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Tracing Value-added and Double Counting in Gross Exports
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ABSTRACT

This paper proposes a framework for gross exports accounting that breaks up a country's gross exports into various value-added components by source and additional double counted terms. By identifying which parts of the official trade data are double counted and the sources of the double counting, it bridges official trade (in gross value terms) and national accounts statistics (in value added terms). Our parsimonious framework integrates all previous measures of vertical specialization and value-added trade in the literature into a unified framework. To illustrate the potential of such a method, we present a number of applications including re-computing revealed comparative advantages and the magnifying impact of multi-stage production on trade costs.

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1. Introduction

As different stages of production are now regularly performed in different countries, intermediate inputs cross borders multiple times. As a result, traditional statistics on trade values become increasingly less reliable as a gauge of the value contributed by any particular country. This paper integrates and generalizes the many attempts in the literature at tracing value added by country and measuring vertical specialization in international trade. We provide a unified conceptual framework that is more comprehensive than the current literature. By design, this is an accounting exercise, and does not directly examine the causes and the consequences of global production chains. However, an accurate and well-defined conceptual framework to account for value added by source country from available data is a necessary step toward a better understanding of all these issues.

Supply chains can be described as a system of value-added sources and destinations. Within a supply chain, each producer purchases inputs and then adds value, which is included in the cost of the next stage of production. At each stage in the process, as goods cross an international border, the value-added trade flow is equal to the value added paid to the factors of production in the exporting country. However, as all official trade statistics are measured in gross terms, which include both intermediate inputs and final products, they "double count" the value of intermediate goods that cross international borders more than once. Such a conceptual and empirical shortcoming of gross trade statistics, as well as their inconsistency with the System of National Accounts (SNA) accounting standards, has long been recognized by both the economics profession and policy makers. [1]

Case studies on global value chains based on detailed micro data for a single product or a single sector in industries such as electronics, apparel, and motor vehicles have provided detailed examples of the discrepancy between gross and value-added trade. According to a commonly cited study of the Apple iPod (Dedrick, Kraemer, and Linden, 2008), while the Chinese factory gate price of an assembled iPod is $144, only $4 constitutes Chinese value added. Other case studies of specific products show similar discrepancies. Case studies, while enhancing our intuitive understanding of global production chains in particular industries, cannot offer a comprehensive picture of the gap between value-added and gross trade, and an economy's participation in cross-border production chains. Several researchers have examined the issue of

[1] See, for example, Leamer et al. (2006), Grossman and Rossi-Hasberg(2008), and Lamy (October 2010).

vertical specialization on a systematic basis, including the pioneering effort of Hummels, Ishii, and Yi (2001) (HIY in subsequent discussion). They suggested that a country can participate in vertical specialization in two ways: (a) uses imported intermediate inputs to produce exports; (b) exports intermediate goods that are used as inputs by other countries to produce goods for export. HIY proposed to measure the imported foreign content in a country's exports based on a country's Input-Output (IO) table, which they label as VS (vertical specialization). For a sample of 11 OECD and 3 non-OECD countries, they calculated that the average share of foreign content in exports was about 21% in 1990.

There are two key assumptions in HIY's foreign content (VS) estimation: the intensity in the use of imported inputs is the same between production for exports and production for domestic sales; and imports are 100% foreign sourced. The first assumption is violated in the presence of processing exports, which is a significant portion of exports for a large number of developing countries (Koopman, Wang and Wei, 2008 and 2012). The second assumption does not hold when there is more than one country exporting intermediate goods.

There is a growing literature in recent years to estimate value-added trade with the advent of global Inter-Country Input-Output (ICIO) tables based on the Global Trade Analysis Project (GTAP) and World Input-Output database (WIOD)[2], such as Daudin, Rifflart, and Schweisguth (2011), Johnson and Noguera (2012) and Foster, Stehrer and de Vries (2011). These papers discuss the connections between their works with HIY, but they are more closely related to the factor content trade literature. Our paper is the only one in this recent literature to consistently generalize HIY's original concepts to a global setting and make HIY a special case of a more general framework. By integrating the literature on vertical specialization and the literature on value added trade, this paper expands upon the previous literature in the following five aspects:

First, we provide a unified and transparent mathematical framework to completely decompose gross exports into its various components, including exports of value added, domestic value added that returns home, foreign value added, and other additional double counted terms. Measures of vertical specialization and value-added trade in the existing literature

[2] Though usefully global in scope, the GTAP database does not separate imported intermediate and final goods in bilateral trade flows, so improvements have to be made. WIOD is a European Commission sponsored research project to produce better and more up-to-date global ICIO tables, based on a compilation of single-country supply and use tables and detailed bilateral trade statistics for the years 1995-2009. The framework in this paper can be applied to generate a time series decomposition of gross trade flows into their value added and double counted components based on WIOD World IO tables.

all can be derived from this framework and expressed as some linear combinations of these components. We show why some of the existing measures are special cases of the generalized measures in our framework and why some of them have to be modified from their original definitions in a more general multi-country framework with unrestricted intermediate trade.

Second, rather than simply excluding double counted items from official trade statistics, we provide an accounting formula that quantifies different types of double counted items for the first time in the literature. Knowing the structure of the double counted items in a country's gross exports can help us to gauge the depth and pattern of that country's participation in global production chains. In other words, the relative importance of the various double-counted terms in addition to value-added trade estimates often contains useful and important information. For example, in some sectors, China and the United States may have a similar amount of value added exports. However, the composition of the double counted terms can be very different for the two countries. For China, the double counted terms may show up primarily in the form of the use of foreign components (e.g., foreign product designs or machinery) in the final goods that China exports. For the United States, the double counted terms may show up primarily in the form of domestic value added finally returned and consumed at home (e.g., product designs by Apple that is used in the final Apple products produced abroad but sold in the U.S. market). The structure of these double counted items in addition to their total sums offer additional information about the U.S. and China's respective positions in the global value chain.

In addition, we differentiate the double counting terms relative to value-added exports in a country's gross exports into different types according to whether they should be accounted as part of a country's GDP and show how they can be quantified, and explain the role they play in the subtle differences among three related concepts (domestic content in a country's exports, value added in exports, and exports of value added) that so far have not always been clearly distinguished in the literature.

Third, our accounting framework establishes a formal and precise relationship between value-added measures of trade and official trade statistics, thus providing an observable benchmark for value-added trade estimates, as well as a workable way for national and international statistical agencies to remedy the missing information in current official trade statistics without dramatically changing the existing data collection practices of national customs authorities.

Fourth, our estimated global ICIO table may better capture the international source and use of intermediate goods than in previous databases in two ways. In estimating intermediate goods in bilateral trade, we use end-use classifications (intermediate or final) of detailed import statistics rather than the conventional proportionality assumptions. In addition, we estimate separate input-output coefficients for processing trade in China and Mexico, the two major users of such regimes in the world. While other studies have used a similar correction for Chinese exports, the new Mexican IO table provides improved accuracy in estimates of NAFTA trade flows by distinguishing domestic and Maquiladora production.

Finally, we report a number of applications of our accounting framework and database to illustrate their potential to reshape our understanding of global trade. For example, with gross trade data, the business services sector is a revealed comparative advantage sector for India. In contrast, if one uses our estimated domestic value added (GDP) in exports instead, the same sector becomes a revealed comparative disadvantage sector for India. The principal reason for this is how the indirect exports of business services are counted in high-income countries. Consider Germany. Most of its manufacturing exports embed lots of German domestic business services. In comparison, most of Indian goods exports use comparatively little Indian business services. Once indirect exports of domestic business services are taken into account, Indian's business service exports become much less impressive relative to Germany and many other developed countries. As another example, the value added decomposition shows that a significant portion of China's trade surplus to the United States in gross trade terms reflects indirect value added exports that China does on behalf of Japan, Korea and Taiwan. While such stories have been understood in qualitative terms, our framework offers a way to quantify these effects.

This paper is organized as follows. Section 2 presents the conceptual framework of gross exports accounting. Section 3 discusses database construction methods. In particular, we show how the required inter-country IO model can be estimated from currently available data sources and report major empirical decomposition results for the year 2004. Section 4 presents a number of applications that help to illustrate how our gross exports accounting framework may alter our understanding of issues in international trade and in open-economy macroeconomics. Section 5 concludes.

2. Gross Exports Accounting: Concepts and Measurement

In this section, we first lay out the main measures of vertical specialization and trade in value-added as they are originally proposed in the literature. We highlight a key conceptual difference that separates some measures from others, namely when it is appropriate to include double-counted items for some purposes but not for others.

We then propose a way to fully decompose a country's gross exports into the sum of components that include both the country's value added exports and various double-counted components. We further differentiate these double counted terms into different types.

2.1 Concepts

Four measures have been proposed in the vertical specialization and value-added trade literature:

1. HIY (2001) proposed a measure of vertical specialization from the import side, which is the imported content in a country's exports. We follow HIY and label it as VS. It includes both the direct and indirect imported input content in exports. However, HIY only considered the case in which the Home country does not export intermediary goods though it imports intermediary goods from the rest of the world. In mathematical terms, a country's VS in total exports at the sector level can be expressed as[3] :

$$VS = A^M (I - A^D)^{-1} E \tag{1}$$

2. HIY (2001) also proposed a second measure of vertical specialization from the export side (which they call VS1). It measures the value of intermediate exports sent indirectly through third countries to final destinations. However, they did not provide a mathematical definition for VS1 as they did for VS.

3. Daudin et al (2011) singled out a particular subset of VS1, the value of a country's exported goods that are used as imported inputs by the rest of the world to produce final goods and shipped back to home. They call it VS1*.

4. Johnson and Noguera (2012) defined value-added exports as value-added produced in source country s and absorbed in destination country r and proposed using value-added to gross export ratio, the "VAX ratio" as a summary measure of the value-added content of trade.

[3] *D. Hummels et al.* , *Journal of International Economics* 54 (2001) page 80.

By definition, as value-added is a "net" concept, double counting is not allowed. As the first three measures of vertical specialization all involve values that show up in more than one country's gross exports, they, by necessity, have to include some double-counted portions of the official trade statistics. More border crossings by intermediate goods (more double counting) means a larger difference between trade in value-added and these vertical specialization measures. This implies that these two type measures are not equal to each other in general because double counting is only allowed in one of them. They equal each other only in some special cases as we will show later both analytically and numerically.

In addition, these existing measures are all proposed as stand-alone indicators. No common mathematical framework proposed in the literature provides a unified accounting for them and spells out their relationships explicitly. More importantly, as noted earlier, the most widely used HIY measure (VS) needs two strong assumptions and is only valid in special cases; there is no mathematically specified measure for indirect exports through third countries, and all four measures proposed so far do not identify all components in gross exports.

We provide below a unified framework that breaks up a country's gross exports into the sum of various well defined components. The value added exports, VS, VS1, and VS1* are linear combinations of these components. In addition, we show how one may generalize the VS measure without the restrictive assumption made by HIY (no two-way trade in intermediate goods). By properly including various double counted terms, our accounting is complete in the sense that the sum of these well-identified components yields 100% of the gross exports.

For ease of understanding, we start with a discussion of a two-country one sector case in Section 2.2. We relate the components of our decomposition formula with the existing measures in the literature in Section 2.3. We provide several numerical examples in Section 2.4 to show intuitively how our gross exports accounting equation works. Finally, we present the most general G-country M-sector case.

2.2 Two-country case

Assume a two-country (home and foreign) world, in which each country produces in a single tradable sector. The good in that sector can be consumed directly or used as an intermediate input, and each country exports both intermediate and final goods to the other.

All gross output produced by country *r* must be used as either an intermediate good or a final good, either at home or abroad. So country s's gross output, x_s, has to satisfy the following accounting relationship:

$$x_s = a_{ss}x_s + a_{sr}x_r + y_{ss} + y_{sr}\,, r,s = 1,2 \qquad (2)$$

Where y_{sr} is the final demand in country *r* for the final good produced in Country *s*, and a_{sr} is the input-output (IO) coefficient, describing units of intermediate goods produced in *s* used in the production of one unit gross output in Country r. The two-country production and trade system can be written as an inter-country input-output (ICIO) model as follows

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} y_{11} + y_{12} \\ y_{21} + y_{22} \end{bmatrix}, \qquad (3)$$

With rearranging, we have

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} I - a_{11} & -a_{12} \\ -a_{21} & I - a_{22} \end{bmatrix}^{-1} \begin{bmatrix} y_{11} + y_{12} \\ y_{21} + y_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}. \qquad (4)$$

Since equation (4) takes into account both the direct and indirect use of a country's gross output as intermediate goods in the production of its own and foreign final goods, the coefficients in the B matrix (Leontief inverse) are referred to as "total requirement coefficients" in the input-output literature. Specifically, b_{11} is the total amount of Country 1's gross output needed to produce an extra unit of the final good in Country 1 (which is for consumption in both Countries 1 and 2); b_{12} is the total amount of Country 1's gross output needed to produce an extra unit of the final good in Country 2 (again for consumption both at home and abroad). Similar interpretations can be assigned to the other two coefficients in the B matrix.

We can break up each country's gross output according to where it is ultimately absorbed by rearranging both countries' final demand into a matrix format by source and destination, and rewrite equation (4) as follows:

$$\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} b_{11}y_{11} + b_{12}y_{21} & b_{11}y_{12} + b_{12}y_{22} \\ b_{21}y_{11} + b_{22}y_{21} & b_{21}y_{12} + b_{22}y_{22} \end{bmatrix} \qquad (5)$$

where y_{sr} is as defined in equation (2), giving the final goods produced in country s and consumed in country r. This final demand matrix in the middle of equation (5) is a 2 by 2 matrix, summing along each row of the matrix equals y_s, which represents the global use of the final goods produced in each country as specified in equation (4).

We label the 2 by 2 matrix on the left hand side of equation (5) the “gross output decomposition matrix.” It fully decomposes each country’s gross outputs according to where it is absorbed. Each element x_{sr} in this matrix is the gross output in source country s necessary to sustain final demand in destination country r. Summing along its row equals total gross output in country s, x_s *as* specified in equation (2). For example, it breaks up country 1’s gross output x_1 into two parts: $x_1 = x_{11} + x_{12}$. While x_{11} is the part of Country 1’s gross output that is ultimately absorbed in country 1, x_{12} is the part of Country 1’s gross output that is ultimately absorbed in Country 2.

The RHS of equation (5) further decomposes x_{11} itself into two parts: $x_{11}=b_{11}y_{11} + b_{12}y_{21}$. The first part, $b_{11}y_{11}$ is the part of Country 1’s gross output required to produce Country 1’s final good that is consumed in Country 1. The second part, $b_{12}y_{21}$, is the part of Country 1’s gross output that is exported as an intermediate good, and eventually returns home as part of Country 1’s imports from abroad (embedded in foreign final goods).

Similarly, x_{12} can also be decomposed into two parts: $x_{12} = b_{11}y_{12} + b_{12}y_{22}$. The first part, $b_{11}y_{12}$ is the part of Country 1’s gross output that is used to produce exported final good that is consumed abroad. $b_{12}y_{22}$ is the part of Country 1’s gross output that is exported as an intermediate good and used in country 2 to produce final good that is consumed there. Of course, $x_1 = x_{11} + x_{12}$ is nothing but Country 1’s total output. By assumption, they are produced by the same technology and therefore have the same share of domestic value added.

By the same interpretation, Country 2’s gross output also can be first broken up into two parts: $x_2 = x_{21} + x_{22}$. x_{21} is Country 2's gross output that is ultimately absorbed in Country 1, which can be in turn broken up to $b_{21}y_{11}+b_{22}y_{21}$. x_{22} is Country 2’s domestic absorption of its own gross output, and can be further broken up to $b_{21}y_{12}+b_{22}y_{22}$.

This conceptual decomposition of a country’s gross output according to where it is absorbed and further breaking them out in terms of each country's final demand reflect the basic and uncontroversial Leontief insight, thus is a very useful stepping stone for thinking of a country’s export of value added.

By the definition of the input-output coefficients, to produce 1 unit of Country 1’s good, a_{11} units of domestic intermediate good is used, and a_{21} units of imported intermediate good is used. Therefore, the fraction of domestic output that represents the domestic value added in Country 1 is

$$v_1 = 1 - a_{11} - a_{21}$$

Similarly, the share of domestic value added in Country 2's gross output is:

$$v_2 = 1 - a_{12} - a_{22}$$

We define V, the 2×2 matrix,

$$V = \begin{bmatrix} v_1 & 0 \\ 0 & v_2 \end{bmatrix}. \quad (6)$$

Multiplying these direct value-added shares with the Leontief inverse B produces the 2×2 value-added share (VB) matrix, our basic measure of value-added shares by source of production.

$$VB = \begin{bmatrix} v_1 b_{11} & v_1 b_{12} \\ v_2 b_{21} & v_2 b_{22} \end{bmatrix}. \quad (7)$$

Within VB, v_1b_{11} and v_2b_{22} denote the domestic value-added share of domestically produced products for country 1 and country 2 respectively, v_2b_{21} and v_1b_{12} denote the share of foreign country's value-added in the same goods.[4] Because all value added must be either domestic or foreign, the sum along each column is unity:

$$v_1 b_{11} + v_2 b_{21} = v_1 b_{12} + v_2 b_{22} = 1. \quad (8)$$

Given the assumption on the input-output coefficients, there is no difference in the share of domestic value added in Country 1's production for goods absorbed at home versus its production for exports.[5] Therefore, the total domestic value added in Country 1's gross output is simply v_1x_1; it is country 1's GDP by definition.

The total value added in Country 1's gross outputs can be easily broken up into two parts based on where it is ultimately absorbed: $v_1x_1 = v_1x_{11} + v_1x_{12}$, where v_1x_{11} is the domestic value added that is ultimately absorbed at home, and v_1x_{12} is the domestic value added that is ultimately absorbed abroad.

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[4] Note that the VB matrix is not any arbitrary share matrix, but rather the one that reflects the underlying production structure embedded in the ICIO model specified in equations (2) and (3). It contains all the needed information on value-added production by source.

[5] Such an assumption is maintained by HIY (2001), Johnson and Noguera (2011), and Daudin et al (2011). One might allow part of the production for exports (processing exports) to take on different input-output coefficients. Such a generalization is pursued by Koopman, Wang, and Wei (2012), who have worked out a generalized formula for computing the share of domestic value added in a country's gross exports when processing trade is prevalent. However, they have not pursued a total decomposition of a country's gross exports that allows one to compute the structure of double counted items.

The last part, v_1x_{12}, is also Country 1's exports of value added. It is instructive to decompose the last item further. Since $v_1x_{12} = v_1b_{11}y_{12} + v_1b_{12}y_{22}$, Country 1's exports of value added has two components: Country1's value added embedded in Country 1's exports of final good that is absorbed in Country 2 ($v_1b_{11}y_{12}$); and Country 1's value added in its exports of intermediate good that is used by Country 2 to produce final good that is ultimately locally consumed ($v_1b_{12}y_{22}$).

Note that v_1x_{12} is conceptually the same as Country 1's value added exports as defined by Johnson and Noguera (2012) except that we express it as the sum of two components related only to the final demand in the two countries. To summarize, Country 1's and 2's "value-added exports" are, respectively:

$$VT_{12} \equiv v_1x_{12} = v_1b_{11}y_{12} + v_1b_{12}y_{22}$$
$$VT_{21} \equiv v_2x_{21} = v_2b_{21}y_{11} + v_2b_{22}y_{21} \quad (9)$$

Intuitively, there are at least two reasons for a country's exports of value added to be smaller than its gross exports to the rest of the world. First, the production for its exports may contain foreign value added or imported intermediate goods (a). Second, part of the domestic value added that is exported may return home after being embodied in the imported foreign goods rather than being absorbed abroad (b). In other words, exports of value added are a net concept; it has to exclude from the gross exports both foreign value added and the part of domestic value added that is imported back to home.

