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On the Political Economy of Urban Growth: Homeownership versus Affordability*

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Abstract

We study the equilibrium properties of an overlapping-generation economy where agents choose where to locate, and how much housing to own, and city residents vote on the number of new building permits every period. Under-supply of housing persists in equilibrium under conditions we characterize. City residents invest in housing because they expect their investment to be protected by a majority of voters opposed to urban growth. They vote against growth because they have invested in local housing. This vicious cycle between ownership and urban growth generates a tension between the common housing policy objectives of affordability for all and homeownership for most. Homeownership subsidies increase resistance to urban growth. Capturing the value of new building permits and distributing the proceeds to residents may move the economy away from a welfare-dominated no-growth equilibrium.

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Housing policy aims primarily at making housing available to all households and homeownership possible for most. National governments expend significant resources pursuing both objectives. At the same time, many urban growth restrictions enacted by local governments work against these objectives and raise the cost of housing.

Local policies that restrict urban growth cannot survive without local political support. We ask whether and under what circumstances the local political economy of housing supply stands in the way of efficiency. We find that inefficiently low housing supply may persist when residents vote on the issuance of building permits in their jurisdiction, therefore making housing unaffordable for too many households. The effect of overly restrictive urban growth policies is exacerbated when households have incentives to own their homes.

Our results point to a fundamental tension between the general objectives of homeownership and affordability. The political economy of urban growth at the local level calls into question the architecture of housing policy in many countries.

In an influential book entitled *The Homevoter Hypothesis*, Fischel (2001, p. 5) suggests that: "concern for home values is the central motivator of local government behavior." While the idea that homeowners use the political process to protect home values is appealing, it opens up a number of questions. On the one hand, what determines households' location choice and housing investment in the first place? And how do these decisions relate to their support for urban growth policies? On the other hand, to what extent does "homevoter" behavior affect the welfare of local residents and other people? If it generates welfare losses by overly restricting housing supply, what elements of the housing market or the policy process limit or amplify this loss? More generally, how could everyone's welfare be improved?

We seek to understand who participates in the urban growth decision process, the stakes of the participants, and the way that participants' preferences translate into policies. In the process, we need, at a minimum, a *location choice model* to determine who lives in a particular area, a *housing investment model* to predict which real estate assets the residents own, and a *collective choice model* to map the identities and preferences of local residents into political decisions on urban growth.

Our goal is to provide a first step toward a theory that encompasses the three elements: housing consumption, housing investment, and collective choice as to urban growth. Each element is quite complex in and of itself. Each is linked to the others through multiple channels. Our aim is a parsimonious and tractable framework to gain insights into the basic issues at play and to link areas of research that have traditionally been separate.

Our theory builds upon the standard urban location choice model. Agents are free to locate in the countryside where they earn nothing or in the city where they earn a wage that depends on the city's technology and their own productivity. Living in the countryside is free; living in the city requires consuming one unit of housing. In equilibrium, city housing rents guarantee there is no excess demand for the use of city housing units.

To add housing investment considerations, we extend this static framework to a dynamic stochastic environment. To provide microeconomic foundations for the choice between owning and renting, we need agents who live for multiple periods. Housing rents fluctuate endogenously in response to shocks to the city production technology. Owning housing therefore exposes households to housing price risk. Households are risk-averse. They trade off the cost of homeownership against the premium between the returns to homeownership and the returns to renting a property.[1] The homeownership premium is meant to capture factors such as the favorable tax treatment of homeownership, intrinsic preferences for homeownership, and transaction costs in the rental market.[2]

The critical innovations in our approach are (1) that adding a house to a city requires a building permit, and (2) that local residents have a say (vote) on the number of permits to be issued every period.

The issue of building permits affects voters through three channels. First, new housing construction reduces their housing rents for the remainder of their lives. Second, lower future rents imply an immediate drop in the price of their housing investment. Third, voters may capture some benefits from any windfall gains deriving from the new construction permits if permits are sold to developers and the revenues are used for local services (for example).[3]

Voters also understand that new construction changes the stakes of current residents and allows new residents to move in, hence bringing about potential shifts in future voting outcomes. Future voting outcomes matter to current voters because they affect future housing rents and thus current housing prices. This dynamic dimension of the voting game is critical to understanding the economic forces at play. We propose to handle it by working with an equilibrium concept akin to stationary sub-game perfection in order to focus on the question: What is the smallest city size that can be sustained in a political equilibrium?

Householder age is a key variable to characterize the individual trade-off between lower future rents and lower housing prices. As housing ownership weakly increases with age, older agents suffer more of a loss from any drop in housing prices. They also benefit less from any drop

[1] Tenure choice in a dynamic stochastic environment has been studied (for example) in Ortalo-Magné and Rady (2002), Sinai and Souleles (2005), and Davidoff (2006).

[2] See, for example, Henderson and Ioannides (1983).

[3] To our knowledge, there is little systematic evidence on the distribution of windfall gains. In her comprehensive review of housing supply in Britain, Barker (2003, Chapter 5) argues that: (1) Developers hold option agreements on large tracts of land currently without building permission; (2) Developers have significant local market power; (3) While local authorities have a legal avenue to demand a fee for issuing building permits, the amount obtained in this way is quite low (of the order of £2000/8000 per unit built – see Table 8.2 in Barker). These three facts taken together seem to indicate that most of the windfall gains accrue to developers. Things may be different in Hong Kong, where the government uses an auction mechanism to sell land for development (see Or and Ogden, 2006).

in future rents because their lives are shorter and there are fewer periods over which they will consume housing. This is one reason people support urban growth less as they age.

The median voter theorem applies. The median voters are the members of the cohort with the median age. Whether the median voters oppose urban growth or not depends on how much housing they own, which depends in turn on the homeownership premium and voters' expectations for future housing rents and thus future voting outcomes.

We find there are stationary equilibria, where a city is below its optimal size, yet the median cohort and all older agents oppose urban growth because they have made sizeable housing investments. Agents also continue to invest heavily in housing, because they expect the majority to oppose urban growth in the future. Before they made purchase decisions, all agents would have preferred a larger city, so that housing would have been cheaper and more people could have located there but the equilibrium is sustained because the median voter is someone who already owns housing.

The minimal city size that can be sustained in equilibrium depends on model parameters meant to capture institutional features of the housing market. A higher homeownership premium will increase the housing investment of the median voter, and hence reduces the minimum city size. In this sense, the local political economy of urban growth creates tension between the two common housing policy objectives of (1) housing affordability for all and (2) homeownership for most. Subsidizing homeownership encourages households to invest in housing *and* to vote against urban growth at the local level; the result is more expensive housing.

The equilibrium city size also depends on how permits are allocated. The minimal size is low when the windfall gains generated by the issuance of building permits go elsewhere than to residents. An effective way to remove opposition to growth is to create institutions that channel more of developers' profits to residents.

We build on our basic model to study a number of extensions. Eliminating the assumption that households may invest in a divisible amount of housing does not change our results. Allowing all households in the economy to vote (both city and countryside residents) lessens the inefficiency problems because it adds to the voting population a group of households that sees only benefits to urban growth.

When we consider a geography with more than one city, a critical variable becomes the mobility of agents across cities. When agents are mobile, urban growth in one city implies the ensuing rent reduction is shared across all cities, and the local rent reduction for that particular level of local urban growth is not as high as in the one-city case. This lower rent effect may imply stronger support for urban growth.

The model highlights the critical role of local demographics in urban growth policy. Older agents have a shorter lifetime to benefit from any lower future cost of housing, but are affected by lower future rents on the value of their housing investment the same way as all other homeowners. This insight may provide guidance for empirical work aimed at understanding the determinants of urban growth restrictions.

