Price Competition under Subsidization: Applications to Medicare Reform

Awi Federgruen and Lijian Lu

Graduate School of Business, Columbia University, New York, NY 10027

February 6, 2013

Abstract

We consider price competition models for oligopolistic markets, in which a significant part of the product or service price is paid by a third party, as a subsidy. The consumer is, therefore, impacted by the net price, defined as the difference between the *nominal* price and the subsidy, while the firms earn the full nominal price, partially paid by the subsidizing third party and the remainder by the consumer. When choosing among the various competing options, the consumer trades off the net price paid with various other product or service attributes, as in standard price competition models. The subsidy may be *exogenously* specified and pre-announced to the competing firms. Alternatively, it may be *endogenously* determined, as a function of the set of nominal prices selected by the competing firms, for example the lowest or the second lowest price.

We first characterize the equilibrium behavior under a general subsidy scheme of the above type; this in a base model, where we assume that the consumer choice model is of the general MultiNomialLogit (MNL) type. We also derive comparison results for the price equilibria that arise under alternative subsidy schemes. We proceed to apply our results to the Medicare insurance market, both in terms of its existing structure, as well as in terms of various proposals to redesign the program, in particular the Wyden-Ryan plan. We show that implementation of the latter plan in 2010 would have reduced the capitation rates, on average by 18.5% and enabled savings of 16.2% in the governments' costs. These numbers are significantly larger than traditional estimates obtained under the assumption that the plans' premia and market shares would not be affected by the new capitation rate scheme. For beneficiaries continuing to opt for the traditional Medicare plan, the average monthly cost is roughly $64. Finally, we discuss extension of our model to Mixed MultiNominal Logit demand systems and prospect theoretical price competition models.

1 Introduction and Summary

We consider price competition models for oligopolistic markets, in which a significant part of the product or service price is paid by a third party, as a subsidy. The consumer, therefore, only incurs the *net* price, defined as the difference between the *nominal* price and the subsidy, while the firms earn the full nominal price, partially paid by the consumer and the remainder by the subsidizing third party. When choosing among the various competing options, the consumer trades off the net price paid with various other product or service attributes, as in standard price competition models. In some settings, the subsidy may exceed the nominal price for some of the firms, in which case the consumer receives a rebate equal to part or all of the difference. More broadly, the impact of the net price on the consumer's valuation of the product, may be represented by a general non-linear "response function".

The subsidy may be *exogenously* specified and pre-announced to the competing firms. Alternatively, it may be *endogenously* determined, as a function of the set of nominal prices selected by the competing firms, for example the lowest price, the second lowest price, or the average or median price. In particular with endogenously specified subsidies, price setting may be organized via a closed bid auction in which all qualified providers or suppliers select their prices, simultaneously. Unlike classical auctions, this type does not result in a unique winner capturing all of the business; rather it is a mechanism to arrive at a competitive and endogenously determined subsidy. It is reasonable to conjecture that endogenously specified subsidy levels result in lower equilibrium subsidies as well as lower nominal prices, compared with exogenous subsidies, thus stimulating the intensity of the price competition while lowering the subsidy costs incurred by the third party. Examples of such oligopoly markets include the Medicare insurance market (both as currently structured and in various reform proposals), and the market for solar panels where the federal government and most of the states offer very significant subsidies. Much of this paper is devoted to an analysis of the Medicare insurance market as an application of a general theoretical model developed in its first part.

We first characterize the equilibrium behavior under a general subsidy scheme of the above type; this in a base model, where we assume that the consumer choice model is of the general MultiNomialLogit (MNL) type. Here, we show that a pure strategy Nash equilibrium exists under very general conditions for the structure of the subsidy scheme and the utility functions underlying the MNL model. Establishing the existence of an equilibrium is a challenge even under exogenously specified subsidies, and, a fortiori, when the subsidy is endogenously determined. Moreover, while it is well known that an equilibrium exists, in general MNL models without price subsidization, this may fail to apply to various generalizations of the basic MNL model, for example Mixed MNL models (MMNL) where the market is segmented and the structure of the utility functions varies by segment.(In the latter case, Allon et al. 2012 have shown that an equilibrium may fail to exist while providing specific market share conditions under which the existence question can be answered in the affirmative.)

We obtain additional characterizations for what are arguably, the most important special subsidy structures, i.e., the case of exogenous subsidies and that where the subsidy is specified as the lowest of the selected prices. Here it is possible, under mild conditions, to guarantee that the equilibrium is globally stable: whatever the initial price vector, when firms adjust their price choices, iteratively, by selecting best responses to their competitors' prices, this dynamic scheme converges to an equilibrium. While a stronger form of equilibrium, global stability ensures that the game has a componentwise smallest and componentwise largest equilibrium, and it also provides a simple algorithm to compute an equilibrium. Moreover, the scheme can be used to verify whether the equilibrium is unique. We show in addition, that both the smallest and the largest price equilibrium under the lowest bid subsidy, are (component-wise) exceeded by the smallest and largest equilibrium under an exogenous subsidy, as long as the latter stays below a given threshold.

We proceed to apply our results to an empirical study of the Medicare insurance market. The goal of most empirical industrial organization studies is to estimate the parameters in a very specific consumer choice model with detailed assumptions about the various product attributes which impact the utility measures, as well as, the specific structural forms of the various interdependencies. Our objective is to *predict* the impacts of various reform proposals or equilibrium premia, out-of-pocket costs for the beneficiaries and government spending. These proposals include, in particular, the Wyden-Ryan and Domenici-Rivlin plans. To this end, a far more aggregate or parsimonious model representation suffices, in which, in each county's market all parameters can be determined to *match* the observed premia and enrollment data and to satisfy the equilibrium conditions under the existing subsidy scheme. With these parameters specified, we proceed with counterfactual studies to predict the consequences of the above reform plans.

Medicare provides health insurance coverage to all US citizens and permanent residents, ages 65 and older, as well as younger people with specific disabilities. The current Medicare system was put in place in 2003, with the adoption of the Medicare Modernization Act (MMA), and the adoption of Medicare Advantage (MA), formerly known as Medicare Choice or Medicare Part C. In 2012, Medicare has covered approximately 48 million individuals, at an annual cost close to half a trillion dollars. Moreover, without any restructuring, Medicare costs are estimated to grow at twice the rate of the GDP, the result of the upcoming retirement of many baby boomers, increased longevity, as well as the escalating costs of healthcare. The Congressional Budget Office has estimated that the government's healthcare liabilities, as a percentage of the GDP, would grow from 5% to 12% in the next 40 years, in the absence of a fundamental restructuring of the system. It is generally understood that this would bankrupt the Medicare system.

Medicare Advantage has allowed private insurance companies to offer private plans, as an alternative to the traditional Medicare option, which continues to be run by the Federal government. In 2003, private MA plans captured only 13% of the potential market; however, their share has steadily grown to 27% in 2012. Under the current structure, the government announces a county-specific capitation rate or premium subsidy to all participating insurance companies.[1] In advance of any new calendar year, all

[1]Individuals have a specific risk score based on their prior medical history. This score is an estimate of the individual's

insurance companies are permitted to submit, by a given deadline, one or several plans to the Centers for Medicare & Medicaid Services (CMS), each covering one or a collection of counties.(Collusion is of course illegal.) For a plan to be eligible it must satisfy various criteria: In particular, it must provide benefits that are actuarially at least equivalent to those in the traditional Medicare plan, even though the specific menu of services and devices covered, as well as any associated copayments, etcetera, may be varied freely. In an open enrollment period, beneficiaries choose one of the available alternative plans, i.e., the traditional Medicare plan or one of the private MA plans, with full knowledge of the associated net premia and rebates.

The Wyden-Ryan (W-R) plan advocates modifying the competitive bidding process to one in which the capitation rate is no longer exogenously specified, but determined by the *second lowest* bid.[2] Those adopting the lowest premium plan would receive a rebate in the amount of 75% of the difference between the second lowest and the lowest bid.

The impact of the W-R plan is widely discussed and has been front and center in the 2012 election campaign. The *New England Journal of Medicine* published two articles, Antos (2012) and Aaron and Frakt (2012), presenting diametrically opposing arguments regarding the merits of the competitive bidding proposal. As mentioned, our results now establish that prices go down when the capitation rate is determined as the lowest bid. Song et al. (2012a) have estimated how much beneficiaries would have had to pay in 2009, under the W-R plan, were they to stay with (or adopt) the traditional Medicare plan option, however under the assumption that the premia bid by the various insurance plans would have remained *unaltered*, as would the various plans' market shares. (Song et al. 2012a estimate that, on average, a beneficiary would have paid $64, monthly, if they opted to stay with the traditional Medicare plan: this represents 9% of the cost of this plan.)[3]

Feldman et al. (2012) provide a widely cited estimate of the government's cost savings due to the W-R proposed competitive bidding scheme. Their estimate is, again, based on national 2009 data, and, as in Song et al. (2012a), the assumption that the premia bid by the insurers and their market shares would remain *unchanged*. The authors conclude that a cost reduction of 9.8% would have been achieved, which translates into an aggregate saving of close to $600 billion until the year 2020. The

expected covered costs, as a fraction of the average individual's cost. The actual subsidy received for any given individual is determined as the "normalized' capitation rate , multiplied with the individual's risk score.

[2]The actual formula is somewhat more complex in that it specifies the minimum of the second lowest bid and the traditional Medicare nominal premium. However, the latter is rarely lower than the second lowest bid.

[3]Much larger estimates of annual costs of $6000 or more, for those reaching the age of 65 in the year 2030, have been propagated by the Obama campaign. However these numbers are based on a different provision in the W-R plan, where any given year's capitation rate, in any county, is capped at the prior year's value multiplied by the growth rate of the GDP plus one percentage point. These estimates assume that traditional Medicare costs will continue to grow at a rate significantly in excess of the growth rate of the country's GDP. In any case, the impact of this provision in the W-R plan is beyond the scope of this paper.

Patient Protection and Affordable Care Act (PPACA), popularly referred to as the Obama Care Act, it self contains measures to reduce the capitation rates to lower but still exogenously specified levels. The authors estimate that these measures, if and when enacted, would result in a saving of 4.2%.[4] This estimate is, again, based on the assumption that the new capitation rates would have no impact on the premia bid.

