{\partial p_k}$ ($j = m+1, m+2, ..., m+n$).[8] There exists at least one nonzero (non-trivial) solution for this equation system if and only if the augmented coefficient matrix of this system is singular, i.e.,

$$\det M = 0, \tag{22}$$

where

$$M = \begin{pmatrix} p_1 U_{m+1,m+1} - p_{m+1} U_{1,m+1}, & \cdots, & p_1 U_{m+1,m+n,} - p_{m+1} U_{1,m+n}, & 0 \\ \vdots & \vdots & \vdots & \vdots \\ p_1 U_{k,m+1} - p_k U_{1,m+1}, & \cdots, & p_1 U_{k,m+n} - p_k U_{1,m+n}, & U_1 \\ \vdots & \vdots & \vdots & \vdots \\ p_1 U_{m+n,m+1} - p_{m+n} U_{1,m+1}, & \cdots, & p_1 U_{m+n,m+n} - p_{m+n} U_{1,m+n}, & 0 \\ p_{m+1}, & \cdots, & p_{m+n}, & -c_k \end{pmatrix}. \tag{23}$$

Define

$$H = \begin{pmatrix} p_1 U_{m+1,m+1} - p_{m+1} U_{1,m+1}, & \cdots, & p_1 U_{m+1,m+n,} - p_{m+1} U_{1,m+n}, \\ \vdots & \vdots & \vdots \\ p_1 U_{k,m+1} - p_k U_{1,m+1}, & \cdots, & p_1 U_{k,m+n} - p_k U_{1,m+n}, \\ \vdots & \vdots & \vdots \\ p_1 U_{m+n,m+1} - p_{m+n} U_{1,m+1}, & \cdots, & p_1 U_{m+n,m+n} - p_{m+n} U_{1,m+n}, \end{pmatrix}. \tag{24}$$

[8] The $n+1$ equations include eqns. (20) (one equation for each $i \in \{m+1, ..., m+n\}$) and eqn. (21).

Using the Laplace Expansion to expand the determinant in eqn. (22) by the last column (and the cofactor of U_1 by the last row), yields

$$(-1)^{k-m+n+1} U_1 \sum_{j=m+1}^{m+n} (-1)^{n+j-m} p_j H_{j-m,k-m} - c_k \det H = 0, \tag{25}$$

where $H_{j-m,k-m}$ is the $j-m, k-m$ minor of H. Substituting eqn. (19) into (25), we have

$$p_1 \sum_{j=m+1}^{m+n} (-1)^{k+j} U_j H_{j-m,k-m} = -c_k \det H. \tag{26}$$

Assume first that H is non-singular and notice that

$$\frac{(-1)^{k+j} H_{j-m,k-m}}{\det H} \tag{27}$$

is the $(j-m, k-m)$ component of $\left(H^T\right)^{-1}$. Denoting

$$\nabla U = (U_{m+1}, U_{m+2}, ..., U_{m+n}) \quad \text{and} \quad \mathbf{c} = (c_{m+1}, c_{m+2}, ..., c_{m+n}), \tag{28}$$

it follows from eqn. (26) that

$$p_1 \left(H^T\right)^{-1} (\nabla U) = -\mathbf{c} \Leftrightarrow p_1 \nabla U = -H^T \mathbf{c}, \tag{29}$$

or equivalently

$$\sum_{j=m+1}^{m+n} (p_1 U_{ji} - p_j U_{1i}) c_j = -p_1 U_i \quad (i = m+1, m+2, ..., m+n). \tag{30}$$

The determinant $\det H$ is proportional to the bordered Hessian of U when considered as a function of the last n variables. The strict quasiconcavity of U implies by Theorem VI of Bernstein and Toupin (1962) that $\det H \neq 0$ on a dense set, so that (30) holds on a dense set. By continuity (30) holds everywhere. Notice that for any $i \in \{m+1, m+2, ..., m+n\}$,

$$\frac{\partial}{\partial c_i} \frac{\sum_{j=m+1}^{m+n} c_j U_j}{U_1} = \frac{\left(U_i + \sum_{j=m+1}^{m+n} c_j U_{ij}\right) U_1 - U_{1i} \sum_{j=m+1}^{m+n} c_j U_j}{(U_1)^2}. \tag{31}$$

It follows from eqn. (30) and the first order conditions that

$$\sum_{j=m+1}^{m+n} (U_{ij} U_1 - U_j U_{1i}) c_j = -U_i U_1 \quad (i = m+1, m+2, ..., m+n). \tag{32}$$

Substituting eqn. (32) into (31), it follows that

$$\frac{\partial}{\partial c_i} \frac{\sum_{j=m+1}^{m+n} c_j U_j}{U_1} = 0. \tag{33}$$

Therefore, we have

$$\frac{\sum_{j=m+1}^{m+n} c_j U_j}{U_1} = h^{(1)}\left(c_1, ...c_m\right), \tag{34}$$

where $h^{(1)}\left(c_1, ...c_m\right)$ is an arbitrary positive function. Similarly, it can be proved that for $\forall i \in \{1, 2, ..., m\}$

$$\frac{\sum_{j=m+1}^{m+n} c_j U_j}{U_i} = h^{(i)}\left(c_1, ...c_m\right), \tag{35}$$

for certain positive functions $h^{(i)}\left(c_1, ...c_m\right)$. By Frobenius integrability conditions $h^{(i)} \frac{\partial h^{(j)}}{\partial c_i} = h^{(j)} \frac{\partial h^{(i)}}{\partial c_j}$ for $i, j \in \{1, 2, ..., m\}$. Integrating the above over-determined system yields

$$U\left(c_1, ..., c_{m+n}\right) = f\left(g\left(c_1, ..., c_m\right) c_{m+1}, g\left(c_1, ..., c_m\right) c_{m+2}, ..., g\left(c_1, ..., c_m\right) c_{m+n}\right), \tag{36}$$

and g can be obtained from the following integrable system of equations

$$\frac{g}{g_i} = h^{(i)}\left(c_1, ...c_m\right) \qquad \left(i \in \{1, 2, ..., m\}\right). \tag{37}$$

