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Tracing Value-Added and Double Counting in Gross Exports†

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This paper proposes an accounting framework that breaks up a country's gross exports into various value-added components by source and additional double-counted terms. Our parsimonious framework bridges a gap between official trade statistics (in gross value terms) and national accounts (in value-added terms), and integrates all previous measures of vertical specialization and value-added trade in the literature into a unified framework. To illustrate the potential of such a method, we present a number of applications including re-computing revealed comparative advantages and the magnifying impact of multi-stage production on trade costs. (*JEL* E01, E16, F14, F23, L14)

As different stages of production are now regularly performed in different countries, intermediate inputs cross borders multiple times. As a result, traditional trade statistics become increasingly less reliable as a gauge of the value contributed by any particular country. This paper integrates and generalizes the many attempts in the literature at tracing value added by country and measuring vertical specialization in international trade. We provide a unified conceptual framework that is more comprehensive than the current literature. By design, this is an accounting exercise, and does not directly examine the causes and the consequences of global production chains. However, an accurate and well-defined conceptual framework to account for value added by sources is a necessary step toward a better understanding of all these issues.

Supply chains can be described as a system of value-added sources and destinations. Within a supply chain, each producer purchases inputs and then adds value, which is included in the cost of the next stage of production. At each stage, the value added equals the value paid to the factors of production in the exporting country. However, as all official trade statistics are measured in gross terms, which

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include both intermediate inputs and final products, they "double count" the value of intermediate goods that cross international borders more than once. Such a conceptual and empirical shortcoming of gross trade statistics, as well as their inconsistency with the System of National Accounts (SNA) accounting standards, has long been recognized by both the economics profession and policy makers.[1]

Case studies on global value chains based on detailed micro data for a single product or a single sector in industries such as electronics, apparel, and motor vehicles have provided detailed examples of the discrepancy between gross and value-added trade. While enhancing our intuitive understanding of global production chains in particular industries, they do not offer a comprehensive picture of the gap between value-added and gross trade, and an economy's participation in cross-border production chains. Several researchers have examined the issue of vertical specialization on a systematic basis, including the pioneering effort of Hummels, Ishii, and Yi (2001)—henceforth, HIY. They suggested that a country can participate in vertical specialization in two ways: (i) uses imported intermediate inputs to produce exports; (ii) exports intermediate goods that are used as inputs by other countries to produce goods for export. HIY proposed to measure the imported foreign content in a country's exports based on a country's Input-Output (IO) table.

There are two key assumptions in HIY's foreign content (VS) estimation: the intensity in the use of imported inputs is the same between production for exports and production for domestic sales; and imports are 100 percent foreign sourced. The first assumption is violated in the presence of processing exports, which is a significant portion of exports for a large number of developing countries (Koopman, Wang, and Wei 2008 and 2012). The second assumption does not hold when there is more than one country exporting intermediate goods.

A growing recent literature aims to estimate value-added trade using global Inter-Country Input-Output (ICIO) tables based on the Global Trade Analysis Project (GTAP, Narayanan and Walmsley 2008) and World Input-Output database (WIOD, Timmer 2012), such as Daudin, Rifflart, and Schweisguth (2011), Johnson and Noguera (2012), and Stehrer, Foster, and de Vries (2012). Most of these papers discuss the connections between their works with HIY, but are more closely related to the factor content trade literature, except Koopman et al. (2010). By integrating the literature on vertical specialization and the literature on trade in value added, this paper makes the following contributions:

First, we provide a unified and transparent mathematical framework to completely decompose gross exports into its various components, including exports of value added, domestic value added that returns home, foreign value added, and other additional double-counted terms. Measures of vertical specialization and trade in value added in the existing literature can be derived from this framework and expressed as some linear combinations of these components. We show why some existing measures are special cases of the generalized measures in our framework and why some have to be modified from their original definitions in a more general multi-country framework with unrestricted intermediate trade.

[1] See, for example, National Research Council (2006) and Grossman and Rossi-Hansberg (2008).

Second, rather than simply excluding double-counted items from official trade statistics, we provide an accounting formula that quantifies different types of double-counted items for the first time in the literature. Knowing the relative importance of different double-counted items in a country's gross exports can help us to gauge the depth and pattern of that country's participation in global production chains. For example, in some sectors, China and the United States may have a similar amount of value-added exports. However, the composition of the double-counted terms can be very different. For China, the double-counted terms may show up primarily in the form of the use of foreign components (e.g., foreign product designs or machinery) in the final goods that China exports. For the United States, the double-counted terms may show up primarily in the form of domestic value added finally returned and consumed at home (e.g., product designs by Apple that are used in the final Apple products produced abroad but sold in the US market). The structure of these double-counted items in addition to their total sums offer additional information about the United States and China's respective positions in the global value chain.

Third, our accounting framework establishes a precise relationship between value-added measures of trade and official trade statistics, thus providing an observable benchmark for value-added trade estimates, as well as a workable way for national and international statistical agencies to remedy the missing information in current official trade statistics without dramatically changing the existing data collection practices of national customs authorities.

Fourth, our estimated global ICIO table may better capture the international source and use of intermediate goods than in previous databases in two ways. In estimating intermediate goods in bilateral trade, we use end-use classifications (intermediate or final) of detailed import statistics rather than the conventional proportionality assumptions. In addition, we estimate separate input-output coefficients for processing trade in China and Mexico, the two major users of such regimes in the world.

Finally, we report a number of applications to illustrate the potential of our approach to reshape our understanding of global trade. For example, with gross trade data, the business services sector is a revealed comparative advantage sector for India. In contrast, if one uses our estimated domestic value added in exports instead, the same sector becomes a revealed comparative disadvantage sector for India in 2004. The principal reason for this is how the indirect exports of business services are counted in high-income countries. Consider Germany. Most of its manufacturing exports embed German domestic business services. In comparison, most of Indian goods exports use comparatively little Indian business services. Once indirect exports of domestic business services are taken into account, Indian's business service exports become much less impressive relative to Germany and many other developed countries. As another example, the value-added decomposition shows that a significant portion of China's trade surplus to the United States in gross trade terms reflects indirect value-added exports that China does on behalf of Japan, Korea and Taiwan. While such stories have been understood in qualitative terms, our framework offers a way to quantify these effects.

This paper is organized as follows. Section I presents the conceptual framework of gross exports accounting. Section II discusses database construction methods, especially on how to estimate the required inter-country IO model from currently available data sources. Section III presents a number of illustrative applications. Section IV concludes.

I. Gross Exports Accounting: Concepts and Measurement

A. *Concepts*

Four measures have been proposed in the existing literature:

(i) The first measure of vertical specialization, proposed by HIY (2001), refers to the imported content in a country's exports. This measure, labeled as VS, includes both the directly and indirectly imported input content in exports. In mathematical terms, it can be expressed as

$$\mathbf{VS} = \mathbf{A}^{M}(\mathbf{I} - \mathbf{A}^{D})^{-1}\mathbf{E}. \tag{1}$$

(ii) A second measure, also proposed by HIY (2001) and labeled as VS1, looks at vertical specialization from the export side, and measures the value of intermediate exports sent indirectly through third countries to final destinations. However, HIY did not provide a mathematical definition for VS1.

(iii) A third measure is the value of a country's exported goods that are used as imported inputs by the rest of the world to produce final goods that are shipped back to home. This measure was proposed by Daudin, Rifflart, and Schweisguth (2011). Because it is a subset of VS1, they call it VS1*.

(iv) A fourth measure is value-added exports, which is value added produced in source country *s* and absorbed in destination country *r*. Johnson and Noguera (2012) defined this measure and proposed to use the ratio of value-added exports to gross exports, or the "VAX ratio" as a summary measure of value-added content of trade.

By definition, as value added is a "net" concept, double counting is not allowed. As the first three measures of vertical specialization all involve values that show up in more than one country's gross exports, they, by necessity, have to include some double-counted portions of the official trade statistics. This implies that these two types of measures are not equal to each other in general because double counting is only allowed in one of them. They become equal only in some special cases as we will show later both analytically and numerically. In addition, these existing measures are all proposed as stand-alone indicators. No common mathematical framework proposed in the literature provides a unified accounting for them and spells out their relationships explicitly.

We provide below a unified framework that breaks up a country's gross exports into the sum of various components. The value-added exports, VS, VS1, and VS1* are linear combinations of these components. In addition, we show how one may generalize the VS measure without the restrictive assumption made by HIY (no two-way trade in intermediate goods). By properly including various double-counted terms, our accounting is complete in the sense that the sum of these well-identified components yields 100 percent of the gross exports.

For ease of understanding, we start with a discussion of a two-country one sector case in Section II. We relate the components of our decomposition formula with the existing measures in the literature in Section III. We provide several numerical examples in Section IV to show intuitively how our gross exports accounting equation works. Finally, we present the most general *G*-country *N*-sector case.

B. *Two-Country Case*

Assume a two-country (home and foreign) world, in which each country produces in a single tradable sector. The good in that sector can be consumed directly or used as an intermediate input, and each country exports both intermediate and final goods to the other.

All gross output produced by Country s must be used as either an intermediate good or a final good, either at home or abroad. So Country s' gross output, x_s, has to satisfy the following accounting relationship:

$$x_s = a_{ss}x_s + a_{sr}x_r + y_{ss} + y_{sr}, \qquad r,s = 1, 2, \tag{2}$$

where y_{sr} is the final demand in Country r for the final good produced in Country s, and a_{sr} is the input-output (IO) coefficient, describing units of intermediate goods produced in s used in the production of one unit gross output in Country r. The two-country production and trade system can be written as an inter-country input-output (ICIO) model as follows:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} y_{11} + y_{12} \\ y_{21} + y_{22} \end{bmatrix}. \tag{3}$$

With rearranging, we have

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} I - a_{11} & -a_{12} \\ -a_{21} & I - a_{22} \end{bmatrix}^{-1} \begin{bmatrix} y_{11} + y_{12} \\ y_{21} + y_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}. \tag{4}$$

Since equation (4) takes into account both the direct and indirect use of a country's gross output as intermediate goods in the production of its own and foreign final goods, the coefficients in the B matrix (Leontief inverse) are referred to as "total requirement coefficients" in the input-output literature. Specifically, b_{11} is the total amount of Country 1's gross output needed to produce an extra unit of the final good in Country 1 (which is for consumption in both Countries 1 and 2); b_{12} is the total amount of Country 1's gross output needed to produce an extra unit of the final good in Country 2 (again for consumption both at home and abroad). Similar interpretations can be assigned to the other two coefficients in the B matrix.

We can break up each country's gross output according to where it is ultimately absorbed by rearranging both countries' final demand into a matrix format by source and destination, and rewrite equation (4) as follows:

$$(5)\quad \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} b_{11}y_{11} + b_{12}y_{21} & b_{11}y_{12} + b_{12}y_{22} \\ b_{21}y_{11} + b_{22}y_{21} & b_{21}y_{12} + b_{22}y_{22} \end{bmatrix},$$

where y_{sr} is as defined in equation (2), giving the final goods produced in Country s and consumed in Country r. This final demand matrix in the middle of equation (5) is a 2×2 matrix, summing along each row of the matrix equals y_s, which represents the global use of the final goods produced in each country as specified in equation (4).

We label the 2×2 matrix on the left-hand side of equation (5) the "gross output decomposition matrix." It fully decomposes each country's gross outputs according to where it is absorbed. Each element x_{sr} is the gross output in source Country s necessary to sustain final demand in destination Country r. Summing along its row equals total gross output in Country s, x_s as specified in equation (2). For example, it breaks up Country 1's gross output x_1 into two parts: $x_1 = x_{11} + x_{12}$. While x_{11} is the part of Country 1's gross output that is ultimately absorbed in Country 1, x_{12} is the part of Country 1's gross output that is ultimately absorbed in Country 2.

The RHS of equation (5) further decomposes x_{11} itself into two parts: $x_{11} = b_{11}y_{11} + b_{12}y_{21}$. The first part, $b_{11}y_{11}$ is the part of Country 1's gross output required to produce Country 1's final good that is consumed in Country 1. The second part, $b_{12}y_{21}$, is the part of Country 1's gross output that is exported as an intermediate good, and eventually returns home as part of Country 1's imports from abroad (embedded in foreign final goods).