The key trade-off that determines household policy preferences is the difference between a household's expected future stream of housing expenditures and the stream of rents embedded in the value of its housing investment. In the data, age may provide a useful measure of where a household falls along this trade-off, as in the model, but more refined approaches could easily incorporate our general understanding of the life-cycle profile of housing expenditures and investments and households' propensity to move, two factors we abstract from in our theory.[4]

Dubin, Kiewiet and Noussair (1992) analyze voting data on urban growth control measures on the San Diego ballot in 1988. They take advantage of cross-sectional differences in the socioeconomic makeup of precincts to tease out the factors correlated with support for growth controls. They find strong support for the hypothesis that homeowners are more likely to favor growth controls. Local homeowners are presumably more invested in local housing than local renters.

Our results also shed new light on the relative contribution of nature and regulations to urban growth. Our finding of persistent undersupply raises the possibility that a strong empirical correlation between the presence of natural barriers and housing prices may not be an indication that high prices are due exclusively to the natural barriers. Instead, the vicious cycle we identify between homeownership and supply may very well play a role. This result finds support in the empirical analysis proposed by Saiz (forthcoming): that restrictive urban growth policies are more likely in metropolitan areas that face natural land constraints such as water bodies, and steep slopes.

An increasing body of evidence points out that urban growth restrictions are critical to housing market dynamics. Glaeser, Gyourko, and Saks (2005a) report that changes in regulatory regimes explain the scarcity of land for housing development in what have become the most expensive U.S. housing markets. Green, Malpezzi, and Mayo (2005) find that housing supply regulations are the key driver of differences in housing supply elasticities across U.S. metropolitan areas. In the United Kingdom, Barker (2003, 2005) identifies regulatory constraints on the release of land for housing development as the primary reason behind the unresponsiveness of housing supply to price increases.[5]

A number of authors analyze urban growth controls in a static setting. Brueckner (1995),

[4] See, for example, Fernandez-Villaverde and Krueger (2007), Sinai and Souleles (2008), and Shore and Sinai (forthcoming).

[5] For further evidence on the critical impact of housing regulations on housing supply, see Ellickson (1977), Ozanne and Thibodeau (1983), Rose (1989), Malpezzi, Chun and Green (1998), Mayer and Somerville (2000), Glaeser and Gyourko (2003), Evans and Hartwich (2005), Glaeser, Gyourko and Saks (2005b), Quigley and Raphael (2005), and Glaeser and Ward (2006).

and Helsley and Strange (1995) consider supply restriction in an economy where land in the city is owned by absentee landlords. Brueckner and Lai (1996) assumes resident landowners share the city with a group of renters (the landowners own more housing than they use for themselves). All three papers feature static models where city size is decided by maximizing the utility of a particular constituency; e.g., the homogeneous resident landowners in Brueckner and Lai (1996). Like these authors, we focus on one – stylized – form of growth restriction. We do not consider regulations pertaining to height, development density, and use in the interest of parsimony.[6]

Like Glaeser, Gyourko, and Saks (2005a) we are convinced that progress in our understanding of the determinants of housing supply restrictions requires explicit modeling of how household preferences translate into policy. Glaeser, Gyourko, and Saks study the effects of changing judicial tastes, reduced ability to bribe regulators, rising incomes and demand for public amenities, and improvements in the ability of homeowners to organize and influence local decisions. They find that the increased ability of local residents to block new projects is the main driver for the rise in urban growth restrictions. They conclude that cities have changed from urban growth machines to sort of homeowners' cooperatives.

Our paper forgoes most of the political process complexity of Glaeser, Gyourko, and Saks in order to push the literature along another dimension. In our model, the composition of the local population, households' tenure decision, and hence their preferences for urban growth are determined endogenously. We also assume that when householders vote, they understand the consequences for the composition of future electorates and policy outcomes. This feature matters because current housing prices reflect agents' expectations about future rents, which depend on future voting outcomes.

A different but complementary political economy literature focuses on another critical policy instrument of local housing supply regulation: minimum house size requirements. Calabrese, Epple, and Romano (2007) consider a one-shot representative democracy where households vote after having purchased housing in the community of their choice. They show how a minimum house size requirement may yield welfare gains by allowing meaningful community differentiation.

The closest research to ours is by Coate (2010). Like us, he explores the inefficiencies in housing supply that may arise in dynamic political environments, although we explore two distinct sources of distortion. Coate considers a heterogeneous housing stock and examines the effect of zoning rules that affect the quantity as well as the quality of housing. He demonstrates that owners of smaller homes have an incentive to impose a minimum house size to force the relative supply of such homes to shrink over time in their city. Scarcity increases the relative price of small houses and also the local property tax base by forcing an increase in the percentage of larger houses. He finds that a small difference in initial housing stocks from the efficient

[6] See, for example, Wheaton (1998), and Bertaud and Brueckner (2004).

steady-state equilibrium housing stocks yields equilibrium paths that do not converge to the efficient steady-state.

We focus instead on a homogeneous housing stock, and allow agents to choose between renting and owning. We therefore focus on another source of dynamic distortion in housing supply, the effect of ownership incentives. Analysis of the interactions between ownership subsidies and size restrictions is left to future research.

We introduce the model in Section 1. Section 2 characterizes the equilibrium. Section 3 discusses welfare implications and explores the tension between the goal of encouraging ownership and the goal of making housing more affordable. Section 4 discusses extensions to indivisible housing investment, national voting, and multiple cities.

1 Model

We focus on the interaction between affordability and homeownership, and build a parsimonious model to discuss this trade-off. One of our key objectives is to endogenize the choice between buying and renting. The evidence that we reviewed earlier points to the importance of risk considerations in real estate investment decisions. A reasonable model of endogenous tenure must therefore contain uncertainty and, as a consequence, a dynamic component.

As a multi-period investment model is already quite complex, we abstract from other important themes in urban economics. We do not deal with externalities, such as congestion or public good, nor do we discuss taxation. The relation between homeownership and affordability turns out to depend on a simple median voter argument, however, so the trade-off we focus on is likely to figure even if the model is enriched with other public economics considerations.

1.1 Population and Geography

We assume an overlapping-generation economy where a mass 1 of agents is born in every period. Agents live for $S+1$ periods; they face choices at ages s, $s=0,...,S-1$. At age S, agents simply consume their wealth and die without voting. It is convenient to assume that S is an odd number so that $\frac{S-1}{2}$ is the age of the median cohort when cohorts are ranked by age from age 0 to age $S-1$.

There are two locations in the economy: the countryside and the city. In the countryside, there is an unlimited supply of houses available at a cost normalized to zero. In the city, the number of houses is determined endogenously. Each house accommodates at most one agent. For convenience, we let n_t denote the number of houses available per cohort. City size is then $S \times n_t$. Geographic constraints imply that the number of houses in the city cannot exceed $S\bar{n} \leq S$; that is, not everyone can live in the city.

At birth each agent draws a productivity parameter, ε. For convenience, we assume that ε is distributed uniformly over the unit interval. Upon learning its city productivity ε, the agent decides whether to live in the city or the countryside. The decision is final; agents cannot move later in life.

The economy is also populated with a set of competitive and risk neutral real estate investment trusts (REITs). The REITs are owned by outsiders. They may buy properties and rent them to agents.

1.2 Credit Market

All agents and the REITs can borrow and lend at the risk free rate $1 - 1/\beta$.

1.3 Production

The income of an agent who lives in the countryside is normalized to zero. An agent who lives in the city earns

$$y_t + \varepsilon_i,$$

where y_t represents the city productivity level.

City productivity follows the stochastic process

$$y_t = y_{t-1} + \tau_t,$$

where τ_t is an i.i.d. shock, normally distributed with mean zero and variance σ^2.

1.4 City Housing Market

To earn income in the city, an agent must live in the city. This requires having exclusive use of a house. A key feature of our model is that the agent can own or rent (or part-own and part-rent) the house he lives in. However, for accounting purposes, we assume that the agent rents the whole house. If he also owns it (or owns a share of it), he will pay a market rent to himself, with no transaction costs.

The market rent of homes in the city is denoted r_t and the market price of homes is denoted p_t. Both are determined in equilibrium.

An agent born at time t may invest in fractional amounts of housing every period, independently of where he lives. We denote the housing investment of this agent at time $t+s$ as $a_{t,t+s}$. The city homes that are not owned by agents are owned by the REITs.