Finally, we show that $f, g \in \mathcal{U}$. Since $g > 0$, from the first order condition, it can be easily verified that U being strictly increasing is equivalent to f and g being strictly increasing. Since U is strictly quasiconcave in each of its argument, for any $\mathbf{c}' = \left(c_1', c_2', ..., c_{m+n}'\right)$, $\mathbf{c}'' = \left(c_1'', c_2'', ..., c_{m+n}''\right)$ and $0 < \alpha < 1$, one has

$$U\left(\alpha \mathbf{c}' + (1-\alpha)\mathbf{c}''\right) > \min\left(U\left(\mathbf{c}_1'\right), U\left(\mathbf{c}_1''\right)\right). \tag{38}$$

Assuming that the $m+1$ to $m+n$ components in $\mathbf{c}'$ and $\mathbf{c}''$ are the same (denoted by $\left(c_{m+1}, c_{m+2}, ..., c_{m+n}\right)$) and denoting

$${}_m\mathbf{c}' = \left(c_1', c_2', ..., c_m'\right) \quad \text{and} \quad {}_m\mathbf{c}'' = \left(c_1'', c_2'', ..., c_m''\right) \tag{39}$$

it follows that

$$U\left(\alpha \mathbf{c}_1' + (1-\alpha)\mathbf{c}_1''\right) = f\left(g\left((\alpha)_m \mathbf{c}' + (1-\alpha)_m \mathbf{c}''\right) c_{m+1}, ..., g\left((\alpha)_m \mathbf{c}' + (1-\alpha)_m \mathbf{c}''\right) c_{m+n}\right)$$

$$> \min\left(f\left(g\left({}_m\mathbf{c}'\right) c_{m+1}, ..., g\left({}_m\mathbf{c}'\right) c_{m+n}\right), f\left(g\left({}_m\mathbf{c}''\right) c_{m+1}, ..., g\left({}_m\mathbf{c}''\right) c_{m+n}\right)\right). \tag{40}$$

Since f is strictly increasing in each of its argument, it follows that

$$g\left((\alpha)_m \mathbf{c}' + (1-\alpha)_m \mathbf{c}''\right) > \min\left(g\left({}_m\mathbf{c}'\right), g\left({}_m\mathbf{c}''\right)\right), \tag{41}$$

implying that g is strictly quasiconcave, so that $g \in \mathcal{U}$. One can prove $f \in \mathcal{U}$ similarly. ■

Given the equivalence between the utility (6) and optimal consumption being myopic, it will prove convenient throughout the rest of this paper to adopt the convention of referring to (6) as myopic utility or corresponding to myopic preferences.

Based on Proposition 1, it is now possible to answer both questions raised in Section 1. First relating to whether additive separability of U is necessary for myopic behavior, it is clear from eqn. (6) that this is not the case.[9] Concrete non-additive examples are given below. Second, Proposition 1 demonstrates that it is not necessary for demand to be independent of all future prices as in the log additive case. It is possible for demand in period t to be a function of prices in periods 1 to $t+1$ but be independent of subsequent prices by appropriately combining periods 1 to $t+1$ to form a long period and then applying Proposition 1.

As summarized next, if there is only one good in certain periods, then the utility function given in Proposition 1 can take simpler forms.

Remark 2 *For simplicity, assume that there are two periods and at most two goods in each period. Then the following summarizes the forms of utility implied by Proposition 1.*

i *Suppose there is only one good in periods one and two. Optimal period one demand c_1 is myopic if and only if $U(c_1, c_2)$ takes the form*

$$U(c_1, c_2) = f(g(c_1)\, c_2), \tag{42}$$

which is ordinally equivalent to

$$U(c_1, c_2) = h(c_1) + \ln c_2, \tag{43}$$

where $f, g, h \in \mathcal{U}$ and $g > 0$.

ii *Suppose there are two goods in period one and one good in period two. Optimal period one demands c_{11} and c_{12} are myopic if and only if $U(c_{11}, c_{12}, c_2)$ takes the form*

$$U(c_{11}, c_{12}, c_2) = f(g(c_{11}, c_{12})\, c_2), \tag{44}$$

which is ordinally equivalent to

$$U(c_{11}, c_{12}, c_2) = h(c_{11}, c_{12}) + \ln c_2, \tag{45}$$

where $f, g, h \in \mathcal{U}$ and $g > 0$.

[9] It should be noted that in the special case of two periods with one good per period, (ordinal) additive separability is necessary for demand to be myopic – see eqn. (43) below.

iii *Suppose there is one good in period one and two goods in period two. Optimal period one demand c_1 is myopic if and only if $U(c_1, c_{21}, c_{22})$ takes the form*

$$U(c_1, c_{21}, c_{22}) = f(g(c_1)\, c_{21}, g(c_1)\, c_{22}), \tag{46}$$

where $f, g \in \mathcal{U}$ and $g > 0$.

The following Examples illustrate cases (ii) and (iii) above, respectively.