Similarly, x_{12} can also be decomposed into two parts: $x_{12} = b_{11}y_{12} + b_{12}y_{22}$. The first part, $b_{11}y_{12}$ is the part of Country 1's gross output that is used to produce exported final good that is consumed abroad. $b_{12}y_{22}$ is the part of Country 1's gross output that is exported as an intermediate good and used in Country 2 to produce final good that is consumed there. Of course, $x_1 = x_{11} + x_{12}$ is nothing but Country 1's total output. By assumption, they are produced by the same technology and therefore have the same share of domestic value added.

By the same interpretation, Country 2's gross output also can be first broken up into two parts: $x_2 = x_{21} + x_{22}$. x_{21} is Country 2's gross output that is ultimately absorbed in Country 1, which can be in turn broken up to $b_{21}y_{11} + b_{22}y_{21}$. x_{22} is Country 2's domestic absorption of its own gross output, and can be further broken up to $b_{21}y_{12} + b_{22}y_{22}$.

By definition, to produce one unit of Country 1's good, a_{11} units of domestic intermediate good is used, and a_{21} units of imported intermediate good is used. Therefore, the fraction of domestic output that represents the domestic value added in Country 1 is $v_1 = 1 - a_{11} - a_{21}$. Similarly, the share of domestic value added in

Country 2's gross output is: $v_2 = 1 - a_{12} - a_{22}$. Therefore, we can define a 2×2 direct value-added coefficient matrix as

$$\text{(6)} \qquad \mathbf{V} = \begin{bmatrix} v_1 & 0 \\ 0 & v_2 \end{bmatrix}.$$

Multiplying **V** with the Leontief inverse **B** produces the 2×2 value-added share (**VB**) matrix, our basic measure of value-added shares by source of production:

$$\text{(7)} \qquad \mathbf{VB} = \begin{bmatrix} v_1 b_{11} & v_1 b_{12} \\ v_2 b_{21} & v_2 b_{22} \end{bmatrix}.$$

Within **VB**, $v_1 b_{11}$ and $v_2 b_{22}$ denote the domestic value-added share of domestically produced products for Country 1 and Country 2 respectively; $v_2 b_{21}$ and $v_1 b_{12}$ denote the share of foreign country's value added in the same goods. Because all value added must be either domestic or foreign, the sum along each column is unity[2]:

$$\text{(8)} \qquad v_1 b_{11} + v_2 b_{21} = v_1 b_{12} + v_2 b_{22} = 1.$$

Given the assumption on the input-output coefficients, there is no difference in the share of domestic value added in Country 1's production for goods absorbed at home versus its production for exports.[3] Therefore, the total domestic value added in Country 1's gross output is simply $v_1 x_1$; it is Country 1's GDP by definition.

The total value added in Country 1's gross outputs can be easily broken up into two parts based on where it is ultimately absorbed: $v_1 x_1 = v_1 x_{11} + v_1 x_{12}$where $v_1 x_{11}$ is the domestic value added that is ultimately absorbed at home, and $v_1 x_{12}$ is the domestic value added that is ultimately absorbed abroad.

The last part, $v_1 x_{12}$, is also Country 1's exports of value added. It is instructive to decompose the last item further. Since $v_1 x_{12} = v_1 b_{11} y_{12} + v_1 b_{12} y_{22}$, Country 1's exports of value added has two components: Country 1's value added embedded in Country 1's exports of final good that is absorbed in Country 2 $(v_1 b_{11} y_{12})$; and Country 1's value added in its exports of intermediate good that is used by Country 2 to produce final good that is ultimately locally consumed $(v_1 b_{12} y_{22})$.

Note that $v_1 x_{12}$ is conceptually the same as Country 1's value-added exports as defined by Johnson and Noguera (2012) except that we express it as the sum of two components related only to the final demand in the two countries. To summarize, Country 1's and 2's "value-added exports" are, respectively,

$$\text{(9)} \qquad VT_{12} \equiv v_1 x_{12} = v_1 b_{11} y_{12} + v_1 b_{12} y_{22}$$

$$VT_{21} \equiv v_2 x_{21} = v_2 b_{21} y_{11} + v_2 b_{22} y_{21}.$$

[2] Note that the VB matrix is not any arbitrary share matrix, but rather the one that reflects the underlying production structure embedded in the ICIO model specified in equation (3). It contains all the needed information on value-added production by source.

[3] Such an assumption is maintained by HIY (2001), Johnson and Noguera (2012), and Daudin, Rifflart, and Schweisguth (2011). One might allow part of the production for exports (processing exports) to take on different input-output coefficients. Such a generalization is pursued by Koopman, Wang, and Wei (2012), who have worked out a generalized formula for computing the share of domestic value added in a country's gross exports when processing trade is prevalent.

Intuitively, there are at least two reasons for a country's exports of value added to be smaller than its gross exports to the rest of the world. First, the production for its exports may contain foreign value added or imported intermediate goods (a). Second, part of the domestic value added that is exported may return home after being embodied in the imported foreign goods rather than being absorbed abroad (b). In other words, exports of value added are a net concept; it has to exclude from the gross exports both foreign value added and the part of domestic value added that is imported back to home.

Identifying and estimating these double-counted terms in gross exports have important implications for measuring each country's position in global value-chains. For example, two countries can have identical ratios of value-added exports to gross exports but very different ratios of (a) and (b). Those countries that are mainly upstream in global production chains, such as product design, tend to have a large value of (b) but a small value of (a). In comparison, those countries mainly specializing in assembling imported components to produce final products tend to have a small value of (b) but a big value of (a). However, the existing literature lacks a uniform and transparent framework to compute exports of value added, part (a) and part (b) simultaneously. We venture to do this next. Without loss of generality, let us work with Country 1's gross exports first:

$$e_{12} = y_{12} + a_{12}x_2. \tag{10}$$

It says that Country 1's exports consist of final goods and intermediate goods. Combining (10) with equation (8), we have

$$\begin{aligned} e_{12} &= (v_1 b_{11} + v_2 b_{21})(y_{12} + a_{12}x_2) \\ &= v_1 b_{11} y_{12} + v_2 b_{21} y_{12} + v_1 b_{11} a_{12} x_2 + v_2 b_{21} a_{12} x_2 \\ &= v_1 b_{11} y_{12} + v_2 b_{21} y_{12} + v_1 b_{12} y_{22} + v_1 b_{12} y_{21} \\ &\quad + v_1 b_{12} a_{21} x_1 + v_2 b_{21} a_{12} x_2. \end{aligned} \tag{11}$$

A step-by-step proof of $v_1 b_{11} a_{12} x_2 = v_1 b_{12} y_{22} + v_1 b_{12} y_{21} + v_1 b_{12} a_{21} x_1$ can be found in online Appendix A.

Here we give the economic intuition behind it. The total value of Country 1's intermediate exports must include two types of value. First, it must include all value added by Country 1 in its imports from Country 2. To see this, we note that in order for exported value produced by Country 1 to come back through its imports, it must have first been embodied in Country 1's intermediate exports, which is $v_1 b_{12} y_{21} + v_1 b_{12} a_{21} x_1$. Second, it must include all value added generated in Country 1 that is absorbed in Country 2 after being used as intermediate inputs by Country 2, which is $v_1 b_{12} y_{22}$.

Note that multiplying the Leontief inverse with intermediate goods exports leads to some double counting of gross output and thus some value terms in exports.

However, in order to account for 100 percent of the value of Country 1's intermediate goods exports and to identify what is double counted, we have to include them into the accounting equation first. By decomposing the last two terms in equation (11) further, we can see precisely what is double counted. Using the gross output identity (equation (2)) $x_1 = y_{11} + a_{11}x_1 + e_{12}$ and $x_2 = y_{22} + a_{22}x_2 + e_{21}$, it is easy to show that

$$(12) \qquad x_1 = (1 - a_{11})^{-1} y_{11} + (1 - a_{11})^{-1} e_{12}$$

$$x_2 = (1 - a_{22})^{-1} y_{22} + (1 - a_{22})^{-1} e_{21}.$$

$(I - a_{11})^{-1} y_{11}$ is the gross output needed to sustain final goods that are both produced and consumed in Country 1, using domestically produced intermediate goods; deducting it from Country 1's total gross output, what is left is the gross output needed to sustain Country 1's production of its gross exports e_{12}. Therefore the two terms in the right-hand side of equation (12) both have straightforward economic meanings. We can further show that $b_{12} a_{21} (1 - a_{11})^{-1} y_{11} = b_{11} y_{11} - (1 - a_{11})^{-1} y_{11}$ (see proof, also in online Appendix A), which is the total gross output needed to sustain final goods both produced and consumed in Country 1, but using intermediate goods that originated in Country 1 and shipped to Country 2 for processing before being re-imported by Country 1 (gross output sold indirectly in the domestic market). These two parts plus $b_{12} y_{21}$sum to x_{11} in equation (5), which is the gross output of Country 1 absorbed in Country 1 to sustain its domestic final demand both directly and indirectly. It indicates that Country 1's domestic final demand is satisfied by three production channels: (i) $(1 - a_{11})^{-1} y_{11}$ is part of Country 1's gross output sold directly in the domestic market that is consumed there; (ii) $b_{12} y_{21}$ is part of Country 1's gross output used as intermediate goods by Country 2 to produce final goods that are consumed in Country 1; (iii) $b_{12} a_{21} (1 - a_{11})^{-1} y_{11}$ is part of Country 1's gross output used as intermediate goods by Country 2 to produce intermediate goods that are exported to country 1 to produce final goods in Country 1 that are consumed there. They are all needed to sustain the domestic final demand in Country 1, but they differ in terms of how they participate in international trade.

Replacing x_1 by $(1 - a_{11})^{-1} y_{11} + (1 - a_{11})^{-1} e_{12}$ and x_2 by $(1 - a_{22})^{-1} y_{22} + (1 - a_{22})^{-1} e_{21}$ in equation (11), and rearranging terms, we can fully decompose Country 1's gross exports into its various value-added and double-counted components as follows:

$$(13) \quad e_{12} = v_1 b_{11} e_{12} + v_2 b_{21} e_{12} = [v_1 b_{11} y_{12} + v_1 b_{12} y_{22}]$$

$$+ [v_1 b_{12} y_{21} + v_1 b_{12} a_{21} (1 - a_{11})^{-1} y_{11}] + v_1 b_{12} a_{21} (1 - a_{11})^{-1} e_{12}$$

$$+ [v_2 b_{21} y_{12} + v_2 b_{21} a_{12} (1 - a_{22})^{-1} y_{22}] + v_2 b_{21} a_{12} (1 - a_{22})^{-1} e_{21}.$$

While the algebra to arrive at equation (13) may be a bit tedious, expressing a country's gross exports as the sum of these eight terms on the right-hand side of equation (13) is very useful. We go over their economic interpretations systematically.

The first two terms in equation (13) (or the two terms in the first square bracket) are value-added exports, i.e., Country 1's domestic value added absorbed outside Country 1.

The third term, $v_1 b_{12} y_{21}$ is Country 1's domestic value added that is initially embodied in its intermediate exports but is returned home as part of Country 1's imports of the final good. The fourth term, $v_1 b_{12} a_{21}(1 - a_{11})^{-1} y_{11}$, is also Country 1's domestic value added that is initially exported by Country 1 as part of its intermediate goods to Country 2, but then is returned home via its intermediate imports from Country 2 to produce final goods that are absorbed at home. Both the third and the fourth terms are domestic value added produced in Country 1, exported to Country 2, but then return to and stay in Country 1. Both are counted at least twice in trade statistics as they first leave Country 1 for Country 2, and then leave Country 2 for Country 1 (and ultimately stay in Country 1). Note that the value represented by these two terms can be embodied in trade transactions that cross borders back and forth more than twice as long as they originate in Country 1 and are ultimately consumed in Country 1.

The fifth term, $v_1 b_{12} a_{21}(1 - a_{11})^{-1} e_{12}$, may be called a "pure double-counted term." The reason for labeling it as such will become clear after we present a similar dissection of Country 2's gross exports and a further decomposition of this term. This term only occurs when both countries export intermediate goods. If at least one country does not export intermediate goods (i.e., no two-way trade in intermediate goods), this terms disappears.