Identifying and estimating these double counted terms in gross exports in addition to value-added exports have important implications for measuring each country's position in global value-chains. For example, two countries can have identical ratios of value added exports to gross exports but very different ratios of (a) and (b). Those countries that are mainly upstream in global production chains, such as product design, tend to have a large value of (b) but a small value of (a). In comparison, those countries mainly specializing in assembling imported components to produce final products tend to have a small value of (b) but a big value of (a). However, the existing literature lacks a uniform and transparent framework to compute exports of value added, (a) and (b) simultaneously. Because one needs a gross exports accounting (decomposition) framework to identify these conceptually different components from official gross exports statistics, we venture to do this next. Without loss of generality, let us work with country 1's gross exports first:

$$e_{12} = y_{12} + a_{12}x_2 \tag{10}$$

It says that country 1's exports consist of final goods and intermediate goods. Combining (10) with equation (8), we have

$$\begin{aligned} e_{12} &= (v_1 b_{11} + v_2 b_{21})(y_{12} + a_{12}x_2) = v_1 b_{11} y_{12} + v_2 b_{21} y_{12} + v_1 b_{11} a_{12} x_2 + v_2 b_{21} a_{12} x_2 \\ &= v_1 b_{11} y_{12} + v_2 b_{21} y_{12} + v_1 b_{12} y_{22} + v_1 b_{12} y_{21} + v_1 b_{12} a_{21} x_1 + v_2 b_{21} a_{12} x_2 \end{aligned} \tag{11}$$

A step by step proof of $v_1 b_{11} a_{12} x_2 = v_1 b_{12} y_{22} + v_1 b_{12} y_{21} + v_1 b_{12} a_{21} x_1$ can be found in appendix A. Here we give the economic intuition behind it. The total value of country 1's intermediate exports must include two types of value. First, it must include all value added by Country 1 in its imports from Country 2. To see this, we note that in order for exported value produced by Country 1 to come back through its imports, it must have first been embodied in Country 1's intermediate exports, which is $v_1 b_{12} y_{21} + v_1 b_{12} a_{21} x_1$. Second, it must include all value added generated in Country 1 that is absorbed in Country 2 after being used as intermediate inputs by Country 2, which is $v_1 b_{12} y_{22}$.

Note that multiplying the Leontief inverse with intermediate goods exports leads to some double counting of gross output and thus some value terms in exports. However, in order to account for 100% of the value of country 1's intermediate goods exports and to identify what is double counted, we have to include them into the accounting equation first. By decomposing the last two terms further, we can see precisely what is double counted. Using the gross output identity (equation (2)) $x_1 = y_{11} + a_{11}x_1 + e_{12}$ and $x_2 = y_{22} + a_{22}x_2 + e_{21}$, it is easy to show that

$$x_1 = (1 - a_{11})^{-1} y_{11} + (1 - a_{11})^{-1} e_{12} \qquad x_2 = (1 - a_{22})^{-1} y_{22} + (1 - a_{22})^{-1} e_{21} \tag{12}$$

$(I - a_{11})^{-1} y_{11}$ is the gross output needed to sustain final goods that are both produced and consumed in country 1, using domestically produced intermediate goods; deducting it from country 1's total gross output, what is left is the gross output needed to sustain country 1's production of its gross exports e_{12}. Therefore the two terms in right hand side of equation (12) both have straightforward economic meanings. We can further show that $b_{12} a_{21} (1 - a_{11})^{-1} y_{11} = b_{11} y_{11} - (1 - a_{11})^{-1} y_{11}$ (see proof, also in Appendix A), which is the total gross output needed to sustain final goods both produced and consumed in country 1, but using intermediate goods that originated in Country 1 and shipped to Country 2 for processing before being re-imported by Country 1 (gross output sold indirectly in the domestic market). These two parts plus $b_{12}y_{21}$ sum to x_{11} in equation (5), which is the gross output of country 1 absorbed in country 1 to sustain its domestic final demand both directly and indirectly. It indicates that Country 1's domestic final demand is satisfied by three production channels: (1) $(I-a_{11})^{-1}y_{11}$ is part of country 1's gross output sold directly in the domestic market that is consumed there; (2) $b_{12}y_{21}$ is part of country 1's gross output used as intermediate goods by country 2 produce final goods that is consumed in country 1; (3) $b_{12}a_{21}(1-a_{11})^{-1}y_{11}$ is part of country 1's gross output used as intermediate goods by country 2 to produced intermediate goods that is exported to country 1 to produce final goods in country 1 that is consumed there. They are all needed to sustain the domestic final demand in country 1, but they differ in terms of how they participate in international trade.

Replacing x_1 by $(1-a_{11})^{-1}y_{11}+(1-a_{11})^{-1}e_{12}$ and x_2 by $(1-a_{22})^{-1}y_{22}+(1-a_{22})^{-1}e_{21}$ in equation (11), and rearranging terms, we can fully decompose Country 1's gross exports into its various value-added and double counted components as follows:

$$\begin{aligned} e_{12} &= v_1b_{11}e_{12}+v_2b_{21}e_{12}=[v_1b_{11}y_{12}+v_1b_{12}y_{22}] \\ &+[v_1b_{12}y_{21}+v_1b_{12}a_{21}(1-a_{11})^{-1}y_{11}]+v_1b_{12}a_{21}(1-a_{11})^{-1}e_{12} \\ &+[v_2b_{21}y_{12}+v_2b_{21}a_{12}(1-a_{22})^{-1}y_{22}]+v_2b_{21}a_{12}(1-a_{22})^{-1}e_{21}. \end{aligned} \tag{13}$$

While the algebra to arrive at equation (13) may be a bit tedious, expressing a country's gross exports as the sum of these eight terms on the right hand side of equation (13) is very useful. We go over their economic interpretations systematically.

The first two terms in equation (13) (or the two terms in the first square bracket) are value-added exports, i.e. country 1's domestic value-added absorbed outside country 1.

The third term, $v_1b_{12}y_{21}$ is country 1's domestic value-added that is initially embodied in its intermediate exports but is returned home as part of Country 1's imports of the final good. The fourth term, $v_1b_{12}a_{21}(1-a_{11})^{-1}y_{11}$, is also Country 1's domestic value added that is initially exported by Country 1 as part of its intermediate goods to Country 2, but then is returned home via its intermediate imports from country 2 to produce final goods that is absorbed at home. Both the third and the fourth terms are domestic value added produced in Country 1, exported to Country 2, but then return to and stay in Country 1. Both are counted at least twice in trade statistics as they first leave Country 1 for Country 2, and then leave Country 2 for Country 1 (and ultimately stay in Country 1). Note that the value represented by these two terms can be embodied in trade transactions that cross borders back and forth for more than twice as long as they originate in Country 1and are ultimately consumed in Country 1.

The fifth term, $v_1 b_{12} a_{21} (1 - a_{11})^{-1} e_{12}$, may be called a "pure double counted term." The reason for labeling it as such will become clear after we present a similar dissection of Country 2's gross exports and a further decomposition of this term. This term only occurs when both countries export intermediate goods. If at least one country does not export intermediate goods (i.e., no two-way trade in intermediate goods), this terms disappears.

The sixth term, $v_2 b_{21} y_{12}$, is the foreign value-added in country 1's gross exports of final goods, and the seventh term, $v_2 b_{21} a_{12} (1 - a_{22})^{-1} y_{22}$, is the foreign value-added in country 1's gross exports of intermediate goods, they both ultimately go back to the foreign country and consumed there.

The eighth (and the last) term is another pure double counted item in country 1's gross exports. Similar to the 5th term, this term would disappear if at least one country does not export intermediate goods.

In a similar way, we can express Country 2's gross exports as the sum of eight terms:

$$\begin{aligned} e_{21} &= v_1 b_{12} e_{21} + v_2 b_{22} e_{21} = [v_2 b_{21} y_{11} + v_2 b_{22} y_{21}] \\ &+ [v_2 b_{21} y_{12} + v_2 b_{21} a_{12} (1 - a_{22})^{-1} y_{22}] + v_2 b_{21} a_{12} (1 - a_{22})^{-1} e_{21} \\ &+ [v_1 b_{12} y_{21} + v_1 b_{12} a_{21} (1 - a_{11})^{-1} y_{11}] + v_1 b_{12} a_{21} (1 - a_{11})^{-1} e_{12} \end{aligned} \tag{14}$$

Comparing equations (13) and (14), there are a few noteworthy features. First, the 3rd, 4th and 5th terms in Country 1's gross exports (13) are identical to the 6th, 7th, and 8th terms in Country 2's exports (14), and vice versa. This means that, the value added that is initially produced and exported by Country 1 but then re-imported by Country 1, is exactly the same as foreign value added in Country 2's gross exports to Country 1. Symmetrically, the foreign value added in Country 1's gross exports to Country 2, is the same as Country 2's value added, initially produced and exported by Country 2, but re-appears as part of Country 1's gross exports to Country 2.

Second, while the 1st and 2nd terms in equations (13) and (14) constitute value-added exports, all other terms are double counted components in a country's official exports statistics. However, there are conceptually interesting differences among the 3rd and the 4th terms as the first group, the 6th and the 7th terms as the second group, and the 5th and 8th terms as the third group. The differences are revealed when comparing them to the two countries' GDP. More precisely, a country's GDP is the sum of its value-added exports plus its domestic value-added consumed at home:

$$GDP_1 = v_1 x_1 = v_1 (b_{11} y_{12} + b_{12} y_{22} + b_{12} y_{21} + b_{11} y_{11})$$
$$= v_1 \{ b_{11} y_{12} + b_{12} y_{22} + [b_{12} y_{21} + b_{12} a_{21} (1 - a_{11})^{-1} y_{11}] \} + v_1 (1 - a_{11})^{-1} y_{11} \quad (15)$$

$$GDP_2 = v_2 x_2 = v_2 (b_{21} y_{11} + b_{22} y_{21} + b_{21} y_{12} + b_{22} y_{22})$$
$$= v_2 \{ b_{21} y_{11} + b_{22} y_{21} + [b_{21} y_{12} + b_{21} a_{12} (1 - a_{22})^{-1} y_{22}] \} + v_2 (1 - a_{22})^{-1} y_{22} \quad (16)$$

The last term in each GDP equation is value-added produced and consumed at home that are not related to international trade; while the first four terms in the bracket in each GDP equation are exactly the same as the first four terms in equations (13) and (14). It is easy to show that the sum of global GDP always equals global final demand:

$$GDP_1 + GDP_2 = v_1 x_1 + v_2 x_2 = (1 - a_{11} - a_{21}) x_1 + (1 - a_{12} - a_{22}) x_2$$
$$= x_1 - a_{11} x_1 - a_{12} x_2 + x_2 - a_{21} x_1 - a_{22} x_2 = y_1 + y_2 \quad (17)$$

Equations (15) and (16) show that the 3rd and 4th terms in equations (13) and (14) are counted as part of the home country's GDP (even though they are not part of the home country's exports of value added). Because they represent a country's domestic value-added that is initially exported but imported back and consumed in the initial producing country, they are part of the value-added created by domestic production factors. The 6th and 7th terms represent the foreign value added in a Country's exports that are ultimately absorbed in the foreign country. They are counted once as part of the foreign country's GDP in equations (15) and (16). In comparison, because a combination of the part of GDP that is consumed at home and exports of value added yields 100% of a country's GDP, the 5th and 8th terms are not part of either country's GDP. In this sense, they are "pure double counted terms."

Subtracting global GDP from global gross exports using equations (13), (14), (15) and (16) yields the following:[6]

$$e_{12} + e_{21} - GDP_1 - GDP_2 = v_1 [b_{12} y_{21} + b_{12} a_{21} (1 - a_{11})^{-1} y_{11}] + v_2 [b_{21} y_{12} + b_{21} a_{12} (1 - a_{22})^{-1} y_{22}]$$
$$+ 2 v_1 b_{12} a_{21} (1 - a_{11})^{-1} e_{12} + 2 v_2 b_{21} a_{12} (1 - a_{22})^{-1} e_{21} - [v_1 (1 - a_{11})^{-1} y_{11} + v_2 (1 - a_{22})^{-1} y_{22}] \quad (18)$$

Equation (18) shows clearly that besides the value added produced and consumed at home (in the last square bracket), which is not a part of either country's gross exports, the 6th and

[6] A step by step derivation is provided in Appendix A.

7th terms in equations (13), $v_2[b_{21}y_{12} + b_{21}a_{12}(1-a_{22})^{-1}y_{22}]$, and the 6th and 7th terms in equation (14) $v_1[b_{12}y_{21} + b_{12}a_{21}(1-a_{11})^{-1}y_{11}]$, double counted only once as foreign value-added in the other country's gross exports. Because the 3rd and 4th terms in (13) and (14) reflect part of the countries' GDP, they are not double counted from the global GDP point of view. In comparison, both the 5th and 8th terms are over-counted twice relative to the global GDP since they are not a part of either country's GDP.

Third, the nature of the 5th and 8th terms can be understood further if we break them up further. In particular, with a bit of algebra, they can be expressed as linear combinations of components of the two countries' final demand:

$$v_1b_{12}a_{21}(1-a_{11})^{-1}e_{12} = b_{12}a_{21}[v_1b_{11}y_{12} + v_1b_{12}y_{22} + v_1b_{12}y_{21} + v_1b_{12}a_{21}(1-a_{11})^{-1}y_{11}] \quad (19)$$

$$v_2b_{21}a_{12}(1-a_{22})^{-1}e_{21} = b_{21}a_{12}[v_2b_{21}y_{11} + v_2b_{12}y_{21} + v_2b_{21}y_{12} + v_2b_{21}a_{12}(1-a_{22})^{-1}y_{22}] \quad (20)$$ [7]

The four terms inside the square bracket on the RHS of equation (19) are exactly the same as the first four terms in the gross exports accounting equation (13). A similar statement can be made about the four terms inside the square bracket in equation (20) to gross export accounting equation (14). This means that the 5th and the 8th terms double counted a fraction of both a country's value added exports and its domestic value added that has been initially exported but are eventually returned home. Just to belabor the point, unlike the other terms in Equation (13) that are parts of some countries' GDP, the 5th and the 8th terms over-count the values that are already captured by other terms in gross exports. Again, this feature suggests that they are "pure double counted terms."

Note, if Country 2 does not export any intermediate goods, then $a_{21} = b_{21} = 0$, and the entire RHS of equations (20) and (19) would vanish. Alternatively, if Country 1 does not export intermediate goods, then $a_{12} = b_{12} = 0$, the entire RHS of equations (19) and (20) would also vanish. In other words, the 5th and 8th terms exist only when two-way trade in intermediate goods exist so that some value added is shipped back and forth as a part of intermediate trade between the two countries. Because the eight components of equations (13) and (14) collectively constitute 100% of a country's gross exports, missing any part, including the two pure double counting terms, the accounting would not be complete.

[7] Proofs of equations (19) and (20) are provided in Appendix A.

Finally, while the 5th and 8th terms in Equation (13) are (double counted) values in intermediate goods trade that are originated in Countries 1 and 2, respectively, we cannot directly see where they are absorbed. By further partitioning the 5th and 8th terms, we can show where they are finally absorbed and interpret the absorption by input/output economics. With a bit of algebra, we can show[8]:

$$\begin{aligned} & v_1 b_{12} a_{21} (1-a_{11})^{-1} e_{12} + v_2 b_{21} a_{12} (1-a_{22})^{-1} e_{21} \\ & = a_{12}[b_{21} y_{12} + b_{21} a_{12} (1-a_{22})^{-1} y_{22}] + a_{21}[b_{12} y_{21} + b_{12} a_{21} (1-a)^{-1} y_{11}] \end{aligned} \quad (21)$$

Therefore, the sum of 8th and 5th terms in equation (13) is equivalent to the sum of the four terms in the RHS of equation (21) . From equation (5), it is easy to see that the terms in the square bracket in equation (21) are parts of x_{22} and x_{11}, *which are trade-related portions of gross output that are both originally produced and finally consumed in the source country;* therefore they are part of each country's gross intermediate exports ($a_{12}x_2$ and $a_{21}x_1$). Since the value added embodied in those intermediate goods are already counted once in the production of each country's GDP (the 3rd and 4th, terms in equations (13) and (14)), they are double counted in value-added (GDP) terms. As we pointed earlier, the exact same terms will also appear in Country 2's official exports statistics.

Due to the presence of these types of conceptually different double counting in a country's gross exports, we may separately define “domestic value added in exports,”(“part of a country’s GDP in its exports”) and “domestic content in exports.”The former excludes the pure double counted intermediate exports that return home; whereas the latter is the former plus the pure double counted term.

2.3 *Using the accounting equation to generate measures of vertical specialization*

We can relate the definitions of the three concepts to components in Equation (13). Country 1’s exports of value added are the sum of the 1st and the 2nd term. It takes into account both where the value is created and where it is absorbed. Country 1’s value added in its exports is the sum of its exports of value added and the 3rd and the 4th terms. This concept takes into account where the value is created but not where it is absorbed. Obviously, Country 1’s “value added in its exports” is generally greater than its “exports of value added.” Finally, the domestic content in Country 1’s exports is Country 1’s value added in its exports plus the 5th term, the

[8] A step by step proof is in appendix A.

double counted intermediate goods exports that are originated in Country 1. This last concept also disregard where the value is ultimately absorbed. By assigning the 5th term to the domestic content in Country 1's exports, and the 8th term to the foreign content in Country 1's exports, we can achieve the property that the sum of the domestic content and the foreign content yields the total gross exports. We will justify these definitions in what follows. We will also argue that one of these measures may be more appropriate than others, depending on particular economic applications.

We already show that the first two terms in Equation (13) correspond to value added exports as proposed in Johnson and Noguera (2012). We now link other measures in the literature to linear combinations of the components in the same gross exports accounting equation. Following HIY's original ideas, Koopman, Wang and Wei (2008 and 2012) have shown that gross exports can be decomposed into domestic content and foreign content/vertical specialization (VS) in a single country IO model without two-way international trade in intermediate goods.

If we were to maintain HIY's assumption that Country 1 does not export intermediate good (i.e., $a_{12} = b_{12} = 0$), then the two pure double counted terms, or the 5th and the 8th terms in equation (13) are zero. In this case, we can easily verify that the sum of the last four terms in equation (13) is identical to the VS measure in the HIY (2001) paper.

To remove the restriction that HIY imposed on intermediate trade, we have to determine how to allocate the two pure double counted terms. We choose to allocate the double counted intermediate exports according to where they are originally produced. That is, even though the 5th term in equation (13) reflects pure double counting, it nonetheless is originally produced in Country 1 and therefore can be treated as part of Country 1's domestic content. Similarly, we allocate the 8th term to the foreign content in Country 1's exports. Such a definition is consistent with HIY's original idea that a country's gross exports consist of either domestic or foreign content and the major role of their VS measure is to quantify the extent to which intermediate goods cross international boarder more than once. It is also computationally simple because the share of domestic and foreign content can be obtained directly from the VB matrix.

However, since the 5th term reflects double counted intermediate goods in a country's gross exports, we may wish to exclude it if we are to consider which part of Country 1's GDP is exported. In particular, we define "domestic value added in Country 1's gross exports" (regardless of where the exports are ultimately absorbed) as the sum of the first four terms on the RHS of equation (13). This variable can be shown to equal to $v_1(1-a_{11})^{-1}e_{12}$, part of equation (12), Country 1's gross output identity.

$$\begin{aligned}
DV_1 &= v_1(1-a_{11})^{-1}e_{12} = v_1(1-a_{11})^{-1}(y_{12}+a_{12}x_2) \\
&= v_1(1-a_{11})^{-1}[y_{12}+a_{12}(b_{21}y_{11}+b_{22}y_{21}+b_{21}y_{12}+b_{22}y_{22})] \\
&= v_1(1-a_{11})^{-1}[(1+a_{12}b_{21})y_{12}+a_{12}(b_{21}y_{11}+b_{22}y_{21}+b_{22}y_{22})] \\
&= v_1(1-a_{11})^{-1}\{(1-a_{11})b_{11}y_{12}+(1-a_{11})b_{12}(y_{21}+y_{22})+[(1-a_{11})b_{11}-1]y_{11}\} \\
&= v_1b_{11}y_{12}+v_1b_{12}y_{22}+v_1b_{12}y_{21}+v_1[b_{11}-(1-a_{11})^{-1}]y_{11} \\
&= v_1b_{11}y_{12}+v_1b_{12}y_{22}+v_1[b_{12}y_{21}+b_{12}a_{21}(1-a_{11})^{-1}y_{11}]
\end{aligned} \tag{22}$$

The derivation uses the property of inverse matrix $(1-a_{11})b_{11}=1+a_{12}b_{21}$ and $(1-a_{11})b_{12}=a_{12}b_{22}$.