Example 1 *Assume a two period setting, where c_{11} and c_{12} are the quantities of the period one goods and c_2 is the quantity of the period two good. The consumer's utility takes the following special myopic form of eqn. (45)*

$$U(c_{11}, c_{12}, c_2) = -\frac{c_{11}^{-\delta}}{\delta} - \frac{c_{12}^{-\delta}}{\delta} + \ln c_2, \tag{47}$$

which is maximized subject to

$$p_{11}c_{11} + p_{12}c_{12} + p_2c_2 = I, \tag{48}$$

where $\delta > -1$ and $\delta \neq 0$. Combining the first order conditions with the budget constraint yields

$$p_{11}c_{11} + p_{12}\left(\frac{p_{11}}{p_{12}}\right)^{\frac{1}{1+\delta}} c_{11} + p_{11}c_{11}^{1+\delta} = I, \tag{49}$$

implying that c_{11} depends only on p_{11} and p_{12}. Since

$$c_{12} = \left(\frac{p_{11}}{p_{12}}\right)^{\frac{1}{1+\delta}} c_{11}, \tag{50}$$

c_{12} also depends only on p_{11} and p_{12}. Hence the optimal period one consumption vector (c_{11}, c_{12}) is myopic.

Example 2 *Assume a two period setting, where c_1 is the quantity of the period one good and c_{21} and c_{22} are the quantities of the period two goods. The consumer's utility takes the following special myopic form of eqn. (46)*

$$U(c_1, c_{21}, c_{22}) = (c_1c_{21})^{\frac{1}{4}} + \sqrt{c_1c_{22}}, \tag{51}$$

which is maximized subject to

$$p_1c_1 + p_{21}c_{21} + p_{22}c_{22} = I. \tag{52}$$

Combining the first order conditions with the budget constraint yields

$$c_1 = \frac{I}{2p_1}, \tag{53}$$

implying that period one optimal consumption c_1 is independent of p_{21} and p_{22}.

Remark 3 *The utility (51) is of particular interest, since it is neither additively separable nor homothetic but still results in myopic period one demand.*

Next we derive the necessary and sufficient condition for consumption vectors in multiple periods, including the first, to be myopic. The key tactic is to reformulate the problem so that Proposition 1 can be applied recursively. Since the notation for the general T period case is quite messy, without loss of generality, we state the following Result for three periods. The argument for more general cases proceeds in a similar manner.

Result 1 *Assume there are three periods. In periods one, two and three, the quantity of goods is denoted by* (c_{11}, c_{12}), (c_{21}, c_{22}, c_{23}) *and* (c_{31}, c_{32}), *respectively. The optimal consumption vector in each period is myopic if and only if* $U(c_{11}, c_{12}, ..., c_{32})$ *takes the form*

$$U = f^{(1)}\left(g^{(2)}(c_{11}, c_{12}, c_{21}, c_{22}, c_{23})\, g^{(1)}(c_{11}, c_{12})\, c_{31}, g^{(2)}(c_{11}, c_{12}, c_{21}, c_{22}, c_{23})\, g^{(1)}(c_{11}, c_{12})\, c_{32}\right), \tag{54}$$

where

$$g^{(2)}(c_{11}, c_{12}, c_{21}, c_{22}, c_{23}) = f^{(2)}\left(g^{(1)}(c_{11}, c_{12})\, c_{21}, g^{(1)}(c_{11}, c_{12})\, c_{22}, g^{(1)}(c_{11}, c_{12})\, c_{23}\right), \tag{55}$$

$f^{(1)}, f^{(2)}, g^{(1)} \in \mathcal{U}$ *and* $f^{(2)}, g^{(1)} > 0$.

Proof. To apply Proposition 1, combine the first and second periods into a single long period. Then the necessary and sufficient condition for the optimal consumption vector in this combined period to be myopic is that the utility function U takes the form

$$U = f^{(1)}\left(g^{(2)}(c_{11}, c_{12}, c_{21}, c_{22}, c_{23})\, g^{(1)}(c_{11}, c_{12})\, c_{31}, g^{(2)}(c_{11}, c_{12}, c_{21}, c_{22}, c_{23})\, g^{(1)}(c_{11}, c_{12})\, c_{32}\right). \tag{56}$$

This form of utility ensures that the optimal period two demands (c_{21}, c_{22}, c_{23}) are myopic.[10] For the optimal consumption vector in period one to be myopic, Proposition 1 can be applied again to $g^{(2)}(c_{11}, c_{12}, c_{21}, c_{22}, c_{23})$. The necessary and sufficient condition for the optimal period one consumption vector (c_{11}, c_{12}) to be myopic is

$$g^{(2)}(c_{11}, c_{12}, c_{21}, c_{22}, c_{23}) = f^{(2)}\left(g^{(1)}(c_{11}, c_{12})\, c_{21}, g^{(1)}(c_{11}, c_{12})\, c_{22}, g^{(1)}(c_{11}, c_{12})\, c_{23}\right). \tag{57}$$

[10] Note that, in general, $g^{(1)}$ can be combined with $g^{(2)}$ if one only requires (c_{21}, c_{22}, c_{23}) to be myopic. However, in order to ensure the myopia of (c_{11}, c_{12}) as well as (c_{21}, c_{22}, c_{23}), $g^{(1)}$ and $g^{(2)}$ need to be separated.