The sixth term,$v_2 b_{21} y_{12}$, is the foreign value added in Country 1's gross exports of final goods; and the seventh term, $v_2 b_{21} a_{12}(1 - a_{22})^{-1} y_{22}$, is the foreign value added in Country 1's gross exports of intermediate goods. They both ultimately go back to the foreign country and are consumed there.

The eighth (and the last) term is another pure double-counted item in Country 1's gross exports. Similar to the fifth term, this term would disappear if at least one country does not export intermediate goods.

In a similar way, we can express Country 2's gross exports as a sum of eight terms:

$$(14)\quad e_{21} = v_1 b_{12} e_{21} + v_2 b_{22} e_{21} = [v_2 b_{22} y_{21} + v_2 b_{21} y_{11}]$$
$$+ [v_2 b_{21} y_{12} + v_2 b_{21} a_{12}(1 - a_{22})^{-1} y_{22}] + v_2 b_{21} a_{12}(1 - a_{22})^{-1} e_{21}$$
$$+ [v_1 b_{12} y_{21} + v_1 b_{12} a_{21}(1 - a_{11})^{-1} y_{11}] + v_1 b_{12} a_{21}(1 - a_{11})^{-1} e_{12}.$$

Comparing equations (13) and (14), there are a few noteworthy features. First, the third, fourth, and fifth terms in Country 1's gross exports (13) are identical to the sixth, seventh, and eighth terms in Country 2's exports (14), and vice versa. This means that the value added, that is initially produced and exported by Country 1 but then re-imported by Country 1, is exactly the same as foreign value added in Country 2's gross exports to Country 1. Symmetrically, the foreign value added in Country 1's gross exports to Country 2, is the same as Country 2's value added, initially produced and exported by Country 2, but reappears as part of Country 1's gross exports to Country 2.

Second, while the first and second terms in equations (13) and (14) constitute value-added exports, all other terms are double-counted components in a country's official exports statistics. However, there are conceptually interesting differences among the third and the fourth terms as the first group, the sixth and the seventh terms as the second group, and the fifth and eighth terms as the third group. The differences are revealed when comparing them to the two countries' GDP. More precisely, a country's GDP is the sum of its value-added exports plus its domestic value added consumed at home:

$$(15)\quad \begin{aligned} GDP_1 = v_1x_1 &= v_1(b_{11}y_{12} + b_{12}y_{22} + b_{12}y_{21} + b_{11}y_{11}) \\ &= v_1\{b_{11}y_{12} + b_{12}y_{22} + [b_{12}y_{21} + b_{12}a_{21}(1 - a_{11})^{-1}y_{11}]\} \\ &\quad + v_1(1 - a_{11})^{-1}y_{11} \end{aligned}$$

$$(16)\quad \begin{aligned} GDP_2 = v_2x_2 &= v_2(b_{21}y_{11} + b_{22}y_{21} + b_{21}y_{12} + b_{22}y_{22}) \\ &= v_2\{b_{21}y_{11} + b_{22}y_{21} + [b_{21}y_{12} + b_{21}a_{12}(1 - a_{22})^{-1}y_{22}]\} \\ &\quad + v_2(1 - a_{22})^{-1}y_{22}. \end{aligned}$$

The last term in each GDP equation is value-added produced and consumed at home that are not related to international trade; while the first four terms in the bracket in each GDP equation are exactly the same as the first four terms in equations (13) and (14). It is easy to show that the sum of global GDP always equals global final demand:

$$(17)\quad \begin{aligned} GDP_1 + GDP_2 = v_1x_1 + v_2x_2 &= (1 - a_{11} - a_{21})x_1 + (1 - a_{12} - a_{22})x_2 \\ &= x_1 - a_{11}x_1 - a_{12}x_2 + x_2 - a_{21}x_1 - a_{22}x_2 = y_1 + y_2. \end{aligned}$$

Equations (15) and (16) show that the third and fourth terms in equations (13) and (14) are counted as part of the home country's GDP (even though they are not part of the home country's exports of value added). Because they represent a country's domestic value added that is initially exported but imported back and consumed in the initial producing country, they are part of the value added created by domestic production factors. The sixth and seventh terms represent the foreign value added in a country's exports that are ultimately absorbed in the foreign country. They are counted once as part of the foreign country's GDP in equations (15) and (16). In comparison, because a combination of the part of GDP that is consumed at home and exports of value added yields 100 percent of a country's GDP, the fifth and eighth terms are not part of either country's GDP. In this sense, they are "pure double-counted terms."

Subtracting global GDP from global gross exports using equations (13), (14), (15), and (16) yields the following:

$$(18)\quad \begin{aligned} e_{12} + e_{21} - GDP_1 - GDP_2 &= v_1[b_{12}y_{21} + b_{12}a_{21}(1 - a_{11})^{-1}y_{11}] \\ &\quad + v_2[b_{21}y_{12} + b_{21}a_{12}(1 - a_{22})^{-1}y_{22}] \\ &\quad + 2v_1b_{12}a_{21}(1 - a_{11})^{-1}e_{12} + 2v_2b_{21}a_{12}(1 - a_{22})^{-1}e_{21} \\ &\quad - [v_1(1 - a_{11})^{-1}y_{11} + v_2(1 - a_{22})^{-1}y_{22}]. \end{aligned}$$

Equation (18) shows clearly that besides the value added produced and consumed at home (in the last square bracket), which is not a part of either country's gross exports, the sixth and seventh terms in equation (13), $v_2[b_{21}y_{12} + b_{21}a_{12}(1 - a_{22})^{-1}y_{22}]$, and the sixth and seventh terms in equation (14) $v_1[b_{12}y_{21} + b_{12}a_{21}(1 - a_{11})^{-1}y_{11}]$, are double counted only once as foreign value added in the other country's gross exports. Because the third and fourth terms in (13) and (14) reflect part of the countries' GDP, they are not double counted from the global GDP point of view. In comparison, both the fifth and eighth terms are counted twice relative to the global GDP since they are not a part of either country's GDP.

Third, the nature of the fifth and eighth terms can be better understood if we break them up further. In particular, with a bit of algebra, they can be expressed as linear combinations of components of the two countries' final demand:

$$(19)\qquad v_1b_{12}a_{21}(1 - a_{11})^{-1}e_{12}$$
$$= b_{12}a_{21}[v_1b_{11}y_{12} + v_1b_{12}y_{22} + v_1b_{12}y_{21} + v_1b_{12}a_{21}(1 - a_{11})^{-1}y_{11}]$$

$$(20)^4 v_2b_{21}a_{12}(1 - a_{22})^{-1}e_{21}$$
$$= b_{21}a_{12}[v_2b_{21}y_{11} + v_2b_{22}y_{21} + v_2b_{21}y_{12} + v_2b_{21}a_{12}(1 - a_{22})^{-1}y_{22}].$$

The four terms inside the square bracket on the RHS of equation (19) are exactly the same as the first four terms in the gross exports accounting equation (13). A similar statement can be made about the four terms inside the square bracket in equation (20) to gross export accounting equation (14). This means that the fifth and the eighth terms double count a fraction of both a country's value-added exports and its domestic value added that has been initially exported but are eventually returned home. Just to belabor the point, unlike the other terms in equation (13) that are parts of some countries' GDP, the fifth and the eighth terms over-count the values that are already captured by other terms in gross exports. Again, this feature suggests that they are "pure double-counted terms."

Note, if Country 2 does not export any intermediate goods, then $a_{21} = b_{21} = 0$, and the entire RHS of equations (20) and (19) would vanish. Alternatively, if Country 1 does not export intermediate goods, then $a_{12} = b_{12} = 0$, the entire RHS of equations (19) and (20) would also vanish. In other words, the fifth and eighth terms exist only when two-way trade in intermediate goods exist so that some value added is shipped back and forth as a part of intermediate trade between the two countries. Because the eight components of equations (13) and (14) collectively constitute 100 percent of a country's gross exports, missing any part, including the two pure double-counted terms, would yield an incomplete accounting of the gross exports.

Finally, while the fifth and eighth terms in equation (13) are (double-counted) values in intermediate goods trade that are originated in Countries 1 and 2, respectively, we cannot directly see where they are absorbed. By further partitioning the fifth and

[4] Proofs of equations (18), (19), and (20) are provided in online Appendix A.

eighth terms, we can show where they are finally absorbed and interpret the absorption by input/output economics. With a bit of algebra, we can show[5]

$$(21)\quad v_1 b_{12} a_{21}(1 - a_{11})^{-1} e_{12} + v_2 b_{21} a_{12}(1 - a_{22})^{-1} e_{21}$$
$$= a_{12}[b_{21} y_{12} + b_{21} a_{12}(1 - a_{22})^{-1} y_{22}]$$
$$+ a_{21}[b_{12} y_{21} + b_{12} a_{21}(1 - a_{11})^{-1} y_{11}].$$

Therefore, the sum of fifth and eighth terms in equation (13) is equivalent to the sum of the four terms in the RHS of equation (21). From equation (5), it is easy to see that the terms in the square bracket in equation (21) are parts of x_{22} and x_{11}, *which are trade-related portions of gross output that are both originally produced and finally consumed in the source country*; therefore they are part of each country's gross intermediate exports ($a_{12}x_2$ and $a_{21}x_1$). Since the value added embodied in those intermediate goods are already counted once in the production of each country's GDP (the third and fourth, terms in equations (13) and (14)), they are double counted in value-added (GDP) terms. As we pointed earlier, the exact same terms will also appear in Country 2's official exports statistics.

Due to the presence of these types of conceptually different double counting in a country's gross exports, we may separately define "domestic value added in exports"(part of a country's GDP in its exports) and "domestic content in exports." The former excludes the pure double-counted intermediate exports that return home; whereas the latter is the former plus the pure double-counted term.

C. *Using the Accounting Equation to Generate Measures of Vertical Specialization*

We can relate the definitions of the three concepts to components in equation (13). Country 1's exports of value added are the sum of the first and the second term. It takes into account both where the value is created and where it is absorbed. Country 1's *value added in its exports* is the sum of its exports of value added and the third and the fourth terms. This concept takes into account where the value is created but not where it is absorbed. Obviously, Country 1's "value added in its exports" is generally greater than its "exports of value added." Finally, *the domestic content in Country 1's exports* is Country 1's value added in its exports plus the fifth term, the double-counted intermediate goods exports that are originated in Country 1. This last concept also disregards where the value is ultimately absorbed. By assigning the fifth term to the domestic content in Country 1's exports, and the eighth term to the foreign content in Country 1's exports, we can achieve the property that the sum of the domestic content and the foreign content yields the total gross exports. We will justify these definitions in what follows.

We already show that the first two terms in equation (13) correspond to value-added exports as proposed in Johnson and Noguera (2012). We now link other measures in

[5] A step-by-step proof is in online Appendix A.

the literature to linear combinations of the components in the same gross exports accounting equation. Following HIY's original ideas, Koopman, Wang, and Wei (2008 and 2012) have shown that gross exports can be decomposed into domestic content and foreign content/vertical specialization (VS) in a single country IO model without two-way international trade in intermediate goods. If we were to maintain HIY's assumption that Country 1 does not export intermediate good (i.e., $a_{12} = b_{12} = 0$), then the two pure double-counted terms, or the fifth and the eighth terms in equation (13) are zero. In this case, we can easily verify that the sum of the last four terms in equation (13) is identical to the VS measure in the HIY (2001) paper.

To remove the restriction that HIY imposed on intermediate trade, we have to determine how to allocate the two pure double-counted terms. We choose to allocate the double-counted intermediate exports according to where they are originally produced. That is, even though the fifth term in equation (13) reflects pure double counting, it nonetheless is originally produced in Country 1 and therefore can be treated as part of Country 1's domestic content. Similarly, we allocate the eighth term to the foreign content in Country 1's exports. Such a definition is consistent with HIY's original idea that a country's gross exports consist of either domestic or foreign content and the major role of their VS measure is to quantify the extent to which intermediate goods cross international borders more than once. It is also computationally simple because the share of domestic and foreign content can be obtained directly from the **VB** matrix.