Applying the above price competition model, we have, based on the 2010 county by county data, *computed* what equilibrium prices would emerge from the competitive bidding schemes, in each of the counties. We have used these new equilibrium prices to estimate both the cost savings to the government and the net premia to be paid by beneficiaries who opt to stay with the traditional Medicare plan. As could be expected, we observe a *significant reduction* of the equilibrium premia, compared to those selected under the prevailing, exogenously specified capitation rates. As a consequence, we estimate that the W-R plan would result in a saving of approximately 18.5% in the capitation rate and of 16.2% in the governments' costs. Thus, if the W–R plan had been implemented in 2012, it would have saved close to \$80 billion in this calender year along; this, compared with a total of \$68 billion from 2012–2016, to be saved under the Affordable Care Act, due to its mandated reduction of capitation rates. For beneficiaries continuing to opt for the traditional Medicare plan, the average monthly cost is roughly \$64, comparable to those estimated in Song et al. (2012a), under the assumption of unaltered premia and market shares.

We conduct our analysis by assuming that the demand for the various insurance options is specified by a MNL model, with utility measures that are increasing convexly with the net premium paid by the beneficiary. The structures proposed by the W–R plan corresponds with a piece-wise linear convex response function. Indeed, most of the economic studies of the Medicare market, have used an MNL model of this type, as the underlying consumer choice model, except for choosing utility measures that are *linear* in the *nominal* premium. See, for example, Dowd et al. (2003), Hall (2007), Lustig (2008) and Nosal (2012).[5]

The equilibrium behavior of MNL based price competition models is well understood when the utility measure depends on the nominal prices only, i.e., in the absence of subsidies, see e.g., Anderson et al. (2001), Bernstein and Federgruen (2004), and Gallego et al. (2006). We assume that the observed

[4]In its original form, the Patient Protection and Affordable Care Act of 2010 (Pub. L. No. 111-148, 124 Stat. 119 [2010]) specified that capitation rates would be specified as a weighted average of the premia bid for the various MA plans. These provisions were to take effect in 2012. However, the Health Care and Education Reconciliation Act of 2010 (Pub. L. No. 111-152, 124 Stat. 1029 [2010]), which was passed only days after the main bill, stripped the competitive-pricing provisions from health reform and replaced them with *exogenously* specified capitation rates tied to a percentage of the cost of the traditional Medicare plan in the relevant county. It has been widely reported that this reflected a deal struck to pass the PPACA, after much pushback by the insurance industry. Feldman et al. (2012)'s estimates of the cost savings, resulting from the passed legislation, reflect these final capitation rules.

[5]Some of these consumer choice models, treat some of the coefficients in the utility measures as random, or introduce other segmentations. This gives rise to so-called MMNL models, a generalization we address in Section 8.

premia in 2010 are the equilibrium prices in the competition model generated by the prevailing exogenous capitation rates. This allows us to derive the unknown parameters in the MNL model, including the cost per beneficiary under the various insurance plans in each county. With the parameters of the MNL model specified, we compute the new equilibrium that would arise under a proposed bidding scheme.

The remainder of the paper is organized as follows. In Section 2, we present the general model. Sections 3-5 characterize the equilibrium behavior when the subsidy is exogenously specified, or endogenously determined as the lowest price, or the n-th lowest price, respectively. The above discussed comparison results of the price equilibria under various subsidy schemes are derived in Section 6. Section 7 applies our model to the Medicare market and reports our estimates for the impact of proposed competitive bidding schemes, such as the scheme in the W–R plan. Section 8 discusses the generalization of our model to one with heterogeneous customers, represented by a MMNL model; this section pays specific attentions to the incorporation of switching costs. Section 9 concludes our paper with a brief discussion of other settings, for example prospect theoretical models in behavioral economics, where the customer's utility measures depend on the *differential* between the nominal price and a given reference value. Appendix A contains all proofs except that of Theorem 1.

2 Model

Consider an oligopolistic market with N competing single-product firms each selling a product or service. The firms differentiate themselves via an arbitrary collection of observable product characteristics, as well as their price. A significant part of the product or service price is paid by a third party, as a *subsidy* (sometimes referred to as *capitation rate* or *benchmark rate*). Examples of such oligopoly markets include the Medicare insurance market, both as currently structured and in various reform proposals, and the market for solar panels where the federal government and most of states offer very significant subsidies. The consumers are assumed to purchase at most one unit of the product or service within a given time period. For example, in the Medicare industry, every eligible beneficiary enrolls in at most one of the available plans, for a given calender year. Customers pay or receive an amount which is based on the net price, defined as the difference between the *nominal* price and the subsidy. The subsidy may be an *exogenously* specified constant, which is pre-announced to the competing firms. Alternatively, it may be *endogenously* determined, as a function of the set of nominal prices selected by the competitors, for example the lowest, the second lowest, the average or median price. Each firm's cost structure is assumed to be affine. For each firm $i = 1, 2, \ldots, N$, let

$$\begin{aligned}
c_i &= \text{the marginal cost rate of providing product } i \\
p_i &= \text{nominal price of the product or service provided by firm } i\text{, to be selected from an interval } [p_i^{min}, p_i^{max}] \\
p_{-i} &= \text{price vector for all firms other than } i\text{, i.e., } p_{-i} = (p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_N) \\
p_{(n)}^{-i} &= \text{the } n\text{th smallest price excluding } p_i \text{ if } n \geq 1\text{, and } c_i \text{ if } n = 0 \\
p_{(n)} &= \text{the } n\text{th smallest price, } n \geq 1 \\
g(\mathbf{p}) &= \text{the subsidy} \\
\Delta p_i &= \textit{net} \text{ price of product } i = p_i - g(p) \\
d_i &= \text{market share of firm } i
\end{aligned}$$

We choose $p_i^{min} = c_i, i = 1, 2, \ldots, N$. Each customer j assigns a utility measure to each of the N available products, as follows

$$u_{ij} = a_i - b_i \cdot f\left(p_i - g(\mathbf{p})\right) + \epsilon_{ij}, \quad i = 1, 2, \ldots, N, \; j = 1, 2, \ldots. \tag{1}$$

Here, the intercept a_i denotes the aggregate impact of all of the product's observable attributes, with the exception of to price. The, generally non-linear, function $f(\cdot)$ characterizes how the net price (i.e., gross price minus subsidy) impacts on the utility measure. A non-linear choice for this function may often be necessary; for example, in the MA industry, if the net price is negative, the customer receives less than (the absolute value of) this net price as a rebate, in accordance with a specific rebate percentage. In addition, the marginal disutility due to an extra \$10 of out-of-pocket expenses, may not be constant. We allow for the price sensitivity coefficient b_i to be product specific. Finally, the last term ϵ_{ij} in (1) represents a random unobserved component of the customer j's utility for product i, which varies by customer.

We represent the no-purchasing option as a product 0, with a utility measure

$$u_{0j} = a_0 - b_0 f\left(p_0 - g(\mathbf{p})\right) + \epsilon_{0j}, \quad j = 1, 2, \ldots, \tag{2}$$

where a_0 is a constant and ϵ_{0j} is a random unobserved component. This representation is motivated by the Medicare market application discussed in Section 7, where beneficiaries may choose not to enroll in any of the competing Medicare Advantage plans, but to opt for the traditional Medicare program, whose premium is exogenously given. In other applications, consumers may choose not to purchase any variant of the product, in which case the utility measure is best described without the second term in (2), i.e., selecting $b_0 = 0$. While a special case of the general specification in (2), some of our results are predicated on the assumption $b_0 = b_1 = \cdots = b_N = b$, i.e., an identical price sensitivity coefficient for all products. We therefore return to the case $u_{0j} = a_j + \epsilon_{0j}$ in the last section, Section 9.

To complete the specification of the utility functions (1), the random variables $\{\epsilon_{ij}\}$ for the unobserved utility components are assumed to be i.i.d across firms and customers, following standard type 1–extreme value or Gumbel distribution, i.e., $\mathbb{P}(\epsilon_{ij} \leq x) = \exp(-\exp(-x+\gamma))$ where γ is Euler's constant (0.5772). The mean and variance of ϵ_{ij} are $E[\epsilon_{ij}] = 0$ and $var[\epsilon_{ij}] = \pi^2/6$. This give rise to a variant of the famous MNL model, with the following expected demand functions:

$$d_i(\mathbf{p}) = \frac{\exp(a_i - b_i \cdot f(p_i - g(\mathbf{p})))}{\exp(a_0 - b_0 f(p_0 - g(\mathbf{p}))) + \sum_{j=1}^{N} \exp(a_j - b_j \cdot f(p_j - g(\mathbf{p})))}, \tag{3}$$

see for e.g., Anderson et al. (2001). The above specification treats the utilities of all customers as identically distributed. Often, the market needs to be segmented into several customer classes, each with its own specification of the utility measures. This generalization is referred as a Mixed-MultiNomialLogit (MMNL) model, and will be discussed in Section 8.

Assumption 1 $f(\cdot)$ *is increasing, continuously differentiable everywhere, with the possible exception of a countable set* $\mathbb{P}$.

In case the nominal price of a firm exceeds the subsidy, the firm earns the full price, partially paid by the third party and the remainder by its consumer. When the nominal price of a firm falls bellow the subsidy, the third party pays the firm its nominal price plus a rebate which equals part or all of the net price; however, the rebate must be returned to the customers by extra benefits or cash payment. Thus, the firm earns its selected nominal price, under all circumstances, for each of its customers, and its profit function is given by

$$\pi_i(p_i, p_{-i}) = (p_i - c_i) d_i(\mathbf{p}). \tag{4}$$

In the next three sections, we characterize the equilibrium behavior in the above competition model, under various specifications of the subsidy function $g(\cdot)$. The subsidy is determined as follows

$$g(\mathbf{p}) = \begin{cases} C, & \text{exogenous subsidy} \\ p_{(1)}, & \text{subsidy based on the lowest price} \\ p_{(n)}, & \text{subsidy based on the } n\text{-th lowest price, } n = 1, 2, \ldots \end{cases} \tag{5}$$

Here C is a constant that is specified and pre-announced. Taking the Medicare insurance market as an example, the *exogenous subsidy* corresponds to the traditional Medicare system where the subsidy (benchmark) is specified based on the FFS medicare cost in the previous years and it is pre-announced before insurance company bid. The Widden-Ryan plan, as well as the earlier Domenici-Rivlin plan, specify the subsidy to be the *second-lowest bid.*

In Section 3-5, we characterize the price behavior in the competition model, for each of the three subsidies (5), i.e., exogenous subsidy, subsidy based on the lowest or n-th lowest bid, respectively.

For any function $H(x)$, we denote the left-limit and right-limit at a particular point x_0 by $H_-(x_0) \equiv \lim_{x \nearrow x_0} H(x)$ and $H_+(x_0) \equiv \lim_{x \searrow x_0} H(x)$, respectively. Similarly, we write $\frac{\partial_- H}{\partial x}(x_0) = \lim_{u \nearrow x_0} \frac{\partial H}{\partial x}(u)$ and $\frac{\partial_+ H}{\partial x}(x_0) = \lim_{u \searrow x_0} \frac{\partial H}{\partial x}(u)$, respectively, whenever these limits exist.