Since U takes the form

$$U = f\left(g^{(1)}(c_{11}, c_{12})\, c_{21}, g^{(1)}(c_{11}, c_{12})\, c_{22}, g^{(1)}(c_{11}, c_{12})\, c_{23}, g^{(1)}(c_{11}, c_{12})\, c_{31}, g^{(1)}(c_{11}, c_{12})\, c_{32}\right), \tag{58}$$

where f is a function determined by $f^{(1)}$ and $f^{(2)}$, it follows from Proposition 1 that the optimal period one consumption vector (c_{11}, c_{12}) is myopic. In conclusion, the optimal consumption vector in each period is myopic in this three period setting if and only if the utility function U takes the form (56), where $g^{(2)}$ is defined in (57). Without loss of generality, assume that $f^{(2)}, g^{(1)} > 0$. Applying the similar argument as in the proof of Lemma 1, it can be shown that $f^{(1)}, f^{(2)}, g^{(1)} \in \mathcal{U}$. ■

If there are three periods and in each period there is only one commodity, then using Remark 2 and applying Proposition 1 recursively (as discussed in Result 1), optimal consumption in every period is myopic if and only if U takes the following form up to an increasing monotone transformation

$$U(c_1, c_2, c_3) = f(g(c_1)\, c_2) + \ln(g(c_1)\, c_3), \tag{59}$$

where $f, g \in \mathcal{U}$ and $g > 0$.[11] It should be noted that if $f(x) = \ln x$ and $g(x) = \sqrt{x}$, we have

$$U(c_1, c_2, c_3) = \ln c_1 + \ln c_2 + \ln c_3, \tag{60}$$

which is the well-known log additive (or ordinally equivalent Cobb-Douglas) utility. The stronger restriction on preferences (60) guarantees that demand in each period is not only independent of future prices (as in the myopia Definition 1), but also of past prices. Thus for instance, optimal c_2 will depend on both p_1 and p_2 in general if U takes the form of (59) but only on p_2 if U is given by (60).

On the other hand, in contrast to demand being myopic in each period as in (59), it follows from Remark 2 that optimal consumption will be myopic in period one if and only if U takes the form

$$U(c_1, c_2, c_3) = f(g(c_1)\, c_2, g(c_1)\, c_3), \tag{61}$$

where in general there is no requirement for the functions f and g in (61) to be related to f and g in (59). However, the utility functions (59) and (61) will give the same optimal c_1 if and only if $g(c_1)$ is the same. Moreover, it follows from (59) and (61) that given period one demand is myopic, period two demand will also be myopic if and only if the utility U is ordinally additively separable in c_3.

[11] More generally in a four period setting (with one good per period), optimal consumption in each period is myopic if and only if $U(c_1, c_2, c_3, c_4)$ takes the form

$$U(c_1, c_2, c_3, c_4) = f^{(1)}\left(f^{(2)}\left(g^{(1)}(c_1)\, c_2\right) g^{(1)}(c_1)\, c_3\right) + \ln\left(f^{(2)}\left(g^{(1)}(c_1)\, c_2\right) g^{(1)}(c_1)\, c_4\right),$$

where $f^{(1)}, f^{(2)}, g^{(1)} \in \mathcal{U}$ and $f^{(2)}, g^{(1)} > 0$.

Remark 4 *Although our focus in this paper is on the consumer's period one optimization and not her decisions in subsequent periods, the comparison of utilities (59) and (61) prompts an observation on decisions over time. Whereas the assumption that the consumer is myopic in every period is clearly stronger than assuming myopia in the first period (or some subset of periods), there is a sense in which the former assumption may be viewed as more natural. If the consumer is assumed to be myopic just in the first period, then why when period one ends doesn't she become myopic in the second period – retaining the same intertemporal pattern? If that were the case and her preferences in period one were represented by the utility form in eqn. (61) and not (59), then her utility in period two would have to take the form of eqn. (43),* $U(c_2, c_3) = h(c_2) + \ln c_3$. *In this case the MRS (marginal rate of substitution) between* c_2 *and* c_3 *would change from period one to period two and the consumer would be inconsistent. On the other hand, if the consumer is myopic in every period, implying that her preferences in the first period can be represented by the utility in eqn. (59), then the same utility with the given* c_1 *can be assumed in the second period and her period two demands will still be myopic, i.e., independent of the period three price. Since her MRS between* c_2 *and* c_3 *remains the same when changing periods, the consumer will not revise her plan and thus is consistent.*[12] *Whereas some may find this argument for assuming the consumer is myopic in every period to be persuasive at the individual demand and preference level, we will see in Examples 3 and 4 below that the equilibrium implications of being myopic in each period are considerably stronger than those associated with the assumption of being myopic in just the first period.*

3 Interest Rate Implications

In this Section, we investigate the implications of preferences being myopic for equilibrium interest rates. A standard certainty representative agent equilibrium model is assumed (see, for example, Kocherlakota 2001).[13] In period one, assume a single good, denoted by c_1, and $T-1$ zero coupon bonds, where b_t $(t = 2, 3, ...T)$ denotes the quantity of zero coupon bonds purchased in period one and maturing at the beginning of period t and paying one unit of c_t.[14] The period one price of the zero coupon bond is denoted q_t, where subscript t indicates that the bond matures at the

[12] For a detailed discussion of changing tastes and consistency, see Selden and Wei (2012).

[13] Since a static setting is assumed, all bond prices are observed at the beginning of the current time period and all interest rates are current spot rates. We do not consider implied forward rates or future spot interest rates.