Following the same frame of thinking, we label only the sum of the 6th and 7th terms, a subset of the foreign content, as the "foreign value added in Country 1's exports," and define country 1's VS as

$$VS_1 = v_2b_{21}e_{12} = v_2[b_{21}y_{12}+b_{21}a_{12}(1-a_{22})^{-1}y_{22}]+v_2b_{21}a_{12}(1-a_{22})^{-1}e_{21} = (1-v_1b_{11})e_{12} \tag{23}$$

The first two terms are foreign value-added or GDP in country 1's exports, and VS share of country 1 equals v_2b_{21}. Therefore, such a measure of foreign content is a natural extension of HIY's VS measure in a two-country world with unrestricted intermediate goods trade. Because the VS share is defined this way, it is natural to define the domestic content share in country 1's exports as 1- VS share or v_1b_{11}.

In a two-country world, Country 1's VS1 is identical to its VS1* and Country 2's VS:

$$\mathrm{VS1}_1 = VS1^*_1 = v_1b_{12}e_{21} = v_1[b_{12}y_{21}+b_{12}a_{21}(1-a_{22})^{-1}y_{11}]+v_1b_{12}a_{21}(1-a_{11})^{-1}e_{12} \tag{24}$$

which is the sum of the 3rd, 4th, and 5th terms in equation (13). However, in a multi-country setting to be discussed later, VS1* will only be a subset of VS1, and the latter will also include some third country terms.

2.4 Numerical examples

To enhance the intuition for our formula, we provide some numerical examples. *Example 1: One of the countries does not export intermediate goods, and does not have domestic value added in its gross exports.*

Country A has a gross output of 150, which is produced by combining domestic intermediate goods of 50 and domestic value added of 100. It exports 70 units of its output to Country B, consisting of 50 units of intermediate goods and 20 units of final goods. (Country A also supplies 50 units at home as intermediate goods, and another 30 units at home as final goods.)

Country B does something extremely simple: it produces a gross output of 50 units by using 50 units of imported intermediate goods from Country A and adding no domestic value. Country B then exports the entire 50 units of its output to Country A as final goods.

The two country's production and trade relationship can be summarized by the following inter-country input-output model:

$$\begin{bmatrix} x_A \\ x_B \end{bmatrix} = \begin{bmatrix} 0.33 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 150 \\ 50 \end{bmatrix} + \begin{bmatrix} 30+20 \\ 50+0 \end{bmatrix} = \begin{bmatrix} 150 \\ 50 \end{bmatrix},$$

In this extremely simple and therefore transparent example, it is clear the entire 50 units of Country B's gross exports consists of foreign value added (or imported intermediate goods from Country A), and no domestic value added is exported since no value is added by Country B's production. So its VS share = 100%, and VAX ratio =0%.

Intuitively, out of the total of 70 units of Country A's gross exports, 50 units return home (as Country B's exports to A). Therefore, Country A's exports of value added that are ultimately absorbed in Country B are 20 units. So, VS share =0%, and VAX ratio =20/70.

We can easily check that our gross exports accounting formula correctly back out these terms. Since

$$A = \begin{bmatrix} 0.33 & 1 \\ 0 & 0 \end{bmatrix}$$

It is obvious v_1= 0.67, and v_2=0 and easy to work out that

$$B = \begin{bmatrix} 1.5 & 1.5 \\ 0 & 1 \end{bmatrix} \qquad VB = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

This indicates that country 1's share of domestic value-added in its exports is 100% and country 2's share is 0%.

$$\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} 1.5 & 1.5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 30 & 20 \\ 50 & 0 \end{bmatrix} = \begin{bmatrix} b_{11}y_{11} + b_{12}y_{21} & b_{11}y_{12} + b_{12}y_{22} \\ b_{21}y_{11} + b_{22}y_{21} & b_{21}y_{12} + b_{22}y_{22} \end{bmatrix} = \begin{bmatrix} 45+75 & 30+0 \\ 0+50 & 0+0 \end{bmatrix}$$

For Country A, the accounting equation indicates that the exports of domestic value added are 0.67*30=20 units (20 for the first term, and zero for the second term). The domestic value added that returns home as embedded in imported intermediates is 0.67*75=50 units (consisting of 50 for the 3rd term, and zero for the 4th term). Our formula returns a value of zero for the 6th and 7th terms, which correspond to zero foreign value added in Country A's exports. The two pure double counted terms take a value of zero, which is consistent with our theoretical discussion that these terms will disappear if at least one of the countries does not export intermediate goods.

For country B, our formula produces a zero value for exports of value added, and indicates that 50 units (or 100%) of the country's export come from foreign value added. All other terms are zero. These are exactly consistent with our intuition.

To sum up, in this transparently simple example, we can work out a decomposition of a country's gross exports in our head, and understand the economic meaning of each of these terms. We can verify easily that our gross exports accounting formula correctly generates these items.

Example 2: Two countries, one sector, and one of the countries does not export intermediate good

Consider a world consisting of two countries (USA and CHN) and a single sector of electronics. The two countries have identical gross exports and identical value added exports (and hence identical VAX ratios). The point of this example is to show that the structure of the "double counted" values in gross exports contains useful information.

Both USA and CHN have a gross output of 200. USA's total output consists of 150 units of intermediate goods (of which, 100 units are used at home and 50 units are exported) and 50 units of final goods (of which 30 are consumed at home and 20 are exported).

CHN's total output consists of 50 units of intermediate goods (all used at home) and 150 units of final goods (of which, 70 units are exported and 80 units are used at home).

By construction, both countries export 70 units of their output (50 units of intermediate goods + 20 units of final goods exported by USA, and 70 units of final goods exported by CHN).

The domestic value added in USA's output is therefore 100 (=value of gross output 200 – value of domestic intermediate good of 100). Note no foreign value is used in USA's production. The domestic value added in CHN's output is also 100 (= value of gross output 200 -value of domestic intermediate goods of 50 – value of imported intermediate goods of 50). The input-output relationship can be summarized as follows:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 200 \\ 200 \end{bmatrix}, \qquad A = \begin{bmatrix} 0.5 & 0.25 \\ 0 & 0.25 \end{bmatrix}, \qquad V = \begin{bmatrix} 0.5 & \\ 0 & 0.5 \end{bmatrix}$$

The two-country production and trade system can be written as an inter-country input output (ICIO) model as follows

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.25 \\ 0 & 0.25 \end{bmatrix} \begin{bmatrix} 200 \\ 200 \end{bmatrix} + \begin{bmatrix} 30+20 \\ 70+80 \end{bmatrix},$$

The Leontief inverse matrix and the VB matrix can be computed easily as

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix}, \quad VB = \begin{bmatrix} 1 & 0.33 \\ 0 & 0.67 \end{bmatrix}$$

We can break up each country's gross output according to where it is ultimately absorbed by rearrange each country's final demand as follows:

$$\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 20 \\ 70 & 80 \end{bmatrix} = \begin{bmatrix} 60+46.69 & 40+53.3 \\ 0+93.33 & 0+106.67 \end{bmatrix} = \begin{bmatrix} 106.7 & 93.3 \\ 93.3 & 106.7 \end{bmatrix}$$

It is easy to verify that $x_1 = x_{11} + x_{12}$ and $x_2 = x_{21} + x_{22}$. USA's value-added export is 46.7 ($v_1 x_{12}$=0.5*93.33), where 20 (=$v_1 b_{11} y_{12}$=0.5*2*20, first term in equation (13)) is the amount of USA's value added in its export of final goods that is absorbed in CHN, and 26.7(=$v_1 b_{12} y_{22}$ =0.5*0.667*80, second term in (13)) is the amount of USA's value added in the export of intermediate goods that is also absorbed in CHN. The amount of USA's value added that is embedded in its export of intermediate goods but returns home as part of CHN's export of final goods is 23.3 (=$v_1 b_{12} y_{21}$=0.5*0.667*70, the 3rd term in (13)). Using the terminology of Daudin et al, USA's VS1*=23.3, the same as the VS1* estimates obtained from our accounting equation (13).

By construction, foreign value added in USA's exports equals zero (since CHN does not export intermediate goods). It is easy to verify that the sum of USA's exports of value added (46.7) and the amount of returned value added (23.3) is 70, which is the value of its gross exports.

For CHN, its value added exports (that are absorbed in USA) are also 46.7 (=$v_2 b_{22} y_{21}$ =0.5*1.333*70, first term in equation (14)). It is entirely embedded in CHN's exports of final good to USA as CHN does not export intermediate good by assumption. However, CHN does import intermediate goods from USA to produce both goods consumed in CHN and goods exported to USA. The amount of USA's value added used in the production of CHN's exports is 23.3(=$v_1 b_{12} y_{21}$=0.5*0.667*70, the 6th term in equation (14)). The sum of CHN's exports of value added (46.7) and the foreign value added in its exports (23.3) is 70, which is the same as CHN's gross exports.

In this example, both countries have identical gross exports and exports of value added, and therefore identical VAX ratios. However, the reasons underlying why value added exports deviate from the gross exports are different. For USA, the VAX ratio is less than one because some of the value added that is initially exported returns home after being used as an intermediate good by CHN in the latter's production for exports. For CHN, the VAX ratio is less than one because its production for exports uses intermediate goods from USA which embeds USA's value added.

In addition, the double counted items are also value added at some stage of production. More precisely, the VS in CHN's exports (23.3) is simultaneously a true value added from USA's viewpoint as it is value added by USA in its exports to CHN (that returned home)[9], but a "double counted item" from CHN's viewpoint as it is not part of CHN's value added.

Since we assume that CHN does not export intermediate goods (or $a_{21}=0$ *and* $b_{21} = 0$), there is no channel for CHN's gross output to be used by USA in its production and for CHNs gross output to be first exported and then returned home. Therefore, our gross exports accounting equation produces the same estimate for VS as the HIY's formula, i.e:

$$v_2(1-b_{11}) = v_2 b_{21} e_{12} = 0.5 \times 0.667 \times 70 = 23.3 = a_{21}(1-a_{11})^{-1} e_{12} = 0.25 \times (1-0.25)^{-1} \times 70 = 23.3$$

[9] Because there is no foreign value-added in country 1's production, the 30 unit of domestic final demand are 100% its own value-added, just as its exports, so its GDP equal to 100. For country 2, the value-added in its exports and domestic final consumption also sum to 100.

Domestic value-added share for CHN's exports equals (70-23.3)/70 = 0.667 = $v_2 b_{22}$=0.667. There is no difference between HIY's measure and our method in such a case since there are no pure double counting terms due to two way trade in intermediate goods. Both VS in USA and VS1* in CHN are zero. But this equality will not hold when we remove the assumption of $a_{21}=0$ (which will see in next example).

However, because there is domestic value-added embodied in USA's intermediate goods exports that is eventually returned home, the domestic value-added share in USA's exports equals to 1. As a result, using the VAX ratio (0.667) as a metric of the share of home country's domestic value-added in its gross exports would produce an underestimate.

Example 3: Both countries export intermediate good in an inter country supply chain[10]

We now consider an example in which both countries export (and import) intermediate goods in an inter-country supply chain. This example will show our accounting equation can decompose a country's gross exports into various value-added and double counted components in a way that is consistent with one's intuition. We will also illustrate why and how our estimate of VS1* in such a case differs from Daudin et al, why and how our estimate of the share of domestic value-added (GDP) in exports differs from Johnson and Noguera's value-added to gross exports ratio, and why and how our estimate of foreign value-added (GDP) in exports differs from HIY's VS measure but our foreign content in exports generalizes it.

Suppose the world production and trade take place in five stages (in a year) as summarized by Table 1. In Stage 1, perhaps a design stage, Country 1 uses labor to produce a unit of Stage-1 output. This is exported to Country 2 as an input to Stage-2 production. In Stage 2, Country 2 adds a unit of labor to produce 2 units of Stage-2 output which are shipped back to country 1 as an input to Stage-3 production. Country 1 adds another unit of labor to produce 3 units of Stage-3 output which are then exported to country 2 as an input to Stage-4 production. In Stage 4, country 2 adds a unit of labor to produce 4 units of Stage-4 output which are shipped back to country 1 as an input to Stage-5 production. The Stage-5 output is the final good. 3 units of the final good are exported to country 2, and 2 units are absorbed domestically in country 1.

[10] We are grateful to Peter Dixon and Maureen Rimmer for helping us to develop this instructive example.

Suppose each unit of intermediate and final goods is worth \$1. The total output in country 1 is \$9, in country 2 is \$6, the total value added (labor inputs) in the two countries is \$3 and \$2 respectively. The total exports from 1 to 2 and from 2 to 1 are \$7 and \$6, respectively; and the exports of final goods from 1 to 2 and 2 to 1 are \$3 and \$0 respectively.

For this simple example of an international supply chain, we can decompose both countries' gross exports into value-added and double counted components by intuition without using any equations. The intuitive decomposition is summarized in Table 2. We will then verify that our exports decomposition formula produces exactly the same results.

We proceed as follows: Starting from the last stage (Stage 5), each country contributes \$2 of value-added with their (previously produced) intermediate inputs, and Country 1 contributes an additional \$1 of labor input to produce a total of 5 units of the final good. We assume labor is homogenous across countries. Since 2 units of the final good stay in Country 1 and 3 units are consumed in Country 2, all the value-added embodied in intermediate inputs that are eventually absorbed by each country should be split as 40% for country 1 and 60% for country 2, in proportion to the units of the final good consumed by the two countries. Therefore, the total value added exports from Country 1 to 2 are 0.6*\$3=\$1.8 (which is recorded in the cell in row "total" and column 2a of Table 2). Similarly, Country 2's exports of value added to 1 are 0.4*\$2=\$0.8 (which is recorded in the cell in row "total" and column 2b). Out of Country 1's \$7 of gross exports, the total amount of double counting, or the difference between its gross exports and its value-added exports is \$5.2 (=\$7-\$1.8). This is recorded in the cell in row "total" and column 7a. Similarly, out of Country 2's \$6 of gross exports, the total amount of double counting is \$6-\$0.8=\$5.2, which is recorded in the cell in row "total" and column (7b).

The beauty of this simple example is that we can work out the structure of the double counted values by intuition. Given what happens in Stage 5, we can split a country's value added in production in each of the earlier stages into the sum of value-added exports in that stage (that is ultimately absorbed abroad) and the value added that is exported in that stage but returns home next stage as part of its imports from the foreign country. Then the amount of exports in each of the first 4 stages that are double counted can be computed as each stage's gross output minus value added exports in that stage. In Stage 1, Country 1's domestic value added is \$1 (recorded in the cell (S1, 1a)). Since we know by Stage 5, 40% of the final good stays in Country 1, and 60% is exported to Country 2, we can split the \$1 of domestic value added into \$0.6 of

Country 1's exports of value added (recorded in the cell (S1, 2a)) and $0.4 of the domestic value added that returns home in the next stage and eventually consumed at home in Stage 5 (recorded in (S1, 3a)). Out of Country 1's gross exports of $1 in Stage 1, the total double counted amount is the difference between its gross exports and value added exports, or $1-$0.6=$0.4, as recorded in (S1, 7a).

In Stage 2, Country 2 uses $1 of intermediate good from Country 1 as an input together with its additional $1 of labor to produce $2 exports. Its domestic value added is $1 (recorded in (S2, 1b)). Again, since we know the split of the final good consumption in the two countries in Stage 5, we can split Country 2's domestic value added into $0.4 of its exports of value added (recorded in (S2, 2b)) and $0.6 of domestic value added that will return home in Stage 3 and eventually consumed at home in Stage 5(recorded in S2, 4b)). Recall that out of $1 of intermediate good that Country 2 imports from Country 1, $0.4 will go back to Country 1 and be consumed there eventually. This is recorded in (S2, 5b), which is numerically identical to (S1, 3a). The remaining $0.6 is double counted intermediate goods, and is recorded in (S2, 6b). This can also be verified in the following way. Since we know Country 2's gross exports in Stage 2 is $2 but its value added exports are only $0.4, the total amount of double counting in this stage's gross exports must be the difference between the two, or $1.6 as recorded in (S2, 7b).Therefore, the "pure double counted" portion of foreign intermediate good has to equal $1.6 (S2, 7b) -$0.6 (S2, 3b) - $0.4 (S2, 5b), which equals to $0.6, as recorded in (S2, 6b). This amount represents the part of Country 1's Stage 1 intermediate good exports that cross borders more than twice before it can be embed in the final goods for consumption.

In Stage 3, Country 1 uses $2 of imported intermediate goods from Country 2 as an input with its additional $1 of labor to produce $3 exports. Country 1's domestic value added is $1 (S3, 1a). Again, because 60% of the final good will be eventually absorbed in the foreign country, the $1 of domestic value added can be split into $0.6 of Country 1's exports of value added (S3, 2a) and $0.4 of the domestic value added that is exported in Stage 3 but will return in Stage 4 and eventually consumed there in Stage 5(S3, 3a). Furthermore, the Stage 3 production does use imported intermediate good from the previous stage. The amount of foreign value added embedded in its intermediate good imported from Country 2 that is not pure double counting should be the same as Country 2's domestic value added that is sent to Country 1 in Stage 2 but returns home and will be eventually absorbed there. We know that amount is $0.6 (S2, 3b).

Therefore, the amount of foreign value added that is used in Country 1's Stage 3 production for exports and that will be eventually absorbed in Country 2 should be the same as $0.6 in (S3, 5a).

Because the value of Country 1's stage 1 exports ($1) is already counted three times by the time Stage 3 exports take place, we record that amount as a pure double counting item in (S3, 4a). Since we know out of $3 of Country 1's gross exports in Stage 3, only $0.6 is exports of value added that will eventually be absorbed abroad, $3-$0.6=$2.4 represents the total amount of double counting in this stage's gross exports, and is recorded in (S3, 7a). Out of the $1 foreign value added from Stage 2, since the amount that will go back to the foreign country and is absorbed there is 0.6 (S3, 5a), the amount of pure double counting must be $1-$0.6=$0.4, as recorded in (S3, 6a).

One way to check the sensibility of our reasoning is to compare the total amount of double counting in Stage-3 gross exports with the sum of the double counted components. Out of Country 1's $3 of gross exports in Stage 3, we know the total amount of double counting is $2.4 (recorded in (S3, 7a)). We can check that the sum of the double counted components in Country 1's exports in this stage (the sum of (S3, 3a), (S3, 4a), (S3, 5a), and (S3, 6a)) is also $2.4.

We now move to Stage 4, when Country 2 combines $1 of domestic value (recorded in (S4, 1b)) with $3 of intermediate goods imported from Country 1 in the previous stage, and exports $4 of intermediate goods in gross terms to Country 1. Given that 40% of the final good will be absorbed in Country 1 by stage 5, we can split Country 2's $1 domestic value added in this stage into $0.4 which is Country 2's value added exports (S4, 2b), and $0.6 which is the amount of its domestic value added that will return home in Stage 5 and be absorbed at home (S4, 3b). Country 2's gross exports in this stage also contain 40% of County 1's value added from the previous stage, recorded as $0.4 in (S4, 5b).

By symmetry, the pure double counting amount in (S4, 4b) must be the same as (S3, 4a), which is $1. Let us next work out the pure double counting term in (S4, 6b). First, out of Country 2's $4 gross exports in Stage 4, only $0.4 is value added exports, we know the total amount of double counting must be $3.6, which is recorded in (S4, 7b). Second, we also know $3.6 of the total amount of double counting must be equal to the sum of the double counted components, or the sum of (S4, 3b), (S4, 4b), (S4, 5b) and (S4, 6b). This implies that (S4, 6b) should be $1.6. The economic meaning of (S4, 6b) is repeated double counting of the intermediate goods that have been double counted in previous rounds of trade.

We now go to Stage 5. Because this is the final stage in which the final good is produced by Country 1 but distributed 40% and 60% in Countries 1 and 2, respectively, we record the values somewhat differently from the earlier stages (when the entire production was exported). While Country 1's domestic value added in the production is \$1 in this stage, only 60% of the final good is exported. So we record the amount of domestic value-added in Country 1's exports as \$0.6 (S5, 1a). The amount of Country 1's value added exports (that is absorbed in Country 2) is also \$0.6, as recorded in (S5, 2a).

Since Stage 5 production uses imported intermediate good from the previous stage, it embeds foreign value added from Stage 4. The amount of foreign value added from Stage 4 that is used in Country 1's Stage 5 production and eventually absorbed in the foreign country is proportional to the amount of the final good that is exported from Country 1 to 2. This means (S5, 5a) is \$0.6. This of course is the same value as in (S4, 3b).