However, since the fifth term reflects double-counted intermediate goods in a country's gross exports, we may wish to exclude it if we are to consider which part of Country 1's GDP is exported. In particular, we define "domestic value added in Country 1's gross exports" (regardless of where the exports are ultimately absorbed) as the sum of the first four terms on the RHS of equation (13). This variable can be shown to equal $v_1(1 - a_{11})^{-1} e_{12}$, part of equation (12), Country 1's gross output identity.

$$
\begin{aligned}
(22)\quad DV_1 &= v_1(1 - a_{11})^{-1} e_{12} = v_1(1 - a_{11})^{-1}(y_{12} + a_{12}x_2) \\
&= v_1(1 - a_{11})^{-1}[y_{12} + a_{12}(b_{21}y_{11} + b_{22}y_{21} + b_{21}y_{12} + b_{22}y_{22})] \\
&= v_1(1 - a_{11})^{-1}[(1 + a_{12}b_{21})y_{12} + a_{12}(b_{21}y_{11} + b_{22}y_{21} + b_{22}y_{22})] \\
&= v_1(1 - a_{11})^{-1}\{(1 - a_{11})b_{11}y_{12} + (1 - a_{11})b_{12}(y_{21} + y_{22}) \\
&\qquad + [(1 - a_{11})b_{11} - 1]y_{11}\} \\
&= v_1 b_{11} y_{12} + v_1 b_{12} y_{22} + v_1 b_{12} y_{21} + v_1[b_{11} - (1 - a_{11})^{-1}]y_{11} \\
&= v_1 b_{11} y_{12} + v_1 b_{12} y_{22} + v_1[b_{12}y_{21} + b_{12}a_{21}(1 - a_{11})^{-1}y_{11}].
\end{aligned}
$$

The derivation uses the property of inverse matrix $(1 - a_{11})b_{11} = 1 + a_{12}b_{21}$ and $(1 - a_{11})b_{12} = a_{12}b_{22}$.

Following the same frame of thinking, we label only the sum of the sixth and seventh terms, a subset of the foreign content, as the "foreign value added in Country 1's exports," and define Country 1's VS as

$$VS_1 = v_2 b_{21} e_{12} = v_2[b_{21} y_{12} + b_{21} a_{12}(1 - a_{22})^{-1} y_{22}] + v_2 b_{21} a_{12}(1 - a_{22})^{-1} e_{21} = (1 - v_1 b_{11}) e_{12}. \tag{23}$$

The first two terms are foreign value added or GDP in Country 1's exports, and VS share of Country 1 equals $v_2 b_{21}$. Therefore, such a measure of foreign content is a natural extension of HIY's VS measure in a two-country world with unrestricted intermediate goods trade. Because the VS share is defined this way, it is natural to define the domestic content share in Country 1's exports as 1 − VS share or $v_1 b_{11}$.

In a two-country world, Country 1's VS1 is identical to its VS1* and Country 2's VS:

$$VS1_1 = VS1_1^* = v_1 b_{12} e_{21} = v_1[b_{12} y_{21} + b_{12} a_{21}(1 - a_{22})^{-1} y_{11}] + v_1 b_{12} a_{21}(1 - a_{11})^{-1} e_{12}, \tag{24}$$

which is the sum of the third, fourth, and fifth terms in equation (13). However, in a multi-country setting to be discussed later, VS1* will only be a subset of VS1, and the latter will also include some third country terms.

D. *Numerical Examples*

To enhance the intuition for our approach, we provide some numerical examples.

Example 1 (Two Countries, One Sector, and Only One Country Exports Intermediate Good):

Consider a world consisting of two countries (USA and CHN) and a single sector of electronics. The two countries have identical gross exports and identical value-added exports (and hence identical VAX ratios). The point of this example is to show that the structure of the "double-counted" values in gross exports contains useful information.

Both USA and CHN have a gross output of 200. USA's total output consists of 150 units of intermediate goods (of which, 100 units are used at home and 50 units are exported) and 50 units of final goods (of which 30 are consumed at home and 20 are exported).

CHN's total output consists of 50 units of intermediate goods (all used at home) and 150 units of final goods (of which, 70 units are exported and 80 units are used at home). By construction, both countries export 70 units of their output (50 units of intermediate goods + 20 units of final goods exported by USA, and 70 units of final goods exported by CHN).

The domestic value added in USA's output is therefore 100 (= value of gross output 200-value of domestic intermediate good of 100). Note no foreign value is used in USA's production. The domestic value added in CHN's output is also 100 (= value of gross output 200-value of domestic intermediate goods of 50 + value of imported intermediate goods of 50). The input-output relationship can be summarized as follows:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 200 \\ 200 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0.5 & 0.25 \\ 0 & 0.25 \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

The two-country production and trade system can be written as an inter-country input output (ICIO) model as follows:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.25 \\ 0 & 0.25 \end{bmatrix} \begin{bmatrix} 200 \\ 200 \end{bmatrix} + \begin{bmatrix} 30 + 20 \\ 70 + 80 \end{bmatrix}.$$

The Leontief inverse matrix and the **VB** matrix can be computed easily as

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33, \end{bmatrix}, \quad \mathbf{VB} = \begin{bmatrix} 1 & 0.33 \\ 0 & 0.67 \end{bmatrix}.$$

We can break up each country's gross output according to where it is ultimately absorbed by rearranging each country's final demand as follows:

$$\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 20 \\ 70 & 80 \end{bmatrix} = \begin{bmatrix} 60 + 46.69 & 40 + 53.3 \\ 0 + 93.33 & 0 + 106.67 \end{bmatrix} = \begin{bmatrix} 106.7 & 93.3 \\ 93.3 & 106.7 \end{bmatrix}.$$

It is easy to verify that $x_1 = x_{11} + x_{12}$ and $x_2 = x_{21} + x_{22}$. USA's value-added export is 46.7 ($v_1 x_{12} = 0.5 \times 93.33$), where 20 ($= v_1 b_{11} y_{12} = 0.5 \times 2 \times 20$, first term in equation (13)) is the amount of USA's value added in its export of final goods that is absorbed in CHN, and 26.7($= v_1 b_{12} y_{22} = 0.5 \times 0.667 \times 80$, second term in (13)) is the amount of USA's value added in its export of intermediate goods that is also absorbed in CHN. The amount of USA's value added that is embedded in its export of intermediate goods but returns home as part of CHN's export of final goods is 23.3 ($= v_1 b_{12} y_{21} = 0.5 \times 0.667 \times 70$, the third term in (13)). Using the terminology of Daudin, Rifflart, and Schweisguth (2011), USA's $VS1^* = 23.3$, the same as the estimates from our accounting equation (13).

By construction, foreign value added in USA's exports equals zero (since CHN does not export intermediate goods by assumption). It is easy to verify that the sum of USA's exports of value added (46.7) and the amount of returned value added (23.3) is 70, which is the value of its gross exports.

For CHN, its value-added exports (that are absorbed in USA) are also 46.7 ($= v_2 b_{22} y_{21} = 0.5 \times 1.333 \times 70$, first term in equation (14)). It is entirely embedded in CHN's exports of final goods to USA. However, CHN does import intermediate goods from USA to produce both goods consumed in CHN and goods exported to USA. The amount of USA's value added used in the production of CHN's exports is 23.3 ($= v_1 b_{12} y_{21} = 0.5 \times 0.667 \times 70$, the sixth term in equation (14)). The sum of CHN's exports of value added (46.7) and the foreign value added in its exports (23.3) is 70, which is the same as CHN's gross exports.

In this example, both countries have identical gross exports and exports of value added, and therefore identical VAX ratios. However, the reasons underlying why value-added exports deviate from the gross exports are different. For USA, the VAX ratio is less than one because some of the value added that is initially exported returns home after being used as an intermediate good by CHN in the latter's production for exports. For CHN, the VAX ratio is less than one because its production for exports uses intermediate goods from USA which embeds USA's value added.

In addition, the double-counted items are also value added at some stage of production. More precisely, the VS in CHN's exports (23.3) is simultaneously a true value added from USA's viewpoint as it is value added by USA in its exports to CHN (that returned home), but a "double-counted item" from CHN's viewpoint as it is not part of CHN's value added.

Since we assume that CHN does not export intermediate goods (or $a_{21} = 0$ and $b_{21} = 0$), there is no channel for CHN's gross output to be used by USA in its production and for CHNs gross output to be first exported and then returned home. Therefore, our gross exports accounting equation produces the same estimate for VS as the HIY's formula, i.e.,

$$v_2(1 - b_{11}) = v_2 b_{21} e_{12} = 0.5 \times 0.667 \times 70 = 23.3$$

$$= a_{21}(1 - a_{11})^{-1} e_{12} = 0.25 \times (1 - 0.25)^{-1} \times 70 = 23.3.$$

Domestic value-added share for CHN's exports equals $(70 - 23.3)/70 = 0.667 = v_2 b_{22} = 0.667$. There is no difference between HIY's measure and our method in such a case since there are no pure double-counting terms due to two way trade in intermediate goods. Both VS in USA and VS1* in CHN are zero. But this equality will not hold when we remove the assumption of $a_{21} = 0$.

However, because there is domestic value added embodied in USA's intermediate goods exports that is eventually returned home, the domestic value-added share in USA's exports equals to 1. As a result, using the VAX ratio (0.667) as a metric of the share of home country's domestic value added in its gross exports would produce an underestimate. The corresponding ICIO table and more computation details can be found in online Appendix C.

Example 2 (Different Sequences of Trade for Same Final Goods Consumption and Same Value-Added Contribution from Each Source Country):

Imagine the United States trades in a good that has $1 million of US value added plus $100,000 of value added from each of other five countries before the final good, priced at \$1.5 million, is consumed in the United States. Let us consider two different production sequences of the supply chains.

Case 1: If each of the five countries sends the parts sequentially between them for assembly before they enter the United States, then total trade will be \$100K + \$200K + \$300K + \$400K + \$500K = \$1.5 million.

Case 2: If the United States exports its part first, then the production proceeds sequentially and the final goods produced by the production chain are shipped back to the United States, the total trade will be \$1 million + \$1.1 million + \$1.2 million + \$1.3 million + \$1.4 million + \$1.5 million = \$7.5 million.

Both cases yield the same value added but different amounts of gross trade. However, the two sequences of trade will correspond to two different inter-country input-output (ICIO) tables. We can attach concrete labels to the final product and the inputs to get more economic intuition. Let us think of the final product as a Barbie doll, and the inputs as hair (made by the United States), head, body, hat, clothes, and shoes, respectively. The information about the sequence of the production is embedded in the direct input-output matrix A. In the first case, if the hair is attached by the United States at the last stage of production, it is an input to the "doll" industry. While in the second case, if the hair is first exported to the country producing "head," attached to the "head" there, and re-exported to the next stage of production, it becomes an input to the "head" industry. The two different organizations of the spatial and temporal connections of hair to head show up in changed entries to the ICIO matrix, with inputs of hair to the "doll" industry falling in the former recipient country ICIO coefficient in the first case, but inputs of hair rising in the second country's "head" industry and measured inputs of heads (now bulked up with the hair value) rising in all head-using industries in all countries downstream from the initial head-producing country to the final "doll" industry that uses heads. While a change in the first country's hair inputs from the final country's doll industry to the second country's head industry in the direct I-O matrix A, the inverse (direct plus indirect request) matrix B will still capture that the hair arrived via the heads. Due to the less direct route of hair to dolls is raising measured inputs in all subsequent stages, as well as in the trade statistics, this case clearly shows a rise in gross imports and exports for those downstream of the head industry with only a redirection (not an increase) in exports of hair even though value added is constant by assumption.

In online Appendix C, we show how ICIO tables can be constructed for each of the two cases. In each case, our decomposition formula decomposes the gross exports into value added by each source country plus additional double-counted terms. The results are reported in Table 1. Because these are very simple examples, we can also decompose the gross exports in each case by economic intuition (see Table 2). The results are identical to what our decomposition formula generates. However, our analytical formula not only clearly shows the structure of a country's value-added exports (exports in final goods (v1), in intermediate goods (v2), and indirect exports via third countries (v3)), but also further splits the overall double counted amount to what is domestic value added initially exported and eventually

TABLE 1— DECOMPOSITION OF GROSS EXPORTS IN DIFFERENT ORGANIZATION OF GLOBAL PRODUCTION CHAIN FOR SAME FINAL GOODS DELIVERY AND VALUE-ADDED BY SOURCE COUNTRIES

Terms in accounting equation	Term 1 (v1)	Term 2 (v2)	Term 3 (v3)	Term 4 and 5 (v4, v5)	Term 6 (v6)	Term 7 and 8 (v7, v8)	Term 9 (v9)	Total gross exports
Case 1								
C1	0	0	1	0	0	0	0	1
C2	0	0	1	0	0	0	1	2
C3	0	0	1	0	0	0	2	3
C4	0	0	1	0	0	0	3	4
C5	0	1	0	0	0	0	4	5
USA	0	0	0	0	0	0	0	0
Case 2								
USA	0	0	0	10	0	0	0	10
C1	0	0	1	0	0	0	10	11
C2	0	0	1	0	0	0	11	12
C3	0	0	1	0	0	0	12	13
C4	0	0	1	0	0	0	13	14
C5	1	0	0	0	0	14	0	15

Note: The unit is $100K.