3 Exogenous Subsidy

In this section, we study the price competition model with the subsidy exogenously set as a constant, i.e., $g(\mathbf{p}) = C$. The market-share given by (3) for each MA plan $i = 1, 2, \ldots, N$ is

$$d_i(\mathbf{p}) = \frac{\exp\left(a_i - b_i f\left(p_i - C\right)\right)}{\exp\left(a_0 - b_0 f\left(p_0 - C\right)\right) + \sum_{k=1}^{N} \exp\left(a_k - b_k f\left(p_k - C\right)\right)}. \tag{6}$$

Taking derivatives with respect to the price variables, we get

$$\frac{\partial d_i}{\partial p_i} = -b_i f'\left(p_i - C\right) d_i (1 - d_i), \quad \text{if } \Delta p_i = p_i - C \notin \mathbb{P}, \tag{7}$$

$$\frac{\partial d_i}{\partial p_j} = b_j f'\left(p_j - C\right) d_i d_j, \quad \text{if } \Delta p_j = p_j - C \notin \mathbb{P} \text{ for any } j = 1, 2, \ldots, N, j \neq i. \tag{8}$$

When Δp_i or Δp_j is an element of $\mathbb{P}$, formulae, similar to (7) and (8), apply to the left-derivatives and the right-derivatives of d_i with respect to (w.r.t) p_i and p_j, respectively. Note that each firm's demand is decreasing in its own price and increasing in any of its competitors' prices, as in the classical MNL model.

Theorem 1 *Under an exogenously specified subsidy, the competition model is log-supermodular. In particular, there exists a pure Nash equilibrium price vector p^*, and the set of all price equilibria is a lattice and, therefore, has a componentwise largest and smallest element, $\bar{p}^*$ and $\underline{p}^*$, respectively.*

Proof. We first prove the theorem when f is piecewise linear; we then give the proof for a general function f that satisfies Assumption 1, by approximating this function by a sequence of piecewise linear functions,

Part 1: Assume, the function $f(\cdot)$, is piecewise linear with $M + 1$ segments, characterized by M break points $(x_1, x_2, \ldots, x_M)$ and $M + 1$ non-negative slopes $(\beta_1, \beta_2, \ldots, \beta_{M+1})$ as follows:

$$f(x) = \begin{cases} f(x_1) - \beta_1(x_1 - x), & x \leq x_1 \\ f(x_1) + \beta_2(x - x_1), & x \in [x_1, x_2] \\ \vdots & \vdots \\ f(x_{M-1}) + \beta_M(x - x_{M-1}), & x \in [x_{M-1}, x_M] \\ f(x_M) + \beta_{M+1}(x - x_M), & x \geq x_M \end{cases}$$

For any $i = 1, 2, \ldots, N$, and fixed prices $p_{i1} > p_{i2}$, we show that the difference in the logarithms of the profit function for firm i, $\log(\pi_i(p_{i1}, p_{-i})) - \log(\pi_i(p_{i2}, p_{-i}))$, is non-decreasing in any competitor's price $p_j, j \neq i$ for any p_{-i}. It suffices to show this property is satisfied in the following two cases: (i) $\Delta p_{i2} < \Delta p_{i1}$ are contained in the same line segment, i.e., $x_{m-1} \leq \Delta p_{i2} < \Delta p_{i1} \leq x_m$ for some $m = 1, 2, \ldots, M+1$; (ii) $\Delta p_{i2}, \Delta p_{i1}$ are separated by exactly one break point x_m, i.e., $\Delta p_{i2} < x_m < \Delta p_{i1}$ for some $m = 1, 2, \ldots, M$. (If $\Delta p_{i2}, \Delta p_{i1}$ are separated by more than one break point, the difference $\log(\pi_i(p_{i1}, p_{-i})) - \log(\pi_i(p_{i2}, p_{-i}))$ can be written as the sum of differences involving pairs of price levels that are separated by a single break point.) [6]

(i) $\underline{x_{m-1} \leq \Delta p_{i2} < \Delta p_{i1} \leq x_m}$. Note that

$$\log(\pi_i(p_{i1}, p_{-i})) - \log(\pi_i(p_{i2}, p_{-i})) = \int_{p_{i2}}^{p_{i1}} \frac{\partial \log(\pi_i)}{\partial p_i}(p_i, p_{-i}) dp_i,$$

since for all $p_i \in (p_{i1}, p_{i2})$, $\Delta p_i = p_i - C$ is in the interior of the same line segment, where the function $f(\cdot)$ is differentiable. Hence, it is sufficient to show that $\frac{\partial \log \pi_i(p_i, p_{-i})}{\partial p_i}$ is non-decreasing in $p_j, j \neq i$, for any $p_i \in (C + x_{m-1},\ C + x_m)$. For any $p_i \in (C + x_{m-1},\ C + x_m)$, we have

$$\frac{\partial \log(\pi_i)}{\partial p_i} = \frac{1}{p_i - c_i} + \frac{\partial d_i}{\partial p_i} / d_i = \frac{1}{p_i - c_i} - b_i f'(p_i - C)(1 - d_i).$$

Since $f'(p_i - C) = \beta_m$ for any $p_i \in (C + x_{m-1},\ C + x_m)$ and d_i is non-decreasing in $p_j, j \neq i$, it follows that $\frac{\partial \log(\pi_i)}{\partial p_i}$ is non-decreasing in $p_j, j \neq i$.

(ii) $\underline{C + x_{m-1} < p_{i2} < C + x_m < p_{i1} < C + x_{m+1}}$. One has, for $\delta > 0$ sufficiently small, that

$$\begin{aligned} \log \pi_i(p_{i1}, p_{-i}) - \log \pi_i(p_{i2}, p_{-i}) \quad = \quad & \overbrace{\log \pi_i(p_{i1}, p_{-i}) - \log \pi_i(C + x_m + \delta, p_{-i})} \\ & + \underline{\log \pi_i(C + x_m + \delta, p_{-i}) - \log \pi_i(C + x_m - \delta, p_{-i})} \\ & + \underbrace{\log \pi_i(C + x_m - \delta, p_{-i}) - \log \pi_i(p_{i2}, p_{-i})}. \end{aligned}$$

By case (i), it suffices to show that there exists $\delta > 0$ small enough such that

$$\log \pi_i(C + x_m + \delta, p_{-i}) - \log \pi_i(C + x_m - \delta, p_{-i}) \text{ is non-decreasing in } p_j, j \neq i. \tag{9}$$

We will show this by considering two cases

Case (ii.a): For any $\Delta p_j \notin \mathbb{P}$, let

$$\Delta_j(\delta) = \frac{\partial \left[\log \pi_i(C + x_m + \delta, p_{-i}) - \log \pi_i(C + x_m - \delta, p_{-i})\right]}{\partial p_j}.$$

[6] $x_0 = -\infty, x_{M+1} = +\infty$.

In the following, we will show for any $j \neq i$,

$$\Delta_j(\delta) \geq 0 \quad \text{for any } \delta > 0. \tag{10}$$

By the definition of $\Delta_j(\delta)$, one has

$$\begin{aligned}
\Delta_j(\delta) &= \frac{\partial\left[\log \pi_i(C+x_m+\delta, p_{-i}) - \log \pi_i(C+x_m-\delta, p_{-i})\right]}{\partial p_j} \\
&= \frac{\partial \log d_i(C+x_m+\delta, p_{-i})}{\partial p_j} - \frac{\partial \log d_i(C+x_m-\delta, p_{-i})}{\partial p_j} \\
&= \frac{b_j f'(p_j-C)\exp\left(a_j - b_j f(p_j-C)\right)}{\exp\left(a_i - b_i f(x_m) - b_i\beta_{m+1}\delta\right) + \exp\left(a_j - b_j f(p_j-C)\right) + \sum_{k\neq i,j}^{N} \exp\left(a_k - b_k f\left(p_k - C\right)\right)} \\
&\quad - \frac{b_j f'(p_j-C)\exp\left(a_j - b_j f(p_j-C)\right)}{\exp\left(a_i - b_i f(x_m) + b_i\beta_m\delta\right) + \exp\left(a_j - b_j f(p_j-C)\right) + \sum_{k\neq i,j} \exp\left(a_k - b_k f\left(p_k - C\right)\right)} \\
&= \frac{b_j f'(p_j-C) e^{a_j - b_j f(p_j-C)} \cdot \left[e^{a_i - b_i f(x_m) + b_i\beta_m\delta} - e^{a_i - b_i f(x_m) - b_i\beta_{m+1}\delta}\right]}{\left[e^{a_i - b_i f(x_m) - b_i\beta_{m+1}\delta} + \sum_{k\neq i} e^{a_k - b_k f(p_k-C)}\right] \cdot \left[e^{a_i - b_i f(x_m) + b_i\beta_m\delta} + \sum_{k\neq i} e^{a_k - b_k f(p_k-C)}\right]} \\
&= \frac{b_j f'(p_j-C) e^{a_j - b_j f(p_j-C)} e^{a_i - b_i f(x_m) + b_i\beta_m\delta} \cdot \left[1 - e^{-b_i(\beta_m+\beta_{m+1})\delta}\right]}{\left[e^{a_i - b_i f(x_m) - b_i\beta_{m+1}\delta} + \sum_{k\neq i} e^{a_k - b_k f(p_k-C)}\right] \cdot \left[e^{a_i - b_i f(x_m) + b_i\beta_m\delta} + \sum_{k\neq i} e^{a_k - b_k f(p_k-C)}\right]} \\
&\geq 0, \qquad \text{for any } \delta > 0.
\end{aligned}$$

Case (ii.b). In this case, we show (9) holds when $\Delta p_j \in \mathbb{P}$. Fixing p_{-ij}, we simplify the notation by writing $d_i(p_i, p_j)$ and $\pi_i(p_i, p_j)$ instead of $d_i(p_i, p_{-i})$ and $\pi_i(p_i, p_{-i})$. For $\delta > 0$ small enough, we will show

$$\begin{aligned}
&\log \pi_i(C+x_m+\delta, C+x_o+\delta) - \log \pi_i(C+x_m+\delta, C+x_o-\delta) \\
&\geq \log \pi_i(C+x_m-\delta, C+x_o+\delta) - \log \pi_i(C+x_m-\delta, C+x_o-\delta).
\end{aligned} \tag{11}$$