To determine the value in (S5, 4a), we note that the total value added from Country 1 in the first and the 3rd stages are \$1. Both values are counted as part of Country 2's intermediate exports in Stage 4. Since only 60% of the final good are exported, the pure double counting associated with the domestically produced intermediate goods in the previous stages is \$2*0.6 = \$1.2.

To determine the value of (S5, 6a), we first note that the total amount of double counting in Stage 5 exports is the difference between the value of gross exports in that stage (\$3) and the value added exports in that stage (\$0.6), which is \$2.4, as recorded in (S5, 7a). The value in (S5, 6a) would simply be the difference between \$2.4 and the sum of the values in (S5, 2a), (S5, 4a), and (S5, 5a), which yields \$0.6. The amount in (S5, 6a) represents the value that is originally created in Country 2 but has been counted multiple times beyond the value added of Country 2 already assigned to Countries 1 and 2.

We can check the sensibility of the discussion by summing over the values across the five stages. For example, when we sum up the values over all stages in Column (2a), we obtain 1.8, which is exactly the amount of Country 1's value added exports that we intuitively think should be. Summing up the values in Column (7a) across the five stages yields \$5.2, which is the same as what we obtain intuitively earlier.

Separately, we can apply our decomposition formula and generate the measurements of the same set of economic concepts. To do so, we note that the five stages in this example are best represented by 5 sectors (e.g., car windows, paint on a car, rubber tires on a car and a whole car are considered in separate sectors). Applying a multi-sector version of our gross exports accounting equations (13) and (14), we obtain estimates of the various components of the gross trade and summarize them in Table 3. (The computation details can be found in Appendix C.) It can be checked easily that the numbers in Table 3 generated by our formula match exactly with the corresponding ones that one can intuitively work out in Table 2. In particular, Country 1's value added exports (that are absorbed abroad) from our formula in Table 3 are \$1.8, exactly as that in Table 2. In comparison, the total domestic value added in Country 1's exports (that does not exclude exported value added that returns home but does exclude the pure double counted term) is \$2.6. This example confirms our theoretical discussion that value-added exports are generally smaller than domestic value-added (GDP) in exports and domestic content in exports. If one is interested in the share of domestic value added in a country's exports, then the VAX ratio is not the right metric.

From Table 3, the VS measure produced by our decomposition formula (13) is 2.2. The intuitive discussion in connection with Table 2 illustrates why we argue that the VS measure is not a 'net' concept and is not equal to foreign value added in a country's gross exports. The fundamental reason is that the VS measure has to include some pure double counted terms. (Again, these pure double counting terms would disappear if we use the HIY assumption that at least one of the countries does not export intermediate good.)

The more intermediate trade crosses border, the larger these double counted foreign intermediates imports are. With two-way intermediate trade, the part of foreign GDP that is embodied in the home country's gross exports will always be smaller than the VS measure. Relative to the original VS measure, our generalized measure includes double counted intermediate exports produced by the foreign country that may cross border several times (v8). The numerical results also show HIY's convention that a country's gross exports is equal to domestic content plus vertical specialization is also maintained by our accounting equation (as long as one defines domestic content and vertical specialization appropriately).

Finally, this example also shows that if one only considers returning domestic value-added in final goods, while excluding domestic content returning home via intermediate goods imports, such as Daudin et al (2011), then one would under-estimate VS1*. In this example, if one applies Daudin et al's narrow definition of VS1*, it would be zero as indicated by v3 in

Table 3. If one also includes returning domestic value added in intermediate good and a pure double counting term, VS1* would become \$3 instead. Our redefined measure of VS1* is more complete.

2.5 The General Case of G Countries and N Sectors

We now discuss the general case with any arbitrary number of countries and sectors. The ICIO model, gross output decomposition matrix, value-added by source shares matrix, are given succinctly by block matrix notations:

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_G \end{bmatrix} = \begin{bmatrix} I-A_{11} & -A_{12} & \cdots & -A_{1G} \\ -A_{21} & I-A_{22} & \cdots & -A_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ -A_{G1} & -A_{G2} & \cdots & I-A_{GG} \end{bmatrix}^{-1} \begin{bmatrix} \sum_r^G Y_{1r} \\ \sum_r^G Y_{2r} \\ \vdots \\ \sum_r^G Y_{Gr} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1G} \\ B_{21} & B_{22} & \cdots & B_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ B_{G1} & B_{G2} & \cdots & B_{GG} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_G \end{bmatrix} \quad (25)$$

$$\begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1G} \\ X_{21} & X_{22} & \cdots & X_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ X_{G1} & X_{G2} & \cdots & X_{GG} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1G} \\ B_{21} & B_{22} & \cdots & B_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ B_{G1} & B_{G2} & \cdots & B_{GG} \end{bmatrix} \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1G} \\ Y_{21} & Y_{22} & \cdots & Y_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{G1} & Y_{G2} & \cdots & Y_{GG} \end{bmatrix} \quad (26)$$

$$VB = \begin{bmatrix} V_1B_{11} & V_1B_{12} & \cdots & V_1B_{1G} \\ V_2B_{21} & V_2B_{22} & \cdots & V_2B_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ V_GB_{G1} & V_GB_{G2} & \cdots & V_GB_{GG} \end{bmatrix} \quad (27)$$

With G countries and N sectors, A, and B are GN×GN matrices; V and VB are G×GN matrices. V_s denotes a 1 by N row vector of direct value-added coefficient, A_{sr} is a N×N block input-output coefficient matrix, B_{sr} denotes the N×N block Leontief inverse matrix, which is the total requirement matrix that gives the amount of gross output in producing country s required for a one-unit increase in final demand in destination country r. X_{sr} is a N×1 gross output vector give gross output produced in s and absorbed in r. $X_s = \sum_r^G X_{sr}$ is also a N×1 vector that gives country s' total gross output. Y_{sr} is a N×1 vector give final goods produced in s and consumed in r. $Y_s =$

$\sum_{r}^{G} Y_{sr}$ is also a N×1 vector that gives the global use of s' final goods. Both the gross output decomposition and final demand matrix in equation (26) are GN×G matrices.

Let $\hat{V}_s$ be a N by N diagonal matrix with direct value-added coefficients along the diagonal. (Note $\hat{V}_s$ has a dimension that is different from V_s). We can define a GN by GN diagonal value-added coefficient matrix as

$$\hat{V} = \begin{bmatrix} \hat{V}_1 & 0 & \cdots & 0 \\ 0 & \hat{V}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{V}_G \end{bmatrix} \tag{28}$$

Using the similar intuition as we used to derive equation (9) in the two country one sector case, we can obtain domestic value-added in a country's gross output by multiplying this value-added coefficient matrix with the right hand side of equation (26), the gross output decomposition matrix. This will result in a GN by G value-added production matrix $\hat{V}BY$ as

$$\hat{V}BY = \begin{bmatrix} \hat{V}_1 & 0 & \cdots & 0 \\ 0 & \hat{V}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{V}_G \end{bmatrix} \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1G} \\ X_{21} & X_{22} & \cdots & X_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ X_{G1} & X_{G2} & \cdots & X_{GG} \end{bmatrix} = \begin{bmatrix} V_1 \sum_{r}^{G} B_{1r} Y_{r1} & V_1 \sum_{r}^{G} B_{1r} Y_{r2} & \cdots & V_1 \sum_{r}^{G} B_{1r} Y_{rG} \\ V_2 \sum_{r}^{G} B_{2r} Y_{r1} & V_2 \sum_{r}^{G} B_{2r} Y_{r2} & \cdots & V_2 \sum_{r}^{G} B_{2r} Y_{rG} \\ \vdots & \vdots & \ddots & \vdots \\ V_G \sum_{r}^{G} B_{Gr} Y_{r1} & V_G \sum_{r}^{G} B_{Gr} Y_{r2} & \cdots & V_G \sum_{r}^{G} B_{Gr} Y_{rG} \end{bmatrix} \tag{29}$$

Elements in the diagonal columns give each country's production of value-added absorbed at home. As in the two country case, exports of value-added can be defined as the elements in the off-diagonal columns of this GN by G matrix as

$$VT_{sr} \equiv V_s X_{sr} = V_s \sum_{g}^{G} B_{sg} Y_{gr} \tag{30}$$

Obviously, it excludes value-added produced by the home country that returns home after being processed abroad. A country's total value-added exports to the world equal:

$$VT_{s*} = \sum_{r \neq s}^{G} VX_{sr} = V_s \sum_{r \neq s}^{G} \sum_{g=1}^{G} B_{sg} Y_{gr} \tag{31}$$

By rewriting equation (31) into three groups according to where and how the value-added exports are absorbed, we obtain decomposition as follows:

$$VT_{s*} = V_s \sum_{r \neq s}^{G} B_{ss} Y_{sr} + V_s \sum_{r \neq s}^{G} B_{sr} Y_{rr} + V_s \sum_{r \neq s}^{G} \sum_{t \neq s,r}^{G} B_{sr} Y_{rt} \qquad (32)^{11}$$

This is the value-added export decomposition equation in terms of all countries' final demands. The first term is value-added in the country's final goods exports; the second term is value-added in the country's intermediate exports used by the direct importer to produce final goods consumed by the direct importer; the third term is value-added in the country's intermediate exports used by the direct importing country to produce final goods for third countries. Comparing with equation (9), we can see clearly what is missing in our two country case: it is the re-export of value-added via third countries, the last term of the RHS of equation (32), because the distinction between value-added exports from direct and indirect sources only can be made in a three or more country setting.

Define a country's gross exports to the world as:

$$E_{s*} = \sum_{r \neq s}^{G} E_{sr} = \sum_{r \neq s}^{G} (A_{sr} X_r + Y_{sr}) \qquad (33)$$

Using the logic similar to the derivation of equations (11), we can first decompose a country's gross exports to its various components as follows:

$$uE_{s*} = V_s B_{ss} E_{s*} + \sum_{r \neq s}^{G} V_r B_{rs} E_{s*}$$
$$= VT_{s*} + \{V_s \sum_{r \neq s}^{G} B_{sr} Y_{rs} + V_s \sum_{r \neq s}^{G} B_{sr} A_{rs} X_s\} + \{\sum_{t \neq s}^{G} \sum_{r \neq s}^{G} V_t B_{ts} Y_{sr} + \sum_{t \neq s}^{G} \sum_{r \neq s}^{G} V_t B_{ts} A_{sr} X_r\} \qquad (34)^{12}$$

Based on the gross output identity for each country $X_s = Y_{ss} + A_{ss} X_s + E_{s*}$, we have,

$$X_s = (I - A_{ss})^{-1} Y_{ss} + (I - A_{ss})^{-1} E_{s*} \qquad X_r = (I - A_{rr})^{-1} Y_{rr} + (I - A_{rr})^{-1} E_{r*} \qquad (35)$$

Replace X_s and X_r in equation (34), insert equation (32), we obtain the G country, N sector generalized version of gross exports accounting equation as follows:

[11] This value-added exports decomposition also could be done at the bilateral level, however, it is different from equation (15) in Johnson and Noguera (2012). They split bilateral gross exports to three groups.
[12] The step by step proof is provided in online Appendix B.

$$uE_{s*} = \{V_s \sum_{r \neq s}^{G} B_{ss} Y_{sr} + V_s \sum_{r \neq s}^{G} B_{sr} Y_{rr} + V_s \sum_{r \neq s}^{G} \sum_{t \neq s,r}^{G} B_{sr} Y_{rt}\}$$
$$+ \{V_s \sum_{r \neq s}^{G} B_{sr} Y_{rs} + V_s \sum_{r \neq s}^{G} B_{sr} A_{rs} (I - A_{ss})^{-1} Y_{ss}\} + V_s \sum_{r \neq s}^{G} B_{sr} A_{rs} (I - A_{ss})^{-1} E_{s*}$$
$$+ \{\sum_{t \neq s}^{G} \sum_{r \neq s}^{G} V_t B_{ts} Y_{sr} + \sum_{t \neq s}^{G} \sum_{r \neq s}^{G} V_t B_{ts} A_{sr} (I - A_{rr})^{-1} Y_{rr}\} + \sum_{t \neq s}^{G} V_t B_{ts} A_{sr} \sum_{r \neq s}^{G} (I - A_{rr})^{-1} E_{r*} \quad (36)$$

Equation (36) has nine terms. It is very similar to equations (13) and (14) in the two country case with only one difference. It has an additional term, representing indirect value-added exports via third countries, in its value-added exports besides the two value-added terms that directly absorbed by the direct importer. Therefore, the first three terms are value-added exports (only two terms in the two country case). The 4th and 5th term in the second bracketed expression, includes the source country's value-added in both its final and intermediate goods imports, which are first exported but eventually returned and consumed at home, both of which are parts of the source country's GDP but represent a double counted portion in official gross export statistics, and have a similar economic interpretation with the 3rd and 4th terms in equations (13) and (14). They differ from the two country case in that we have to account for the domestic value-added returning home from each of the G-1 countries here, not just from Country 2. The 7th and 8th terms in the third bracketed expression, represent foreign value-added (GDP) in the source country's gross exports, including foreign GDP embodied in both final and intermediate goods. They differ from the 6th and 7th terms in equation (13) in the two country case in that equation (36) further partitions each of the foreign value added (GDP) by individual country sources. There are also two pure double counted terms, (the 6th and the 9th terms) in equation (36) as equations (13) and (14), but they sum up the double counted portion of two way intermediate trade from all bilateral routes, not just between Country 1 and 2. The complete gross exports accounting made by equations (36) is also diagrammed in Figure 1.

Similar to the two country case, measures of vertical specialization can be expressed as linear combination of the various components identified by equations (36) as follows:

$$DV_s = V_s (I - A_{ss})^{-1} E_{s*} = VT_{s*} + V_s \sum_{r \neq s}^{G} B_{sr} Y_{rs} + V_s \sum_{r \neq s}^{G} B_{sr} A_{rs} (I - A_{ss})^{-1} Y_{ss} \geq \sum_{r}^{G} VT_{s*} \quad (37)$$

This is the first five terms in equation (36) and clearly shows that a country's domestic value-added in its exports is generally greater than its value-added exports in aggregate. The two measures equal each other only in the case where there is no returned domestic value-added in imports, i.e. when both $V_s \sum_{r \neq s}^{G} B_{sr} Y_{rs}$ and $\sum_{r \neq s}^{G} B_{sr} A_{rs} (I - A_{ss})^{-1} Y_{ss}$ are zero.

$$\begin{aligned} VS_s &= \sum_{t \neq s}^{G} \sum_{r \neq s}^{G} V_t B_{ts} Y_{sr} + \sum_{t \neq s}^{G} \sum_{r \neq s}^{G} V_t B_{ts} A_{sr} (I - A_{rr})^{-1} Y_{rr} + \sum_{t \neq s}^{G} V_t B_{ts} A_{sr} \sum_{r \neq s}^{G} (I - A_{rr})^{-1} E_{r*} \\ &= \sum_{r \neq s}^{G} V_r B_{rs} E_{s*} = \sum_{t \neq s}^{G} \sum_{r \neq s}^{G} V_t B_{ts} Y_{sr} + \sum_{t \neq s}^{G} \sum_{r \neq s}^{G} V_t B_{ts} A_{sr} X_r \end{aligned} \tag{38}$$

which is composed of the last three terms in equation (36).

We can verify that equation (38) is reduced to more familiar expressions in some special cases. Using a single country IO model, Koopman, Wang and Wei (2008, 2012) have shown

$$VS\ share = u - A_v (I - A^D)^{-1} = uA^M (I - A^D)^{-1} \tag{39}$$

In the G-country world,

$$\text{VS share} = \sum_{r \neq s}^{G} V_r B_{rs} = u - V_s B_{ss} = u - V_s (I - A_{ss})^{-1} - \sum_{r \neq s}^{G} V_s B_{sr} A_{rs} (I - A_{ss})^{-1} \qquad (40)^{13}$$

The last term in the last step can be interpreted as the adjustment made for domestic content returned to the source country. Therefore, our foreign content measure of gross exports is a natural generalization of HIY's VS measure in a multi-country setting with unrestricted intermediate goods trade. Because $\sum_{r \neq s}^{G} V_r B_{rs} + V_s B_{ss} = u$, it is natural to define a country's domestic content in its exports as:

$$DC_s = V_s B_{ss} E_{s*} = VT_{s*} + V_s \sum_{r \neq s}^{G} B_{sr} Y_{rs} + V_s \sum_{r \neq s}^{G} B_{sr} A_{rs} X_s \geq DV_s \geq \sum_{r}^{G} VT_{sr} \tag{41}$$

This is the sum of first 6 terms in equation (36).[14] It shows that a country's domestic content in its exports is generally greater than the part of its GDP in exports therefore is also greater than its value-added exports in the aggregate. The three measures equal each other only in the case where there is no returned domestic value in imports, i.e. when both $V_s \sum_{r \neq s}^{G} B_{sr} Y_{rs}$ and $V_s \sum_{r \neq s}^{G} B_{sr} A_{rs} X_s$ are zero.

[13] See online appendix for the step by step proof.
[14] Note that the third term can be further decomposed into two terms (the 5th and 6th terms) in equation (36).

The second HIY measure of vertical specialization (VS1) measures the value of the exported goods that are used as imported inputs by other countries to produce their exports. Although an expression for such indirectly exported products has not been previously defined mathematically in the literature, it can be specified based on some of the terms in our gross exports accounting equations.

$$VS1_s = V_s \sum_{r \neq s}^{G} B_{sr} E_{r*} = V_s \sum_{r \neq s}^{G} \sum_{t \neq s,r}^{G} B_{sr} Y_{rt} + V_s \sum_{r \neq s}^{G} \sum_{t \neq s,r}^{G} B_{sr} A_{rt} X_t + V_s \sum_{r \neq s}^{G} B_{sr} Y_{rs} + V_s \sum_{r \neq s}^{G} B_{sr} A_{rs} X_s \qquad (42)$$

This means HIY's VS1 can be expressed as the third term in (36) plus the third and fifth terms in equation (34). The second term in equation (42) measures how much domestic content in exported goods from the source country is used as imported inputs to produce other countries' intermediate goods exports. It also shows clearly that HIY's VS1 measure is generally greater than indirect value-added exports because the latter only includes the first term of (42), but excludes domestic content that is returned home and the value embodied in intermediate goods exports via third countries (they are already counted as other countries' foreign content in these third countries' exports), i.e.

$$IV_s = V_s \sum_{r \neq s}^{G} \sum_{t \neq s,r}^{G} B_{sr} Y_{rt} \leq VS1_s \qquad (43)$$

Compared with equation (24) of the definition of VS1 in the two-country case, two additional terms (the first two terms) appear on the RHS of (42) because of the third country effect.

$$VS1^*_s = \sum_{s \neq t} V_s B_{sr} E_{rs} = V_s \sum_{r \neq s}^{G} B_{sr} Y_{rs} + V_s \sum_{r \neq s}^{G} B_{sr} A_{rs} X_s \qquad (44)$$

As equation (44) shows, we define VS1* as a subset of VS1 similar to Daudin et al. (2011), but our definition differs from theirs as they include only domestic value-added returned home in final goods imports (the first term in equation (44)) but exclude domestic content returned home by being embodied in the imports of intermediate goods, the second term in equation (44).[15] If one omits the second term, then VS1* would be inconsistent with the core idea of measuring vertical specialization from the export side, as it would fail to account for the source country's exports used by third countries to produce their export of intermediate goods. It would therefore consistently under-estimate actual vertical specialization. To put it differently, the same domestic content embodied in a country's intermediate goods exports manifests itself in

[15] Note that the second term can be further decomposed into two terms (the 5th and 6th terms) in equation (36).

international trade flows in two ways: (a) as foreign content in other countries exports, and (b) as the source country's indirect exports of domestic content via a third country. In other words, the foreign content in one country's exports is the domestic content of another country embodied in its indirect exports. As an example, the Japanese content in the form of Japanese-made computer chips used in China's exports of electronic toys to the United States represents foreign content in China's exports, and it is also simultaneously Japan's indirect exports of its domestic content to the United States. While these two perspectives produce the identical numbers when aggregating across all countries at the global level, their values for a given country can be very different. Therefore, when measuring a country's participation in vertical specialization it is useful to be able trace the two perspectives separately. This is also why HIY proposed two measures of vertical specialization, because a complete picture of vertical specialization and a county's position in a vertical integrated production network has to involve both measures. Indeed, for a given country, the ratio of the two measures provides insight for the country's position in global value chains. Downstream countries tend to have a higher share of vertical specialization from the import side, i.e higher foreign content (VS) in their exports, while upstream countries tend to have a higher share of vertical specialization from export side (VS1), higher share of exports via third countries. In addition, as we show in both equations (13) and (36), ignoring domestic content returning home via intermediate goods imports, part of the 9 components (or part of the 8 terms in the case of two countries) in a country's gross exports, would leave the gross export accounting incomplete.