[14] It should be noted that in order to investigate the impact of myopia on the term structure, the choice of a zero coupon bond versus a standard nonzero coupon bond is not innocuous. For example, consider a three period setting in which the two period bond pays a coupon rate of ξ per

beginning of period t. And as is standard, the net interest rate r_{t-1} during period $t-1$ associated with the zero coupon bond purchased in period one and maturing at date t is given by

$$q_t = \frac{1}{(1+r_{t-1})^{t-1}}. \tag{62}$$

The representative agent is endowed in period one with a fixed supply $\overline{c}_1$ of period one consumption and $\overline{b}_2, \overline{b}_3, ..., \overline{b}_T$ zero coupon bonds[15] and has preferences over consumption streams $(c_1, ..., c_T)$ represented by U.[16] The optimization problem is given by

$$\max_{c_1,b_2,...,b_T} U(c_1, ..., c_T) \qquad S.T.\ c_1 + \sum_{t=2}^{T} q_t c_t = \overline{c}_1 + \sum_{t=2}^{T} q_t \overline{b}_t \tag{63}$$

$$S.T.\ c_t = b_t \quad \forall t \in \{2, 3, ..., T\}. \tag{64}$$

It is clear that in this optimization, if U takes one of the myopic forms of utility derived in the prior Section, optimal demands will not be independent of future prices since prices will always enter into the demand functions through total income or wealth, i.e.,

$$I = \overline{c}_1 + \sum_{t=2}^{T} q_t \overline{b}_t. \tag{65}$$

As a result, the presence of endowments precludes using Definition 1 to define myopic demand. Endowments introduce a third term into the classic Slutsky equation typically referred to as the endowment (income) effect (see Arrow and Hahn, 1971, p. 225 and Varian, 1992, p. 145). However, the myopic forms of utility will ensure that the corresponding demand function income and substitution effects discussed in Remark 1 continue to exactly offset each other. Thus in the present exchange economy setting, when we say that c_t is myopic we will mean that this equality of minus the income effect and the substitution effect holds and the endowment effect

cent at the end of periods one and two. Then period two consumption is given by $c_2 = b_2 + \xi b_3$, which is a function of both b_2 and b_3. Thus, it would not be appropriate to say that c_2 is myopic if and only if it is independent of the two period bond price q_3. The advantage of the zero coupon bond assumption is that all of the information concerning the bond is incorporated in its price and this difficulty can be avoided.

[15] Here we assume that $\overline{b}_2, \overline{b}_3, ...\overline{b}_T \neq 0$. Such an assumption is not atypical. It could for instance be associated with the debt being issued by a government which is outside the model (see, for example, Parlour, et al., 2011 and the literature cited therein). Alternatively, our assumption of nonzero supplies of bonds could be dropped if we were to allow for endowments in the form of period two and three income (in units of consumption). This would change none of the conclusions, only making the notation more complicated.

[16] It should be noted that because we do not assume additive utility, our analysis will not include the typical period discount factors present in standard equilibrium interest rate models.

is ignored. And in this sense, each of the preference conditions derived in Section 2 can be applied to our equilibrium analysis. Indeed our goal in this Section is to investigate the implications of these forms of utility for equilibrium interest rates.

To examine the implications of myopic preferences, assume the first and second time periods are combined to form a new "long" period with consumption c_1 and c_2 (or b_2). It follows from Proposition 1 that in a pure demand setting (without endowments) the following myopic utility

$$U(c_1, ..., c_T) = f(g(c_1, c_2)\, c_3, g(c_1, c_2)\, c_4, ..., g(c_1, c_2)\, c_T)\,. \tag{66}$$

results in optimal demands for both c_1 and c_2 (or b_2) being independent of the interest rates on the 3- through T-period zero coupon bonds. It is natural to ask what are the implications of the myopic utility (66) for equilibrium interest rates. One natural conjecture might be that myopia is necessary and sufficient for the equilibrium period one interest rate r_1 to be independent of the supplies $\bar{b}_t$ $(t = 3, 4, ..., T)$. Indeed it can be verified that for the utility (66), one always has

$$1 + r_1 = \frac{1}{q_2} = \frac{\partial U/\partial c_1}{\partial U/\partial c_2} = \left.\frac{\partial g(c_1, c_2)/\partial c_1}{\partial g(c_1, c_2)/\partial c_2}\right|_{(c_1,c_2)=\left(\bar{c}_1,\bar{b}_2\right)}, \tag{67}$$

which is independent of $\bar{b}_t$ $(t = 3, 4, ..., T)$. However, r_1 being independent of $\bar{b}_t$ $(t = 3, 4, ..., T)$ although necessary is not sufficient for myopic utility. The interest rate r_1 is also independent of $\bar{b}_t$ $(t = 3, 4, ..., T)$ when utility takes the additively separable form, as we show below. And as we have seen additive separability does not imply myopic utility. In the remainder of this paper, we derive the different equilibrium interest rate implications corresponding to preferences being additively separable, homothetic or myopic. It should be noted that the most widely assumed form of myopic utility the log additive form (60) not only exhibits myopia and additive separability but also homotheticity.

Beginning with additive separability, it is possible by applying a classic result of Samuelson (1947) to the current representative agent equilibrium setting, to derive the restrictions on equilibrium interest rates that are equivalent to preferences being additively separable.