There is a premium between the return to housing investment for the owner-occupier and the return to housing for other owners; for every property a landlord manages, she receives the rent r_t minus a fixed management cost, κ. This premium is meant to capture a variety of factors such as the true management costs of rental properties, moral hazard in the rental market, and the implicit subsidy embedded in the favorable tax treatment of homeownership. Agents do not incur such a management cost on any home equity they have if they live in the home.

It is easy to see that no agent buys housing to rent it out. Agents are risk averse; they compete in the housing investment market against the REITs which are risk neutral. The only agents who buy housing are agents who may take advantage of the premium κ: the agents who live in the city and decide to own some or all of their home. Note that this implies no city agent ever chooses to own more than one unit of housing, because all city dwellers consume exactly one unit of housing in order to work in the city.

1.5 Preferences

Agents enjoy constant absolute risk aversion (CARA) utility from consumption of numeraire at the end of life, at age $s = S$:

$$U = -\exp\left(-\gamma w_S\right),$$

where w_S denotes wealth just before death.

1.6 Voting on Housing Supply

Building a new house in the city requires a building permit. The number of building permits to be issued is decided by city residents through majority voting (one can think of Downsian electoral competition, where the only dimension is the number of permits to be issued). The vote is held at the end of the period. That is, the vote is held after new agents have decided on their location choice, and after all agents have optimized their portfolios. Houses cannot be destroyed, so agents may only consider an increase in city size.

It takes time to build houses. We assume that construction authorized at t starts at $t+1$ and is spread evenly over S periods. Given a vote to increase city size by an amount Sg, city size increases by g every period for the next S period. This assumption guarantees that, following a vote, the number of homes available to the newborn cohort in every future period remains constant, even if overall city size grows slowly for S periods. This structure of supply increase allows the economy to jump from one steady-state to the next following a vote to increase housing supply.[7]

Who captures the value from the building permits is critical to the equilibrium outcome of the model. We assume that new permits are assigned to a set of measure zero of developers (who hence do not affect the vote). The city government imposes a fee $\psi \geq 0$ on every new permit. In line with the timing of construction over time, the net present value of the fees to be collected (following a vote to issue a total of Sg permits but evaluated at the time of the vote,

[7] Results would be similar with alternative assumptions, such as immediate construction, but the characterization would be complex because some generations would be over-represented in the city.

or one period ahead of first construction) is:

$$\sum_{s=1}^{S} \beta^s g\psi = \beta \frac{1-\beta^S}{(1-\beta)} g\psi.$$

This parametrization allows us to control how much of the surplus goes to the developers; the higher the parameter ψ, the less of the surplus they capture.

To start with, we assume that the net present value of the fees collected is shared equally by the residents of the city at the time of the vote.

1.7 Timing

The timing of events for an agent born at t is as follows:

1. At t, the agent learns his city productivity, ε, and then chooses whether to live in the countryside or in the city.
2. At every $t+s$, with $s = 0, ..., S-1$, an agent who lives in the city:
   (a) Learns the value of the shock τ_{t+s};
   (b) Revises housing investment $a_{t,t+s}$;
   (c) Pays rent r_{t+s} for the house he occupies and collects rents $a_{t+s}r_{t+s}$ on other houses owned;
   (d) Votes on the number of new permits to issue.
3. At time $t+S$, the agent leaves the city, liquidates the housing assets if any, and consumes all his accumulated wealth.

2 Analysis

Infinite-horizon overlapping-generation voting models can have a large number of equilibria, some of them difficult to characterize. We start by defining a restrictive but appealing class of political economy equilibria. This approach allows us to ask what the smallest city size is that can be sustained in a political equilibrium. This is the question that most interests us because we are keen to understand when the political economy of housing supply stands in the way of an efficient organization of economic activity over space.

2.1 Equilibrium Concept

We say that a sub-game perfect equilibrium of this game is *stationary* if housing supply is constant on every continuation game. In other words, equilibrium prescribes that voters oppose new construction on the equilibrium path and also that they oppose it in every other possible subgame.

Stationary equilibria are a very specific subset of Markov-perfect equilibria. As this game has two states, y_t and n_t, the voting outcome in a Markov-perfect equilibrium can be expressed as $g_t(y_t, n_t)$, the number of new housing permits issued per period for the next S periods when income is y_t and the current city size is n_t. In stationary equilibria, we have that, given a starting size n_0:

$$g_t(y_t, n_t) = 0 \text{ for all } y_t \text{ and for all } n_t \geq n_0.$$

The policy variable, g, cannot be negative, so the number of houses n_t can only go up. Hence, there is no need to consider city sizes below n_0.

On a stationary equilibrium path, city size is constant, $n_t = n_0$ for all t. If a deviation occurs and generation t picks $g_t > 0$, our restriction to stationary equilibria dictates that the next generations continue to choose $g_{t+s} = 0$.

The stationarity condition is illustrated in Figures 1 and 2. Figure 1 represents the equilibrium path, with time on the x-axis and city size on the y-axis. The initial city size is n_0. At every election, residents could vote to increase the city size – an option represented, in a discrete way, by the upward-sloping segments. Yet, they always vote in favor of keeping the current size.

Figure 2 depicts what would happen in the case of a deviation. Suppose that at time 1 citizens deviate from equilibrium and authorize new construction. The city is now larger, but the stationary equilibrium in this continuation game still dictates that citizens vote against new construction. Of course, the citizens could deviate once, and again equilibrium in the new continuation game would dictate a stationary path.

Stationarity imposes a restriction on the set of potential equilibria. Without stationarity, there could be equilibria with an extremely small city. Intuitively, even a small size increase today could create the "expectation" of large increases in the future. Any deviation today would trigger a collapse in house prices. Hence, current generations do not modify the city size, even when it is extremely low. By assuming stationarity, we make it more difficult to prove the result that city size may be inefficiently low in equilibrium.

We ask whether stationary equilibria exist and, if so, what the lowest sustainable city size n^* is in a stationary equilibrium.

We proceed in two steps. First, we characterize the equilibrium decisions of individuals for any exogenous stationary path: $n_t = n$ for all n. Second, we move to the political economy problem and we analyze under what conditions stationary paths are politically sustainable.