Therefore, equation (36) provides a new way of thinking about the gross exports statistics. It demonstrates that various double counted items in gross exports can be used to gauge the depth of a country's participation in global production chains and provide useful quantitative information to construct various measures of vertical specialization. In other words, the relative importance of the various double-counted terms in addition to the value-added trade estimates contain useful and important information on how a country participates in the global production chains and vertical specialization. Simply stripping away double counted items and focusing just on value added trade would miss such useful information. We have already provided numerical examples in an earlier sub-section to illustrate the intuition of this point; now we have laid out the accounting equation in a more general multi-country setting. It provides a transparent framework that allows various value-added and double counted components in a country's official gross exports statistics to be correctly identified and estimated.

Because our gross exports accounting approach can simultaneously produce estimates of the domestic/foreign content in exports, which is a natural extension of HIY's measures to the global setting, estimates of value added exports, and estimates of various double counted measures in gross exports, which reflect the depth of a country's participation in vertical specialization, our approach can have many useful applications.

Equation (36) (or Figure 1) also integrates the older literature on vertical specialization with the newer literature on trade in value added, while ensuring that the sum of value-added components from all sources and the additional double counted intermediate goods exports (due to back-and-forth trade in intermediate goods) yields total gross exports. The previous vertical specialization literature only decomposes gross exports into two components: domestic and foreign content. In comparison, Equation (36) shows that a country's domestic content can be further broken up into sub-components that reveal the destinations for a country's exported value added, including its own value-added that returns home in its imports and what is double counted due to cross border intermediate goods trade. Similarly, equation (36) also traces out the foreign content in a country's exports to its sources.

On the other hand, the trade in value-added literature emphasizes estimation of value-added exports by eliminating double counting. However, without knowing the structure of the double counted components in gross exports, it would be hard to fully address what the vertical specialization literature intends to do, such as finding an explanation for why the growth of global trade is much faster than the growth of global GDP after World War II (Yi, 2003). Our gross exports accounting method integrates the major concepts in the literature into a unifying framework on the one hand, and clearly distinguishes them on the other hand.

Finally, please note that a single subscript is used for the domestic content measure and two subscripts are used for the trade in value-added measure. This is to suggest that the trade in value-added measure holds for both aggregate and bilateral trade, while the gross export accounting equation we propose only holds for a country's total exports to the world.

3. Data and results

3.1 The construction of an Inter-Country Input-output (ICIO) table and its data sources

To implement the above accounting method, we need an inter-country input output (ICIO) table, that is, a database detailing international production and use for all flows of value added. The database should specify (a) transactions of intermediate products and final goods within and between each country at the industry level, (b) the direct value-added in production of each industry in all countries, and (c) the gross output of each industry in all countries. Such an ICIO table goes beyond a collection of single-country IO tables. It specifies the origin and destination of all transaction flows by industry as well as every intermediate and/or final use for all such flows. For example, an ICIO table would describe the number of electronics components produced in Japan that were shipped to China. It would also distinguish the number that were used as intermediate inputs in each Chinese sector and the number that were used in Chinese private household consumption and capital formation. However, these tables are not available on a global basis, and in fact are rarely available at the regional level. The available global databases, such as the GTAP Multi-Country Input-Output (MCIO) tables, do not have enough detail on the cross-border supply and the use of goods to be directly used to implement our methodological framework.

To provide a workable dataset and empirically conduct our gross export decomposition, we construct a global ICIO table for 2004 based on version 7 of the GTAP database as well as detailed trade data from UN COMTRADE, and two additional IO tables for major emerging economies where processing exports are a large portion of their external trade. We integrate the GTAP database and the additional information using a quadratic mathematical programming model that (a) minimizes the deviation of the resulting new data set from the original GTAP data, (b) ensures that supply and use balance for each sector and every country, and (c) keeps all sectoral bilateral trade flows in the GTAP database constant.[16] The new database covers 26 countries and 41 sectors and is used as the major data source of this paper.[17]

ICIO tables specify country *r*'s use in sector *i* of imports from sector *j* of source country *s*. To estimate these detailed inter-industry and inter-country intermediate flows, we need to (i) separate gross bilateral trade flows at the sector level in the GTAP database into intermediate,

[16] Please refer to Tsigas, Wang and Gehlhar (2012) for details on how such a database can be constructed from the GTAP database.
[17] See online Appendix table C2 for countries included in each region and their concordance to GTAP regions.

consumption and investment goods trade flows, and (ii) allocate intermediate goods from a particular country source to each sector it is used within all destination countries. We address the first task by concording the three end-use categories defined by UN Broad Economic Categories (BEC) to the 6-digit HS level bilateral trade data in COMTRADE[18]. This differs from Johnson and Noguera (2012) and Daudin, Rifflart, and Schweisguth (2011), who also transform the MCIO table in the GTAP database into an ICIO table. However, they do not use detailed trade data to identify intermediate goods in each bilateral trade flow. Instead, they apply a proportionality method directly to the GTAP trade data; i.e., they assume that the proportion of intermediate to final goods is the same for domestic supply and imported products.

However, no additional information is currently available to properly allocate intermediates of a particular sector from a specific source country to its use industries at the destination economy. Thus, sector *j*'s imported intermediate inputs of a particular product are initially allocated to each source country by assuming they are consistent with the aggregate source structure of that particular product.[19]

The use of end-use categories to distinguish imports by use is becoming more widespread in the literature and potentially avoids some noted deficiencies in the proportionality method. Feenstra and Jensen (2009) use a similar approach to separate final goods from intermediates in U.S. imports in their recent re-estimation of the Feenstra-Hanson measure of material off-shoring. Dean, Fung, and Wang (2011) show that the proportionality assumption underestimates the share of imported goods used as intermediate inputs in China's processing trade. Nordas (2005) stated that the large industrial countries have a higher share of intermediates in their exports than in their imports, while the opposite is true for large developing countries. These results imply that the intermediate content of imports differs systematically from the intermediate content in domestic supply.

In theory, less distorted intermediate share estimates will provide a better row total control for each block matrix of A_{sr} in the ICIO coefficient matrix *A*, thus improving the accuracy of the

[18] Both the zero/one and a weighting scheme can be used with BEC. We used a zero/one classification. Shares based on additional information could be applied to dual use products to further improve the allocation. These are areas for future research.

[19] For example, if 20% of U.S. imported intermediate steel comes from China, we then assume that each U.S. industry obtains 20% of its imported steel from China. Such an assumption ignores the heterogeneity of imported steel in different sectors. It is possible that 50% of the imported steel used by the U.S. construction industry may come from China, while only 5% of the imported steel used by auto makers may be Chinese.

most important parameters (the inter country IO coefficients) in an ICIO model. This is why distinguishing imports by BEC may potentially improve ICIO database quality over the proportionality assumption. When it comes to actual data, it is possible that the BEC method represents only a numerically small improvement at the aggregate level or introduces its own errors. To precisely assess whether and how much the BEC method improves over the proportionality approach, one would need the true inter-country IO coefficients as a “reference point”, which unfortunately do not exist on a global scale. We will provide some evidence that the two alternative ways to identify intermediate goods from bilateral trade flows in the literature have a significant impact on trade cost when the magnification effects of multi-stage production are taken into account in one of our application examples.

3.2 Complete accounting of gross exports

Table 4 presents a complete accounting of each country’s gross exports to the world in 2004 using the 9 basic value-added and double counted components specified in equation (36). The column numbers correspond to the order of each item in the equations and also the box numbers in Figure 1. The first three columns also correspond to the three terms in the RHS of equation (32).

We compute all these nine terms independently according to our gross exports accounting equation and verify that they sum to exactly 100 percent of gross exports. The resulting estimates constitute the first such decomposition in a global setting and clearly highlight what is double counted in the official trade statistics. Column (15) reports the percentage of double counting by adding columns (4) to (9). At the global level, only domestic value added in exports absorbed abroad are value-added exports. In addition to foreign content in exports, domestic content that returns home from abroad is also a part of double counting in official trade statistics, since it crosses borders at least twice. Such returned value added has to be separated from domestic value-added absorbed abroad in order to fully capture multiple counting in official trade statistics. Therefore, for any country’s gross exports, the double counting portion equals the share of gross exports in excess of the value-added exports. This share is about 25.6% for total world exports in 2004 based on our ICIO database.

The accounting results reported in table 4 also provide a more detailed breakdown of domestic content in exports than has been previously available in the literature. The variations in the relative size of different components across countries provide a way to gauge the differences in the role that countries play in global production networks. For example, for the United States, the share of foreign value-added in its exports was only 9% (columns 7+8), while its own GDP first exported then finally return home is large at 11.3% (columns 4+5), indicating that most of its exports reflect its own domestic value added. In comparison, for China’s and Mexico's processing exports, the share of foreign GDP in their gross exports return to abroad is 46.5% and 55.8% respectively, with an additional 10.1% and 7.6% of their gross exports coming from intermediate goods produced in foreign countries (column 9), indicating domestic value added accounts for less than half the value of both countries processing exports. More importantly, about half of the double counting in U.S. exports – reflected as 1-VAX ratio (column 11) in column 15 - comes primarily from its own domestic content returning home via imports (12.4% over 25.4%). In contrast, almost all of the double counting in China's and Mexico's processing exports comes from imported foreign content (56.6% over 56.9% and 63.4 over 63.7 respectively). These calculations highlight U.S. export producers and Chinese and Mexican processing exporters' respective positions at the head and tail of global production chains in 2004.

The structure of double counted terms in each country's gross exports listed in columns (4) through (9) offers additional information on how each country participates in vertical specialization and its relative position in global production chains. For example, for Western EU countries, about 40% of its double counted gross exports come from domestic content returned home (7.4/18.9). In comparison, for most developing countries, the foreign content tends to dominate, with only a very tiny portion of their domestic content returning home. Within foreign content, Maquiladora producers in Mexico, export processing zones in China and Viet Nam, tend to have a large portion embodied in their final goods exports (37.6%, 34.1% and 24.4%, respectively), reflecting their position as the assemblers of final goods in global production chains. For developed and newly industrialized economies, the shares in intermediate goods exports and the pure double counted portion due to multiple border crossing intermediate goods trade are much higher. Similarly, upstream natural resource producers such as Australia & New Zealand, Russia, Indonesia and Philippines, have a significant portion of their intermediate exports used by other countries to produce their intermediate goods exports. This is also true for upstream producers of manufacturing intermediates such as Japan.

To reiterate the connection of the 9 basic value-added components reported in the left-hand panel of table 4 to measures in the existing literature by numerical estimates, column (11) reports the value-added to gross exports (VAX) ratio proposed by Johnson and Noguera (2012) by adding up columns (1), (2), and (3); column (12) list HIY's VS share by summing column (7) , (8) and (9). Column (14) lists the share of domestic content extensively discussed in the vertical specialization literature by summing columns (1) through (6); Finally, column (16) gives the share of vertical trade by adding columns (12) and (13), which is an indicator of how intensively a country is participating in the global production chain.

Comparing domestic content share estimates (Column 14) and Johnson & Noguera's VAX ratio (Column 11) reported in table 1, we see interesting differences between high-income countries and emerging market economies. For most emerging market economies, the numerical difference of these two measures is quite small. This means that only a tiny part of domestic content returns home for most countries. In comparison, for the United States, Western Europe and Japan, the difference between domestic content share and the value added export share is more significant. This reflects the fact that advanced economies export relatively more upstream components, and some of the value added embedded in these intermediate goods returns home as part of other countries' exports to the advanced economies. Such differences between high-income countries and emerging market economies would not be apparent if one does not compute the domestic content share and the VAX ratio separately.

4. Broad implications for a better understanding of global trade

The decomposition results have implications for a variety of research and policy questions. In this section, to illustrate the potential importance of the decomposition, we briefly discuss a few applications.

4.1 Revealed Comparative Advantage index based on gross and value-added trade

The concept of revealed comparative advantage (RCA for short), proposed by Balassa (1965), has proven useful in many research and policy applications. In standard applications, it is defined as the share of a sector in a country's total gross exports relative to the world average of the same sector in world exports. When the RCA exceeds one, the country is said to have a revealed comparative advantage in that sector; when the RCA is below one, the country is said to have a revealed comparative disadvantage in that sector. The problem of double counting of certain value added components in the official trade statistics suggests that the traditional computation of RCA could be noisy and misleading. The gross exports accounting excise we proposed in this paper provides a way to remove the distortion of double counting by focusing on domestic value-added in exports. Because domestic value-added or GDP in a country's exports describes the characteristics of a country's production (total domestic factor content in output); it does not depend on where the output is absorbed. In comparison, the concept of exports of value added (that is ultimately absorbed abroad) depends on where a given value added is absorbed as well as it is produced. For those applications in which a production-based RCA is the right measure, we can use domestic value-added in exports to compute RCA.

We re-compute the RCA index at the country-sector level for all the countries and sectors in our database. Due to space constraints, we select two sectors and compare the country rankings of RCAs using both gross exports and domestic value-added in gross exports. In Figure 2, we report the two sets of RCA indices for the finished metal products sector. Using gross exports data, both China and India show a strong revealed comparative advantage (ranked the first and fourth, respectively, among the set of countries in our database, and with the absolute values of RCA at 1.94 and 1.29, respectively). However, when looking at domestic value added in that sector's exports, both countries ranking in RCA drop precipitously to 7th and 15th place, respectively.[20] In fact, for India, the sector has switched from being labeled as a comparative advantage sector to a comparative disadvantage sector. Unsurprisingly, the ranking for some other countries move up. For example, for the United States, not only does its RCA ranking move up from 10th place under the conventional calculation to the 3rd place under the new calculation, but its finished metal products industry also switches from being labeled as a comparative disadvantage sector to a comparative advantage sector.

Another example is the "real estate and business services" sector. Using data on gross exports, India exhibits a strong revealed comparative advantage in that sector on the strength of its unusually high share of business services exports in its overall exports. However, once we compute RCA using domestic value-added in exports, the same sector becomes a comparative disadvantage sector for India! One key reason for the change is that business services in

[20] Sectoral value added here includes value produced by the factors of production employed in the finished metal products sector and then embodied in gross exports of all downstream sectors, rather than the value added employed in upstream sectors that are used to produce finished metal products in the exporting country. This distinction is particularly important in the business services sector, discussed next.

advanced countries are often exported indirectly by being embedded in these countries manufacturing exports. Indeed, the RCA rankings for this sector in the United States, the European Union and Japan all move up using data on the domestic value-added in exports. Therefore, compared to the share of this sector in other countries' exports (after taking into account indirect value added exports), the Indian share of the sector in its exports becomes much less impressive.

These examples illustrate the possibility that our understanding of trade patterns and revealed comparative advantage could be modified substantially once we have the right data on domestic value added in exports.

4.2 Magnification of trade costs from multi-stage production

As noted by Yi (2003, 2010), multi-stage production magnifies the effects of trade costs on world trade. There are two separate magnification forces. The first exists because goods that cross national borders multiple times incur tariffs and transportation costs multiple times. The second exists because tariffs are applied to gross imports, even though value added by the direct exporter may be only a fraction of this amount. Different ways of participating in global production chain affects the extent to which different countries are affected by such cost magnification. However, Yi (2003) does not actually measure the magnification of tariffs, though it is important to his simulation exercise.

Our gross export accounting method provides an ideal way to re-examine the magnification issue. In Table 5, we first report standard tariffs (on a country's exports) in columns (1a). These are trade-weighted tariff rate applied by a country's trading partners (in ad-valorem equivalent). Column (2a) reports the share of imported content in final goods exports. These imported intermediate inputs are used to produce final goods exports, and so incur multiple tariffs charges. These tariff rates on the imported inputs (as a share of f.o.b. export value) are presented in columns (3a); they are trade-weighted average tariffs for intermediate inputs from the other 25 countries/regions in our database that are used in the exporting country to produce final goods exports. The sum of the two tariffs is reported in Column (4a).

Columns (5a) reports our illustrative calculation of the first order magnification effect of using imported intermediate inputs to produce exports. It represents the magnification effect if tariffs were the only factor that augments the trading costs. For instance, one additional stage of production increases trade costs of Vietnam's merchandise production by 80% of its standard tariff.

Although the number is already quite high for a number of countries, these values still represent only the lower bound of the true multi-stage tariff charge. First, in this illustration, we only consider two stages of production, while in the real world, these inputs may have already crossed multiple borders before reaching the final exporter. Second, we ignore transport costs in this example, but transport costs are also magnified as intermediate goods cross multiple borders.

The second magnification force occurs because tariffs are applied to gross export values instead of the value added in the direct exporting country. Table 2 also reports the magnification ratio of the "effective" tariff rate to the standard tariff rate. Column (6a) reports the effective tariff rate, which equals the standard tariff rate in column (2a) divided by the domestic content share (which is 1 minus column (2a)) and weighted by trade. Column (7a) reports the implied magnification ratio due to the presence of vertical specialization. These effects are generally larger than the tariff magnification factor reported in column (5a).

Generally speaking, tariffs play a large role in the magnification of trade costs in the presence of GVCs for emerging market economies, while they play a smaller role for most developed countries. The fact that the domestic value added share in emerging economies' merchandise exports is usually lower than that in developed countries tends to amplify the effective trade cost for developing countries. As an implication, reducing tariffs and nontariff barriers in manufacturing sectors globally is fully consistent with the interest of emerging market economies because it lowers the cost of GVC participation for developing countries. Lowering "own" tariffs on intermediate inputs for domestic manufacturing production would significantly reduce the magnification effects as demonstrated in column (5), while lowering such tariffs in other countries would significantly reduce the effective rate of protection, as seen in columns (6) and (7), due to the lower domestic value-added share in most developing countries' manufacturing exports.

To see if the end-use classifications and the proportionality assumption produce different results, we go through the same set of calculation but using the proportionality assumption to construct our data set. All the estimates in Columns (1b), (2b), …,(7b) are the direct counterparts to Columns (1a), (2a), …, (7a). In Column 8, we report the difference in terms of % of each country's gross exports for the magnification factor computed using the two different databases.

For Indonesia, Malaysia, and China, the BEC method produces a larger magnification effect. In comparison, for Canada, India, and Mexico, the BEC method produces a smaller magnification effect. In general, which method we use makes a difference.

4.3 Bilateral trade imbalance

Because a country's gross exports embed value added from other countries, its bilateral trade balance in value added terms can be very different from its bilateral balance in gross trade terms. This point is already well understood qualitatively. The decomposition results in this paper allow us to quantify the difference.

Figure 3 provides a scatter plot of the trade balance in value added terms against the trade balance in standard trade statistics for all bilateral country pairs in our ICIO database. Without loss of generality, the two countries in any pair are always ordered in such a way that the trade balance in gross terms is non-negative. A negative value-added to gross BOT ratio indicates there is a sign change between BOT measured in gross and value-added terms. All observations that lie below the 45 degree line have their bilateral trade imbalance smaller in value-added terms than those in gross terms, and vice visa for observations that lie above the 45 degree line.

Value-added flows give a much different picture of the contributions of China and Japan to the U.S. and Western EU countries' trade deficits. Because China is the final assembler in a large number of global supply chains, and it uses components from many other countries, especially East Asian countries, its trade surplus with US and Western EU countries measured in value-added term is 41% and 49% less than that measured in gross terms. In contrast, Japan's trade surplus with the U.S. and Western EU countries are 40% and 31% larger measured in value-added terms, because Japan exports parts and components to countries throughout Asia that are eventually assembled into final products and exported to the United States and Western EU countries.[21]Zooming in near the origin shows that the trade balances of a number of country pairs even have opposite signs measured in value-added and gross terms. For example, Japan's trade balance vis-à-vis China is switched from a surplus in gross trade terms to a deficit in value added terms. This is consistent with the notion that a significant part of Japan's exports to China are components used by China-based firms for exports to the United States, the European Union and other markets. This further illustrates potentially misleading nature of gross bilateral trade imbalances.