TABLE 2—INTUITIVE ACCOUNTING FOR THE GROSS EXPORTS FLOWS IN NUMERICAL EXAMPLE 2

		Value-added input	Intermediate input	Gross output	Value-added exports	Gross exports	Double counting
Case 1							
Stage 1	country 1	100,000	0	100,000	100,000	100,000	0
Stage 2	country 2	100,000	100,000	200,000	100,000	200,000	100,000
Stage 3	country 3	100,000	200,000	300,000	100,000	300,000	200,000
Stage 4	country 4	100,000	300,000	400,000	100,000	400,000	300,000
Stage 5	country 5	100,000	400,000	500,000	100,000	500,000	400,000
Stage 6	USA	1,000,000	500,000	1,500,000	0	0	0
Stage 7	USA consumes all the final goods from stage 6, its own production						
Total		1,500,000	1,500,000	3,000,000	500,000	1,500,000	1,000,000
Case 2							
Stage 1	USA	1,000,000	0	1,000,000	0	1,000,000	1,000,000
Stage 2	country 1	100,000	1,000,000	1,100,000	100,000	1,100,000	1,000,000
Stage 3	country 2	100,000	1,100,000	1,200,000	100,000	1,200,000	1,100,000
Stage 4	country 3	100,000	1,200,000	1,300,000	100,000	1,300,000	1,200,000
Stage 5	country 4	100,000	1,300,000	1,400,000	100,000	1,400,000	1,300,000
Stage 6	country 5	100,000	1,400,000	1,500,000	100,000	1,500,000	1,400,000
Stage 7	USA consumes all the final goods from stage 6 − country 5's exports						
Total		1,500,000	6,000,000	7,500,000	500,000	7,500,000	7,000,000

returning home (v4), what is foreign value added (v7), and what is double counted due to intermediate goods imported from foreign countries (v9). The additional detail provides more information about the structure of both the value-added and the double-counted trade. A more detailed two-country supply chain example is provided in online Appendix D with additional economic intuition.

E. *The General Case of G Countries and N Sectors*

We now discuss the general case with any arbitrary number of countries and sectors. The ICIO model, gross output decomposition matrix, value added by source shares matrix, are given succinctly by block matrix notations:

$$(25)\quad \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_G \end{bmatrix} = \begin{bmatrix} \mathbf{I}-\mathbf{A}_{11} & -\mathbf{A}_{12} & \cdots & -\mathbf{A}_{1G} \\ -\mathbf{A}_{21} & \mathbf{I}-\mathbf{A}_{22} & \cdots & -\mathbf{A}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ -\mathbf{A}_{G1} & -\mathbf{A}_{G2} & \cdots & \mathbf{I}-\mathbf{A}_{GG} \end{bmatrix}^{-1} \begin{bmatrix} \sum_r^G \mathbf{Y}_{1r} \\ \sum_r^G \mathbf{Y}_{2r} \\ \vdots \\ \sum_r^G \mathbf{Y}_{Gr} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} & \cdots & \mathbf{B}_{1G} \\ \mathbf{B}_{21} & \mathbf{B}_{22} & \cdots & \mathbf{B}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{G1} & \mathbf{B}_{G2} & \cdots & \mathbf{B}_{GG} \end{bmatrix} \begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \\ \vdots \\ \mathbf{Y}_G \end{bmatrix}$$

$$(26)\quad \begin{bmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} & \cdots & \mathbf{X}_{1G} \\ \mathbf{X}_{21} & \mathbf{X}_{22} & \cdots & \mathbf{X}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{X}_{G1} & \mathbf{X}_{G2} & \cdots & \mathbf{X}_{GG} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} & \cdots & \mathbf{B}_{1G} \\ \mathbf{B}_{21} & \mathbf{B}_{22} & \cdots & \mathbf{B}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{G1} & \mathbf{B}_{G2} & \cdots & \mathbf{B}_{GG} \end{bmatrix} \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & \cdots & \mathbf{Y}_{1G} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} & \cdots & \mathbf{Y}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y}_{G1} & \mathbf{Y}_{G2} & \cdots & \mathbf{Y}_{GG} \end{bmatrix}$$

$$(27)\quad \mathbf{VB} = \begin{bmatrix} \mathbf{V}_1\mathbf{B}_{11} & \mathbf{V}_1\mathbf{B}_{12} & \cdots & \mathbf{V}_1\mathbf{B}_{1G} \\ \mathbf{V}_2\mathbf{B}_{21} & \mathbf{V}_2\mathbf{B}_{22} & \cdots & \mathbf{V}_2\mathbf{B}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_G\mathbf{B}_{G1} & \mathbf{V}_G\mathbf{B}_{G2} & \cdots & \mathbf{V}_G\mathbf{B}_{GG} \end{bmatrix},$$

with G countries and N sectors, $\mathbf{A}$, and $\mathbf{B}$ are $GN \times GN$ matrices; $\mathbf{V}$ and $\mathbf{VB}$ are $G \times GN$ matrices. $\mathbf{V}_s$ denotes a $1 \times N$ row vector of direct value-added coefficient, $\mathbf{A}_{sr}$ is a $N \times N$ block input-output coefficient matrix, $\mathbf{B}_{sr}$ denotes the $N \times N$ block Leontief inverse matrix, which is the total requirement matrix that gives the amount of gross output in producing country s required for a one-unit increase in final demand in destination country r. $\mathbf{X}_{sr}$ is a $N \times 1$ gross output vector that gives gross output produced in s and absorbed in r. $\mathbf{X}_s = \sum_r^G \mathbf{X}_{sr}$ is also a $N \times 1$ vector that gives country s' total gross output. $\mathbf{Y}_{sr}$ is a $N \times 1$ vector give final goods produced in s and consumed in r. $\mathbf{Y}_s = \sum_r^G \mathbf{Y}_{sr}$ is also a $N \times 1$ vector that gives the global use of s' final goods. Both the gross output decomposition and final demand matrix in equation (26) are $GN \times G$ matrices.

Let $\hat{\mathbf{V}}_s$ be a $N \times N$ diagonal matrix with direct value-added coefficients along the diagonal. (Note $\hat{\mathbf{V}}_s$ has a dimension that is different from $\mathbf{V}_s$.) We can define a $GN \times GN$ diagonal value-added coefficient matrix as

$$\hat{\mathbf{V}} = \begin{bmatrix} \hat{\mathbf{V}}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{V}}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \hat{\mathbf{V}}_G \end{bmatrix}. \tag{28}$$

Using the similar intuition as we used to derive equation (9) in the two-country one sector case, we can obtain domestic value added in a country's gross output by multiplying this value-added coefficient matrix with the right-hand side of equation (26), the gross output decomposition matrix. This will result in a $GN \times G$ value-added production matrix $\hat{\mathbf{V}}\mathbf{BY}$ as

$$\begin{bmatrix} \hat{\mathbf{V}}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{V}}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \hat{\mathbf{V}}_G \end{bmatrix} \begin{bmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} & \cdots & \mathbf{X}_{1G} \\ \mathbf{X}_{21} & \mathbf{X}_{22} & \cdots & \mathbf{X}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{X}_{G1} & \mathbf{X}_{G2} & \cdots & \mathbf{X}_{GG} \end{bmatrix} \tag{29}$$

$$= \begin{bmatrix} \hat{\mathbf{V}}_1 \sum_r^G \mathbf{B}_{1r}\mathbf{Y}_{r1} & \hat{\mathbf{V}}_1 \sum_r^G \mathbf{B}_{1r}\mathbf{Y}_{r2} & \cdots & \hat{\mathbf{V}}_1 \sum_r^G \mathbf{B}_{1r}\mathbf{Y}_{rG} \\ \hat{\mathbf{V}}_2 \sum_r^G \mathbf{B}_{2r}\mathbf{Y}_{r1} & \hat{\mathbf{V}}_2 \sum_r^G \mathbf{B}_{2r}\mathbf{Y}_{r2} & \cdots & \hat{\mathbf{V}}_2 \sum_r^G \mathbf{B}_{2r}\mathbf{Y}_{rG} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\mathbf{V}}_G \sum_r^G \mathbf{B}_{Gr}\mathbf{Y}_{r1} & \hat{\mathbf{V}}_G \sum_r^G \mathbf{B}_{Gr}\mathbf{Y}_{r2} & \cdots & \hat{\mathbf{V}}_G \sum_r^G \mathbf{B}_{Gr}\mathbf{Y}_{rG} \end{bmatrix}.$$

Elements in the diagonal columns give each country's production of value added absorbed at home. As in the two-country case, exports of value added can be defined as the elements in the off-diagonal columns of this $GN \times G$ matrix as

$$VT_{sr} = \hat{\mathbf{V}}_s \mathbf{X}_{sr} = \hat{\mathbf{V}}_s \sum_g^G \mathbf{B}_{sg} \mathbf{Y}_{gr}. \tag{30}$$

Obviously, it excludes value added produced by the home country that returns home after being processed abroad. A country's total value-added exports to the world equal:

$$(31) \qquad VT_{s*} = \sum_{r \neq s}^{G} \mathbf{V}\mathbf{X}_{sr} = \mathbf{V}_s \sum_{r \neq s}^{G} \sum_{g=1}^{G} \mathbf{B}_{sg} \mathbf{Y}_{gr}.$$

By rewriting equation (31) into three groups according to where and how the value-added exports are absorbed, we obtain decomposition as follows:

$$(32)^6 \qquad VT_{s*} = \mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{ss} \mathbf{Y}_{sr} + \mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rr} + \mathbf{V}_s \sum_{r \neq s}^{G} \sum_{t \neq s,r}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rt}.$$

This is the value-added export decomposition equation in terms of all countries' final demands. The first term is value added in the country's final goods exports; the second term is value added in the country's intermediate exports used by the direct importer to produce final goods consumed by the direct importer; and the third term is value added in the country's intermediate exports used by the direct importing country to produce final goods for third countries. Comparing with equation (9), we can see clearly what is missing in our two-country case: it is the re-export of value added via third countries, the last term of the RHS of equation (32), because the distinction between value-added exports from direct and indirect sources only can be made in a setting with three or more countries.

Define a country's gross exports to the world as

$$(33) \qquad \mathbf{E}_{s*} = \sum_{r \neq s}^{G} \mathbf{E}_{sr} = \sum_{r \neq s}^{G} (\mathbf{A}_{sr} \mathbf{X}_r + \mathbf{Y}_{sr}).$$

Using the logic similar to the derivation of equation (11), we can first decompose a country's gross exports to its various components as follows:

$$(34)^7 \quad \mathbf{u}\mathbf{E}_{s*} = \mathbf{V}_s \mathbf{B}_{ss} \mathbf{E}_{s*} + \sum_{r \neq s}^{G} \mathbf{V}_r \mathbf{B}_{rs} \mathbf{E}_{s*}$$

$$= \mathbf{VT}_{s*} + \left\{ \mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rs} + \mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{A}_{rs} \mathbf{X}_s \right\}$$

$$+ \left\{ \sum_{t \neq s}^{G} \sum_{r \neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{Y}_{sr} + \sum_{t \neq s}^{G} \sum_{r \neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{A}_{sr} \mathbf{X}_r \right\}.$$

[6] This value-added exports decomposition could also be done at the bilateral level; however, it is different from equation (15) in Johnson and Noguera (2012), who split bilateral gross exports to three groups.

[7] The step-by-step proof is provided in online Appendix B.