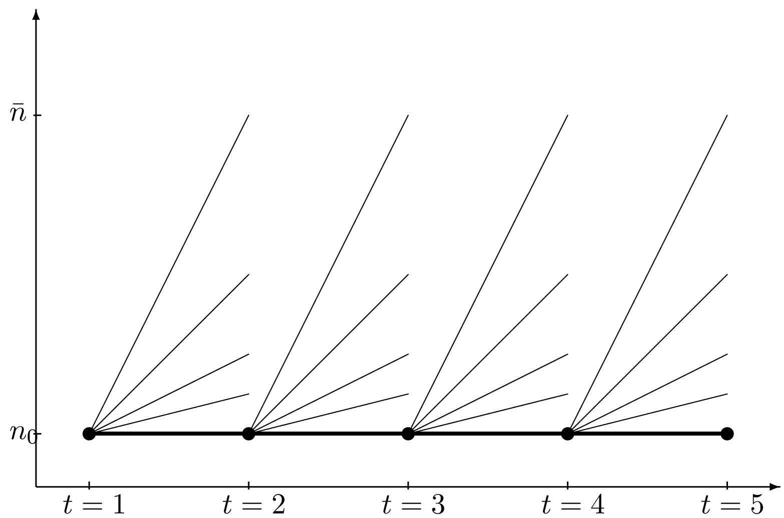

Figure 1: Stationary Equilibrium Path

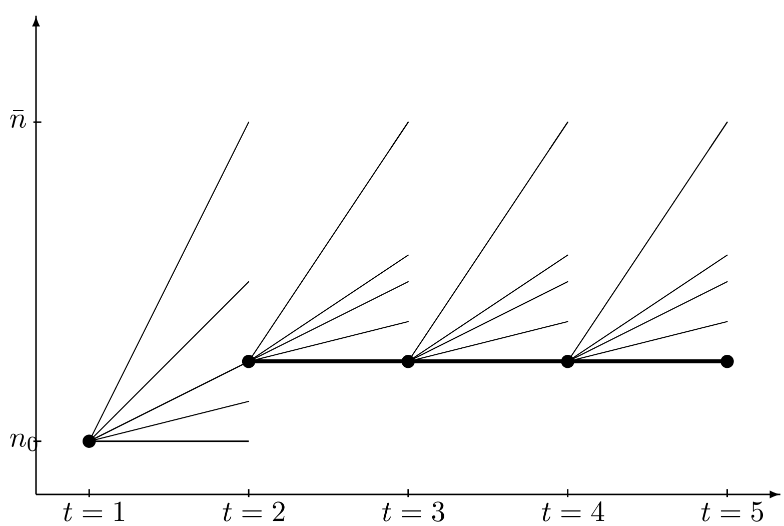

Figure 2: Stationary Equilibrium Path

2.2 Market Equilibrium - Exogenous City Size

Given a city size n, an agent born at date t takes as given the current state of city productivity, y_t, and functions describing how housing rents and prices evolve with city productivity, $r(y_t)$ and $p(y_t)$. Building on work in Ortalo-Magné and Prat (2008), we conjecture that rents are a linear function of city productivity:

$$r_t = y_t + \bar{r}.$$

We conjecture also that some houses will be owned in equilibrium by the risk-neutral competitive absentee REITs, and therefore housing prices are equal to the present value of rents minus management costs discounted at the risk-free rate, or:

$$p_t = \frac{1}{1-\beta}\left(y_t + \bar{r} - \kappa\right).$$

It is convenient to work with these conjectures when solving the agent's problem.

The end-of-life wealth of an agent born in t who lives in the city is:

$$w_t = \frac{1}{\beta^S}\sum_{s=0}^{S}\beta^s v_{t,t+s}$$

where

$$v_{t,t+s} = y_{t,t+s} - r_{t+s} + p_{t+s}a_{t,t+s-1} - \left(p_{t+s} - r_{t+s}\right)a_{t,t+s}$$

is the net cash flow at periods $t+s$ of the agent born at time t for $s = 0, ..., S-1$; the agent receives income $y_{t,t+s}$; pays rent r_{t+s}; invests in housing $a_{t,t+s} - a_{t,t+s-1}$ at price p_{t+s}; and receives rent r_{t+s} on the amount of owned at the end of the period, $a_{t,t+s}$. In the last period of life, the agent liquidates the housing investment:

$$v_{t,t+S} = p_{t+S}\, a_{t,t+S-1}.$$

The contribution of a housing investment $a_{t,t+s}$ to wealth evaluated at $t+s$ is:

$$\begin{aligned} & -p_{t+s} + r_{t+s} + \beta p_{t+s+1} \\ = \;& \frac{-1}{1-\beta}\left(y_{t+s} + \bar{r} - \kappa\right) + y_{t+s} + \bar{r} + \beta\frac{1}{1-\beta}\left(y_{+st} + \bar{r} + \tau_{t+s+1} - \kappa\right) \\ = \;& \frac{\beta}{1-\beta}\tau_{t+s+1} + \kappa. \end{aligned}$$

The agent receives the present value of the innovation in housing rents, τ_{t+s+1}, plus the premium to homeownership, κ. The above equality holds as long as $a_{t,t+s}$ is less than one. Any amount of housing owned in excess of one unit is rented out and thus incurs the management cost κ. As we have noted, no agent wants to own rental housing because agents cannot compete on the housing investment market with the risk-neutral REITs.

We rewrite the end-of-life wealth as

$$w_t = \frac{1}{\beta^S}\sum_{s=0}^{S-1}\beta^s\left(y_{t+s}+\varepsilon-y_{t+s}-\bar{r}\right)+\frac{1}{\beta^S}\sum_{s=0}^{S-1}\beta^s\left(a_{t,t+s}\frac{\beta}{1-\beta}\tau_{t+s+1}+\min\left\{a_{t,t+s},1\right\}\kappa\right),$$

where the min $\{.,.\}$ operator accounts for the fact that agent owning more than the one unit of housing he occupies, would not be able to capture the premium κ on the amount of housing he rents out. This yields

$$w_t = \sum_{s=0}^{S-1}\beta^{s-S}\left(\varepsilon-\bar{r}\right)+\sum_{s=0}^{S-1}\beta^{s-S}\left(a_{t,t+s}\frac{\beta}{1-\beta}\tau_{t+s+1}+\min\left\{a_{t,t+s},1\right\}\kappa\right).$$

The first term denotes the value of working in the city. The agent earns the difference between its individual productivity premium, ε, and the rent premium $\bar{r}$. The second term denotes the earnings from investment in own housing. The agent captures the surprises, τ, and the premium κ on housing owned for personal consumption.

We prove:

Proposition 1 *There is a market equilibrium with n exogenously given and linear prices.*

(a) An agent with productivity parameter ε chooses to live in the city if and only if $\varepsilon \geq 1-n$.

(b) The market rent is given by:

$$r_t = y_t + \bar{r}$$

where

$$\bar{r} = 1-n+K.$$

(c) The house price is

$$p_t = \frac{1}{1-\beta}\left(y_t+\bar{r}-\kappa\right).$$

(d) Agents born at time t and age s hold housing investment:

$$a_{t,t+s} = \min\left\{\frac{1}{2\gamma\sigma^2}\frac{(1-\beta)^2}{\beta^2}\kappa\beta^{S-s},1\right\}$$

where

$$K = \frac{(1-\beta)}{\beta^{-S}-1}\left(\frac{\beta^{\hat{s}-S}-1}{(1-\beta)}\kappa+\hat{s}\frac{(1-\beta)^2}{4\gamma\beta^2\sigma^2}\kappa^2+\gamma\frac{\beta^2}{(1-\beta)^2}\frac{\left(1-\beta^{2(\hat{s}-S)}\right)}{\left(1-\beta^2\right)}\sigma^2\right)$$

and $\hat{s} = \min\left\{S, int\left(S-\ln\left(\frac{2\gamma\sigma^2}{\kappa}\frac{\beta^2}{(1-\beta)^2}\right)/\ln(\beta)\right)+1\right\}$. *$\hat{s}$ is the youngest age at which optimal housing investment is equal to* 1 *or S if optimal housing investment never reaches* 1*.*

Proposition 1 includes four equilibrium conditions: (a) Where agents locate; (b) equilibrium on the rental market; (c) equilibrium on the purchasing market; and (d) individual portfolio optimization.

Condition (i) guarantees that the n agents with highest idiosyncratic productivity parameter ε will locate in the city. As urban life is complementary to individual productivity, efficient sorting requires that more productive agents live in the city.

The market rent will then be determined in a way that makes the n-th most productive agent indifferent between living in the city and the countryside. This indifference condition is somewhat complex because it requires taking into account the potential for real estate gains due to the homeownership premium.

To understand the logic of the result, assume first that there is no homeownership premium: $\kappa = 0$. In that case, the last three conditions of proposition 1 simplify to:

(b) The market rent is given by $r_t = y_t + \bar{r}$, where $\bar{r} = 1 - n$.

(c) The house price is $p_t = \frac{1}{1-\beta}(y_t + \bar{r})$.

(d) Agents born at time t and age s hold housing investment $a_{t,t+s} = 0$.

In such a frictionless world, risk-averse agents do not own housing. All real estate risk is assumed by risk-neutral REITs, which rent to individuals. Price is simply equal to the expected rental income. The rent is equal to the amount that the marginal resident – the one with the n-th highest productivity level – is willing to pay to live in the city. This in turn depends on the current productivity level y_t. In equilibrium the rent and the price are determined by a stochastic process (a random walk) that mirrors the productivity process.