Proposition 2 *Assume the representative agent's optimization problem is characterized by (63) and (64). For all* $t \in \{1, 2, ..., T-1\}$ *and* $T > 2$, r_t *is independent of bond supplies other than* $\bar{b}_{t+1}$ *if and only if the agent's preferences can be represented by an ordinally additively separable utility,*

$$U(c_1, ..., c_T) = \sum_{t=1}^{T} u_t(c_t)\,. \tag{68}$$

Proof. Observing that in a representative agent exchange economy

$$\frac{1}{(1+r_{t-1})^{t-1}} = q_t = \frac{\partial U/\partial c_t}{\partial U/\partial c_1} \quad \forall t \in \{2,3,...,T\}, \tag{69}$$

the proof of this Proposition directly follows from Samuelson (1947) pp. 176-183.[17] ■

Whereas the weak separability corresponding to period one myopia ensures that r_1 only depends on $\bar{b}_2$, additive separability ensures that this same result holds for every period. As can be seen from Example 4 below, myopia in each period differs from additive separability in allowing the equilibrium interest rates r_t ($t = 2,3,...,T-1$) to depend on the supply of bonds in all prior periods and not just $\bar{b}_{t+1}$. Therefore the equilibrium interest rate restrictions implied by additive separability are clearly stronger which is fully consistent with the preference based reservations of Fisher (1930), Hicks (1965) and Lucas (1978) referenced above in Section 1.

Next we isolate the restriction on the equilibrium interest rates corresponding to preferences being homothetic.

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[17] It should be noted that in addition to

$$\frac{\partial q_t}{\partial \bar{b}_i} = 0 \quad (\forall t, i \in \{2,3,...,T\}, i \neq t),$$

implied in our Proposition 2, Samuelson also gives the following in his eqns. (33) (Samuelson 1947, p. 179)

$$\frac{\partial}{\partial \bar{c}_1}\left(\frac{q_t}{q_2}\right) = 0 \quad (\forall t \in \{3,4,...,T\})$$

for the necessary and sufficient condition such that preferences can be represented by an additively separable utility function. As Samuelson states, his condition implies integrability and if this is postulated as a precondition then eqns. (33) cease to all be independent and can be reduced in number. Since we have assumed the existence of U, we don't need to include the above set of equations in Proposition 2. To be more explicit, we can show that this set of equations can be directly derived from $\frac{\partial q_t}{\partial \bar{b}_i} = 0$. Noticing that

$$q_2 = \frac{\frac{\partial U}{\partial c_2}}{\frac{\partial U}{\partial c_1}} \quad \text{and} \quad q_3 = \frac{\frac{\partial U}{\partial c_3}}{\frac{\partial U}{\partial c_1}},$$

one can obtain

$$\frac{\partial q_2}{\partial c_3} = 0 \Leftrightarrow \frac{\partial^2 U}{\partial c_2 \partial c_3}\frac{\partial U}{\partial c_1} - \frac{\partial^2 U}{\partial c_1 \partial c_3}\frac{\partial U}{\partial c_2} = 0$$

and

$$\frac{\partial q_3}{\partial c_2} = 0 \Leftrightarrow \frac{\partial^2 U}{\partial c_2 \partial c_3}\frac{\partial U}{\partial c_1} - \frac{\partial^2 U}{\partial c_1 \partial c_2}\frac{\partial U}{\partial c_3} = 0,$$

implying that

$$\frac{\partial^2 U}{\partial c_1 \partial c_3}\frac{\partial U}{\partial c_2} - \frac{\partial^2 U}{\partial c_1 \partial c_2}\frac{\partial U}{\partial c_3} = 0 \Leftrightarrow \frac{\partial}{\partial c_1}\left(\frac{\frac{\partial U}{\partial c_3}}{\frac{\partial U}{\partial c_2}}\right) = \frac{\partial}{\partial c_1}\left(\frac{q_3}{q_2}\right) = 0.$$

Proposition 3 *Assume the representative agent's optimization problem is characterized by (63) and (64). The interest rate in each period remains the same when the endowments of period one consumption and each zero coupon bond are increased by the same percentage if and only if the agent's preferences are homothetic.*

Proof. First prove sufficiency. Note that homotheticity implies that there exists a homogeneous utility function U such that for $\forall t \in \{2, 3, ..., T\}$

$$\frac{1}{(1 + r_{t-1})^{t-1}} = q_t = \frac{\partial U / \partial c_t}{\partial U / \partial c_1}. \tag{70}$$

Assuming without loss of generality that U is homogeneous of degree 1, Euler's Theorem implies that $\partial U / \partial c_t$ ($\forall t \in \{1, 2, ..., T\}$) is homogeneous of degree 0. It then follows that increases in the endowments of period one consumption and each zero coupon bond by the same percentage imply that the set of equilibrium interest rates is unchanged. Next prove necessity. Since the interest rate (equilibrium price) in each period remains the same when changing all of the consumption and bond endowments by the same percentage, it follows from the budget constraint that if income $I = \bar{c}_1 + \sum_{t=2}^{T} \frac{\bar{b}_t}{(1+r_{t-1})^{t-1}}$ the total wealth increases by a certain percentage, the optimal demands will increase by the same percentage for fixed prices, implying that preferences are homothetic. ■

Given that r_1 being independent of the supplies $\bar{b}_t$ ($t = 3, 4, ..., T$) is only a necessary condition for (c_1, c_2) to be myopic, we next characterize the equilibrium interest rate implications which are both necessary and sufficient for preferences to be representable by the myopic utility (6) in Proposition 1.