Substituting the demand equation (6) into the profit function, we have

$$\begin{aligned}
&\log \pi_i(C+x_m+\delta, C+x_o+\delta) - \log \pi_i(C+x_m+\delta, C+x_o-\delta) \\
&- \left[\log \pi_i(C+x_m-\delta, C+x_o+\delta) - \log \pi_i(C+x_m-\delta, C+x_o-\delta)\right] \\
= \;& \log d_i(C+x_m+\delta, C+x_o+\delta) + \log d_i(C+x_m-\delta, C+x_o-\delta) \\
&- \log d_i(C+x_m+\delta, C+x_o-\delta) - \log d_i(C+x_m-\delta, C+x_o+\delta) \\
= \;& \log\left(\frac{d_i(C+x_m+\delta, C+x_o+\delta)}{d_i(C+x_m+\delta, C+x_o-\delta)}\right) + \log\left(\frac{d_i(C+x_m-\delta, C+x_o-\delta)}{d_i(C+x_m-\delta, C+x_o+\delta)}\right) \\
= \;& \log\left(\frac{A_1 A_2}{B_1 B_2}\right),
\end{aligned}$$

where

$$A_1 = \left(e^{a_i - b_i f(x_m) - b_i \beta_{m+1}\delta} + e^{a_j - b_j f(x_o) + b_j \beta_o \delta} + \sum_{k \neq i,j} e^{a_k - b_k f(p_k - C)} \right),$$

$$B_1 = \left(e^{a_i - b_i f(x_m) - b_i \beta_{m+1}\delta} + e^{a_j - b_j f(x_o) - b_j \beta_{o+1} \delta} + \sum_{k \neq i,j} e^{a_k - b_k f(p_k - C)} \right),$$

$$A_2 = \left(e^{a_i - b_i f(x_m) + b_i \beta_{m}\delta} + e^{a_j - b_j f(x_o) - b_j \beta_{o+1} \delta} + \sum_{k \neq i,j} e^{a_k - b_k f(p_k - C)} \right),$$

$$B_2 = \left(e^{a_i - b_i f(x_m) + b_i \beta_{m}\delta} + e^{a_j - b_j f(x_o) + b_j \beta_{o} \delta} + \sum_{k \neq i,j} e^{a_k - b_k f(p_k - C)} \right).$$

Let

$$\Gamma_i(\delta) \quad \equiv \quad A_1 A_2 - B_1 B_2.$$

Hence (11) is equivalent to $\Gamma_i(\delta) \geq 0$. Since $\Gamma_i(0) = 0$, to show $\Gamma_i(\delta) \geq 0$ for $\delta > 0$ sufficiently small, it suffices to show $\lim_{\delta \searrow 0} \Gamma_i'(\delta) = 0$ and $\lim_{\delta \searrow 0} \Gamma_i''(\delta) > 0$. Indeed, taking derivatives w.r.t δ, it can be shown that

$$\begin{aligned} \Gamma_i'(\delta) \quad = \quad & e^{a_i - b_i f(x_m) + a_j - b_j f(x_o) - (\beta_{m+1} b_i + \beta_{o+1} b_j)\delta} \cdot \Big[- \beta_m b_i e^{(\beta_m + \beta_{m+1}) b_i \delta} - \beta_o b_j e^{(\beta_o + \beta_{o+1}) b_j \delta} \\ & + (\beta_m b_i + \beta_o b_j) e^{((\beta_m + \beta_{m+1}) b_i + (\beta_o + \beta_{o+1}) b_j)\delta} + \beta_{o+1} b_j (-1 + e^{(\beta_m + \beta_{m+1}) b_i \delta}) \\ & + \beta_{m+1} b_i (-1 + e^{(\beta_o + \beta_{o+1}) b_j \delta}) \Big], \end{aligned}$$

so that

$$\begin{aligned} \lim_{\delta \searrow 0} \Gamma_i'(\delta) \quad &= \quad 0, \\ \lim_{\delta \searrow 0} \Gamma_i''(\delta) \quad &= \quad 2 b_i b_j (\beta_o + \beta_{o+1})(\beta_m + \beta_{m+1}) e^{a_i - b_i f(x_m) + a_j - b_j f(x_o)} > 0. \end{aligned}$$

Hence, we have shown that for $\delta > 0$ small enough, (11) holds, so that, $\log(\pi_i)$ is supermodular.

Part 2: Assume now f is a general continuous and increasing function. It is well-known that there exists a sequence of increasing piece-wise linear functions $\{f^{(k)}(\cdot)\}$ such that $\lim_{k \to \infty} f^{(k)}(x) = f(x)$ for any x. For any i, let $\pi_i^{(k)}$ denote firm i's profit function associated with the function $f^{(k)}$. For any given pair of price vectors, $\mathbf{p}, \mathbf{p}'$, such that $\mathbf{p} \geq \mathbf{p}'$, we have by part 1, that:

$$\log \pi_i^{(k)}(p_i, p_{-i}) - \log \pi_i^{(k)}(p_i', p_{-i}) \geq \log \pi_i^{(k)}(p_i, p_{-i}') - \log \pi_i^{(k)}(p_i', p_{-i}'). \tag{12}$$

By a simple continuity argument, we have, for any price vector $\hat{p}$, $\lim_{k \longrightarrow \infty} \log \pi_i^{(k)}(\hat{p}) = \log \pi_i(\hat{p})$. Hence, taking limits in (12) yields

$$\log \pi_i(p_i, p_{-i}) - \log \pi_i(p_i', p_{-i}) \geq \log \pi_i(p_i, p_{-i}') - \log \pi_i(p_i', p_{-i}'),$$

thus, showing that the price game is log-supermodular for a general convex increasing function f. □

An additional implication of the price game being (log-)supermodular is the fact that an equilibrium may be computed with a simple tatônnement scheme: starting with an arbitrary price vector $p^{(0)}$, one iteratively computes a best response price for each of the N firms to the most recently generated prices of the competitors. The scheme is guaranteed to converge to an equilibrium. It can also be shown that

Proposition 1 *Assuming that the response function $f(\cdot)$ is increasing and convex, each firm's profit function is quasi-concave in its own price.*

Therefore, there is a *unique* best response price, for any set of prices selected by the competitors. Moreover, when the scheme is started at $p^{min}[p^{max}]$, it is guaranteed to converge to $\underline{p}^*[\bar{p}^*]$. It is therefore possible to unequivocally determine whether the game has a unique equilibrium by starting the tatônnement scheme, both at p^{min} and at p^{max}, and checking whether the two schemes converge to the same limit point. We are, at this point, unaware of any, a priori, theoretical conditions which guarantee the uniqueness of the price equilibrium.

Finally, we show that for p^{max} sufficiently large, any equilibrium p^* is in the *interior* of the feasible price space $\mathbf{X}_{i=1}^{N}[p_i^{min}, p_i^{max}]$. Note first that

$$\lim_{p_i \searrow c_i} \frac{\partial \log \pi_i(p_i, p_{-i})}{\partial p_i} = \lim_{p_i \searrow c_i} \left[\frac{1}{p_i - c_i} + \frac{\partial \log d_i(p_i, p_{-i})}{\partial p_i} \right] = \lim_{p_i \searrow c_i} \left[\frac{1}{p_i - c_i} - b_i f'(p_i - C)(1 - d_i) \right] = +\infty,$$

since the second term within the squared brackets is bounded in $p_i \searrow c_i$. This implies that, for any equilibrium p^*, $p_i^* > c_i$ for all $i = 1, 2, \ldots, N$. Similarly,

$$\begin{aligned} \lim_{p_i \nearrow \infty} \frac{\partial \log \pi_i(p_i, p_{-i})}{\partial p_i} &= \lim_{p_i \nearrow \infty} \left[\frac{1}{p_i - c_i} - b_i f'(p_i - C)(1 - d_i) \right] \\ &\leq \lim_{p_i \nearrow \infty} \left[-b_i f'_+(0)(1 - d_i(p_i, p_{-i})) \right] \\ &\leq -b_i f'_+(0)(1 - d_i(C, p_{-i})) < 0, \end{aligned}$$

since d_i is decreasing in its own price p_i and f is convex. This implies that, for p^{max} sufficiently large, $p^* < p^{max}$. Thus, if p^{max} is sufficiently large, any equilibrium p^* is an interior point of the feasible price space, so that

$$(p_i^* - c_i) b_i f'\left(p_i^* - C\right)\left(1 - d_i\right) = 1, \qquad \text{if } p_i^* - C \notin \mathbb{P}. \tag{13a}$$

$$\left.\begin{aligned} (p_i^* - c_i) b_i f'_+\left(p_i^* - C\right)\left(1 - d_i\right) \geq 1 \\ (p_i^* - c_i) b_i f'_-\left(p_i^* - C\right)\left(1 - d_i\right) \leq 1 \end{aligned}\right\}, \qquad \text{if } p_i^* - C \in \mathbb{P}. \tag{13b}$$

This set of equations (or inequalities) may be used to infer the firms' cost rates c from the observed price equilibrium in the market. In particular when $p_i^* - C \notin \mathbb{P}$, for all $i = 1, 2, \ldots, N$, the First Order

Conditions (13) reduce to a system of equations, with the unique solutions:

$$c_i = p_i^* - \frac{1}{b_i f'(p_i^* - C)(1 - d_i(p^*))}, \quad i = 1, 2, \ldots, N. \tag{14}$$

When $p_i^* - C \in \mathbb{P}$, for some firm i, the cost rate c_i can be determined only within an interval, the width of which depends on the difference between the right hand and left hand derivatives $[f'_+(p_i^* - C) - f'_-(p_i^* - C)]$:

$$p_i^* - \frac{1}{b_i f'_-(p_i^* - C)(1 - d_i(p^*))} \leq c_i \leq p_i^* - \frac{1}{b_i f'_+(p_i^* - C)(1 - d_i(p^*))}. \tag{15}$$

4 Lowest Price Subsidy

In this section, we study the competition model with a subsidy endogenously determined as the *lowest* among the nominal prices selected by the competing firms, namely, $g(\mathbf{p}) = p_{(1)}$. Some additional structure is needed for the response function $f(\cdot)$:

Assumption 2 *$f(x)$ is increasing, convex and differentiable everywhere with the possible exception of $x = 0$.*

We allow for non-differentiability in $x = 0$, to capture the application of our model to the Medicare market, see Section 7, as well as prospect theoretical models to be discussed in Section 9. We first derive expressions for the sales volumes $\{d_i(p) : \ i = 1, 2, \ldots, N\}$ and their derivatives $\{\frac{\partial d_i}{\partial p_i} : \ i = 1, 2, \ldots, N\}$. To this end, we distinguish among (i) the case where $p_i < p_{(1)}^{-i}$, (ii) the case where $p_i > p_{(1)}^{-i}$, and (iii) $p_i = p_{(1)}^{-i}$.