When instead the homeownership premium is strictly positive ($\kappa > 0$), agents do buy some housing. The amount of individual real investment chosen in equilibrium achieves a balance between risk aversion and the desire to exploit the comparative advantage of individual ownership as opposed to institutional ownership. Investment increases with premium κ and declines with variance γ and risk aversion σ^2.

The market house price is still set to equal the expected rental stream for a REIT, but it now takes into account the fact that the REIT does not benefit from the homeownership premium. This generates a benefit for individuals who live in the city and can become owner-occupiers. Such a benefit makes it more the attractive to live in the city, so it appears in the equilibrium rent.

2.3 Political Equilibrium - Endogenous City Size

We now relax the assumption that city size is exogenous. In every period t, city residents vote on a non-negative amount of housing permits to be issued. This means that, at least off the equilibrium path, the size of the city n_t could vary over time.

As we are interested in a stationary equilibrium, we must ensure that at no time city residents are willing to vote for city expansion. To do this, we must understand what happens if a deviation occurs. So, now assume that at time $\bar{t}$ city residents vote to issue Sg new housing permits.

Our focus on stationary equilibria is helpful. A deviation is followed by stability and we can apply proposition 1. For every $t > \bar{t}$ in the future:

- Newborn agents with productivity ε choose to live in the city if and only if $\varepsilon \geq 1 - n - g$.
- The market rent is given by:
$$r_t = y_t + \bar{r} - g$$
and the house price is:
$$p_t = \frac{1}{1-\beta}\left(y_t + \bar{r} - g - \kappa\right).$$
- Agents born at time $\bar{t} - s$ hold housing investment:
$$a_{\bar{t}-s,\bar{t}} = \min\left\{\frac{1}{2\gamma\sigma^2}\frac{(1-\beta)^2}{\beta^2}\kappa\beta^{S-s}, 1\right\}.$$

Issuing Sg building permits at $\bar{t}$ has three distinct effects that are relevant to the city residents who vote. First, it provides city residents with fee income. The net present value of the total fee payments evaluated the period of the vote is:

$$\sum_{m=1}^{S}\beta^m g\psi = \left(1-\beta^S\right)\psi\frac{\beta}{(1-\beta)}g.$$

Each city resident receives an equal share of the net present value of total fee payments raised:

$$\frac{\beta\left(1-\beta^S\right)}{(1-\beta)}\frac{g\psi}{n}.$$

Second, issuance of Sg permits reduces expected rent in each future period by g units of numeraire consumption. An agent age s at the time of the vote saves g over the remainder of his life, or a total of:

$$\begin{aligned}\beta g + \beta^2 g + ... + \beta^{S-s-1} g &= g\beta\sum_{m=0}^{S-s-2}\beta^m \\ &= \frac{\beta\left(1-\beta^{S-s-1}\right)}{(1-\beta)}g.\end{aligned}$$

Third, permit issuance reduces the expected housing price in the next period by $\frac{g}{1-\beta}$. The capital loss, evaluated at $\bar{t}$, for an agent age s at the time of the vote is:

$$a_{\bar{t}-s,\bar{t}}\frac{\beta}{1-\beta}g.$$

All three effects are linear in the term g. We can thus add them up and focus on the marginal effect of an additional permit (for an individual of generation s with housing investment $a_{\bar{t}-s,\bar{t}}$):

$$\Delta = \frac{\beta}{1-\beta}\left(1-\beta^{S-s-1} + \left(1-\beta^S\right)\frac{\psi}{n} - a_{\bar{t}-s,\bar{t}}\right).$$

The marginal effect Δ will be crucial to determine political equilibrium. We should then understand its properties. The effect Δ does not depend on an agent's individual productivity, ε. It increases weakly with the agent's age (strictly while the agent's housing investment increases with age; i.e., $s < \hat{s}$) for three reasons. The most important one is captured by the first term. As the agent gets older, there are fewer periods of rental expenditures before death. Thus, a major benefit of additional construction – lower rent – goes down with age. The second term, the cash flow to residents deriving from permit payments, is independent of age. The third term, which is negative, increases with age because the investment in real estate rises with age (simply because of the effect of discounting). Thus, opposition to urban growth increases with age.[8]

As $\frac{d\Delta}{ds}$ is always weakly negative, we have monotonic policy preferences. If generation s opposes new permits, all older generations oppose more permits as well. Preferences can thus be represented monotonically on one dimension (age), and the median voter theorem holds: Downsian political competition yields a policy that corresponds to the bliss point of the median voter (see, for instance, Persson and Tabellini, 2002, Chapter 2).

In this case, the median voter is the voter of median age. Therefore, an increase in city size cannot occur if the median generation, $\frac{S-1}{2}$, opposes more permits, namely, if:

$$\Delta|_{s=\frac{S-1}{2}} = \frac{\beta}{1-\beta}\left(1-\beta^{S-s-1}+\left(1-\beta^{S}\right)\frac{\psi}{n}-a_{\bar{t}-s,\bar{t}}\right) \leq 0$$

or

$$\left(1-\beta^{S}\right)\frac{\psi}{n} \leq \left[a_m - \left(1-\beta^{\frac{S-1}{2}}\right)\right] \tag{1}$$

where for convenience we define the housing investment of the median voter $a_m = a_{\bar{t}-\frac{S-1}{2},\bar{t}}$. The term in brackets corresponds to the difference between the capital loss incurred on housing investment and the drop in future housing consumption cost. If housing investment is low enough, the term is negative, and the median voter benefits from an expansion in the city. Otherwise, the median voter weighs the financial cost of housing investment and consumption of housing expansion against his or her share of revenues generated by the sale of the new housing permits $\left(1-\beta^{S}\right)\frac{\psi}{n}$.

This means that limits to urban growth are present if and only if:

$$a_m - \left(1-\beta^{\frac{S-1}{2}}\right) > 0, \tag{2}$$

[8] Formally, we have:

$$\begin{aligned}
\frac{d\Delta}{ds} &= \frac{\beta}{1-\beta}\left(\ln(\beta)\,\beta^{S-s-1}+\ln(\beta)\,\frac{1}{2\gamma\sigma^2}\frac{(1-\beta)^2}{\beta^2}\kappa\beta^{S-s}\right) \\
&= \frac{\beta\ln(\beta)}{1-\beta}\left(\beta^{S-s-1}+\frac{1}{2\gamma\sigma^2}\frac{(1-\beta)^2}{\beta^2}\kappa\beta^{S-s}\right) < 0.
\end{aligned}$$

Note that, given proposition 1, (2) is a condition on the primitives of the problem:

$$\min\left\{\frac{1}{2\gamma\sigma^2}\frac{(1-\beta)^2}{\beta^2}\kappa\beta^{S-m},1\right\}-\left(1-\beta^{\frac{S-1}{2}}\right)>0,$$

which is satisfied when the parameters are such that the min is binding for the median voter – a reasonable case if κ is high enough.

The inequality (1) can be rearranged as:

$$n\geq\frac{\left(1-\beta^S\right)\psi}{a_m-\left(1-\beta^{\frac{S-1}{2}}\right)}.$$

In this case, the median voter opposes any increase in city size if current city size n is already above the right-hand side expression.

For our stationary equilibrium concept to apply, we must check that if a deviation occurred, it would be followed by a stationary equilibrium. Suppose the median voter condition above, equation (2) is satisfied. If a deviation were to occur and citizens vote in favor of more permits at time t, the city would grow. Yet, the inequality above would still be satisfied because the willingness to authorize new permits declines with n, and there would be a stationary equilibrium henceforth.

We therefore obtain the result:

Proposition 2 *Under condition (2), the smallest city size supported by a stationary equilibrium is*

$$n^*=\min\left\{\frac{\left(1-\beta^S\right)\psi}{a_m-\left(1-\beta^{\frac{S-1}{2}}\right)},1\right\}.$$

Otherwise, the median voter always supports an increase in city size, and the only stationary equilibrium is $n^*=1$.

In a stationary equilibrium, voters always prefer the status quo to a one-off expansion of the city. Such an expansion would bring benefits in terms of rent reduction and permit revenues that tend to benefit younger people more than older people. As the benefit of expansion declines with age, a median-voter theorem applies, and a deviation occurs if and only if the median-aged voter benefits from it.