Based on the gross output identity for each country $\mathbf{X}_s = \mathbf{Y}_{ss} + \mathbf{A}_{ss}\mathbf{X}_s + \mathbf{E}_{s*}$, we have,

$$(35) \qquad \mathbf{X}_s = (\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{Y}_{ss} + (\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{E}_{s*}$$

$$\mathbf{X}_r = (\mathbf{I} - \mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + (\mathbf{I} - \mathbf{A}_{rr})^{-1}\mathbf{E}_{r*}.$$

Replace $\mathbf{X}_s$ and $\mathbf{X}_r$ in equation (34), insert equation (32), we obtain the G country, N sector generalized version of gross exports accounting equation as follows:

$$(36) \quad \mathbf{uE}_{s*} = \left\{\mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{ss}\mathbf{Y}_{sr} + \mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{sr}\mathbf{Y}_{rr} + \mathbf{V}_s\sum_{r\neq s}^{G}\sum_{t\neq s,r}^{G}\mathbf{B}_{sr}\mathbf{Y}_{rt}\right\}$$

$$+ \left\{\mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{sr}\mathbf{Y}_{rs} + \mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{sr}\mathbf{A}_{rs}(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{Y}_{ss}\right\}$$

$$+ \mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{sr}\mathbf{A}_{rs}(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{E}_{s*}$$

$$+ \left\{\sum_{t\neq s}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{Y}_{sr} + \sum_{t\neq s}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I} - \mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr}\right\}$$

$$+ \sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}\sum_{r\neq s}^{G}(\mathbf{I} - \mathbf{A}_{rr})^{-1}\mathbf{E}_{r*}.$$

Equation (36) has nine terms. It is very similar to equations (13) and (14) in the two-country case with only one main difference. It has an additional term, representing indirect value-added exports via third countries, in its value-added exports besides the two value-added terms that directly absorbed by the direct importer. Therefore, the first three terms are value-added exports (only two terms in the two-country case). The fourth and fifth terms, in the second bracketed expression, include the source country's value added in both its final and intermediate goods imports, which are first exported but eventually returned and consumed at home, both of which are parts of the source country's GDP but represent a double-counted portion in official gross export statistics, and have a similar economic interpretation as the third and fourth terms in equations (13) and (14). They differ from the two-country case in that we have to account for the domestic value added returning home from each of the G-1 countries here, not just from Country 2. The seventh and eighth terms in the third bracketed expression represent foreign value added (GDP) in the source country's gross exports, including foreign GDP embodied in both final and intermediate goods. They differ from the sixth and seventh terms in equation (13) in the two-country case in that equation (36) further partitions each of the foreign value added (GDP) by individual country sources. There are also two pure double-counted terms, the sixth and the ninth terms in equation (36) as equations (13) and (14), but they sum up the double-counted portions of two way intermediate trade from all bilateral routes, not just between Country 1 and 2. The complete gross exports accounting made by equation (36) is also diagrammed in Figure 1.

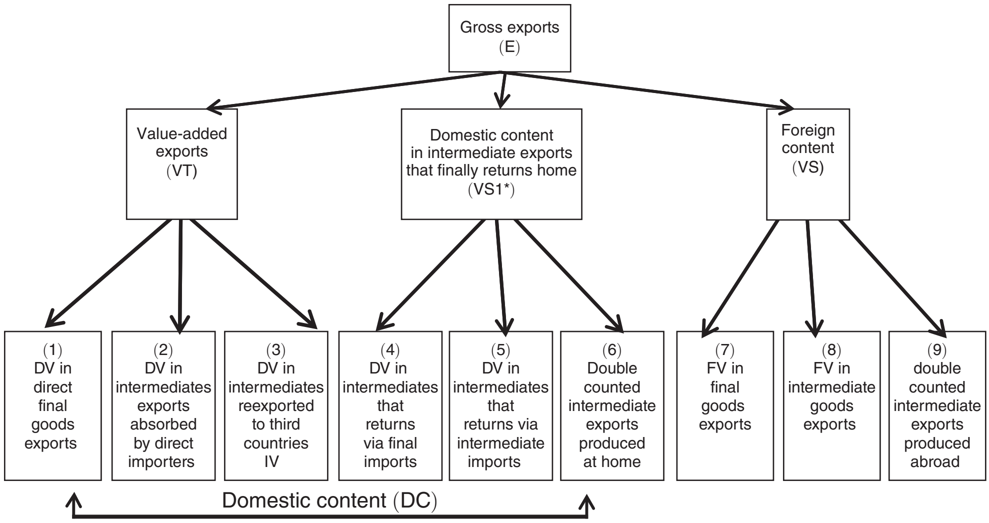

Figure 1. Accounting of Gross Exports: Concepts

Notes: (i) Value-added exports by a country equals (1) + (2) + (3); (ii) GDP in exports equals (1) + (2) + (3) + (4) + (5); (iii) Domestic content in a country's exports equals (1) + (2) + (3) + (4) + (5) + (6); (iv) (7) + (8) + (9) is labeled as VS, and (3) + (4) + (5) + (6) is part of VS1 labeled by HIY (2001); (v) (4) is also labeled as VS1* by Daudin, Rifflart, and Schweisguth (2011); (vi) (4) through (9) involve value added that crosses national borders at least twice, and are the sources of multiple counting in official trade statistics.[8]

Similar to the two-country case, measures of vertical specialization can be expressed as linear combination of the various components identified by equation (36) as follows:

$$(37)\quad DV_s = \mathbf{V}_s(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{E}_{s*}$$
$$= \mathbf{VT}_{s*} + \mathbf{V}_s\sum_{r\neq s}^{\mathbf{G}}\mathbf{B}_{sr}\mathbf{Y}_{rs} + \mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{sr}\mathbf{A}_{rs}(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{Y}_{ss} \geq \sum_{r}^{G} VT_{s*}.$$

These are the first five terms in equation (36) and it clearly shows that a country's domestic value added in its exports is generally greater than its value-added exports in aggregate. The two measures equal each other only in the case where there is no returned domestic value added in imports, i.e., when both $\mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{sr}\mathbf{Y}_{rs}$ and $\sum_{r\neq s}^{G}\mathbf{B}_{sr}\mathbf{A}_{rs}(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{Y}_{ss}$ are zero.

$$(38)\quad VS_s = \sum_{t\neq s}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{Y}_{sr} + \sum_{t\neq s}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I} - \mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr}$$
$$+ \sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}\sum_{r\neq s}^{G}(\mathbf{I} - \mathbf{A}_{rr})^{-1}\mathbf{E}_{r*}$$
$$= \sum_{r\neq s}^{G}\mathbf{V}_r\mathbf{B}_{rs}\mathbf{E}_{s*} = \sum_{t\neq s}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{Y}_{sr} + \sum_{t\neq s}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}\mathbf{X}_r,$$

[8] (3) should not be included in double counting, because when this value crosses a border for the second time, it becomes foreign value in the direct importer's exports. For this reason, it is not included as double counting to avoid an overcorrection.

which is composed of the last three terms in equation (36).

We can verify that equation (38) is reduced to more familiar expressions in some special cases. Using a single country IO model, Koopman, Wang, and Wei (2008, 2012) have shown

$$(39) \qquad VS\ share = \mathbf{u} - \mathbf{A}_v(\mathbf{I} - \mathbf{A}^D)^{-1} = \mathbf{u}\mathbf{A}^M(\mathbf{I} - \mathbf{A}^D)^{-1}.$$

In the *G*-country world,

$$(40)^9 \quad VS\ share = \sum_{r \neq s}^{G} \mathbf{V}_r \mathbf{B}_{rs}$$
$$= \mathbf{u} - \mathbf{V}_s \mathbf{B}_{ss} = \mathbf{u} - \mathbf{V}_s(\mathbf{I} - \mathbf{A}_{ss})^{-1}$$
$$- \sum_{r \neq s}^{G} \mathbf{V}_s \mathbf{B}_{sr} \mathbf{A}_{rs}(\mathbf{I} - \mathbf{A}_{ss})^{-1}.$$

The last term in the last step can be interpreted as the adjustment made for domestic content returned to the source country. Therefore, our foreign content measure of gross exports is a natural generalization of HIY's VS measure in a multi-country setting with unrestricted intermediate goods trade. Because $\sum_{r \neq s}^{G} \mathbf{V}_r \mathbf{B}_{rs} + \mathbf{V}_s \mathbf{B}_{ss} = \mathbf{u}$, it is natural to define a country's domestic content in its exports as:

$$(41) \qquad DC_s = \mathbf{V}_s \mathbf{B}_{ss} \mathbf{E}_{s*}$$
$$= \mathbf{VT}_{s*} + \mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rs} + \mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{A}_{rs} \mathbf{X}_s \geq \mathbf{DV}_s \geq \sum_{r}^{G} \mathbf{VT}_{sr}.$$

This is the sum of the first six terms in equation (36).[10] It shows that a country's domestic content in its exports is generally greater than the part of its GDP in exports, and therefore is also greater than its value-added exports in the aggregate. The three measures equal each other only in the case where there is no returned domestic value in imports, i.e., when both $\mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rs}$ and $\mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{A}_{rs} \mathbf{X}_s$ are zero.

The second HIY measure of vertical specialization (VS1) measures the value of the exported goods that are used as imported inputs by other countries to produce their exports. Although an expression for such indirectly exported products has not been previously defined mathematically in the literature, it can be specified based on some of the terms in our gross exports accounting equations.

$$(42) \qquad VS1_s = \mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{E}_{r*} = \mathbf{V}_s \sum_{r \neq s}^{G} \sum_{t \neq s,r}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rt} + \mathbf{V}_s \sum_{r \neq s}^{G} \sum_{t \neq s,r}^{G} \mathbf{B}_{sr} \mathbf{A}_{rt} \mathbf{X}_t$$
$$+ \mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rs} + \mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{A}_{rs} \mathbf{X}_s.$$

[9] See online Appendix B for the step-by-step proof.
[10] Note that the third term can be further decomposed into two terms (the fifth and sixth terms) in equation (36).

This means HIY's VS1 can be expressed as the third term in (36) plus the third and fifth terms in equation (34). The second term in equation (42) measures how much domestic content in exported goods from the source country is used as imported inputs to produce other countries' intermediate goods exports. It also shows clearly that HIY's VS1 measure is generally greater than indirect value-added exports because the latter only includes the first term of (42), but excludes domestic content that is returned home and the value embodied in intermediate goods exports via third countries (they are already counted as other countries' foreign content in these third countries' exports), i.e.,

$$IV_s = \mathbf{V}_s \sum_{r \neq s}^{G} \sum_{t \neq s,r}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rt} \leq VS1_s. \tag{43}$$

Compared with equation (24) of the definition of VS1 in the two-country case, two additional terms (the first two terms) appear on the RHS of (42) because of the third country effect.

$$VS1_s^* = \sum_{s \neq t} \mathbf{V}_s \mathbf{B}_{sr} \mathbf{E}_{rs} = \mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rs} + \mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{A}_{rs} \mathbf{X}_s. \tag{44}$$

As equation (44) shows, we define $VS1^*$ as a subset of VS1 similar to Daudin, Rifflart, and Schweisguth (2011), but our definition differs from theirs as they include only domestic value added returned home in final goods imports (the first term in equation (44)) but exclude domestic content returned home by being embodied in the imports of intermediate goods, the second term in equation (44).[11] If one omits the second term, then $VS1^*$ would be inconsistent with the core idea of measuring vertical specialization from the export side, as it would fail to account for the source country's exports used by third countries to produce their export of intermediate goods. It would therefore consistently under-estimate actual vertical specialization. To put it differently, the same domestic content embodied in a country's intermediate goods exports manifests itself in international trade flows in two ways: (i) as foreign content in other countries exports, and (ii) as the source country's indirect exports of domestic content via a third country. In other words, the foreign content in one country's exports is the domestic content of another country embodied in its indirect exports. As an example, the Japanese content in the form of Japanese-made computer chips used in China's exports of electronic toys to the United States represents foreign content in China's exports, and it is also simultaneously Japan's indirect exports of its domestic content to the United States. While these two perspectives produce the identical numbers when aggregating across all countries at the global level, their values for a given country can be very different. Therefore, when measuring a country's participation in vertical specialization it is useful to be able to trace the two perspectives separately. This is also why HIY proposed two measures of vertical specialization, because a complete picture of vertical specialization and a county's position in a vertical integrated production network has to involve both measures. Indeed, for a given country, the ratio of the

[11] Note that the second term can be further decomposed into two terms (the fifth and sixth terms) in equation (36).

two measures provides insight for the country's position in global value chains. Downstream countries tend to have a higher share of vertical specialization from the import side, i.e., higher foreign content (VS) in their exports, while upstream countries tend to have a higher share of vertical specialization from export side (VS1), higher share of exports via third countries. In addition, as we show in both equations (13) and (36), ignoring domestic content returning home via intermediate goods imports would leave the gross export accounting incomplete.