[21]Figure 3 also shows that the Korea-China-U.S. triple trade relationship is similar to the Japan-China-U.S. one.

4.4 Other applications

The set of examples discussed so far certainly does not exhaust the possible applications. For example, the Federal Reserve Board and the IMF routinely compute effective exchange rates using trade weights that are based on gross exports and imports. A conceptually better measure should weight trading partners based on the relative importance in value added trade rather than in gross trade terms. Our decomposition results make it feasible to do such computations.

As another example, for some research or policy questions, one might need to look at the response of a country's bilateral or multilateral trade to exchange rate changes. Once one recognizes that there is a potential mismatch between trade in value added and trade in gross terms, one would want to take this into account. Our decomposition allows for a correction.

5. Conclusions

We have developed a unified conceptual framework based on the block-matrix structure of an ICIO model and can fully account for a country's gross exports by its various value-added and double counted components. This new framework incorporates all previous measures of vertical specialization and value-added trade in the literature while adjusting for the back-and-forth trade of intermediates across multiple borders. With a full concordance between value-added and double counted components and official gross trade statistics, it opens the possibility for the System of National Accounts (SNA) to accept the concept of value-added trade without dramatically changing current practice of customs trade data collection. This may in turn provide a feasible way for international statistical agencies to report value-added trade statistics regularly in a relatively low cost fashion.

The creation of a database that encompasses detailed global trade in both gross and value-added terms will allow us to move from a largely descriptive empirical exercise to analysis of the causes and consequences of differences in supply chain participation. We have discussed the use of the accounting results to re-compute revealed comparative advantages and bilateral trade *balance.*

Better information at the sector level can improve our estimation. For instance, current end use classifications, such as the UN BEC, need to be extended to dual use products and services trade. In addition, methods also need to be developed to properly distribute imports to domestic users either based on cross country statistical surveys or based on firm level and Customs transaction-level trade data. We leave such sector level applications to future research.

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Table 1. A two country supply-chain example

	Country 1				Country 2			
	Final Demand	Labor input	Imported input	Output	Imported input	Labor input	Output	Final Demand
Stage 1 in		$1						
Stage 1 out				$1				
Stage 2 in					$1	$1		
Stage 2 out							$2	
Stage 3 in		$1	$2					
Stage 3 out				$3				
Stage 4 in					$3	$1		
Stage 4 out							$4	
Stage 5 in		$1	$4					
Stage 5 out				$5				
	$2							$3
Total	$2	$3	$6	$9	$4	$2	$6	$3

Table 2 Intuitive accounting for the gross export flows in the two country supply chain

	From Country 1's Viewpoint							From Country 2's Viewpoint						
	Domestic Value-added In exports	Value-added exports	Previous Intermediate exports returning home		Foreign intermediate imports		Total double counted intermediate in exports	Domestic Value-added in exports	Value-added exports	Previous Intermediate exports returning home		Foreign intermediate imports		Total double counted interme-diate in exports
			DV	Pure Double counting	FV	Pure Double counting				DV	Pure Double counting	FV	Pure Double counting	
	(1a)	(2a)	(3a)	(4a)	(5a)	(6a)	(7a)	(1b)	(2b)	(3b)	(4b)	(5b)	(6b)	(7b)
Stage 1 (S1): Country 1 exports and Country 2 imports	1	0.6	0.4	0	0	0	0.4							
Stage 2 (S2): Country 2 exports and Country 1 imports								1	0.4	0.6	0	0.4	0.6	1.6
Stage 3 (S3): Country 1 exports and Country 2 imports	1	0.6	0.4	1	0.6	0.4	2.4							
Stage 4 (S4): Country 2 exports and Country 1 imports								1	0.4	0.6	1	0.4	1.6	3.6
Stage 5 (S5): Country 1 exports and Country 2 imports	0.6[a]	0.6	0	1.2	0.6	0.6	2.4							
Total	**2.6**	**1.8**	**0.8**	**2.2**	**1.2**	**1**	**5.2**	**2**	**0.8**	**1.2**	**1**	**0.8**	**2.2**	**5.2**
Terms in Table 3 that correspond to the previous row	DV_1	v1+v2	v3+v4	v5	v6+v7	v8	Sum of v3 to v8	DV_2	v1+v2	v3+v4	v5	v6+ v7	v8	Sum of v3 to v8

Note: a. In stage 5, because Country 1 exports 3 units of the final goods and keeps 2 units at home, 40% of Country 1's domestic value added (or $0.4) in that stage stays home, and 60% of it (or $0.6) is its exports of value added to Country 2. The last row shows the concordance between the second to the last row of this table and the decomposition results reported in Table 3 that are derived from our gross exports accounting equations.
The gross exports accounting equations (13) and (14) provide the final decomposition results as the total row in the table, not the intermediate iteration in each the stage.

Table 3 Gross exports decomposition based on our accounting equation

Terms in accounting equation	E12	E21
v1= $v_1 b_{11} y_{12}$	1.8	0
v2= $v_1 b_{12} y_{22}$	0	0.8
v3= $v_1 b_{12} y_{21}$	0	1.2
v4= $v_1 b_{12} a_{21} (1-a_{11})^{-1} y_{11}$	0.8	0
v5= $v_1 b_{12} a_{21} (1-a_{11})^{-1} e_{12}$	2.2	1
v6= $v_2 b_{21} y_{12}$	1.2	0
v7= $v_2 b_{21} a_{12} (1-a_{22})^{-1} y_{22}$	0	0.8
v8= $v_2 b_{21} a_{12} (1-a_{22})^{-1} e_{21}$	1	2.2
E=Gross exports (sum v1 to v8)	7	6
VT=Value-added exports (sum of v1 and v2)	1.8	0.8
DV=Domestic value-added in gross exports (sum of v1 to v4)	2.6	2
FV=Foreign value-added in gross exports (v6+v7)	1.2	0.8
DC=Domestic content in gross exports (sum of v1 to v5)	4.8	3.0
Double counted home country's intermediate exports	2.2	1
Double counted foreign country's intermediate exports	1	2.2
VS=Vertical specialization(sum v6 to v8) =$v_1 b_{12}$	2.2	3
VS1* measure defined in this paper (sum v3 to v5)	3	2.2
VS1* measure defined in Daudin, et al. (v3 only)	0	1.2
Johnson & Noguera's VAX ratio	0.257	0.133
Share of domestic value-added in gross exports	0.371	0.333
Share of domestic content in gross exports =$v_1 b_{11}$	0.686	0.5

Table 4 Accounting of gross exports, 2004

Country/region	*Value-added exports*				*Domestic VA return home*		*Pure double counting*	*Foreign VA return foreign countries*		*Pure double counting*	*Connection with existing measures*						
	in Billions of US dollars	in direct final exports	in int. absorb by direct importers	in int. re-exports to third countries	in final exports	in int. exports	in int. exports produced in home	in final exports	in int. exports	in int. exports produced abroad	Total	VAX ratio	VS share	VS1 share	DC share	Double count share	Share of Vertical trade
	(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Advanced economies																	
Aus-New Zealand	122.5	27	50.5	10.5	0.2	0.2	0.1	3.7	5.1	2.8	100	88	11.5	27.9	88.5	12	39.4
Canada	323	23.5	43	4	0.5	0.4	0.5	12.8	12	3.3	100	70.5	28.1	12.2	71.9	29.5	40.4
Western EU	1,575.50	38.1	38.1	5	3.1	3.4	0.9	4.8	4.3	2.4	100	81.1	11.4	20.9	88.6	18.9	32.3
Japan	618.9	38.4	34.2	12.2	1.4	1	0.5	4.8	3.7	3.7	100	84.9	12.2	30.8	87.8	15.1	43.1
United States	1,062.30	32.5	36.6	5.5	6.8	4.5	1.2	4.3	4.7	4	100	74.6	12.9	27	87.1	25.4	39.9
Asian NICs																	
Hong Kong	121.7	27.2	35.7	9.1	0.4	0.1	0.1	10.2	9.9	7.4	100	71.9	27.5	19.5	72.5	28.1	47
Korea	283.1	29.5	25.3	10.4	0.3	0.2	0.4	13.1	9.9	10.8	100	65.2	33.9	23.2	66.1	34.8	57
Taiwan	219.8	19.2	25.9	13.1	0.2	0.1	0.5	12.4	12.5	16.1	100	58.2	41.1	27.1	58.9	41.8	68.2
Singapore	150.6	11	20.1	5.2	0.1	0.1	0.4	17	24.2	22.1	100	36.3	63.2	12.8	36.8	63.7	76
Emerging Asia																	
China Normal	334.6	44.2	31.8	8.1	0.4	0.7	0.1	6.8	5.2	2.7	100	84.2	14.6	20.7	85.4	15.8	35.3
China Processing	336	28.8	12.6	1.7	0	0	0.3	34.1	12.4	10.1	100	43.1	56.6	4.3	43.4	56.9	60.9
China total	670.6	36.5	22.2	4.9	0.2	0.3	0.2	20.5	8.8	6.4	100	63.6	35.7	12.5	64.3	36.4	48.1
Indonesia	86.7	20	45.6	10.9	0.2	0.3	0.2	9.2	7.2	6.5	100	76.5	22.9	29	77.1	23.5	51.9
Malaysia	152	16.7	31.5	10.4	0.3	0.1	0.6	12.9	13.6	14	100	58.6	40.5	25	59.5	41.4	65.5
Philippines	50.1	17.6	27.8	12.4	0.1	0	0.2	10.8	14	17.1	100	57.8	41.9	29.4	58.1	42.2	71.2
Thailand	119.4	27.9	24.2	7.9	0.1	0.1	0.2	17.2	11.3	11.2	100	60	39.7	18.5	60.3	40	58.1
Vietnam	32.3	32.9	24.9	4.8	0	0.2	0.2	24.4	7.7	4.9	100	62.6	37.0	14.8	63.0	37.4	51.8
India	99.9	30.2	41.7	7.7	0.1	0.2	0.1	6.4	9.5	4.2	100	79.6	20.1	18.9	79.9	20.4	39
Other emerging																	
Brazil	113	27.4	52.1	7.5	0.1	0.1	0.1	3.9	6.4	2.4	100	87	12.7	19.2	87.3	13	31.9
New EU countries	273.7	28.7	35.2	4.3	0.4	0.3	0.3	13.4	13.2	4.2	100	68.3	30.8	11.4	69.2	31.7	42.1
Mexico Normal	63.6	23.5	52.7	5.9	0.1	0.5	0.1	2.9	11.5	2.8	100	82.1	17.3	18	82.7	17.9	35.3
Mexico Processing	126.9	20.6	13.3	2.4	0	0	0.3	37.6	18.2	7.6	100	36.3	63.4	5.9	36.7	63.7	69.2
Mexico total	190.5	21.6	26.5	3.6	0.1	0.2	0.2	26	16	6	100	51.6	48.0	9.9	52.0	48.4	57.9
Russian	160.2	9.5	70.2	9.4	0.3	0.2	0.1	1.6	5.9	2.7	100	89.1	10.2	31.2	89.8	10.9	41.4
South Africa	61.4	23.1	50	8.5	0.1	0.1	0.1	5.3	8.9	4	100	81.6	18.2	24.2	81.8	18.4	42.4
World average	7,733.40	29.2	38.4	6.8	1.9	1.5	0.6	8.7	7.7	5.1	100	74.4	21.5	21.5	78.5	25.6	43

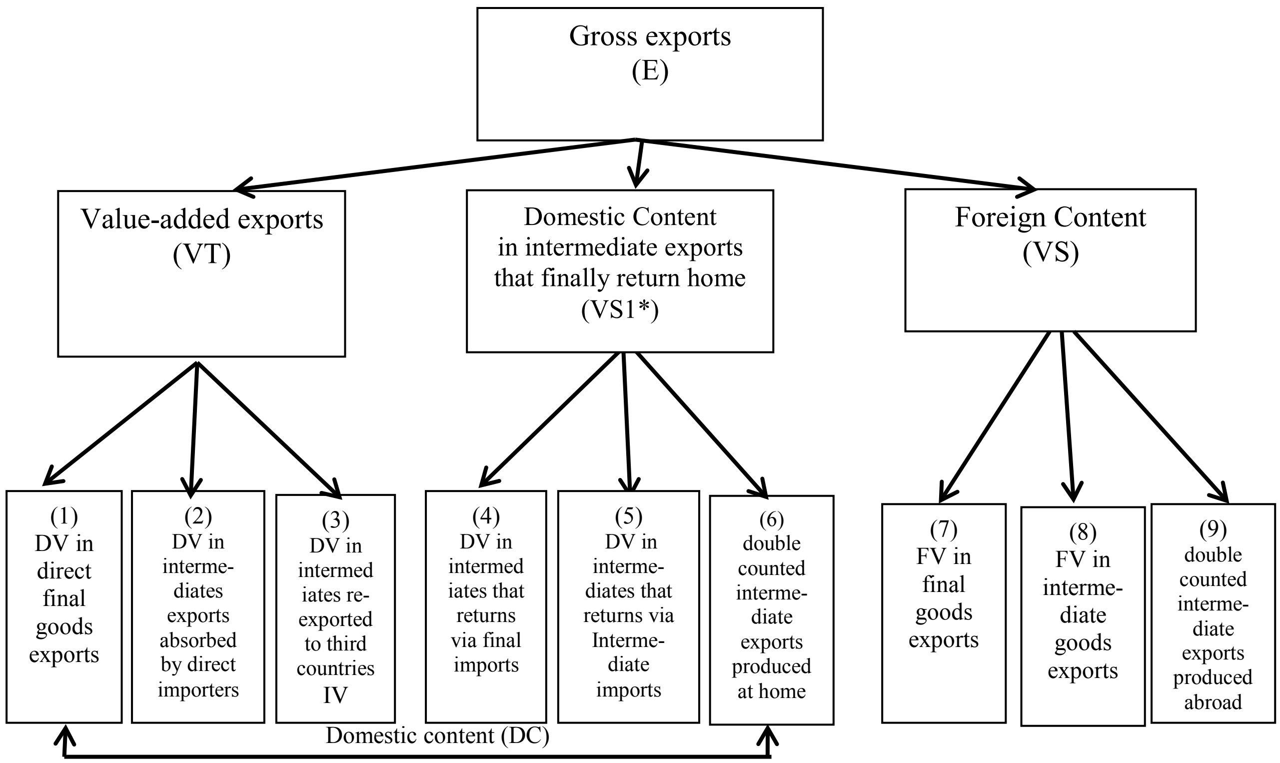
Source: Authors' estimates
Notes: All columns are expressed as a share of total gross exports.
Column (11) = (1)+(2)+(3); column (12) =(7)+(8)+(9);Column (14)= (1)+(2)+(3) +(4)+(5)+(6);Column (15) equal sum from (4) to (9).

Table 5 Magnification of trade costs on final goods exports from vertical specialization, 2004

	Database produced by BEC classification							Database produced by proportion assumption							100* difference	
Country or region	Standard Tariff	Foreign content share (VS)	Tariff on imported inputs	two stage tariffs 1a+3a	Magnifi-cation factor 4a/1a	Effective tariff rate	Magnifi-cation ratio 6a/1a	Standard Tariff	Foreign content share (VS)	Tariff on imported inputs	two stage tariffs 1b+3b	Magnifi-cation factor 4b/1b	Effective tariff rate	Magnifi-cation ratio 6b/1b	Magnifi-cation factor 5a-5b	Magnifi-cation ratio 7a-7b
	(1a)	(2a)	(3a)	(4a)	(5a)	(6a)	(7a)	(1b)	(2b)	(3b)	(4b)	(5b)	(6b)	(7b)	(8)	(9)
Advanced economies																
Aus-New Zealand	15.55	0.13	0.34	15.89	1.02	27.00	1.74	13.48	0.15	0.55	14.03	1.04	26.02	1.93	**-1.9**	**-19.4**
Canada	1.60	0.38	0.24	1.84	1.15	7.05	4.41	1.36	0.38	0.30	1.66	1.22	7.52	5.53	**-7.1**	**-112.3**
Western EU	6.16	0.12	0.24	6.40	1.04	12.09	1.96	6.06	0.13	0.24	6.30	1.04	12.22	2.02	**-0.1**	**-5.4**
Japan	6.22	0.12	0.05	6.27	1.01	11.19	1.80	6.36	0.12	0.06	6.42	1.01	11.42	1.80	**-0.1**	**0.3**
USA	4.38	0.13	0.17	4.55	1.04	9.19	2.10	4.05	0.15	0.21	4.26	1.05	9.26	2.29	**-1.3**	**-18.8**
Asian NICs																
Hong Kong	10.16	0.42	0.00	10.16	1.00	27.91	2.75	10.02	0.40	0.00	10.02	1.00	26.09	2.60	**0.0**	**14.3**
Korea	6.05	0.32	1.46	7.51	1.24	17.32	2.86	6.34	0.35	1.74	8.08	1.27	19.62	3.09	**-3.3**	**-23.2**
Taiwan	4.76	0.42	1.40	6.16	1.29	20.08	4.22	4.45	0.43	1.40	5.85	1.31	19.56	4.40	**-2.0**	**-17.7**
Singapore	3.60	0.70	0.00	3.60	1.00	30.05	8.35	3.22	0.72	0.00	3.22	1.00	30.75	9.55	**0.0**	**-120.2**
Emerging Asia																
China	6.17	0.29	1.91	8.08	1.31	21.42	3.47	6.44	0.29	1.97	8.41	1.31	21.86	3.39	**0.4**	**7.7**
Indonesia	7.53	0.30	1.34	8.87	1.18	24.39	3.24	9.44	0.27	1.28	10.72	1.14	26.65	2.82	**4.2**	**41.6**
Malaysia	3.55	0.46	2.11	5.66	1.59	20.93	5.90	4.38	0.45	2.50	6.88	1.57	23.04	5.26	**2.4**	**63.6**
Philippines	5.57	0.39	1.07	6.64	1.19	22.47	4.03	3.50	0.42	0.94	4.44	1.27	16.52	4.72	**-7.6**	**-68.6**
Thailand	8.16	0.40	4.23	12.39	1.52	36.54	4.48	7.67	0.41	4.36	12.03	1.57	35.05	4.57	**-5.0**	**-9.2**
Vietnam	10.71	0.43	8.62	19.33	1.80	55.10	5.14	10.29	0.45	9.17	19.46	1.89	54.52	5.30	**-8.6**	**-15.4**
India	7.82	0.18	2.98	10.80	1.38	22.08	2.82	6.93	0.19	3.10	10.03	1.45	19.82	2.86	**-6.6**	**-3.6**
Other emerging economies																
Brazil	12.27	0.13	1.22	13.49	1.10	22.77	1.86	11.82	0.13	1.12	12.94	1.09	25.07	2.12	**0.5**	**-26.5**
EU accession	2.41	0.34	0.55	2.96	1.23	12.67	5.26	2.18	0.36	0.57	2.75	1.26	12.24	5.61	**-3.3**	**-35.7**
Mexico	0.88	0.31	1.00	1.88	2.14	6.36	7.23	0.67	0.30	1.02	1.69	2.52	5.73	8.55	**-38.6**	**-132.5**
Russian	5.36	0.18	1.61	6.97	1.30	17.23	3.21	3.64	0.16	1.34	4.98	1.37	14.86	4.08	**-6.8**	**-86.8**
South Africa	7.15	0.20	1.11	8.26	1.16	22.11	3.09	6.75	0.22	1.18	7.93	1.17	20.94	3.10	**-2.0**	**-1.0**

Source: Authors' estimates.

Figure 1 Accounting of gross exports: concepts



Note:

a. value-added exports by a country equals (1) + (2) +(3) .

b. GDP in exports (1) + (2) + (3) + (4) +(5).

c. domestic content in a country's exports equals (1) + (2) + (3) + (4) +(5)+(6).

d. (7)+(8)+(9) is labeled as VS, and (3) + (4)+(5)+(6) is part of VS1 labeled by HIY (2001).

e. (4) are also labeled as VS1* by Daudin et al (2011).

f. (4) through (9) involve value added that crosses national borders at least twice, and are the sources of multiple counting in official trade statistics.[22]

[22] (3) should not be included in double counting, because when this value crosses a border for the second time, it becomes foreign value in the direct importer's exports. For this reason, it is not included as double counting to avoid an over-correction.