The temptation to deviate is reduced with city size because permit revenues have to be shared among more residents. If there are too few residents, they will certainly want to cash in. This means that the city size cannot be too small in a stationary equilibrium.

Of course, the drawback of expanding city size is that house prices will decline. Hence, the temptation to offer more permits depends on how much real estate is owned by the median-aged city resident. As we know from proposition 1, homeownership depends on the homeownership premium.

The minimum city size that can be sustained in equilibrium depends on housing parameters as follows: it declines with homeownership premium κ and increases with permit fees ψ.

Two remarks regarding the properties of the stationary equilibrium are in order. First, our result characterizes the minimum city size. Of course, if the city is, for exogenous reasons, already larger than n^*, then it will stay that way. This model displays path-dependence; cities with identical primitives may end up with different long-term sizes if in the past they had different construction patterns.

Second, if we drop the stationarity assumption, we could get an even lower sustainable minimal city size. One could consider equilibria where future willingness to issue permits depends on having issued permits in the past. Hence, issuing even a small number of permits now means a huge drop in prices because of the expectation that more permits will be issued in the future. The choice therefore is between issuing many permits now, bringing the price down right away, or keeping size constant. It is easy to see that city size can be kept low under these circumstances.

3 Policy Implications: Providing Ownership Subsidies vs. Redistributing Permit Revenues

Our model can be used to analyze different regulatory regimes. We evaluate two types of policies: subsidies to homeowners, and allocating revenues from new building permits to residents and owners. We are interested in the first policy because it is widespread. The second policy is actually rare in practice, but it is suggested by the model.

We begin by characterizing welfare in equilibrium. As before, we focus on stationary equilibria.

Proposition 3 *In a stationary equilibrium, the expected utility of an agent with productivity ε is*

$$U = \begin{cases} 0 & \text{if } \varepsilon < 1 - \hat{n} \\ \frac{\beta^{-S}-1}{1-\beta}\left(\varepsilon - 1 + \hat{n}\right) & \text{if } \varepsilon \geq 1 - \hat{n} \end{cases}.$$

At first glance, agents' expected utility might be affected by a number of factors. The geographic location equilibrium condition, however, pins down the expected utility of agents. The market rent must be such that the marginal city resident is as happy as the countryside residents, whose expected utility does not depend on the city housing market. In turn, the marginal resident's utility determines the expected utility of everybody else in the city through differences in individual productivity. Let $\hat{\varepsilon}$ be the productivity of the marginal resident and $\varepsilon > \hat{\varepsilon}$ the productivity of a more productive resident.

Proposition 3 implies that the difference in the expected utilities of these two agents is determined only by their productivity difference and in a linear manner:

$$U(\varepsilon) - U(\hat{\varepsilon}) = \frac{\beta^{-S} - 1}{1 - \beta} (\varepsilon - \hat{\varepsilon}).$$

A higher $\hat{n}$ means $\hat{\varepsilon}$ is lower and is thus associated with greater expected utility for all agents except the agents who live in the countryside; their utility remains unchanged. Therefore, if agents could step back from their current condition, in particular how much housing they own, they would all support an increase in city size.

The characterization in proposition 3 leads to an observation that may at first appear surprising. A policy improves the long-term welfare of city residents only if it either increases the equilibrium size of the city (higher $\hat{n}$) or makes people in the countryside better off (lower $\hat{\varepsilon}$) This is because the benefits of any policy that affects only city residents will be incorporated in house prices. The price increase will be such that the marginal resident is still indifferent between the city and the countryside.

When we use proposition 3 to make welfare comparisons, we must keep in mind that we are comparing steady-states. We are abstracting from the one-off distributional effects due to capital gains and capital losses that drive the political economy analysis of the previous section. Our exercise can be interpreted in two ways: either as an ex ante exercise, before individual productivities – and hence portfolio decisions – are known, or as an exercise on long-term policy effects.

Let us begin by looking at the effect of a homeownership subsidy λ: every home owner (except REITs) receives λ dollars for every housing unit or fraction thereof owned; the subsidy is paid for by every agent in the economy. The subsidy, therefore, supplements the homeownership premium κ, which is already built in. We have:

Proposition 4 *A homeownership subsidy reduces all citizens' expected utility. It also makes homes less affordable.*

The subsidy has an effect on both the equilibrium city size $\hat{n}$ and the expected utility of people in the countryside $\hat{\varepsilon}$. To understand the two effects, assume first that portfolio decisions as well as city size are exogenously given. A housing subsidy is fully incorporated in an increase of house prices in the city. This creates a capital gain for the current generation that corresponds to the discounted sum of subsidies. Hence, this is a pure transfer from future taxpayers to current homeowners. For future homeowners, the subsidy will be a wash – it will be exactly offset by the additional cost of capital needed to cover the price increase – but all future generations will be burdened by the tax needed to cover the subsidy. So, people in the countryside in particular will be worse off, and $\hat{\varepsilon}$ will go down.

The second effect of the subsidy is indirect. If portfolio decisions are endogenous, a homeownership subsidy will lead to a higher investment in real estate. The median city resident will have a higher real estate investment a_m. Hence, there will be more opposition to city growth. The equilibrium size of the city will be smaller. As we know, this increases house prices in the city and makes everyone weakly worse off (the people who live, or would have lived, in the city are strictly worse off).

The net effect of the subsidy is actually to make homes less affordable. If there were no change in city size, the net cost of buying a home (cost of capital minus subsidy) would be constant. Because of the indirect urban growth effect, the housing cost actually goes up.

While homeownership subsidies appear to have detrimental effects, our model suggests we look for improvements in the way permit revenues are allocated.

Proposition 5 *If the initial fee level is not too high, an increase in the permit fee ψ increases all agents' expected utility.*

An increase in the permit fee ψ does not benefit city residents directly, because the fee benefit is fully incorporated in the house price. Its benefit is indirect and operates through the political channel. City residents are now more supportive of urban growth. The resulting city size increase is welfare improving.

4 Extensions

We have developed three extensions to the baseline model. It is easy to consider what would happen in the model if we restrict housing investments to indivisible housing units. We then ask what happens if we change the geography of the voting process and of the economic environment.

4.1 Indivisible Housing Investment

Removing the assumption that housing investment is perfectly divisible forces agents to choose between owning zero or one housing unit. The countryside residents remain uninterested in any housing investment. Depending on the parameters, some city residents may choose to own a house.

The extent to which the equilibrium outcome of the model is affected depends on whether a majority of city residents (voters) find themselves with more housing ownership in the new regime than when housing was divisible. If they own more housing, they are more sensitive to the capital loss effect of increasing city size and thus more likely to oppose it, and vice versa. The effects of indivisible housing investment on the set of city sizes that are supported in equilibrium is therefore obvious.

4.2 National Vote

In the baseline model, only city residents vote on new housing permits. Rural residents have nothing to lose from urban growth.

If we allow rural residents to vote, we shifts the median voter toward a younger cohort with less housing investment or even toward a rural voter who is not invested in urban real estate. Then a higher minimum city size that can be sustained in equilibrium.

4.3 Competition Among Cities

We can extend the baseline model by adding a second city to our economic environment so there are now three locations: the countryside, and two cities denoted by $l \in \{1, 2\}$, each of initial size n. At birth, agents draw two productivity parameters, one for each city: $(\varepsilon_{l,i})_{l=1,2}$. We take the correlation between the two parameters to be a relevant summary measure of the economic distance between the two cities. We focus on two extremes.

1. Specialized cities. Agents are one of two types, with equal probability. Half the agents have productivity $(\varepsilon_{1,i}, \varepsilon_{2,i}) = (\varepsilon_i, 0)$; the other half have productivity $(\varepsilon_{1,i}, \varepsilon_{2,i}) = (0, \varepsilon_i)$, where ε_i is drawn from a uniform distribution over $[0, 1]$. This is a special case of zero correlation of skills across cities.