Therefore, equation (36) provides a new way of thinking about the gross exports statistics. It demonstrates that various double-counted items in gross exports can be used to gauge the depth of a country's participation in global production chains. Simply stripping away double-counted items and focusing just on value-added trade would miss such useful information. We have already provided numerical examples earlier to illustrate the intuition of this point; now we have laid out the accounting equation in a more general multi-country setting. It provides a transparent framework that allows various value-added and double-counted components in a country's official gross exports statistics to be correctly identified and estimated.

Equation (36) (or Figure 1) also integrates the older literature on vertical specialization with the newer literature on trade in value added, while ensuring that the sum of value-added components from all sources and the additional double-counted intermediate goods exports (due to back-and-forth trade in intermediate goods) yields total gross exports. The previous vertical specialization literature only decomposes gross exports into two components: domestic and foreign content. In comparison, equation (36) shows that a country's domestic content can be further broken up into subcomponents that reveal the destinations for a country's exported value added, including its own value added that returns home in its imports and what is double counted due to cross border intermediate goods trade. Similarly, equation (36) also traces out the foreign content in a country's exports to its sources.

Finally, note that we use a single subscript for the domestic content measure and two subscripts for the trade in value-added measure. This is to suggest that the trade in value-added measure holds for both aggregate and bilateral trade, while the gross export accounting equation we propose only holds for a country's total exports to the world.

II. Data and Results

A. *The Construction of an Inter-Country Input-Output (ICIO) Table and its Data Sources*

To implement the above accounting method, we need an inter-country input output (ICIO) table, that is, a database detailing international production and use for all flows of value added. The database should specify (i) transactions of intermediate products and final goods within and between each country at the industry level, (ii) the direct value added in production of each industry in all countries, and (iii) the gross output of each industry in all countries. Such an ICIO table goes beyond a collection of single-country IO tables. It specifies the origin and destination of all transaction flows by industry as well as every intermediate and/or final use for all such flows. However, these tables are not available on a global basis, and in fact are rarely available at the regional level. The available global databases, such as the GTAP Multi-Country Input-Output (MCIO) tables, do not have enough detail on the cross-border supply and the use of goods to be directly used to implement our methodological framework.

To provide a workable dataset and empirically conduct gross export decomposition, we construct a global ICIO table for 2004 by integrating the GTAP database (version 7) and the additional information from UN COMTRADE using a quadratic mathematical programming model. The model (i) minimizes the deviation of the resulting new dataset from the original GTAP data, (ii) ensures that supply and use balance for each sector and every country, and (iii) keeps all sectoral bilateral trade flows in the GTAP database constant. The new database covers 26 countries and 41 sectors.[12]

To estimate these detailed inter-industry and inter-country intermediate flows, we need to (i) separate gross bilateral trade flows at the sector level in the GTAP database into intermediate, consumption, and investment goods trade flows, and (ii) allocate intermediate goods from a particular country source to each sector it is used within all destination countries. We address the first task by concording the three end-use categories defined by UN Broad Economic Categories (BEC) to the 6-digit HS level bilateral trade data in COMTRADE. This differs from Johnson and Noguera (2012) and Daudin, Rifflart, and Schweisguth (2011), who also transform the MCIO table in the GTAP database into an ICIO table. However, they do not use detailed trade data to identify intermediate goods in each bilateral trade flow, but apply a proportionality method directly to the GTAP trade data; i.e., they assume that the proportion of intermediate to final goods is the same for domestic supply and imported products.

The use of end-use categories to distinguish imports by use is becoming more widespread in the literature and potentially avoids some noted deficiencies in the proportionality method. Feenstra and Jensen (2012) use a similar approach to separate final goods from intermediates in US imports in their recent re-estimation of the Feenstra-Hanson measure of material off-shoring. Dean, Fung, and Wang (2011) show that the proportionality assumption underestimates the share of imported goods used as intermediate inputs in China's processing trade. Nordås (2005) stated that the large industrial countries have a higher share of intermediates in their exports than in their imports, while the opposite is true for large developing countries. These results imply that the intermediate content of imports differs systematically from the intermediate content in domestic supply.

In theory, less distorted intermediate share estimates will provide a better row total control for each block matrix of A_{sr} in the ICIO coefficient matrix A, thus improving the accuracy of the most important parameters (the inter-country IO coefficients) in an ICIO model. This is why distinguishing imports by BEC may improve ICIO database quality over the proportionality assumption. To precisely assess whether and how much the BEC method improves over the proportionality approach, one would need the true inter-country IO coefficients as a "reference point," which unfortunately do not exist on a global scale. We will provide some evidence that the

[12] Tsigas, Wang, and Gehlhar (2012) provide details on the construction of the database. The longer working paper lists the countries/regions and sectors in our sample and their concordance to the GTAP database.

TABLE 3—ACCOUNTING OF GROSS EXPORTS, 2004

	Value-added exports			Domestic VA returns home		Pure double counting	Foreign VA returned to foreign countries	
Country/region	Direct final exports (1)	Intermediate absorb by direct importers (2)	Int. re-exports to third countries (3)	Final exports (4)	Intermediate exports (5)	Intermediate exports produced in home (6)	Final exports (7)	Int. exports (8)
Advanced economies								
Aus-New Zealand	27.0	50.5	10.5	0.2	0.2	0.1	3.7	5.1
Canada	23.5	43.0	4.0	0.5	0.4	0.5	12.8	12.0
Western EU	38.1	38.1	5.0	3.1	3.4	0.9	4.8	4.3
Japan	38.4	34.2	12.2	1.4	1.0	0.5	4.8	3.7
United States	32.5	36.6	5.5	6.8	4.5	1.2	4.3	4.7
Asian NICs								
Hong Kong	27.2	35.7	9.1	0.4	0.1	0.1	10.2	9.9
Korea	29.5	25.3	10.4	0.3	0.2	0.4	13.1	9.9
Taiwan	19.2	25.9	13.1	0.2	0.1	0.5	12.4	12.5
Singapore	11.0	20.1	5.2	0.1	0.1	0.4	17.0	24.2
Emerging Asia								
China normal	44.2	31.8	8.1	0.4	0.7	0.1	6.8	5.2
China processing	28.8	12.6	1.7	0.0	0.0	0.3	34.1	12.4
China total	36.5	22.2	4.9	0.2	0.3	0.2	20.5	8.8
Indonesia	20.0	45.6	10.9	0.2	0.3	0.2	9.2	7.2
Malaysia	16.7	31.5	10.4	0.3	0.1	0.6	12.9	13.6
Philippines	17.6	27.8	12.4	0.1	0.0	0.2	10.8	14.0
Thailand	27.9	24.2	7.9	0.1	0.1	0.2	17.2	11.3
Vietnam	32.9	24.9	4.8	0.0	0.2	0.2	24.4	7.7
India	30.2	41.7	7.7	0.1	0.2	0.1	6.4	9.5
Other emerging								
Brazil	27.4	52.1	7.5	0.1	0.1	0.1	3.9	6.4
New EU countries	28.7	35.2	4.3	0.4	0.3	0.3	13.4	13.2
Mexico normal	23.5	52.7	5.9	0.1	0.5	0.1	2.9	11.5
Mexico processing	20.6	13.3	2.4	0.0	0.0	0.3	37.6	18.2
Mexico total	21.6	26.5	3.6	0.1	0.2	0.2	26.0	16.0
Russian	9.5	70.2	9.4	0.3	0.2	0.1	1.6	5.9
South Africa	23.1	50.0	8.5	0.1	0.1	0.1	5.3	8.9
World average	29.2	38.4	6.8	1.9	1.5	0.6	8.7	7.7

(*Continued*)

two alternative ways to identify intermediate goods from bilateral trade flows in the literature have a significant impact on trade cost when the magnification effects of multi-stage production are taken into account in one of our application examples.

B. *Complete Accounting of Gross Exports*

Table 3 presents a complete accounting of each country's gross exports in 2004 using equation (36). The column numbers correspond to the order of each item in the equation and also the box numbers in Figure 1. The first three columns also correspond to the three terms in the RHS of equation (32).

We compute all nine terms independently and verify that they sum to exactly 100 percent of gross exports. The resulting estimates constitute the first such

Table 3—Accounting of Gross Exports, 2004 (*Continued*)

	Pure double counting	Connection with existing measures						
Country/region	Intermediate exports produced abroad (9)	VAX ratio (10)	VS share (11)	VS1 share (12)	DC share (13)	Double count share (14)	Share of vertical trade (15)	Gross exports in billions of US dollars (16)
Advanced economies								
Aus-New Zealand	2.8	88.0	11.5	27.9	88.5	12.0	39.4	122.5
Canada	3.3	70.5	28.1	12.2	71.9	29.5	40.4	323.0
Western EU	2.4	81.1	11.4	20.9	88.6	18.9	32.3	1,575.5
Japan	3.7	84.9	12.2	30.8	87.8	15.1	43.1	618.9
United States	4.0	74.6	12.9	27.0	87.1	25.4	39.9	1,062.3
Asian NICs								
Hong Kong	7.4	71.9	27.5	19.5	72.5	28.1	47.0	121.7
Korea	10.8	65.2	33.9	23.2	66.1	34.8	57.0	283.1
Taiwan	16.1	58.2	41.1	27.1	58.9	41.8	68.2	219.8
Singapore	22.1	36.3	63.2	12.8	36.8	63.7	76.0	150.6
Emerging Asia								
China normal	2.7	84.2	14.6	20.7	85.4	15.8	35.3	334.6
China processing	10.1	43.1	56.6	4.3	43.4	56.9	60.9	336.0
China total	6.4	63.6	35.7	12.5	64.3	36.4	48.1	670.6
Indonesia	6.5	76.5	22.9	29.0	77.1	23.5	51.9	86.7
Malaysia	14.0	58.6	40.5	25.0	59.5	41.4	65.5	152.0
Philippines	17.1	57.8	41.9	29.4	58.1	42.2	71.2	50.1
Thailand	11.2	60.0	39.7	18.5	60.3	40.0	58.1	119.4
Vietnam	4.9	62.6	37.0	14.8	63.0	37.4	51.8	32.3
India	4.2	79.6	20.1	18.9	79.9	20.4	39.0	99.9
Other emerging								
Brazil	2.4	87.0	12.7	19.2	87.3	13.0	31.9	113.0
New EU countries	4.2	68.3	30.8	11.4	69.2	31.7	42.1	273.7
Mexico normal	2.8	82.1	17.3	18.0	82.7	17.9	35.3	63.6
Mexico processing	7.6	36.3	63.4	5.9	36.7	63.7	69.2	126.9
Mexico total	6.0	51.6	48.0	9.9	52.0	48.4	57.9	190.5
Russian	2.7	89.1	10.2	31.2	89.8	10.9	41.4	160.2
South Africa	4.0	81.6	18.2	24.2	81.8	18.4	42.4	61.4
World average	5.1	74.4	21.5	21.5	78.5	25.6	43.0	7,733.4

Note: All columns are expressed as a share of total gross exports.

Source: Authors' estimates.

decomposition in a global setting and clearly highlight what is double counted in the official trade statistics. Column 15 reports the percentage of double counting by adding columns 4 through 9. At the global level, only domestic value added in exports absorbed abroad are value-added exports. In addition to foreign content in exports, domestic content that returns home from abroad is also a part of double counting in official trade statistics, since it crosses borders at least twice. Such returned value added has to be separated from domestic value added absorbed abroad in order to fully capture multiple counting in official trade statistics. Therefore, for any country's gross exports, the double-counting portion equals the share of gross exports in excess of the value-added exports. We estimate this share to be about 25.6 percent of total world exports in 2004.

The accounting results reported in Table 3 also provide a more detailed breakdown of domestic content in exports than has been previously available in the literature.