(i) For any $\underline{p_i < p_{(1)}^{-i}}$, the subsidy $g(\mathbf{p}) = p_i$. By (3) and the fact that $f(0) = 0$, we have

$$d_i(\mathbf{p}) = \frac{\exp(a_i)}{\exp(a_0 - b_0 f(p_0 - p_i)) + \sum_{k=1}^N \exp(a_k - b_k f(p_k - p_i))}, \tag{16}$$

Taking the derivative with respect to p_i yields

$$\begin{aligned} \frac{\partial d_i}{\partial p_i} &= -\frac{\exp(a_i) \cdot \sum_{k \neq i} \exp(a_k - b_k f(p_k - p_i)) \cdot (b_k f'(p_k - p_i))}{\left(\exp(a_0 - b_0 f(p_0 - p_i)) + \sum_{k=1}^N \exp(a_k - b_k f(p_k - p_i))\right)^2} \\ &= -d_i \sum_{k \neq i} d_k b_k f'(p_k - p_i). \end{aligned} \tag{17}$$

(ii) For any $\underline{p_i > p_{(1)}^{-i}}$, the subsidy $g(\mathbf{p}) = p_{(1)}^{-i}$ and similar to case (i), the market share in (3) satisfies

$$d_i(\mathbf{p}) = \frac{\exp\left(a_i - b_i f\left(p_i - p_{(1)}^{-i}\right)\right)}{\exp\left(a_0 - b_0 f\left(p_0 - p_{(1)}^{-i}\right)\right) + \sum_{k=1}^N \exp\left(a_k - b_k f\left(p_k - p_{(1)}^{-i}\right)\right)}, \tag{18}$$

$$\frac{\partial d_i}{\partial p_i} = -b_i f'\left(p_i - p_{(1)}^{-i}\right) d_i(1 - d_i). \tag{19}$$

(iii) When $\underline{p_i = p^{-i}_{(1)}}$, it is easily verified that both (16) and (18) represent correct expressions for the sales volume $d_i(p)$. Hence, the derivative $\frac{\partial d_i}{\partial p_i}$ may fall to exist for the value $p_i = p^{-i}_{(1)}$. However, the left- and right hand derivatives $\frac{\partial_- d_i}{\partial p_i}$ and $\frac{\partial_+ d_i}{\partial p_i}$ exist; the former is given by (17) and the latter by (19).

We assume the following two conditions hold: For any product $i = 1, 2, \ldots, N$, and any price vector p_{-i},

$$\lim_{p_i \nearrow p^{-i}_{(1)}} \sum_{j \neq i} \frac{\partial d_i}{\partial p_j}(p_i, p_{-i}) \leq |\lim_{p_i \searrow p^{-i}_{(1)}} \frac{\partial d_i}{\partial p_i}(p_i, p_{-i})|, \qquad (D)$$

$$(p_i - c_i) \sum_{j \neq i} \frac{\partial \ln(d_i)}{\partial p_j} \text{ is quasi-convex in } p_i \in [c_i, p^{-i}_{(1)}). \qquad (M)$$

Condition (D) is a variant of the classical dominant-diagonal condition (see e.g. Allon et al. 2012, Bernstein and Federgruen 2004 and Vives 2001). It merely precludes that a uniform price increase by all firms would result in an increase of any of the firms' sales volume. Moreover, the classical dominant-diagonal condition assumes that the inequality in (D) holds for *all* possible prices (see e.g. Bernstein and Federgruen 2004), while our condition (D) is much weaker; the dominant diagonal condition is required only when the firm's price is close to the lowest price offered by the competitors. In Lemma 1 at the end of this section, we show that conditions (D) and (M) apply when f is a piece-wise linear function.

We now show that a pure-strategy price equilibrium exists in the lowest subsidy model by showing that each firm's profit function is quasi-concave in its own price .

Theorem 2 *Assume that Assumption 2 and conditions (D)-(M) apply in the lowest subsidy model. A pure-strategy Nash equilibrium exists.*

The following lemma shows that conditions (D) and (M) apply when function $f(\cdot)$ is piece-wise linear and the price sensitivity coefficient is the same for all firms.

Lemma 1 *(a) The conditions of Theorem 2, i.e., conditions (D)-(M) and Assumption 2 apply, assume (1) $b_i = b$ for all i and (2) f is piece-wise linear, i.e., $f(x) = \alpha x^+ - \beta x^-$ with $\alpha, \beta > 0$.*

(b) The demand of any product i is non-decreasing in the price of any alternative product $j \neq i$.

Lemma 1(b) shows that, when the subsidy is specified as the lowest price, the products act as substitutes. Note also that if the demand for any product i is non-decreasing in the price of any alternative product, the same monotonicity property applies to the associated profit values, and vice versa. The latter monotonicity property (of the profit functions) is often referred to as "the competitive market" property, see e.g., Assumption 1 in Cabral and Villas-Boas (2005).

We now show that under the conditions of Lemma 1, the model is, (log-)supermodular.

Theorem 3 *Assume (1) $b_i = b$ for all $i = 1, 2, \dots, N$ and (2) $f(x) = \alpha x^+ - \beta x^-$ with $\alpha, \beta > 0$. The price competition game with the subsidy specified as the lowest bid is log-supermodular.*

Following the same arguments, provided in the previous section, it is easily verified that any price equilibrium must be an interior point of the feasible price region, provided the upper bounds p^{max} are sufficiently large. Moreover, as shown in (A-2), each profit function $\pi_i(p_i, p_{-i})$ is differentiable everywhere, with the possible exception of the point $p_i = p_{(1)}^{-i}$. Together with (A-3) and (A-4), this implies that any equilibrium p^* satisfies the following system of equations and inequalities:

$$1 - (p_i^* - c_i) \sum_{k \neq i} d_k b_k f'(p_k^* - p_i^*) = 0, \quad \text{if } p_i^* < p_{(1)}^{-i}, \tag{20a}$$

$$1 - (p_i^* - c_i) b_i f'\left(p_i^* - p_{(1)}^{-i}\right)(1 - d_i) = 0, \quad \text{if } p_i^* > p_{(1)}^{-i}, \tag{20b}$$

$$\frac{\partial_+ \log \pi_i(p_{(1)}^{-i}, p_{-i}^*)}{\partial p_i} = 1 - (p_{(1)}^{-i} - c_i) b_i f'_+(0)(1 - d_i) \leq 0, \tag{20c}$$

$$\frac{\partial_- \log \pi_i(p_{(1)}^{-i}, p_{-i}^*)}{\partial p_i} = 1 - (p_{(1)}^{-i} - c_i) \sum_{k \neq i} d_k b_k f'_+\left(p_k^* - p_{(1)}^{-i}\right) \geq 0. \tag{20d}$$

Similar to the system of equations and inequalities in (13), (20) allows us to determine the cost rates $\{c_i\}$ from any observed price equilibrium p^*. For any product with $p_i^* \neq p_{(1)}^{-i}$, the unique corresponding marginal cost rate c_i could be derived from equation (20a) or (20b). If $p_i^* = p_{(1)}^{-i}$, an interval can be determined for the corresponding marginal cost rate c_i by inequalities (20c) and (20d).

5 Subsidy Determined by the n-th Lowest Price

In this section, we study the price competition model when the subsidy is endogenously determined as the nth-lowest price among the set of nominal prices selected by the competing firms, i.e., $g(\mathbf{p}) = p_{(n)}$, for some $n \geq 1$. The Medicare market provides the motivation for this generalization of the lowest price subsidy, considered in the previous section; as mentioned, several bipartisan proposals, in particular the Wyden–Ryan and Domenici–Rivlin's plans, advocate setting the capitation rate as the *second* lowest price. As in Section 4, we assume that Assumption 2 applies.

Note that, the subsidy satisfies the following relationships:

$$g(p_i, p_{-i}) = p_{(n)} = \begin{cases} p_{(n-1)}^{-i}, & p_i \leq p_{(n-1)}^{-i} \\ p_i, & p_{(n-1)}^{-i} < p_i \leq p_{(n)}^{-i} \\ p_{(n)}^{-i}, & p_i > p_{(n)}^{-i} \end{cases} \tag{21}$$

Substituting (21) into (3), we get the following expressions for the sales volumes:

$$d_i(p_i,p_{-i}) = \begin{cases} \frac{\exp\left(a_i - b_i f\left(p_i - p_{(n-1)}^{-i}\right)\right)}{\sum_{k=0}^{N}\exp\left(a_k - b_k f\left(p_k - p_{(n-1)}^{-i}\right)\right)}, & p_i \le p_{(n-1)}^{-i} \\ \frac{\exp(a_i)}{\sum_{k=0}^{N}\exp(a_k - b_k f(p_k - p_i))}, & p_i \in (p_{(n-1)}^{-i}, p_{(n)}^{-i}] \\ \frac{\exp\left(a_i - b_i f\left(p_i - p_{(n)}^{-i}\right)\right)}{\sum_{k=0}^{N}\exp\left(a_k - b_k f\left(p_k - p_{(n)}^{-i}\right)\right)}, & p_i > p_{(n)}^{-i} \end{cases}, \tag{22}$$

As in the previous section, we need the monotonicity property (M) and a variant of the diagonal dominant condition which we now refer to as condition (D′)

$$|\lim_{p_i \searrow p_{(n)}^{-i}} \frac{\partial d_i}{\partial p_i}(p_i,p_{-i})| \ge \lim_{p_i \nearrow p_{(n)}^{-i}} \sum_{j\neq i} \frac{\partial d_i}{\partial p_j}(p_i,p_{-i}), \qquad (D')$$

$$\lim_{p_i \searrow p_{(n-1)}^{-i}} \sum_{j\neq i} \frac{\partial d_i}{\partial p_j}(p_i,p_{-i}) \ge |\lim_{p_i \nearrow p_{(n-1)}^{-i}} \frac{\partial d_i}{\partial p_i}(p_i,p_{-i})|,$$

$$(p_i - c_i)\sum_{j\neq i} \frac{\partial \ln(d_i)}{\partial p_j} \text{ is non-decreasing in } p_i \in (p_{(n-1)}^{-i}, p_{(n)}^{-i}), \qquad (M)$$

The first inequality in (D') is identical to (D). When the response function $f(\cdot)$ is linear, it is easily verified that $\sum_{j=0}^{N} \frac{\partial d_i}{\partial p_j} = 0$, i.e., $|\frac{\partial d_i}{\partial p_i}| = \sum_{j\neq i} \frac{\partial d_i}{\partial p_j}$. Lemma 2, below, provides sufficient conditions for both inequalities in (D').