Figure 2 Value-added-adjusted Revealed Comparative Advantage Indicators

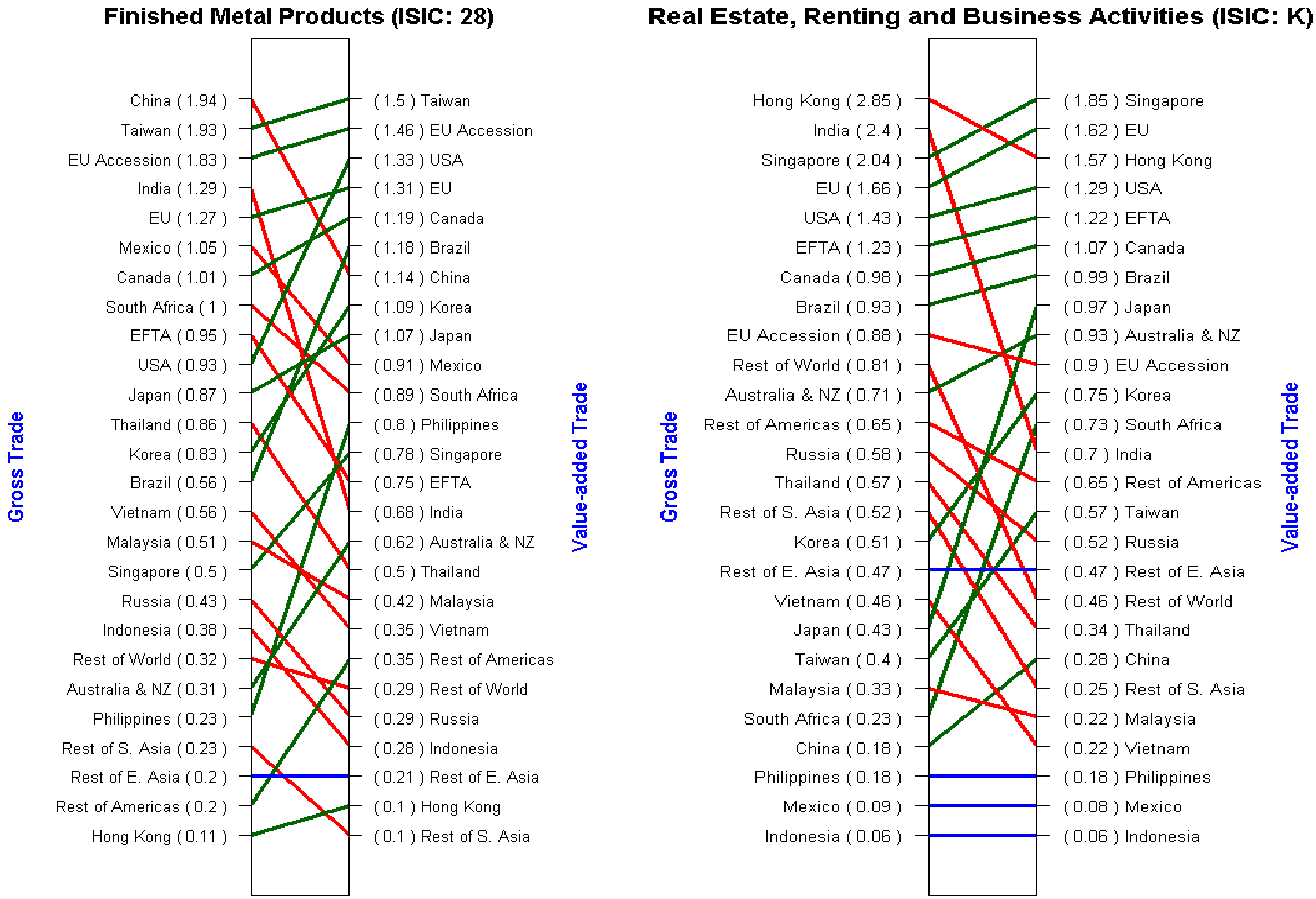
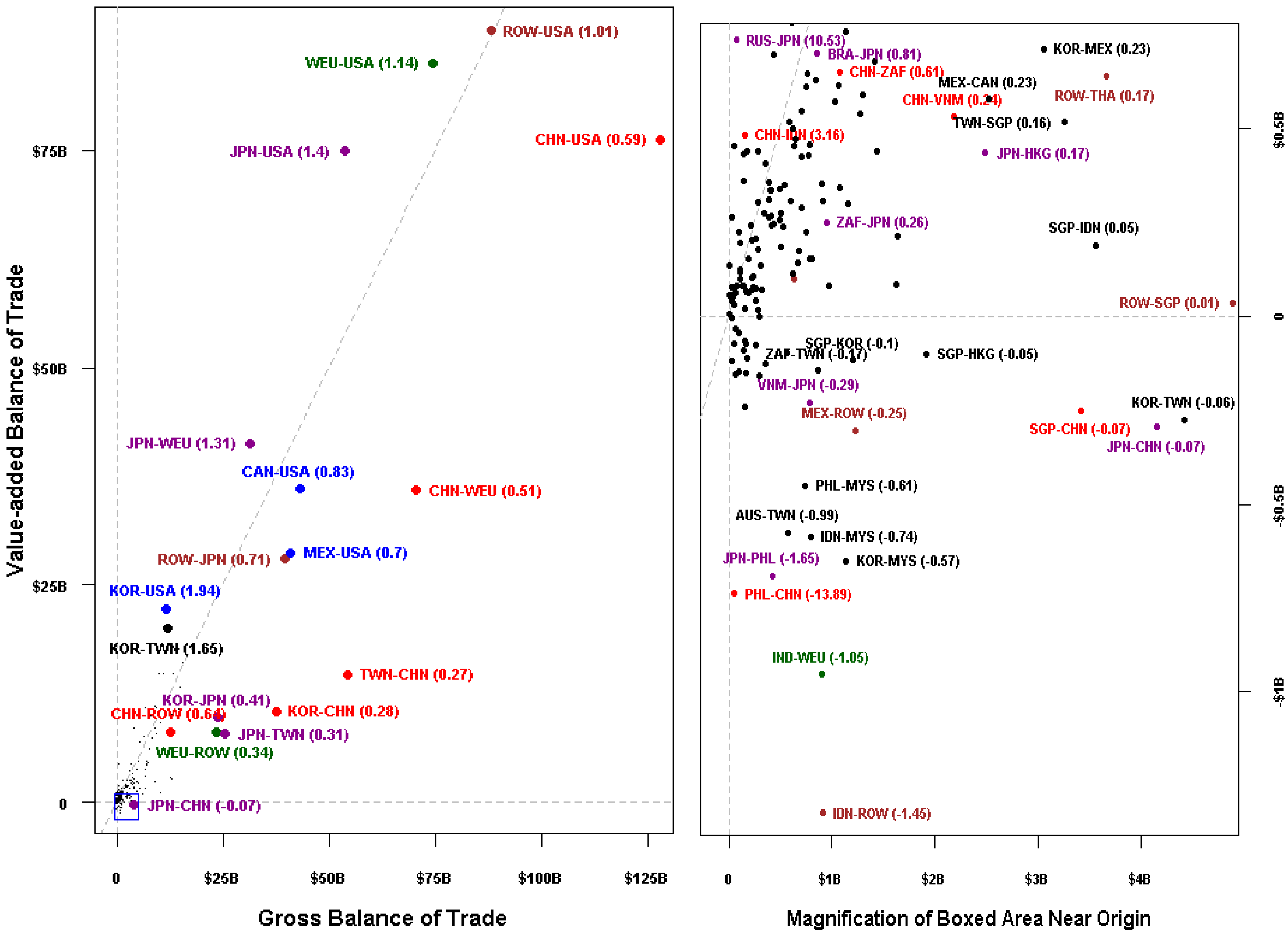


Figure 3: Gross and VA Balance of Trade, 2004



Note: The first country labeled in each pair is the surplus country while the second runs a deficit. Numbers in parentheses are the ratio of value-added to gross surplus.

Appendix A

Based on the property of inverse matrix, we have:

$$\begin{bmatrix} 1-a_{11} & -a_{12} \\ -a_{21} & 1-a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} 1-a_{11} & -a_{12} \\ -a_{21} & 1-a_{22} \end{bmatrix} \tag{A1}$$

Therefore, the following identities hold:

$$(1-a_{11})b_{11}-1=a_{12}b_{21}\,, b_{11}(1-a_{11})-1=b_{12}a_{21}$$

$$a_{21}b_{11}=(1-a_{22})b_{21}\,,\ b_{11}a_{12}=b_{12}(1-a_{22})$$

$$(1-a_{22})b_{21}=a_{21}b_{11}\,, b_{22}a_{21}=b_{21}(1-a_{11})$$

$$a_{21}b_{12}=(1-a_{22})b_{22}-1\,, b_{22}(1-a_{22})-1=b_{21}a_{12}$$

Therefore,

$$b_{11}=(1-a_{11})^{-1}+[b_{11}(1-a_{11})-1](1-a_{11})^{-1}=(1-a_{11})^{-1}+b_{12}a_{21}(1-a_{11})^{-1} \tag{A2}$$

$$b_{22}=(1-a_{22})^{-1}+b_{21}a_{12}(1-a_{22})^{-1}$$

Given (A2), we have

$$b_{11}y_{11}-(1-a_{11})^{-1}y_{11}=b_{12}a_{21}(1-a_{11})^{-1}y_{11} \tag{A3}$$

Using the relationship between gross output x and final demand y specified in equation (5), we have

$$y_2=y_{21}+y_{22}=(1-a_{22})x_2-a_{21}x_1 \tag{A4}$$

Also using $b_{11}a_{12}=b_{12}(1-a_{22})$,

$$\begin{aligned} v_1b_{11}a_{12}x_2 &= v_1b_{12}(1-a_{22})x_2=v_1b_{12}(y_2+a_{21}x_1)=v_1b_{12}a_{21}x_1+v_1b_{12}(y_{21}+y_{22}) \\ &= v_1b_{12}y_{22}+v_1b_{12}y_{21}+v_1b_{12}a_{21}x_1 \end{aligned} \tag{A5}$$

Based on equation (6):

$$x_1=b_{11}y_{11}+b_{12}y_{21}+b_{11}y_{12}+b_{12}y_{22} \tag{A6}$$

From the gross exports identity, we have:

$$x_1=(1-a_{11})^{-1}(e_{12}+y_{11}),\qquad (1-a_{11})^{-1}e_{12}=x_1-(1-a_{11})^{-1}y_{11} \tag{A7}$$

Combining (A6) and (A7), we can easily show that

$$\begin{aligned} & v_1b_{12}a_{21}(1-a_{11})^{-1}e_{12}=v_1b_{12}a_{21}[x_1-(1-a_{11})^{-1}y_{11}]= \\ & v_1b_{12}a_{21}[b_{11}y_{11}+b_{12}y_{21}+b_{11}y_{12}+b_{12}y_{22}-(1-a_{11})^{-1}y_{11}] \\ & =v_1b_{12}a_{21}[b_{11}y_{12}+b_{12}y_{22}+b_{12}y_{21}+b_{12}a_{21}(1-a_{11})^{-1}y_{11}] \end{aligned} \tag{A8}$$

which is the first pure double counted term in Country 1's gross exports accounting equation (13) that is expressed as function of both countries' final demand.

Also based on equation (6):

$$x_2 = b_{21}y_{11} + b_{22}y_{21} + b_{21}y_{12} + b_{22}y_{22} \tag{A9}$$

Also from gross exports identity,

$$x_2 = (1-a_{22})^{-1}(e_{21} + y_{22}), \qquad (1-a_{22})^{-1}e_{21} = x_2 - (1-a_{22})^{-1}y_{22} \tag{A10}$$

Combining (A9) and (A10), we can show that the second pure double counted term in equation (13) can be expressed as:

$$\begin{aligned}
&v_2 b_{21} a_{12}(1-a_{22})^{-1}e_{21} = v_2 b_{21} a_{12}[x_2 - (1-a_{22})^{-1}y_{11}] = \\
&v_2 b_{21} a_{12}[b_{21}y_{11} + b_{22}y_{21} + b_{21}y_{12} + b_{22}y_{22} - (1-a_{22})^{-1}y_{22}] \\
&= v_2 b_{21} a_{12}[b_{21}y_{11} + b_{22}y_{21} + b_{21}y_{12} + b_{22}a_{22}(1-a_{22})^{-1}y_{22}]
\end{aligned} \tag{A11}$$

Alternative way to decompose the two pure double counting terms

Based on (A1), (A7) and (A9),

$$\begin{aligned}
&v_1 b_{12} a_{21}(1-a_{11})^{-1}e_{12} + v_2 b_{21} a_{12}(1-a_{22})^{-1}e_{21} \\
&= (1-a_{11}-a_{21})b_{12}a_{21}(1-a_{11})^{-1}e_{12} + (1-a_{12}-a_{22})b_{21}a_{12}(1-a_{22})^{-1}e_{21} \\
&= [(1-a_{11})b_{12}a_{21} - a_{21}b_{12}a_{21}](1-a_{11})^{-1}e_{12} + [(1-a_{12})b_{21}a_{12} - a_{12}b_{21}a_{12}](1-a_{22})^{-1}e_{21} \\
&= [a_{12}b_{22}a_{21} - a_{21}b_{12}a_{21}](1-a_{11})^{-1}e_{12} + [a_{21}b_{11}a_{12} - a_{12}b_{21}a_{12}](1-a_{22})^{-1}e_{21} \\
&= [a_{12}b_{21}(1-a_{11}) - a_{21}b_{12}a_{21}](1-a_{11})^{-1}e_{12} + [a_{21}b_{12}(1-a_{22}) - a_{12}b_{21}a_{12}](1-a_{22})^{-1}e_{21} \\
&= a_{12}b_{21}e_{12} - a_{21}b_{12}a_{21}(1-a_{11})^{-1}e_{12} + a_{21}b_{12}e_{21} - a_{12}b_{21}a_{12}(1-a_{22})^{-1}e_{21} \\
&= a_{12}b_{21}[e_{12} - a_{12}(1-a_{22})^{-1}e_{21}] + a_{21}b_{12}[e_{21} - a_{21}(1-a_{11})^{-1}e_{12}] \\
&= a_{12}b_{21}\{y_{12} + a_{12}[x_2 - (1-a_{22})^{-1}e_{21}]\} + a_{21}b_{12}\{y_{21} + a_{21}[x_1 - (1-a_{11})^{-1}e_{12}]\} \\
&= a_{12}[b_{21}y_{12} + b_{21}a_{12}(1-a_{22})^{-1}y_{22}] + a_{21}[b_{12}y_{21} + b_{12}a_{21}(1-a)^{-1}y_{11}]
\end{aligned} \tag{A12}$$

Derivation of equation (18)

$$
\begin{aligned}
&e_{12}+e_{21}-GDP_1-GDP_2=e_{12}+e_{21}-v_1x_1-v_2x_2\\
&=v_1[b_{11}y_{12}+b_{12}y_{22}]+2v_1[b_{12}y_{21}+b_{12}a_{21}(1-a_{11})^{-1}y_{11}]+v_2[b_{21}y_{11}+b_{22}y_{21}]\\
&+2v_2[b_{21}y_{12}+b_{21}a_{12}(1-a_{22})^{-1}y_{22}]+2v_1b_{12}a_{21}(1-a_{11})^{-1}e_{12}+2v_2b_{21}a_{12}(1-a_{22})^{-1}e_{21}\\
&-v_1(b_{11}y_{12}+b_{12}y_{22}+b_{12}y_{21}+b_{11}y_{11})-v_2(b_{21}y_{11}+b_{22}y_{21}+b_{21}y_{12}+b_{22}y_{22})\\
&=2v_1[b_{12}y_{21}+b_{12}a_{21}(1-a_{11})^{-1}y_{11}]+2v_2[b_{21}y_{12}+b_{21}a_{12}(1-a_{22})^{-1}y_{22}]\\
&+2v_1b_{12}a_{21}(1-a_{11})^{-1}e_{12}+2v_2b_{21}a_{12}(1-a_{22})^{-1}e_{21}-v_1(b_{12}y_{21}+b_{11}y_{11})-v_2(+b_{21}y_{12}+b_{22}y_{22})\\
&=v_1[b_{12}y_{21}+b_{12}a_{21}(1-a_{11})^{-1}y_{11}]+v_2[b_{21}y_{12}+b_{21}a_{12}(1-a_{22})^{-1}y_{22}]\\
&+2v_1b_{12}a_{21}(1-a_{11})^{-1}e_{12}+2v_2b_{21}a_{12}(1-a_{22})^{-1}e_{21}-(1-a_{11})^{-1}y_{11}-(1-a_{22})^{-1}y_{22} \qquad \text{(A13)}
\end{aligned}
$$

Alternative derivation of gross exports accounting equations

Using the gross output identity $x_1 = y_{11} + a_{11}x_1 + e_{12}$ it is easy to show that

$$x_1 = (1-a_{11})^{-1}y_{11} + (1-a_{11})^{-1}e_{12} \qquad \text{(A14)}$$

For the first part

$$
\begin{aligned}
&v_1(1-a_{11})^{-1}e_{12}=v_1(1-a_{11})^{-1}(y_{12}+a_{12}x_2)\\
&=v_1(1-a_{11})^{-1}[y_{12}+a_{12}(b_{21}y_{11}+b_{22}y_{21}+b_{21}y_{12}+b_{22}y_{22})]\\
&=v_1(1-a_{11})^{-1}[(1+a_{12}b_{21})y_{12}+a_{12}(b_{21}y_{11}+b_{22}y_{21}+b_{22}y_{22})]\\
&=v_1(1-a_{11})^{-1}\{(1-a_{11})b_{11}y_{12}+(1-a_{11})b_{12}(y_{21}+y_{22})+[(1-a_{11})b_{11}-1]y_{11}\} \qquad \text{(A15)}\\
&=v_1b_{11}y_{12}+v_1b_{12}y_{22}+v_1b_{12}y_{21}+v_1[b_{11}-(1-a_{11})^{-1}]y_{11}\\
&=v_1b_{11}y_{12}+v_1b_{12}y_{22}+v_1[b_{12}y_{21}+b_{12}a_{21}(1-a_{11})^{-1}y_{11}]
\end{aligned}
$$

Which give the first four terms in our equation (13) in the text.