Proposition 4 *Assume the representative agent's optimization problem is characterized by (63) and (64). The equilibrium interest rates exhibit the property that for any $t \in \{1, 2, ..., T-1\}$, the present value $\sum_{i=t+1}^{T} \frac{\bar{b}_i}{(1+r_{i-1})^{i-1}}$ is independent of $\bar{b}_j$ ($j \in \{t+1, t+2, ..., T\}$) if and only if preferences are representable by the myopic utility (6) corresponding to optimal period t consumption c_t being myopic.*

Proof. Without loss of generality, we only need to prove the Proposition for optimal period two consumption. First prove sufficiency. It follows from Proposition 1 that period two consumption c_2 is myopic if and only if

$$U(c_1, ..., c_T) = f(g(c_1, c_2)c_3, g(c_1, c_2)c_4, ..., g(c_1, c_2)c_T). \tag{71}$$

In equilibrium, the first order conditions are

$$\frac{g f_j}{g_1 \sum_{i=1}^{T-2} f_i \bar{b}_{i+2}} = \frac{1}{(1 + r_{j+1})^{j+1}}, \tag{72}$$

implying that

$$\sum_{i=3}^{T} \frac{\bar{b}_i}{(1+r_{i-1})^{i-1}} = \sum_{i=1}^{T-2} \frac{\bar{b}_{i+2}}{(1+r_{i+1})^{i+1}} = \frac{g \sum_{i=1}^{T-2} f_i \bar{b}_{i+2}}{g_1 \sum_{i=1}^{T-2} f_i \bar{b}_{i+2}} = \left. \frac{g}{g_1} \right|_{(c_1,c_2)=(\bar{c}_1,\bar{b}_2)} \tag{73}$$

and hence $\sum_{i=3}^{T} \frac{\bar{b}_i}{(1+r_{i-1})^{i-1}}$ is independent of $\bar{b}_j$ ($j \in \{3,4,...,T\}$). Next prove necessity. If $\sum_{i=3}^{T} \frac{\bar{b}_i}{(1+r_{i-1})^{i-1}}$ is independent of $\bar{b}_j$ ($j \in \{3,4,...,T\}$), then we have

$$\frac{\partial}{\partial c_j} \frac{\sum_{i=3}^{T} c_i U_i}{U_1} = 0 \quad \forall j \in \{3,...,T\}, \tag{74}$$

implying that

$$U(c_1,...,c_T) = f(g(c_1,c_2)c_3, g(c_1,c_2)c_4, ..., g(c_1,c_2)c_T). \tag{75}$$

■

The intuition for why Proposition 4 works is that for myopic preferences the set of equilibrium interest rates adjusts so as to keep the present value constant. It should be noted that if preferences take the form associated with consumption being myopic in each period, then we can apply Proposition 4 recursively and conclude that for all $t \in \{1,2,...,T-1\}$, r_t is independent of $\bar{b}_{j+1}$ and $\sum_{i=t+1}^{T} \frac{\bar{b}_i}{(1+r_{i-1})^{i-1}}$ is independent of $\bar{b}_j$ ($j \in \{t+1,t+2,...,T\}$), where the latter is not only necessary but also sufficient.

Remark 5 *The Proposition 4 conclusion that the present value of future bond supplies is independent of changes in the supply of bonds in each period may strike the reader as being reminiscent of the irrelevance of government financial policy in the macroeconomics literature (e.g., Wallace 1981, Bryant 1983 and Stiglitz 1986). There it is assumed in an intertemporal setting that a government exists which both collects taxes and issues debt of differing maturities. If for a given supply of bonds a general equilibrium exists, then modifying the supply of bonds will not affect the equilibrium value of real variables such as consumption although equilibrium interest rates may change. (There is a clear analogy of this result to the famous Modigliani and Miller capital structure irrelevance in corporate finance.) However it should be stressed that the source of "independence" in our setting comes from the form of utility since the government sector in our model is not closed as we do not allow for taxes.*

We conclude this Section with two Examples and a Remark. They illustrate in a four period setting the Proposition 2 and 4 implications on equilibrium interest rates of preferences taking the different forms associated with c_1 and c_2 being myopic versus c_1, c_2, c_3 and c_4 being myopic and the implications of log additive utility.

Example 3 *Assume the representative agent's optimization problem is characterized by (63) and (64), where there are four periods and utility takes the form*

$$U(c_1, c_2, c_3, c_4) = ((\ln c_1 + \ln c_2) c_3)^{\frac{1}{4}} + \sqrt{(\ln c_1 + \ln c_2) c_4}, \tag{76}$$

where $c_1, c_2 > 1$ *and* $c_3, c_4 > 0$ *are assumed to ensure* $U \in \mathcal{U}$*. It follows from Proposition 1 that the optimal consumption vector* (c_1, c_2) *is myopic. Using the representative agent's first order conditions paralleling eqn. (67), straightforward computation results in the following characterization of equilibrium interest rates*

$$\frac{1}{1+r_1} = \frac{\bar{c}_1}{\bar{b}_2}, \tag{77}$$

$$\frac{1}{(1+r_2)^2} = \frac{\bar{c}_1 \left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \left(\left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \bar{b}_3\right)^{-\frac{3}{4}}}{\bar{b}_3 \left(\left(\left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \bar{b}_3\right)^{-\frac{3}{4}} + 2\left(\left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \bar{b}_4\right)^{-\frac{1}{2}}\right)} \tag{78}$$

and

$$\frac{1}{(1+r_3)^3} = \frac{2\bar{c}_1 \left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \left(\left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \bar{b}_4\right)^{-\frac{1}{2}}}{\bar{b}_4 \left(\left(\left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \bar{b}_3\right)^{-\frac{3}{4}} + 2\left(\left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \bar{b}_4\right)^{-\frac{1}{2}}\right)}. \tag{79}$$