Similar to Theorem 2, we show that a pure-strategy price equilibrium exists by showing that each firm's profit function is quasi-concave in its own price for any given price choices of the other alternatives.

Theorem 4 *Assume that conditions $(D') - (M)$ hold. A pure-strategy price equilibrium exists, when the subsidy is determined as the nth-lowest price.*

The following Lemma shows that conditions $(D') - (M)$ hold under the same conditions as in Lemma 1, for (D) and (M), plus an additional assumption which requires the price sensitivity to a rebate to be less than the price sensitivity to an out-of-pocket payment ($\alpha \ge \beta$).

Assumption 3 *(1) $b_i = b$ for all i, (2) f is piece-wise linear, i.e., $f(x) = \alpha x^+ - \beta x^-$ with $\alpha \ge \beta \ge 0$.*

Lemma 2 *Conditions $(D') - (M)$ hold under Assumption 3.*

The assumption $\alpha \ge \beta$ applies in the Medicare insurance market, where each beneficiary pays the full excess of the premium above the subsidy, but receives only part of any shortfall, when the premium is lower than the subsidy. (It also applies to the prospect theoretical competition models discussed in Section 9.)

We conclude that the price competition model has a pure-strategy Nash equilibrium, under minor technical conditions, irrespective of whether the subsidy is exogenously specified or endogenously determined as the lowest price or the n-th lowest price. The question remains whether the same fundamental in variance with respect to the subsidy structure applies to the stronger property of the price competition game being (log-)supermodular. It follows from the proofs of Theorems 1 and 3 that this supermodularity property applies whenever each firm's sales volume increases with its alternatives' prices, that is, when products may be viewed as simple *substitutes*, or, equivalently, markets maybe viewed as competitive, see Assumption 1 in Cabral and Villas-Boas (2005). In the absence of any subsidy, the MNL model clearly represents products that are strictly substitutable. Theorem 1 shows that the same applies under an *exogenously* specified subsidy, as well as under the lowest price subsidy subject to minor technical conditions, see Lemma 1(b). However, under a more complex subsidy structure, such as one based on the second lowest price, products may cease to interact as strict substitutes; in particular, an increase of the second lowest price in the market, now, has two opposite effects: on the one hand, the *net* prices of all other products decrease by the same amount, by itself resulting in an increase of each of their market shares. However, the non-linearity of the function f, even in its simplest form when $f(x) = \alpha x^+ - \beta x^-$, implies a smaller increase of the utility attributed to the cheapest product as supposed to the utility increases for the other alternatives. This has the opposite effect of shifting some of the market share from the lowest priced product toward these other alternatives; the net effect of an increase of the second lowest price, may therefore involve a *decrease* of the market share of the cheapest alternative. Indeed, this phenomenon may well occur depending on the ratio β/α and the relative market shares of the cheapest and the second cheapest products. More specifically, one can show

$$\frac{\partial d_{(1)}(\mathbf{p})}{\partial p_{(2)}} \geq 0 \quad \text{iff} \quad d_{(2)}(\mathbf{p}) \geq \left(1 - \frac{\beta}{\alpha}\right)\left(1 - d_{(1)}(\mathbf{p})\right),$$

where $d_{(n)}(p)$ denotes the sales volume of the (lowest indexed) product with the n-th smallest price. Thus, when the subsidy is based on the second lowest or the n-th lowest price, a higher subsidy is generated, as compared to the lowest price case. Also, such a scheme continues to foster more aggressive price bidding among the competing firms, as compared to the case of pre-specified subsidy. However, this more complicated subsidy structure has the disadvantage of eliminating strict substitutability among all of the alternatives.

6 Comparison of Subsidy Schemes

The Wyden-Ryan and Domenici-Rivlin reform plans for the Medicare insurance industry are based on the conjecture that competitive bidding with a subsidy endogenously specified as, say, the lowest bid, results in lower prices than the current bidding system where the subsidy is pre-specified. While intuitive, two recent publications in the New England Journal of Medicine (Aaron and Frakt (2012) and Antos (2012)) have made opposite predictions for the direction in which the premia and public spending would change if the capitation rate were determined as the second-lowest bid, say.

In this section, we show, for our general model, that the equilibrium prices under the lowest bid subsidy are always exceeded by those arising under the pre-announced fixed subsidies, at least as long as the latter are selected below a threshold. This confirms the conjecture underlying the above reform plans. We achieve our comparison results when the respective competition games can be assumed to be (log-)supermodular. In this section, we therefore assume that Assumption 3 applies, i.e., (1) $b_i = b$ for all $i = 1, 2, \ldots, N$ and (2) $f(x) = \alpha x^+ - \beta x^-$ with $\alpha \geq \beta > 0$. (Recall that, the last assumption $\alpha \geq \beta$ is not required to guarantee that the price competition game is (log-)supermodular, either under an exogenous subsidy or under one determined as the lowest price.) In addition, we need the following *markup* condition:

$\mathbf{P}(\beta)$: (Markup) The subsidy under the exogenous subsidy model is no larger than $c_{min} + \frac{\ln(\alpha)-\ln(\beta)}{b(\alpha-\beta)}$, where $c_{min} = \min_i\{c_i\}$, that is, $C \leq c_{min} + \frac{\ln(\alpha)-\ln(\beta)}{b(\alpha-\beta)}$.

We first derive, for any fixed price vector $\mathbf{p}$, the following ranking of the sales volumes $d^{EXO}(\mathbf{p})$ and $d^{LOW}(\mathbf{p})$.

Lemma 3 *Assume that condition $\mathbf{P}(\beta)$ and Assumption 3 apply. Then,*

$$\alpha\left(1 - d_i^{LOW}(\mathbf{p})\right) \geq \beta\left(1 - d_i^{EXO}(\mathbf{p})\right), \quad \textit{for any } p_i < C. \tag{23a}$$

$$d_i^{LOW}(\mathbf{p}) \leq d_i^{EXO}(\mathbf{p}), \quad \textit{for any } p_i \geq C. \tag{23b}$$

We now show that the price equilibrium under the lowest price subsidy is componentwise smaller than the price equilibrium under an exogenous subsidy.

Theorem 5 *Assume that condition $\mathbf{P}(\beta)$ and Assumption 3 apply.*

(a) For any given price vector of the alternatives, each firm's best response under the lowest price subsidy is smaller than its best response under an exogenous subsidy:

$$p_i^{*LOW}(p_{-i}) \leq p_i^{*EXO}(p_{-i}), \quad \textit{for any firm } i \textit{ and any } p_{-i}. \tag{24}$$

(b) The componentwise largest and smallest price equilibrium under the lowest price subsidy ($\bar{p}^{*LOW}$ and $\underline{p}^{*LOW}$) is componentwise smaller than the corresponding price equilibrium under an exogenous subsidy ($\bar{p}^{*EXO}$ and $\underline{p}^{*EXO}$):

$$\bar{p}_i^{*LOW} \leq \bar{p}_i^{*EXO} \quad \textit{and} \quad \underline{p}_i^{*LOW} \leq \underline{p}_i^{*EXO} \qquad \textit{for any firm } i. \tag{25}$$

Proof.

(a) By (7), one has

$$\begin{aligned}\frac{\partial_+ \log \pi_i^{EXO}(p_i,p_{-i})}{\partial p_i} &= \frac{1}{p_i-c_i} - bf'_+(p_i-C)\left(1-d_i^{EXO}(p_i,p_{-i})\right) \\ &= \begin{cases} \frac{1}{p_i-c_i} - b\alpha\left(1-d_i^{EXO}(p_i,p_{-i})\right), & p_i \geq C \\ \frac{1}{p_i-c_i} - b\beta\left(1-d_i^{EXO}(p_i,p_{-i})\right), & p_i < C \end{cases}\end{aligned}$$

Similarly, by (17) and (19), one has

$$\frac{\partial_+ \log \pi_i^{LOW}(p_i,p_{-i})}{\partial p_i} = \frac{1}{p_i-c_i} - b\alpha\left(1-d_i^{LOW}(p_i,p_{-i})\right).$$

Thus, we obtain

$$\begin{aligned}&\frac{\partial_+ \log \pi_i^{EXO}(p_i,p_{-i})}{\partial p_i} - \frac{\partial_+ \log \pi_i^{LOW}(p_i,p_{-i})}{\partial p_i} \\ =& \begin{cases} b\alpha\left(d_i^{EXO}(p_i,p_{-i}) - d_i^{LOW}(p_i,p_{-i})\right), & p_i \geq C \\ b\alpha\left(1-d_i^{LOW}(p_i,p_{-i})\right) - b\beta\left(1-d_i^{EXO}(p_i,p_{-i})\right), & p_i < C \end{cases} \qquad (26) \\ \geq& \ 0. \quad \text{for any } p_i, p_{-i} \text{ by (23).}\end{aligned}$$

Recall that, both $\log \pi_i^{LOW}(p_i,p_{-i})$ and $\log \pi_i^{EXO}(p_i,p_{-i})$ are quasi-concave by Theorem 2 and Proposition 1. Thus,

$$p_i^{*LOW}(p_{-i}) \equiv \sup\left\{p_i : \frac{\partial_+ \log \pi_i^{LOW}(p_i,p_{-i})}{\partial p_i} \geq 0\right\} \leq \sup\left\{p_i : \frac{\partial_+ \log \pi_i^{EXO}(p_i,p_{-i})}{\partial p_i} \geq 0\right\} \equiv p_i^{*EXO}(p_{-i}).$$

The inequality follows from the fact that the set to the right contains the set to the left, see (26).

(b) Let $\Psi^{EXO} : \mathbb{R}^N \longrightarrow \mathbb{R}^N$ denote the best response operator in the competition game with an exogenous subsidy C that satisfies $\mathbf{P}(\beta)$, i.e., $\Psi_i^{EXO}(p)$ is defined as the unique best response for firm i to the price vector p_{-i} in the game. Similarly, let Ψ^{LOW} denote the best response operator in the competition game when the subsidy is specified as the lowest bid. (Best responses are uniquely determined because, for each firm i, the profit function $\pi_i(p_i,p_{-i})$ is quasi-concave in its own price p_i, see Theorem 2 and Proposition 1.) For any $r \geq 1$, let $\Psi^{EXO(r)}$ $\left[\Psi^{LOW(r)}\right]$ denote the r–fold application of the best response operator Ψ^{EXO} $\left[\Psi^{LOW}\right]$, i.e., $\Psi^{EXO(r)}(p) = \Psi^{EXO}\left(\Psi^{EXO(r-1)}(p)\right)$ $\left[\Psi^{LOW(r)}(p) = \Psi^{LOW}\left(\Psi^{LOW(r-1)}(p)\right)\right]$.