For the second part, from IO model, we have:

$$v_2 + a_{12} + a_{22} = 1, \qquad (v_2 + a_{12})(1-a_{22})^{-1} = 1$$

$$v_1 + a_{11} + a_{21} = 1, \qquad (v_1 + a_{21})(1-a_{11})^{-1} = 1$$

From equation (6),

$$x_1 = b_{11}(y_{12}+y_{11}) + b_{12}(y_{21}+y_{22})$$

$$x_2 = b_{22}(y_{21}+y_{22}) + b_{21}(y_{12}+y_{11})$$

Therefore,

$$VS_1 = (v_2 + a_{12})(1 - a_{22})^{-1} a_{21}[x_1 - (1 - a_{11})^{-1} y_{11}]$$
$$= (v_2 + a_{12})(1 - a_{22})^{-1} a_{21}[b_{11}(y_{11} + y_{12}) + b_{12}(y_{21} + y_{22}) - (1 - a_{11})^{-1} y_{11}]$$
$$= (v_2 + a_{12})(1 - a_{22})^{-1}\{(1 - a_{22})b_{21}(y_{11} + y_{12}) + [(1 - a_{22})b_{22} - 1](y_{21} + y_{22}) - a_{21}(1 - a_{11})^{-1} y_{11}\}$$
$$= (v_2 + a_{12})[b_{21}(y_{11} + y_{12}) + b_{22}(y_{21} + y_{22})] - (y_{21} + y_{22}) - a_{21}(1 - a_{11})^{-1} y_{11}$$
$$= (v_2 + a_{12})b_{21}y_{12} + (1 - a_{22})b_{21}y_{11} - a_{21}(1 - a_{11})^{-1} y_{11} + (1 - a_{22})b_{22}(y_{21} + y_{22}) - (y_{21} + y_{22})$$
$$= (v_2 + a_{12})b_{21}y_{12} + a_{21}[b_{11} - (1 - a_{11})^{-1}]y_{11} + a_{21}b_{12}(y_{21} + y_{22})$$
$$= v_2 b_{21}y_{12} + a_{12}b_{21}y_{12} + a_{21}b_{12}a_{21}(1 - a_{11})^{-1} y_{11} + a_{21}b_{12}y_{21} + (v_2 + a_{12})(1 - a_{22})^{-1} a_{21}b_{12}y_{22}$$
$$= v_2 b_{21}y_{12} + a_{12}b_{21}y_{12} + a_{21}b_{12}a_{21}(1 - a_{11})^{-1} y_{11} + a_{21}b_{12}y_{21} + (v_2 + a_{12})(1 - a_{22})^{-1}[(1 - a_{22})b_{22} - 1]y_{22}$$
$$= v_2 b_{21}y_{12} + a_{12}b_{21}y_{12} + a_{21}b_{12}a_{21}(1 - a_{11})^{-1} y_{11} + a_{21}b_{12}y_{21} + (v_2 + a_{12})[b_{22} - (1 - a_{22})^{-1}]y_{22}$$
$$= v_2 b_{21}y_{12} + a_{12}b_{21}y_{12} + a_{21}b_{12}a_{21}(1 - a_{11})^{-1} y_{11} + a_{21}b_{12}y_{21} + (v_2 + a_{12})[b_{22}(1 - a_{22}) - 1](1 - a_{22})^{-1} y_{22}$$
$$= v_2 b_{21}y_{12} + a_{12}b_{21}y_{12} + a_{21}b_{12}a_{21}(1 - a_{11})^{-1} y_{11} + a_{21}b_{12}y_{21} + (v_2 + a_{12})b_{21}a_{12}(1 - a_{22})^{-1} y_{22}$$
$$= v_2[b_{21}y_{12} + b_{21}a_{12}(1 - a_{22})^{-1} y_{22}] + a_{12}[b_{21}y_{12} + b_{21}a_{12}(1 - a_{22})^{-1} y_{22}] + a_{21}[b_{12}y_{21} + b_{12}a_{21}(1 - a_{11})^{-1} y_{11}]$$

Note that the two pure double counted terms in our equation (13) can be further broken up in terms of intermediate goods trade as follows:

$$v_1 b_{12}a_{21}(1 - a_{11})^{-1} e_{12} + v_2 b_{21}a_{12}(1 - a_{22})^{-1} e_{21}$$
$$= (1 - a_{11} - a_{21})b_{12}a_{21}(1 - a_{11})^{-1} e_{12} + (1 - a_{12} - a_{22})b_{21}a_{12}(1 - a_{22})^{-1} e_{21}$$
$$= [(1 - a_{11})b_{12}a_{21} - a_{21}b_{12}a_{21}](1 - a_{11})^{-1} e_{12} + [(1 - a_{22})b_{21}a_{12} - a_{12}b_{21}a_{12}](1 - a_{22})^{-1} e_{21}$$
$$= [a_{12}b_{22}a_{21} - a_{21}b_{12}a_{21}](1 - a_{11})^{-1} e_{12} + [a_{21}b_{11}a_{12} - a_{12}b_{21}a_{12}](1 - a_{22})^{-1} e_{21}$$
$$= [a_{12}b_{21}(1 - a_{11}) - a_{21}b_{12}a_{21}](1 - a_{11})^{-1} e_{12} + [a_{21}b_{12}(1 - a_{22}) - a_{12}b_{21}a_{12}](1 - a_{22})^{-1} e_{21}$$
$$= a_{12}b_{21}e_{12} - a_{12}b_{21}a_{12}(1 - a_{22})^{-1} e_{21} + a_{21}b_{12}e_{21} - a_{21}b_{12}a_{21}(1 - a_{11})^{-1} e_{12}$$
$$= a_{12}b_{21}[e_{12} - a_{12}(1 - a_{22})^{-1} e_{21}] + a_{21}b_{12}[e_{21} - a_{21}(1 - a_{11})^{-1} e_{12}]$$
$$= a_{12}b_{21}\{y_{12} + a_{12}[x_2 - (1 - a_{22})^{-1} e_{21}]\} + a_{21}b_{12}\{y_{21} + a_{21}[x_1 - (1 - a_{11})^{-1} e_{12}]\}$$
$$= a_{12}b_{21}y_{12} + a_{12}b_{21}a_{12}(1 - a_{22})^{-1} y_{22} + a_{21}b_{12}y_{21} + a_{21}b_{12}a_{21}(1 - a_{11})^{-1} y_{11} \quad \text{(A16)}$$

Therefore the two different ways of derivation give the same analytical results. The four double counted terms in equation (A4) are just a further decomposition in terms of intermediates used up in producing each of the four corresponding homecoming value-added in exports. However, using the property of VB matrix to decompose gross exports simplifies the mathematical proof. For example, in the two-country case, the only proof needed is equation (11); afterwards, we can obtain our core gross export accounting equation by simply inserting each country's gross output identity. This technique is especially useful for the G-country, N-sector setting.

B The derivation of gross exports accounting equation in G country N sector Model

B.1. The G-country, N-sector ICIO Model

Assume a world with G-countries, in which each country produces goods in N differentiated tradable sectors. Goods in each sector can be consumed directly or used as intermediate inputs, and each country exports both intermediate and final goods to all other countries.

All gross output produced by country s must be used as an intermediate good or a final good at home or abroad, or

$$X_s = \sum_r^G (A_{sr} X_r + Y_{sr}), \qquad \text{r,s} = 1,2\ldots\text{ G} \tag{B1}$$

Where X_s is the N×1 gross output vector of country s, Y_{sr} is the N×1 final demand vector that gives demand in country r for final goods produced in s, and A_{sr} is the N×N IO coefficient matrix, giving intermediate use in r of goods produced in s.

The G-country, N-sector production and trade system can be written as an ICIO model in block matrix notation

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_G \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1G} \\ A_{21} & A_{22} & \cdots & A_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ A_{G1} & A_{G2} & \cdots & A_{GG} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_G \end{bmatrix} + \begin{bmatrix} Y_{11} + Y_{12} + \cdots + Y_{1G} \\ Y_{21} + Y_{22} + \cdots + Y_{2G} \\ \cdots\cdots \\ Y_{G1} + Y_{G2} + \cdots + Y_{GG} \end{bmatrix}, \tag{B2}$$

and rearranging,

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_G \end{bmatrix} = \begin{bmatrix} I - A_{11} & -A_{12} & \cdots & -A_{1G} \\ -A_{21} & I - A_{22} & \cdots & -A_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ -A_{G1} & -A_{G2} & \cdots & I - A_{GG} \end{bmatrix}^{-1} \begin{bmatrix} \sum_r^G Y_{1r} \\ \sum_r^G Y_{2r} \\ \vdots \\ \sum_r^G Y_{Gr} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1G} \\ B_{21} & B_{22} & \cdots & B_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ B_{G1} & B_{G2} & \cdots & B_{GG} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_G \end{bmatrix} \tag{B3}$$

where B_{sr} denotes the N×N block Leontief inverse matrix, which is the total requirement matrix that gives the amount of gross output in producing country s required for a one-unit increase in final demand in destination country r. Y_s is a N×1 vector that gives the global use of s' final goods.

B.2. Value-added share by source matrix

Let V_s be the 1×N direct value-added coefficient vector. Each element of V_s gives the ratio of direct domestic value added in total output for country s. This is equal to one minus the intermediate input share from all countries (including domestically produced intermediates):

$$V_s = u(I - \sum_{r}^{G} A_{rs}), \tag{B4}$$

Define V, the G×GN matrix of direct domestic value added for all countries,

$$V = \begin{bmatrix} V_1 & 0 & \cdots & 0 \\ 0 & V_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & V_G \end{bmatrix}. \tag{B5}$$

Multiplying these direct value-added shares with the Leontief inverse matrices produces the G×GN value-added share (VB) matrix as equation (27) in the main text, it has the property:

$$\sum_{s}^{G} V_s B_{sr} = u. \tag{B6}$$

B.3. Decomposition of gross exports

Let E_{sr} be the N×1 vector of gross bilateral exports from s to r.

$$E_{sr} \equiv A_{sr} X_r + Y_{sr} \qquad for \quad s \neq r \tag{B7}$$

A country's gross exports to the world equal

$$E_{s*} = \sum_{r \neq s}^{G} E_{sr} = \sum_{r \neq s}^{G} (A_{sr} X_r + Y_{sr}) \tag{B8}$$

From equation (29) in the main text we know that

$$\sum_{r=1}^{G} \sum_{g=1}^{G} B_{sg} Y_{gr} = \sum_{r=1}^{G} X_{sr} = X_s \tag{B9}$$

Therefore, following identity hold

$$V_s X_s \equiv V_s \sum_{r=1}^{G} \sum_{g=1}^{G} B_{sg} Y_{gr} \tag{B10}$$

Multiplying both sides of (B8) by (B6), we have

$$uE_{s*}(V_s B_{ss} + \sum_{t \neq s}^{G} V_t B_{ts}) E_{s*} = V_s B_{ss} \sum_{r \neq s}^{G} (A_{sr} X_r + Y_{sr}) + \sum_{t \neq s}^{G} V_t B_{ts} \sum_{r \neq s}^{G} (A_{sr} X_r + Y_{sr}) \tag{B11}$$

Now we add and subtract VT_{s*}, defined by equation (32) in the main text, to the first term on RHS of (B11). This gives

$$V_s B_{ss} E_{s*} = VT_{s*} + V_s B_{ss} \sum_{r \neq s}^{G} (A_{sr} X_r + Y_{sr}) - V_s \sum_{r \neq s}^{G} \sum_{g=1}^{G} B_{sg} Y_{gr} \tag{B12}$$

Recall that $X_s = \sum_{r=1}^{G} (A_{sr} X_r + Y_{sr})$ as defined in (B1), insert it together with equation (B9) into (B12) gives

$$V_s B_{ss} E_{s*} = VT_{s*} + V_s B_{ss} (X_s - A_{ss} X_s - Y_{ss}) - V_s (X_s - \sum_{g=1}^{G} B_{sg} Y_{gs}) \tag{B13}$$

Where $X_s - A_{ss} X_s - Y_{ss}$ equals the difference between country s' gross output and gross output sold in domestic market, i.e. what country s' gross exports to the world market; $X_s - \sum_{g=1}^{G} B_{sg} Y_{gs}$ equals the difference between country s' gross output and its gross output finally consumed at domestic market . By rearranging terms,

$$V_s B_{ss} E_{s*} = VT_{s*} + V_s [B_{ss} (I - A_{ss}) - I] X_s + V_s [\sum_{g=1}^{G} B_{sg} Y_{gs} - B_{ss} Y_{ss}] \tag{B14}$$

Substitute $B_{ss}(I - A_{ss}) - I$ in equation (B14) by $\sum_{r \neq s}^{G} B_{sr} A_{rs}$ (the property of inverse matrix, see equation (B19) bellow) we have

$$V_s B_{ss} E_{s*} = V_s \sum_{r \neq s}^{G} \sum_{g=1}^{G} B_{sg} Y_{gr} + V_s \sum_{r \neq s}^{G} B_{sr} Y_{rs} + V_s \sum_{r \neq s}^{G} B_{sr} A_{rs} X_s \tag{B15}$$

Insert (B15) into (B11) and rearrange terms, we obtain equation (34) in the main text.

B.4. Further partition of equation (34)

The term that measures double counting by intermediate goods trade in equation (34) ($V_s \sum_{r \neq s}^{G} B_{sr} A_{rs} X_s$) can be further split into two parts: one is part of the home country's domestic value-added that first exported but finally returns home in its intermediate imports to produce final goods and consumed at home, the other is a pure double counting portion due to two way intermediate trade.

Using the relation $X_s = Y_{ss} + A_{ss} X_s + E_{s*}$, it is easy to show that

$$X_s - (I - A_{ss})^{-1} Y_{ss} = (I - A_{ss})^{-1} E_{s*}. \tag{B16}$$

$(I - A_{ss})^{-1} Y_{ss}$ is the gross output needed to sustain final goods that is both produced and consumed in country s, using domestically produced intermediate goods; deduct it from country s' total gross output, what left is the gross output needed to sustain country s' production of its gross exports. Therefore, the left hand side of equation (B16) has straightforward economic meanings. We can further show that

$$(I - A_{ss})^{-1} Y_{ss} = B_{ss} Y_{ss} - \sum_{r \neq s}^{G} B_{sr} A_{rs} (I - A_{ss})^{-1} Y_{ss} \tag{B17}$$

the last term in RHS of (B17) is the final gross output needed to sustain final goods that is both produced and consumed in country s, but using intermediate goods that was originated in country s but shipped to other countries for processing before being re-imported by the source country in its intermediate goods imports (gross output sold indirectly in domestic market). Given (B17), it easy to see

$$V_s \sum_{r \neq s}^{G} B_{sr} A_{rs} X_s = V_s \sum_{r \neq s}^{G} B_{sr} A_{rs} (I - A_{ss})^{-1} Y_{ss} + V_s \sum_{r \neq s}^{G} B_{sr} A_{rs} (I - A_{ss})^{-1} E_{s*} \tag{B18}$$

Equation (B17) can be proven by using the property of inverse matrix:

$$\begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1G} \\ B_{21} & B_{22} & \cdots & B_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ B_{G1} & B_{G2} & \cdots & B_{GG} \end{bmatrix} \begin{bmatrix} I - A_{11} & -A_{12} & \cdots & -A_{1G} \\ -A_{21} & I - A_{22} & \cdots & -A_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ -A_{G1} & -A_{G2} & \cdots & I - A_{GG} \end{bmatrix} \begin{bmatrix} I & 0 & \cdots & 0 \\ 0 & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I \end{bmatrix}$$

we therefore have

$$B_{ss}(I - A_{ss}) - I = \sum_{r \neq s}^{G} B_{sr} A_{rs} \tag{B19}$$

Using (B19), we have

$$(I - A_{ss})^{-1} Y_{ss} + [B_{ss}(I - A_{ss}) - I](I - A_{ss})^{-1} Y_{ss} = B_{ss} Y_{ss} \tag{B20}$$

This is also the proof of equation (40) in the main text.

Appendix C: Detail computation of numerical example 3 (online posting only)

To apply the export decomposition formula, we have to use a 5-sector version of Equations (13) and (14). Note that an input output table is built on the assumption that all goods within a sector are homogenous, i.e the input-output and direct value-added coefficients are the same for all products within a sector. In our example, because different stages of the production have different direct value-added and IO coefficients, we have to treat the five stages as five different sectors.

The inter-country supply chain data in table 1 can be summarized by the following Input -output IO table:

Table C1: IO table constructed from two-country Supply Chain data

Input		Output	Intermediate use					Final use		Total output
			Country 1			Country 2		Country 1	Country 2	
			Sector 1	Sector 2	Sector 3	Sector 1	Sector 2			
Intermediate input	Country 1	Sector 1				1				1
		Sector 2					3			3
		Sector 3						2	3	5
	Country 2	Sector 1		2						2
		Sector 2			4					4
Value-added			1	1	1	1	1			
Total Input			1	3	5	2	4			

.

From table A, we can obtain following matrixes:

$$A=\begin{bmatrix}0 & 0 & 0 & 1/2 & 0\\ 0 & 0 & 0 & 0 & 3/4\\ 0 & 0 & 0 & 0 & 0\\ 0 & 2/3 & 0 & 0 & 0\\ 0 & 0 & 4/5 & 0 & 0\end{bmatrix}$$

In A， $a_{11}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$， $a_{12}=\begin{bmatrix}1/2 & 0\\ 0 & 3/4\\ 0 & 0\end{bmatrix}$， $a_{21}=\begin{bmatrix}0 & 2/3 & 0\\ 0 & 0 & 4/5\end{bmatrix}$， $a_{22}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

Value-added coefficient：

$v_1=\begin{bmatrix}1 & 1/3 & 1/5\end{bmatrix}$, $v_2=\begin{bmatrix}1/2 & 1/4\end{bmatrix}$

Final goods and exports

$$y_{11}=\begin{bmatrix}0\\0\\2\end{bmatrix},\quad y_{12}=\begin{bmatrix}0\\0\\3\end{bmatrix},\quad y_{21}=\begin{bmatrix}0\\0\end{bmatrix},\quad y_{22}=\begin{bmatrix}0\\0\end{bmatrix},\quad e_{12}=\begin{bmatrix}1\\3\\3\end{bmatrix},\quad e_{21}=\begin{bmatrix}2\\4\end{bmatrix}$$

Leontief inverse B = (I – A) $^{-1}$

$$B=\begin{bmatrix}1 & 1/3 & 1/5 & 1/2 & 1/4\\0 & 1 & 3/5 & 0 & 3/4\\0 & 0 & 1 & 0 & 0\\0 & 2/3 & 2/5 & 1 & 1/2\\0 & 0 & 4/5 & 0 & 1\end{bmatrix}$$

In B， $b_{11}=\begin{bmatrix}1 & 1/3 & 1/5\\0 & 1 & 3/5\\0 & 0 & 1\end{bmatrix}$， $b_{12}=\begin{bmatrix}1/2 & 1/4\\0 & 3/4\\0 & 0\end{bmatrix}$， $b_{21}=\begin{bmatrix}0 & 2/3 & 2/5\\0 & 0 & 4/5\end{bmatrix}$， $b_{22}=\begin{bmatrix}1 & 1/2\\0 & 1\end{bmatrix}$

And $(1-a_{11})^{-1}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$， $(1-a_{22})^{-1}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

Equations (13) and (14) can be converted to a 5-sector version easily by defining each of their terms in a matrix with proper dimensions. The formula in Equations (13) then allows us to decompose Country 1's gross exports as follows:

$$\text{v1}=v_1b_{11}y_{12}=\begin{bmatrix}1 & 1/3 & 1/5\end{bmatrix}\begin{bmatrix}1 & 1/3 & 1/5\\0 & 1 & 3/5\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\\3\end{bmatrix}=\begin{bmatrix}1 & 2/3 & 3/5\end{bmatrix}\begin{bmatrix}0\\0\\3\end{bmatrix}=1.8$$

$$\text{v2}=v_1b_{12}y_{22}=\begin{bmatrix}1 & 1/3 & 1/5\end{bmatrix}\begin{bmatrix}1/2 & 1/4\\0 & 3/4\\0 & 0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}=0$$

$$\text{v3}=v_1b_{12}y_{21}=\begin{bmatrix}1 & 1/3 & 1/5\end{bmatrix}\begin{bmatrix}1/2 & 1/4\\0 & 3/4\\0 & 0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}=0$$

$$v4=v_1b_{12}a_{21}(1-a_{11})^{-1}y_{11}=\begin{bmatrix}1 & 1/3 & 1/5\end{bmatrix}\begin{bmatrix}1/2 & 1/4\\0 & 3/4\\0 & 0\end{bmatrix}\begin{bmatrix}0 & 2/3 & 0\\0 & 0 & 4/5\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\\2\end{bmatrix}$$

$$=\begin{bmatrix}0 & 1/3 & 2/5\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\\2\end{bmatrix}=0.8$$

$$v5 = v_1 b_{12} a_{21} (1 - a_{11})^{-1} e_{12} = \begin{bmatrix} 1 & 1/3 & 1/5 \end{bmatrix} \begin{bmatrix} 1/2 & 1/4 \\ 0 & 3/4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2/3 & 0 \\ 0 & 0 & 4/5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1/3 & 2/5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix} = 2.2$$

$$\text{v6} = v_2 b_{21} y_{12} = \begin{bmatrix} 1/2 & 1/4 \end{bmatrix} \begin{bmatrix} 0 & 2/3 & 2/5 \\ 0 & 0 & 4/5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 & 1/3 & 2/5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} = 1.2$$

$$\text{v7} = v_2 b_{21} a_{12} (1 - a_{22})^{-1} y_{22} = \begin{bmatrix} 1/2 & 1/4 \end{bmatrix} \begin{bmatrix} 0 & 2/3 & 2/5 \\ 0 & 0 & 4/5 \end{bmatrix} \begin{bmatrix} 1/2 & 0 \\ 0 & 3/4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$$

$$v8 = v_2 b_{21} a_{12} (1 - a_{22})^{-1} e_{21} = \begin{bmatrix} 1/2 & 1/4 \end{bmatrix} \begin{bmatrix} 0 & 2/3 & 2/5 \\ 0 & 0 & 4/5 \end{bmatrix} \begin{bmatrix} 1/2 & 0 \\ 0 & 3/4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1/3 & 2/5 \end{bmatrix} \begin{bmatrix} 1/2 & 0 \\ 0 & 3/4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 1$$

We can decompose Country 2’s exports similarly.