The variations in the relative size of different components across countries provide a way to gauge the differences in the role that countries play in global production networks. For example, for the United States, the share of foreign value added in its exports was only 9 percent (columns 7 + 8), while its own GDP first exported then finally return home is large at 11.3 percent (columns 4 + 5), indicating that most of its exports reflect its own domestic value added. In comparison, for China's and Mexico's processing exports, the share of foreign GDP in their gross exports is 46.5 percent and 55.8 percent respectively, with an additional 10.1 percent and 7.6 percent of their gross exports coming from intermediate goods produced in foreign countries (column 9), indicating domestic value added accounts for less than half the value of both countries' processing exports. More importantly, about half of the double counting in US exports reflected as 1-VAX ratio (column 11) in column 15 comes primarily from its own domestic content returning home via imports (12.4 percent over 25.4 percent). In contrast, almost all of the double counting in China's and Mexico's processing exports comes from imported foreign content (56.6 percent over 56.9 percent and 63.4 over 63.7 respectively). These calculations highlight US exporters and Chinese and Mexican processing exporters' respective positions at the head and tail of global production chains.

The structure of double-counted terms in each country's gross exports listed in columns 4 through 9 offers additional information on how each country participates in vertical specialization and its relative position in global production chains. For example, for Western EU countries, about 40 percent of its double-counted gross exports come from domestic content returned home (7.4/18.9). In comparison, for most developing countries, the foreign content tends to dominate, with only a very tiny portion of their domestic content returning home. Within foreign content, Maquiladora producers in Mexico, export processing zones in China and Vietnam, tend to have a large portion embodied in their final goods exports (37.6 percent, 34.1 percent, and 24.4 percent, respectively), reflecting their position as the assemblers of final goods in global production chains. For developed and newly industrialized economies, the shares in intermediate goods exports and the pure double-counted portion due to multiple border crossing intermediate goods trade are much higher. Similarly, upstream natural resource producers such as Australia and New Zealand, Russia, Indonesia, and Philippines, have a significant portion of their intermediate exports used by other countries to produce their intermediate goods exports. This is also true for upstream producers of manufacturing intermediates such as Japan.

To reiterate the connection of the nine basic components reported in Table 3 to measures in the existing literature by numerical estimates, column 11 reports the value-added to gross exports (VAX) ratio proposed by Johnson and Noguera (2012) by adding up columns 1, 2, and 3; column 12 lists HIY's VS share by summing columns 7, 8, and 9. Column 14 lists the share of domestic content extensively discussed in the vertical specialization literature by summing columns 1 through 6; Finally, column 16 gives the share of vertical trade by adding columns 12 and 13, which is an indicator of how intensively a country is participating in the global production chain.

Comparing domestic content share estimates (column 14) and Johnson and Noguera's VAX ratio (column 11) reported in Table 3, we see interesting differences between high-income countries and emerging market economies. For most emerging market economies, the numerical difference of these two measures is quite small. This means that only a tiny part of domestic content returns home for most countries. In comparison, for the United States, Western Europe, and Japan, the difference between domestic content share and the value-added export share is more significant. This reflects the fact that advanced economies export relatively more upstream components, and some of the value added embedded in these intermediate goods returns home as part of other countries' exports to the advanced economies. Such differences between high-income countries and emerging market economies would not be apparent if one does not compute the domestic content share and the VAX ratio separately.

III. Broad Implications for a Better Understanding of Global Trade

The decomposition results have implications for a variety of research and policy questions. In this section, to illustrate the potential importance of the decomposition, we briefly discuss a few applications.

A. *Revealed Comparative Advantage Index Based on Gross and Value-Added Trade*

The concept of revealed comparative advantage (RCA), proposed by Balassa (1965), has proven useful in many research and policy applications. In standard applications, it is defined as the share of a sector in a country's total gross exports relative to the world average of the same sector in world exports. When the RCA exceeds one, the country is said to have a revealed comparative advantage in that sector; when the RCA is below one, the country is said to have a revealed comparative disadvantage in that sector. The problem of double counting of certain value-added components in the official trade statistics suggests that the traditional computation of RCA could be noisy and misleading. The gross exports accounting exercise we proposed in this paper provides a way to remove the distortion of double counting by focusing on domestic value added in exports. Because domestic value added or GDP in a country's exports describes the characteristics of a country's production (total domestic factor content in exports), it does not depend on where the exports are absorbed. For those applications in which a production-based RCA is the right measure, we can use GDP in exports to compute RCA.

We re-compute the RCA index at the country-sector level for all the countries and sectors in our database. Due to space constraints, we select two sectors and compare the country rankings of RCAs using both gross exports and domestic value added in gross exports. In Figure 2, panel A we report the two sets of RCA indices for the finished metal products sector in 2004. Using gross exports data, both China and India show a strong revealed comparative advantage (ranked the first and fourth, respectively, among the set of countries in our database, and with the absolute values of RCA at 1.94 and 1.29, respectively). However, when looking at domestic value added in that sector's exports, both countries ranking in RCA drop precipitously to seventh and fifteenth place, respectively.[13] In fact, for India, the sector has switched

[13] Sectoral value added here includes value produced by the factors of production employed in the finished metal products sector and then embodied in gross exports of all downstream sectors, rather than the value-added

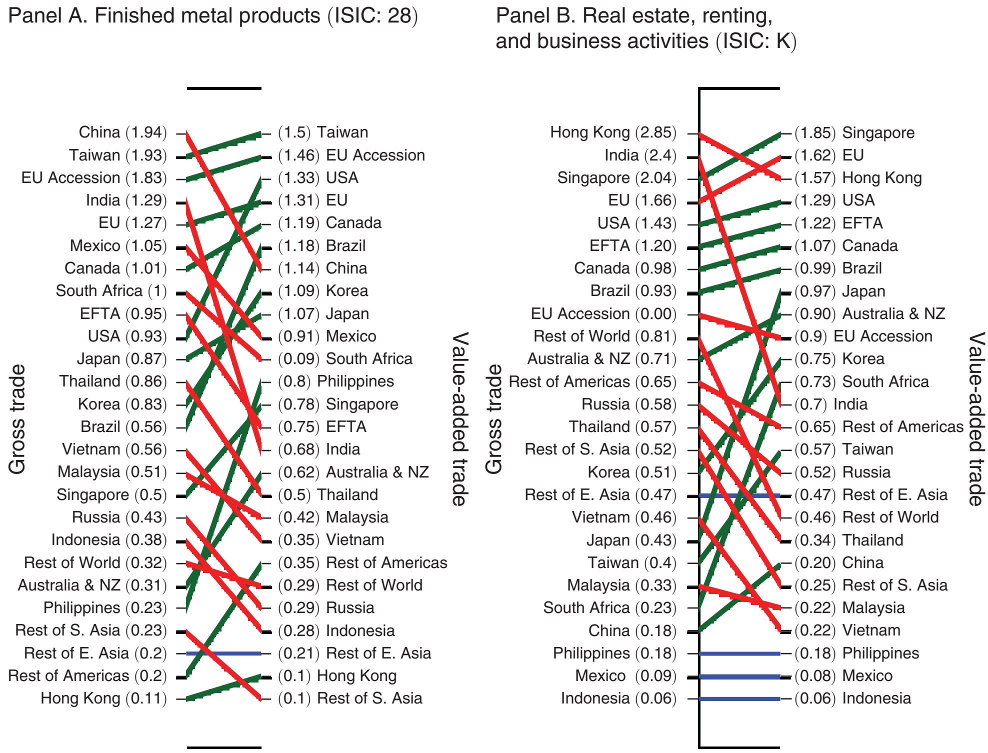

Figure 2. Value-Added-Adjusted Revealed Comparative Advantage Indicators

from being labeled as a comparative advantage sector to a comparative disadvantage sector. Unsurprisingly, the ranking for some other countries move up. For example, for the United States, not only does its RCA ranking move up from tenth place under the conventional calculation to the third place under the new calculation, but its finished metal products industry also switches from being labeled as a comparative disadvantage sector to a comparative advantage sector.

Another example is the "real estate and business services" sector (Figure 2, panel B). Using data on gross exports, India exhibits a strong revealed comparative advantage in that sector on the strength of its unusually high share of business services exports in its overall exports. However, once we compute RCA using domestic value added in exports, the same sector becomes a comparative disadvantage sector for India! One key reason for the change is that business services in advanced countries are often exported indirectly by being embedded in these countries manufacturing exports. Indeed, the RCA rankings for the United States, the European Union, and Japan all move up using data on the domestic value added in exports. Therefore, after taking into account indirect value-added exports, the Indian share of the sector in its exports becomes much less impressive (relative to other countries).

employed in upstream sectors that are used to produce finished metal products in the exporting country. This distinction is particularly important in the business services sector, discussed next.

These examples illustrate the possibility that our understanding of trade patterns and revealed comparative advantage could be modified substantially once we have the right statistics.

B. *Magnification of Trade Costs from Multi-Stage Production*

As noted by Yi (2003, 2010), multi-stage production magnifies the effects of trade costs on world trade. There are two separate magnification forces. The first exists because goods that cross national borders multiple times incur tariffs and transportation costs multiple times. The second exists because tariffs are applied to gross imports, even though value added by the direct exporter may be only a fraction of this amount. Different ways of participating in global production chains affects the extent to which different countries are affected by such cost magnification. However, Yi (2003) does not actually measure the magnification of tariffs, though it is important to his simulation exercise.

Our accounting method provides an ideal way to re-examine the magnification issue. In online Appendix E, we provide an illustrative calculation of the first order magnification effect of using imported intermediate inputs to produce exports. It represents the magnification effect if tariffs were the only factor that augments the trading costs. For instance, we find that one additional stage of production increases trade costs of Vietnam's merchandise production by 80 percent of its standard tariff. Generally speaking, the magnification effect is stronger for emerging market economies than for developed economies due to the lower domestic value-added share in most developing countries' manufacturing exports. In the same Appendix, we also show that global ICIO databases constructed by the BEC end-use classification versus the proportionality assumption produce different trade cost magnification effects for a number of countries.

C. *Other Applications*

The set of examples discussed so far certainly does not exhaust the possible applications. For example, the Federal Reserve Board and the IMF routinely compute effective exchange rates using trade weights that are based on gross exports and imports. A conceptually better measure should weight trading partners based on the relative importance in value-added trade rather than in gross trade terms. Our decomposition results make it feasible to do such computations.

As another example, for some research or policy questions, one might need to look at the response of a country's bilateral or multilateral trade to exchange rate changes. Once one recognizes that there is a potential mismatch between trade in value added and trade in gross terms, one would want to take this into account. Our decomposition allows for a correction.

As a further example, because a country's gross exports embed value added from other countries, its bilateral trade balance in value-added terms can be very different from that in gross trade terms. By our estimation, because China is the final assembler in a large number of global supply chains, and it uses components from many other countries, especially East Asian countries, its trade surplus with the United States and Western EU countries measured in value-added terms is 41 percent and

49 percent less than that measured in gross terms in 2004. In contrast, Japan's trade surplus with the United States and Western EU countries are 40 percent and 31 percent larger measured in value-added terms, because Japan exports parts and components to countries throughout Asia that are eventually assembled into final products there and exported to the United States and Western EU countries.

IV. Conclusions

We have developed a unified conceptual framework that can fully account for a country's gross exports by its various value-added and double-counted components. This new framework incorporates all previous measures of vertical specialization and value-added trade in the literature while adjusting for the back-and-forth trade of intermediates across multiple borders. With a full concordance between value-added and double-counted components and official gross trade statistics, it opens the possibility for the System of National Accounts (SNA) to accept the concept of value-added trade without dramatically changing the current practice of customs trade data collection. This may in turn provide a feasible way for international statistical agencies to report value-added trade statistics regularly in a relatively low cost fashion.

The creation of a database that encompasses detailed global trade in both gross and value-added terms will allow us to move from a largely descriptive empirical exercise to analysis of the causes and consequences of differences in supply chain participation. We show that conventional analyses involving revealed comparative advantage, bilateral trade balance, trade cost or effective real exchange rates can all benefit from our gross exports accounting results.

Better information at the sector level can improve our estimation. For instance, current end use classifications, such as the UN BEC, need to be extended to dual used products and services trade. In addition, methods also need to be developed to properly distribute imports to domestic users either based on cross country statistical surveys or based on firm level and customs transaction-level trade data. We leave such sector level applications to future research.

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