We prove the first inequality in (25), the proof for the second inequality is analogous. Note that,

$$\bar{p}^{*LOW} = \lim_{r \nearrow \infty} \Psi^{LOW(r)}(p^{max}) \leq \lim_{r \nearrow \infty} \Psi^{EXO(r)}(p^{max}) = \bar{p}^{*EXO}.$$

The two equalities follows from Theorem 4.3.2 in Topkis (1998), since the price competition game is (log-)supermodular under both subsidy schemes by Theorems 1 and 3. To prove the inequality, we show, by induction that

$$\Psi^{LOW(r)}(p^{max}) \leq \Psi^{EXO(r)}(p^{max}) \quad \text{for any } r = 1, 2, \ldots.$$

For $r = 1$ the inequality holds by part (a) of this theorem. Assume the inequality holds for some integer $r \geq 1$. Then,

$$\begin{aligned} \Psi^{LOW(r+1)}(p^{max}) &= \Psi^{LOW}\left(\Psi^{LOW(r)}(p^{max})\right) \\ &\leq \Psi^{LOW}\left(\Psi^{EXO(r)}(p^{max})\right) \\ &\leq \Psi^{EXO}\left(\Psi^{EXO(r)}(p^{max})\right) \\ &= \Psi^{EXO(r+1)}(p^{max}). \end{aligned}$$

The first inequality follows from the fact that the best response operator in a (log-)supermodular game is a monotone operator, while the second inequality follows from part (a) of the theorem.

□

Theorem 5 shows, under the conditions specified therein, that all prices in the market go down when an exogenously specified subsidy is replaced by one that is endogenously determined as the lowest bid. (This comparison applies to both the smallest and the largest equilibrium, in case the latter fails to be *unique*.) It has been argued that, a similar across-the-board price reduction can be achieved, simply by reducing the exogenously specified subsidy. Indeed, such a change has been incorporated in the Affordable Care Act, with this implication in mind. However, it is not possible to show that the full vector of prices goes down, componentwise, when an exogenous subsidy is reduced. Such results would have been attainable if each firm's profit function was (log-)supermodular in its own price and subsidy value. However, this structural result fails to hold. More surprisingly, one can show that the *lowest* price in the market *decreases* as a function of the subsidy value, while the *highest* price increases. In other words, the price *spread* increases as a function of the subsidy value, but some of individual prices may not.

7 The Medicare Market: Implications of the Wyden-Ryan plan

We have used our general model to estimate the changes an implementation of the Wyden-Ryan plan, or the Domenici-Rivlin plan, would generate for the Medicare system. For the sake of brevity, we only refer to the former. To our knowledge, all existing estimates of the implications of this and other reform plans, assume that the premia selected by the various insurance companies are not affected by the specific scheme determining the government's subsidy levels. See, for example, the highly quoted Feldman et al. (2012) and Song et al. (2012a). This fundamental assumption appears only to apply to perfectly competitive markets where competitors are forced to set their prices equal to their marginal costs. Clearly, competition in this, as in most, markets is imperfect, and the participating insurance companies are able to extract positive profit margins. A statistical multi-year study by Song et al. (2012b) confirms this, by showing that the insurance companies adjust their premia to changes in the capitation rates, even though the latter are only partially correlated with the actual cost rates per beneficiary. Indeed, their regression model, allowing for many potential explanatory variables, identifies only the capitation rate, and the number of competitors in the market as having a statistically significant impact on the premia. The fact, that the premia decrease significantly as a function of the number of competing plans, is further evidence for the fact that the market is an oligopoly with imperfect competition. This being recognized, the challenge is to estimate *how* premia change as a function of the subsidy scheme.[7] Our model allows for this estimation by comparing the *price equilibrium* that arises before and after a change in the structure or parameters of the subsidy scheme.[8]

Recall from the Introduction that, currently, the federal government specifies, in advance of each calendar year, a capitation rate for each of the 2727 counties in the US. (The capitation rate applies to a beneficiary with an average, normalized risk score of one; the actual amount paid to the company insuring the beneficiary is obtained by multiplying the normalized capitation rate with the beneficiary's specific risk score.) The W-R plan would replace the current *exogenous* capitation rates by a competitive bidding system in which they would be determined *endogenously* as the second lowest premium bid among the various private participating Medicare Advantage plans.(As mentioned, the W-R plan bounds the capitation rates, additionally, by the prior year's rate multiplied by the growth rate of the GDP plus one percentage point; however, in this study we focus on the implications of the competitive bidding system, per se, thus abstracting from these year-to-year growth limits; see, also, Footnote 2.) Beneficiaries enrolled in the *lowest bid* plan would receive a rebate, specified as 75% of the difference between the second lowest and the lowest bid; the same rebate formula would apply to those enrolled in the traditional Medicare (also referred to as the fee-for-service (FFS)) plan, if the premium cost of the latter falls below the capitation rate.

Our study is based on the county-by-county data in 2010. For this calendar year, private insurance

[7]For example, Nosal (2012) comments on the impact of the original PPACA's mandate to determine capitation rates as a weighted average of the premia bid:"Under the new bidding system, plans are paid based on a weighted average of all plans' bids. Since there is little theoretical literature on average-bid auctions, it is not immediately obvious what effect the policy will have on payments. However, claims that the legislation reduces the payment rates are widespread."

[8]Song et al. (2012b) hypothesize that the oligopolistic characteristic of the Medicare market "diverts CMS payments to plan or provider profits and away from Medicare beneficiaries". We show that, while imperfect, the competition in the market results in major premium reductions and savings for the government in response to the introduction of an appropriate competitive bidding scheme.

companies submitted 14576 so-called "contracts", nationwide. This implies that, in an average county, 5.35 companies compete with each other as well as the FFS plan, for the patronage of the county's beneficiaries. (This number varies between 1 and 41.) We have focused on all counties with 2 or more contracts; this represents 2478 out of the total of 2727 counties in the United States, covering approximately 41 million beneficiaries, with a total 14327 contracts. A contract sometimes consists of multiple plans, with somewhat differentiated premia and benefits, in the same county. Enrollment data for each plan-county combination, are publicly available, from the CMS.[9] We treat each county as a separate market, even though many contracts cover multiple counties, in which case they are required to offer uniform premia and benefits throughout the various counties covered. An ideal model would therefore represent all of the US as a single oligopoly market, whose customer population needs to be segmented by county. The reason is that the beneficiaries in each, may elect coverage only from among a small subset of all available plans in the US. This approach would require a very large scale MMNL model, with the different insurance companies selling *multiple* insurance products; computing equilibria in such a large scale model is numerically very challenging and is therefore left to a future exploration.

Focusing on the market in a *given* county, a further simplification is obtained by assuming that each of the prevailing contracts, for this county, consists of a single "representative" plan, thus enabling the market's representation as an oligopoly in which each of the competing firms offers a *single* product. This representation is identical to that adopted in almost all of the recent literature, geared towards an estimation of the demand functions in the Medicare market, see in particular Hall (2007) and Nosal (2012). We have selected the most popular from among the various plans in a given contract-county combination, as the "representative plan"; indeed, this is hardly an approximation, as the vast majority of all beneficiaries enrolled in an MA plan choose the basic and most popular plan within the relevant contract. We now describe the specific model used to characterize the existing Medicare market, as well as the model that would apply after the implementation of the W-R plan. The existing market is characterized as having demand functions of the type specified by (3), with the net premium function $f(\cdot)$, specified as the following piecewise linear function:

$$f(x) = x^+ - 0.75x^-. \tag{27}$$

In other words, the response function satisfies Assumption 3, with $\alpha = 1 > 0.75 = \beta$. Thus, as in most other industrial organization studies, the basic structure of the demand functions in our study, is that of a (variant of the) MNL model. This is also the model choice in the above mentioned recent economics literature on the Medicare market. These papers specify the intercepts $\{a_i : i = 0, \dots, N\}$ as an affine

[9]The enrollment data for each MA plan in each county can be obtained from the Contract/Plan/State/County enrollment data. The number of beneficiaries enrolled in the FFS plan, in each county, are contained in FFS Data 2010.

function of various non-premium related plan characteristics: For example, Nosal (2012) considers, among others, indicator variables describing whether dental and routine eye care, or glasses and drugs are covered, along with the size of the network of health care providers and various co-payment fees. Our estimation procedure is, however, *independent* of the specific choice for the structure of the non-premium related terms in the utility functions, as associated with the different plan options. Only the *aggregate* value of the intercepts in (3) matters, adding a great deal of robustness to our estimation procedure.

Two fundamental differences arise between our model specification and that of the above prior literature: first, we specify the utility measure associated with a given plan as dependent on the *net* rather than the *gross* premium, since the beneficiary only pays (or collects) the net premium (, depending upon whether the net premium is positive or negative). Second, we recognize that the actual out-of-pocket cost may be given by a non-linear function of the net premium, as in (27).

We use firm 0 to represent the traditional Medicare (FFS) plan. Its gross premium is specified exogenously to reflect the actual cost per enrolled beneficiary. In other words, only firms $1, \ldots, N$, representing the Medicare Advantage plans, engage in competitive bidding.

The intercept values can be backed out *unambiguously* from the observed enrollment and premium data, subject to an upfront specification of the parameters $\{b_i : i = 1, \ldots, N\}$, the beneficiaries' price sensitivities with respect to the net premium (rebate) paid or received. While most of our theoretical results apply for general plan dependent price sensitivity coefficients, it stands to reason that, in this setting, these are *uniform* across the different plans, i.e., $b_i = b$ for all $i = 1, \ldots, N$; this is, also, the assumption in all of the existing literature on the Medicare Advantage market, as well as Assumption 3, under which we establish the existence of a pure Nash equilibrium when the subsidy is given by the second lowest bid. We therefore employ the estimate for this price sensitivity coefficient obtained in Nosal (2012), i.e., $b = 0.013$. Similar estimates for this coefficient were obtained in prior MNL models for the MA market, in particular Dowd et al. (2003), with an estimated value $b = 0.019$.