2. Homogeneous cities. The location does not affect the agent's productivity, $\varepsilon_{1,i} = \varepsilon_{2,i} = \varepsilon_i$, where ε_i is drawn from a uniform distribution over $[0, 1]$. This represents the case of perfect correlation of skills across cities.

The specialized cities case is trivial. There is no meaningful economic interaction between the two cities. For all agents, the city where their productivity is zero will be dominated by the countryside because living in the countryside is cheaper than in the city. Therefore, half the population focuses on one city and the countryside, while the other half focuses on the other city and the countryside. Each half of the population is of size one; the results obtained with the one-city model are unchanged. They apply to each city individually.

In the homogeneous cities case, the two cities are identical in the eyes of the agents. The extension corresponds to doubling the size of the city and the population so that market equilibrium outcomes, given initial city sizes n, are identical as before. In particular, any agent with productivity:

$$\varepsilon \geq 1 - n$$

locates in one of the two cities in equilibrium, and all city dwellers are indifferent between living in either of the two cities.

The trade-offs agents face differ when considering issuing new housing permits, however. As before, assume a city issues Sg permits at time $\bar{t}$ and none later, and also that the other city does not issue any permits. Each cohort is of measure 2 (not 1 as before). Before the issuance of new permits, agents with productivity $\varepsilon \geq 1-n$ were locating in the cities. Now, an extra mass g of agents in each newborn cohort moves to the city. Assumption of a uniform distribution of skills over the unit interval, this implies that the newborn agents with the lowest productivity who move to the city have productivity $\varepsilon = 1-n-g/2$. Market equilibrium conditions require this agent be indifferent across the two cities. The rent in both cities will therefore be reduced by $g/2$ by the doubling of city and population size so that now

$$r_t = y_t + \bar{r} - \frac{g}{2}$$

and the house price is

$$p_t = \frac{1}{1-\beta}\left(y_t + \bar{r} - \frac{g}{2} - \kappa\right).$$

The utility of the agents living in the city that is issuing the permits and thus relevant to their vote is now affected as follows:

- Expected housing price in the next period is reduced by $\frac{g}{2(1-\beta)}$. The capital loss, evaluated at $\bar{t}$, for an agent age s at the time of the vote is

$$a_{\bar{t}-s,\bar{t}}\frac{\beta}{1-\beta}\frac{g}{2}.$$

- Expected rent in each future period is reduced by $g/2$ units of numeraire consumption. An agent age s at the time of the vote saves $g/2$ over the remainder of his life, or a total of

$$\frac{1-\beta^{S-s-1}}{(1-\beta)}\frac{g}{2}\beta.$$

- The net present value of the total fee payments evaluated in the period of the vote is as before. Each city resident receives an equal share of the net present value of total fee payments raised:

$$\frac{\beta\left(1-\beta^{S}\right)}{(1-\beta)}\frac{g\psi}{n}.$$

The marginal effect of an extra permits is now:

$$\frac{d\Delta}{dg} = \frac{1}{2}\frac{\beta}{1-\beta}\left(1-\beta^{S-s-1} + 2\left(1-\beta^{S}\right)\frac{\psi}{n} - \frac{1}{2\gamma\sigma^2}\frac{(1-\beta)^2}{\beta^2}\kappa\beta^{S-s}\right).$$

The arguments developed in the one-city case produce the proposition:

Proposition 6 *In the specialized cities case, a sufficient condition for a city size n to be supported by a stationary political economy equilibrium is $n \geq n^*$ where n^* is as defined in proposition 2.*

In the homogeneous cities case, a sufficient condition for a city size n to be supported by a stationary political economy equilibrium is $n \geq 2n^*$.

In other words, because all residents are indifferent between the two cities in the homogeneous cities case, equilibrium rents and prices are always identical across cities. If one city grows in size, the rent and price drops are equal across the cities, but only the residents who voted for the increase in city size receive the benefit from the sale of building permits. These residents fully internalize the revenue effect of their vote, yet suffer only part of the rent-price effect because that effect is shared with the residents of the other city, thanks to the mobility of agents.

By tinkering with the distribution of the individual productivity parameters, $(\varepsilon_{1,i}, \varepsilon_{2,i})$ we can affect the extent to which agents are mobile across cities. The general intuition obtained from the extreme examples will apply. Whenever agents are mobile, their voting decision on rents and prices has less of an effect than in the specialized city (one city) case. Therefore, the sufficient condition for a stationary political equilibrium does not apply for city size as small as in the one-city case.

From a welfare point of view, given our general result that welfare increases with equilibrium city size, competition among cities (or, equivalently, agents' mobility across cities) is good for welfare. The set of city sizes that can be sustained in equilibrium does not include cities as small as when there is no competition among cities (or when agents are not mobile across cities).

5 Conclusion

We have proposed a simple framework to highlight the critical economic forces that determine the regulation of urban growth. All agents would be better off if a city were as large as possible in our model. Nevertheless, in equilibrium, smaller cities may persist when the capital losses on own housing (the present value of the infinite sum of future rents) more than outweigh the gains from lower future housing costs (the present value of the sum of future rents until the death of the agent) and any of the value residents may capture from the issuance of new housing permits.

This result highlights a fundamental tension between what are typically the two primary objectives of housing policy: housing affordability for all, and homeownership for most. That is, encouraging homeownership may result in a political process that blocks any city growth, and thereby keeps housing unaffordable to outsiders.

Would any other policy generate Pareto improvements? Some jurisdictions have tried to capture the value generated by the issuance of housing permits. In our model, moving a city to optimal size requires building houses up to the point where housing permits have no value, hence providing no resources to compensate homeowners whose home would lose value. One could not therefore propose a new arrangement that would move the city to its optimal size without some new fiscal instruments to compensate the losers. We show nevertheless that distributing the value generated by new building permits may move the city away from a bad equilibrium.

We also show that another option with positive welfare potential involves rethinking at what level of jurisdiction urban growth policy is decided, for example including non-residents in the voting process.

Going beyond the model, our findings may suggest a rationale for building housing units targeted at low-skill workers (so-called social housing units). Such housing units provide access to the city to workers with lower earnings without affecting the balance of supply and demand in the housing market relevant to housing for higher-skill workers.

Appendix: Proofs

Proof of Proposition 1

We solve for the optimal sequence of housing investments that maximize the agent's utility, $E[w_t] - \gamma Var[w_t]$, where

$$\begin{aligned} E[w_t] &= \sum_{s=0}^{S-1} \beta^{s-S} \left(\varepsilon - \bar{r} + \min\{a_{t,t+s}, 1\}\kappa\right) \\ Var[w_t] &= \frac{\beta^2}{(1-\beta)^2} \sum_{s=0}^{S-1} \beta^{2(s-S)} Var\left[a_{t,t+s}\tau_{t+s+1}\right] \\ &= \frac{\beta^2}{(1-\beta)^2} \sum_{s=0}^{S-1} \beta^{2(s-S)} a_{t,t+s}^2 \sigma^2 \end{aligned}$$

The first-order condition of utility maximization with respect to $a_{t,t+s}$ yields:

$$\kappa = \gamma \frac{\beta^2}{(1-\beta)^2} \beta^{(s-S)} 2 a_{t,t+s} \sigma^2$$

for $a_{t,t+s} \leq 1$. If $a_{t,t+s}$ is greater than one, then the left-hand side of this equation is zero. This confirms the intuition that no agent wants to own more than one unit of housing.

The optimal housing investment is therefore:

$$a_{t,t+s} = \min\left\{ \frac{1}{2\gamma\sigma^2} \frac{(1-\beta)^2}{\beta^2} \kappa \beta^{S-s}, 1 \right\} \text{ for } s = \{0, \dots, S-1\}$$

Agents' housing investment increases weakly with age, s, and with the homeownership premium κ. It is reduced with the risk aversion parameter γ, and with the variance of productivity shocks, σ^2.