First we can see that r_1 *is independent of* $\left(\bar{b}_3, \bar{b}_4\right)$*, but* r_2 *and* r_3 *depend on all of the bond supplies. Moreover, using (78) and (79) it follows that the present value of period 3 and period 4 bond supplies*

$$\frac{\bar{b}_3}{(1+r_2)^2} + \frac{\bar{b}_4}{(1+r_3)^3} = \bar{c}_1 \left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \tag{80}$$

is independent of $\bar{b}_3$ *and* $\bar{b}_4$*.*

Example 4 *Assume the representative agent's optimization problem is characterized by (63) and (64), where there are four periods and utility takes the form*[18]

$$U(c_1, c_2, c_3, c_4) = \sqrt{\ln(c_1 c_2) c_1 c_3} + \ln(\ln(c_1 c_2)) + \ln c_1 + \ln c_4, \tag{81}$$

where $c_1, c_2 > 1$ *and* $c_3, c_4 > 0$ *are assumed to ensure* $U \in \mathcal{U}$*. It follows from Proposition 1 and Result 1 that optimal consumption in each period is myopic. Using the representative agent's first order conditions paralleling eqn. (67), straightforward computation results in the following characterization of equilibrium interest rates*

$$\frac{1}{1+r_1} = \frac{\bar{c}_1}{\bar{b}_2 \left(1 + \ln\left(\bar{c}_1 \bar{b}_2\right)\right)}, \tag{82}$$

[18] The utility (81) can be obtained from the general four good myopic utility in footnote 11 by assuming

$$f^{(1)}(x) = \sqrt{x}, \quad f^{(2)}(x) = \ln x \text{ and } g^{(1)}(x) = x.$$

It should be noted that if the coefficients in (81) are varied arbitrarily, the resulting utility will not be a special case of the general form and will not result in myopic demand behavior.

$$\frac{1}{(1+r_2)^2} = \frac{\left(\bar{c}_1 \ln\left(\bar{c}_1\bar{b}_2\right)\right)^2}{\left(1+\ln\left(\bar{c}_1\bar{b}_2\right)\right)\left(\bar{c}_1\bar{b}_3 \ln\left(\bar{c}_1\bar{b}_2\right) + 2\sqrt{\bar{c}_1\bar{b}_3 \ln\left(\bar{c}_1\bar{b}_2\right)}\right)} \tag{83}$$

and

$$\frac{1}{(1+r_3)^3} = \frac{2\bar{c}_1 \ln\left(\bar{c}_1\bar{b}_2\right)}{\bar{b}_4\left(1+\ln\left(\bar{c}_1\bar{b}_2\right)\right)\left(2+\sqrt{\bar{c}_1\bar{b}_3 \ln\left(\bar{c}_1\bar{b}_2\right)}\right)}. \tag{84}$$

It is clear that r_1 is independent of $\left(\bar{b}_3, \bar{b}_4\right)$, r_2 is independent of $\bar{b}_4$ and r_3 depends on all bond supplies. Moreover using (82)-(84), it follows that

i

$$\frac{\bar{b}_2}{1+r_1} + \frac{\bar{b}_3}{(1+r_2)^2} + \frac{\bar{b}_4}{(1+r_3)^3} = \bar{c}_1, \tag{85}$$

where the present value is independent of $\bar{b}_2$, $\bar{b}_3$ and $\bar{b}_4$;

ii

$$\frac{\bar{b}_3}{(1+r_2)^2} + \frac{\bar{b}_4}{(1+r_3)^3} = \frac{\bar{c}_1 \ln\left(\bar{c}_1\bar{b}_2\right)}{1+\ln\left(\bar{c}_1\bar{b}_2\right)}, \tag{86}$$

where the present value is independent of $\bar{b}_3$ and $\bar{b}_4$; and

iii

$$\frac{\bar{b}_4}{(1+r_3)^3} = \frac{2\bar{c}_1 \ln\left(\bar{c}_1\bar{b}_2\right)}{\left(1+\ln\left(\bar{c}_1\bar{b}_2\right)\right)\left(2+\sqrt{\bar{c}_1\bar{b}_3 \ln\left(\bar{c}_1\bar{b}_2\right)}\right)}, \tag{87}$$

where the present value is independent of $\bar{b}_4$.

Remark 6 *It is interesting to contrast the equilibrium in Example 4 with that resulting from utility taking the log additive form*

$$U\left(c_1, c_2, c_3, c_4\right) = \ln c_1 + \ln c_2 + \ln c_3 + \ln c_4, \tag{88}$$

where the representative agent is also myopic in each period. In this case, it can be verified that

$$\frac{1}{1+r_1} = \frac{\bar{c}_1}{\bar{b}_2}, \quad \frac{1}{(1+r_2)^2} = \frac{\bar{c}_1}{\bar{b}_3} \quad \textit{and} \quad \frac{1}{(1+r_3)^3} = \frac{\bar{c}_1}{\bar{b}_4}, \tag{89}$$

implying that

$$\frac{\bar{b}_2}{1+r_1} + \frac{\bar{b}_3}{(1+r_2)^2} + \frac{\bar{b}_4}{(1+r_3)^3} = 3\bar{c}_1, \tag{90}$$

$$\frac{\bar{b}_3}{(1+r_2)^2} + \frac{\bar{b}_4}{(1+r_3)^3} = 2\bar{c}_1 \tag{91}$$

and

$$\frac{\bar{b}_4}{(1+r_3)^3} = \bar{c}_1. \tag{92}$$

It can be seen that a change in the endowment $\bar{b}_{t+1}$ *exactly cancels out the interest rate change* $(1+r_t)^t$ *and the present value is always a function of only* $\bar{c}_1$*. Whereas myopia in each period ensures that each present value sum is independent of changes in the respective endowment, additive separability ensures that a change in* $\bar{b}_{t+1}$ *only affects* r_t *and hence implies that the changes in the endowment must be canceled out exactly by the interest rate change.*

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