Inserting the optimal investment solution into the utility function, we obtain:

$$\begin{aligned} U &= \sum_{s=0}^{S-1} \beta^{s-S} \left(\varepsilon - \bar{r} + \min \left\{ \frac{1}{2\gamma\sigma^2} \frac{(1-\beta)^2}{\beta^2} \kappa \beta^{S-s}, 1 \right\} \kappa \right) \\ & -\gamma \left(\frac{\beta^2}{(1-\beta)^2} \sum_{s=0}^{S-1} \beta^{2(s-S)} \left(\min \left\{ \frac{1}{2\gamma\sigma^2} \frac{(1-\beta)^2}{\beta^2} \kappa \beta^{S-s}, 1 \right\} \right)^2 \sigma^2 \right). \end{aligned}$$

Let $\hat{s}$ be the threshold s at which the $\min\{.,1\}$ becomes binding so that for all $s \geq \hat{s}$, optimal investment $a_{t,t+s} = 1$. If the min operators are never binding, we set $\hat{s} = S$ for computational convenience.

The agent's utility can be represented as:

$$\begin{aligned} U &= \sum_{s=0}^{\hat{s}-1} \beta^{s-S} \left(\varepsilon - \bar{r} + \frac{1}{2\gamma\sigma^2} \frac{(1-\beta)^2}{\beta^2} \kappa \beta^{S-s} \kappa \right) \\ & -\gamma \left(\frac{\beta^2}{(1-\beta)^2} \sum_{s=0}^{\hat{s}-1} \beta^{2(s-S)} \left(\frac{1}{2\gamma\sigma^2} \frac{(1-\beta)^2}{\beta^2} \kappa \beta^{S-s} \right)^2 \sigma^2 \right) \\ & +1_{(\hat{s}<S)} \left[\sum_{s=\hat{s}}^{S-1} \beta^{s-S} (\varepsilon - \bar{r} + \kappa) - \gamma \left(\frac{\beta^2}{(1-\beta)^2} \sum_{s=\hat{s}}^{S-1} \beta^{2(s-S)} \sigma^2 \right) \right] \\ &= \frac{\beta^{-S}-1}{(1-\beta)} (\varepsilon - \bar{r}) + \frac{\beta^{\hat{s}-S}-1}{(1-\beta)} \kappa + \hat{s} \frac{(1-\beta)^2}{4\gamma\beta^2\sigma^2} \kappa^2 + \gamma \frac{\beta^2}{(1-\beta)^2} \frac{\left(1-\beta^{2(\hat{s}-S)}\right)}{(1-\beta^2)} \sigma^2. \end{aligned}$$

Agents locate in the city if it provides them more utility than locating in the countryside; i.e., if:

$$\frac{\beta^{-S}-1}{(1-\beta)} (\varepsilon - \bar{r}) + \frac{\beta^{\hat{s}-S}-1}{(1-\beta)} \kappa + \hat{s} \frac{(1-\beta)^2}{4\gamma\beta^2\sigma^2} \kappa^2 + \gamma \frac{\beta^2}{(1-\beta)^2} \frac{\left(1-\beta^{2(\hat{s}-S)}\right)}{(1-\beta^2)} \sigma^2 > 0$$

so that agents with $\varepsilon > \varepsilon^*$ locate in the city where ε^* is defined by

$$\begin{aligned} 0 &= \frac{\beta^{-S}-1}{(1-\beta)} (\varepsilon^* - \bar{r}) + \frac{\beta^{\hat{s}-S}-1}{(1-\beta)} \kappa + \hat{s} \frac{(1-\beta)^2}{4\gamma\beta^2\sigma^2} \kappa^2 + \gamma \frac{\beta^2}{(1-\beta)^2} \frac{\left(1-\beta^{2(\hat{s}-S)}\right)}{(1-\beta^2)} \sigma^2 \\ \varepsilon^* &= \bar{r} - \frac{(1-\beta)}{\beta^{-S}-1} \left(\frac{\beta^{\hat{s}-S}-1}{(1-\beta)} \kappa + \hat{s} \frac{(1-\beta)^2}{4\gamma\beta^2\sigma^2} \kappa^2 + \gamma \frac{\beta^2}{(1-\beta)^2} \frac{\left(1-\beta^{2(\hat{s}-S)}\right)}{(1-\beta^2)} \sigma^2 \right) \end{aligned}$$

or

$$\varepsilon^* = \bar{r} - K$$

where K is defined in the obvious way. So agents locate in the city if their individual city-productivity, ε, is higher than the rent's fixed component, $\bar{r}$.

Agents derive utility from the opportunity to invest in housing in the city and to earn the premium to homeownership, κ. This is why the cutoff ε^* is lower than in the absence of such a premium; the cutoff is lower than $\bar{r}$ whenever $K > 0$. A sufficient condition for this condition to be satisfied is $\kappa > 0$.

Equilibrium in the space market requires that a measure n of agents in each cohort locate in the city. We assume a uniform distribution for ε on $[0,1]$, so market clearing in the space market requires $\varepsilon^* = 1 - n$, or:

$$1 - n = \bar{r} - K.$$

This equation allows us to solve for the rent premium $\bar{r}$:

$$\bar{r} = 1 - n + K.$$

We postulate:

$$p_t = \frac{1}{1-\beta}\left(y_t + \bar{r} - \kappa\right).$$

Replacing the rent premium by its solution in terms of model parameters yields

$$p_t = \frac{1}{1-\beta}\left(y_t + (1-n) - \kappa + K\right).$$

The price reflects the value of city production for the marginal agent, $y_t +$ $(1-n)$, minus the premium κ, plus the value of homeownership to the city residents, K.

By construction, we know that at the rents and prices above, the space market clears; That is, every home in the city is occupied.

By construction, REITs are willing to own any amount of housing. To ensure that the that the housing investment market clears, we therefore just need to check that the city residents do not demand more housing than is available. This condition is satisfied because no city resident wants to own more than one unit of housing.

Proof of Proposition 3

The agents with productivity $\varepsilon < 1 - \hat{n}$ live in the countryside at zero utility. The remaining agents live in the city. Substituting the equilibrium solution we obtain for $\bar{r}$ in their indirect utility function yields:

$$U = \frac{\beta^{-S}-1}{(1-\beta)}\left(\varepsilon - \bar{r}\right) + \frac{\beta^{\hat{s}-S}-1}{(1-\beta)}\kappa + \hat{s}\frac{(1-\beta)^2}{4\gamma\beta^2\sigma^2}\kappa^2 + \gamma\frac{\beta^2}{(1-\beta)^2}\frac{\left(1-\beta^{2(\hat{s}-S)}\right)}{\left(1-\beta^2\right)}\sigma^2$$

and

$$\bar{r} = 1 - n + \frac{(1-\beta)}{\beta^{-S}-1}\left(\frac{\beta^{\hat{s}-S}-1}{(1-\beta)}\kappa + \hat{s}\frac{(1-\beta)^2}{4\gamma\beta^2\sigma^2}\kappa^2 + \gamma\frac{\beta^2}{(1-\beta)^2}\frac{\left(1-\beta^{2(\hat{s}-S)}\right)}{\left(1-\beta^2\right)}\sigma^2\right).$$

We obtain

$$U = \frac{\beta^{-S}-1}{(1-\beta)}\left(\varepsilon - 1 + \hat{n}\right).$$

Proof of Proposition 4

There are two effects of a homeownership subsidy. First, the utility of countryside residents is diminished; their expected payoff is now negative, and it equals the tax increase due to the homeownership subsidy. Second, $\hat{n}$ in proposition 2 goes down because subsidy λ is mathematically equivalent to increasing κ from its initial level to $\kappa + \lambda$, and $\hat{n}$ declines with κ. Both effects reduce the utility of all agents, according to proposition 3.

Proof of Proposition 5

By proposition 2, an increase in ψ leads to an increase in $\hat{n}$. By proposition 3, this increases the expected utility of all agents above a certain threshold and leaves the utility of others unchanged. Of course, ψ must be lower than the price of a new house; hence the first condition.

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