It is well known, and immediate from (3), that, without loss of generality, one of the intercept values $\{a_i\}$ may be normalized at an arbitrary value. We therefore set $a_0 = 0$. With p^* the observed price equilibrium, and $b_i = b$ replaced by an estimate $\hat{b}$, it follows from (3) that $a_i - \hat{b}f(p_i^* - C) + \hat{b}f(p_0 - C) = \ln(d_i) - \ln(d_0),\ i = 2, \ldots, N$ or

$$a_i = \ln(d_i) - \ln(d_0) + \hat{b}f(p_i^* - C) - \hat{b}f(p_0 - C).$$

The cost structure, encountered by the different insurance companies, is affine: In addition to fixed administrative costs, a firm's expenditures grow linearly with the number of enrolled beneficiaries (, after normalizing for differences in risk scores). Thus, the competition model analyzed in Section 3, applies to this setting and, by Theorem 1, has a pure Nash equilibrium, characterized by the system of equations and inequalities (13a)–(13b). Thus, assuming the premia selected by the MA plans represent an equilibrium under the current payment scheme, we use (14) and (15) to determine the cost rates $\{c_i : i = 1, \ldots, N\}$. Unless $p_i^* = C$, the exogenously specified capitation rate, every plan's cost rate is uniquely determined by (14). When $p_i^* = C$, (15) provides an interval in which the plan's cost rate c_i may fall. The width of this interval is determined by the rebate percentage; the closer the rebate percentage is to 100%, the smaller the interval is, approaching a single point value when a full rebate is provided for the cost saving. We have observed that no insurance plan ever specified a premium that was exactly equal to the exogenously specified capitation rate.

In conclusion, the observed premia and market shares provide enough information to determine all of the parameters in the competition model, modulo a *singular* degree of freedom. As mentioned, to remove the latter we have adopted Nosal (2012)'s estimate of the price sensitivity coefficient b. With all model parameters specified, we have computed the price equilibrium that arises when the capitation rate is set endogenously as the lowest and second lowest bid in a closed bid auction, similar to the Wyden-Ryan and Domenici-Rivlin plans. As shown, since the net premium function $f(\cdot)$ is piecewise linear with $\alpha \geq \beta > 0$, Theorems 2 and 4 show that a pure Nash equilibrium exists under either scheme, while the game is guaranteed to be (log-)supermodular under the lowest bid subsidy.

While the above describes our basic model for the Medicare Advantage market, we have also investigated a variant where the market is segmented into two customer classes: The *first* segment consists of those beneficiaries who, in the prior calendar year 2009, subscribed to the traditional FFS plan. The *second* segment consists of the remaining eligible Medicare participants, i.e., those who in 2009 enrolled in a private MA plan. We applied this segmentation to insert a search or inertia cost in the plans' utility measures, whenever an individual considers switching from the FFS plan to a private MA plan or vice versa. The presence of such inertia costs has been widely observed in the marketing literature, for example Dube et al. (2009); it is all the more likely to prevail in the MA market with an elderly population choosing among fairly complex alternatives. Indeed, Nosal (2012) has focused on estimating the magnitude of this inertia effect and has found it to be significant. The segmentation gives rise to a Mixed MultiNomial model (MMNL), of course with the additional complication of utility measures being dependent on *net* rather than *gross* premia via a non-linear response function $f(\cdot)$. In the next section, we explain, how the remaining parameters in the demand functions, as well as the marginal cost rates can be determined in this model, assuming, once again, that the observed price vector is an equilibrium under exogenously specified capitation rates. (Existence of a pure Nash equilibrium in this MMNL model fails to be guaranteed, see Section 8.) With these parameters specified, we, again, compute the equilibrium that arises when the capitation rate is determined as the lowest or second lowest premium bid in a sealed bid auction.

We have computed the equilibria by applying the tatônnement scheme specified in Section 3, starting from a randomly selected price vector in the feasible price space. While, as mentioned, we cannot guarantee, on theoretical grounds, that the equilibrium is unique, we have verified this numerically, by repeating the tatonnement scheme from many randomly selected starting points, observing convergence to a unique price vector, throughout.

7.1 Results

Table 1 reports on the average price results, across all 2478 counties, of the above described equilibrium calculations. We have computed a weighted average of the counties' results, with the counties' number of eligible participants as the weight factor. The first segment of the Table displays the results in the absence of search or inertia costs; the second (third) segment displays the same, assuming this cost value equals 2 and 4, respectively. (Nosal (2012) obtained an estimate of approximately 4 for this parameter, but reports on counterfactual studies based on various values between 0 and 4.) Each segment exhibits, first of all, the actual market results in 2010, which may also be interpreted as the equilibrium under the prevailing exogenous capitation rates. The second and third column in each table segment displays the same results, when the capitation rate is specified endogenously as the lowest and second lowest premium, respectively.

Table 1: Price equilibrium ($/month) for exogenous subsidy v.s. endogenous subsidy

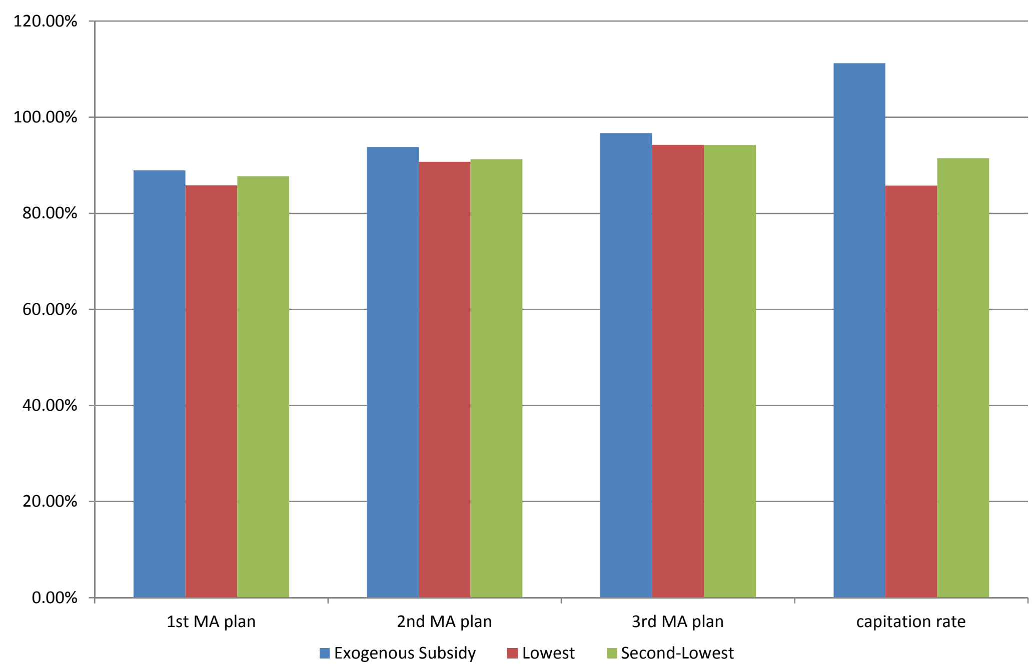
	No inertia cost			With inertia cost					
	$S_1 = 0$			$S_1 = 2$			$S_1 = 4$		
	Exog.	Lowest	Second	Exog.	Lowest	Second	Exog.	Lowest	Second
FFS	752.95	746.87	747.31	752.95	747.83	748.23	752.95	749.62	749.89
1st MA plan	669.71	641.03	655.78	669.71	642.98	660.47	669.71	643.50	662.04
2nd MA plan	706.26	677.69	682.19	706.26	673.45	674.04	706.26	678.31	677.38
3rd MA plan	727.98	703.97	704.02	727.98	705.85	706.06	727.98	707.51	706.37
capitation	837.69	640.69	683.37	837.69	641.53	684.19	837.69	644.09	686.65
ave. price	752.52	740.96	742.48	752.52	742.76	744.15	752.52	746.34	747.44

Note1: The column headings "Exog.", "Lowest" or "Second" are short hand for payment schemes in which the capitation rate is set exogenously, or determined as the lowest or the second-lowest bid, respectively.

Note2: The row headings "1st MA plan", "2nd MA plan", "3rd MA plan" are short hand for the MA plan with the lowest, second-lowest or third-lowest bid, respectively.

Focusing on the case without search/inertia costs, we observe that the average capitation rate is reduced from $838 to $641 or $683, depending upon whether it is specified as the lowest or second lowest bid. Since both the Wyden-Ryan and the Domenici-Rivlin plans adopt the latter scheme, we confine our summary conclusions to the latter. The reduction of the capitation rates amounts to *a cost saving for the government of no less than 18.5%.*[10] (The average capitation rate exceeded the average cost in FFS plans by close to 10%.) Under the existing capitation scheme, the average of the second lowest premia amounted to \$706, a 15.8% reduction compared with the average prevailing capitation rate. This demonstrates that the standard way of estimating the cost savings results in a significant underestimation of the savings potential: As may be expected, insurance companies react to the new subsidy scheme by bidding more aggressively and reducing their premia. Indeed, as shown in Figure 1, the average second lowest premium goes down from 94% to 91% of the average FFS cost value. For those beneficiaries who choose to stay with, or newly enroll in a traditional FFS plan, the average of their out of pocket costs would amount to \$64 per month or \$768 per year, similar to earlier estimates in Song et al.(2012). Similarly, the average out of pocket costs among all Medicare beneficiaries would be \$64 per month or \$768 per year, see Table 3. These results are very similar, when incorporating an assumed search/inertia cost of 2 or 4, the prevalence of an inertia cost component increases the premia, and hence the capitation rate, slightly.

Figure 1: Price as percentage of FFS cost (without inertia cost)



In addition to displaying the average cost value of the FFS plans and the average capitation rates under the three subsidy schemes, Table 1 exhibits the average premium of the lowest, second lowest and third lowest plans, as well as the overall average premium, again under each of the three subsidy

[10]The percentage saved is double that estimated by Feldman et al. (2012).

schemes. In addition to the absolute premium values, it is also of interest to display the various premium values as a percentage of the prevailing FFS cost value. Figure 1 exhibits the weighted average value of these percentages, for the lowest, second lowest and third lowest bid plan, as well as the capitation rate.

Table 2: Market share (%) for exogenous subsidy v.s. endogenous subsidy

	No inertia cost			With inertia cost					
	$S_1 = 0$			$S_1 = 2$			$S_1 = 4$		
	Exog.	Lowest	Second	Exog.	Lowest	Second	Exog.	Lowest	Second
FFS	77.91	72.89	73.26	77.91	74.27	74.56	77.91	76.53	76.66
1st MA plan	2.57	3.77	3.27	2.57	3.53	3.01	2.57	2.76	2.21
2nd MA plan	2.33	3.23	3.09	2.33	2.68	2.59	2.33	2.43	2.54
3rd MA plan	2.63	3.20	3.25	2.63	3.21	3.28	2.63	2.19	2.19
4th MA plan	2.25	3.19	3.20	2.25	2.87	2.93	2.25	2.95	3.00
5th MA plan	2.34	2.86	2.93	2.34	2.71	2.71	2.34	2.44	2.37

Figure 2: Market share for FFS and MA plans (without inertia cost)

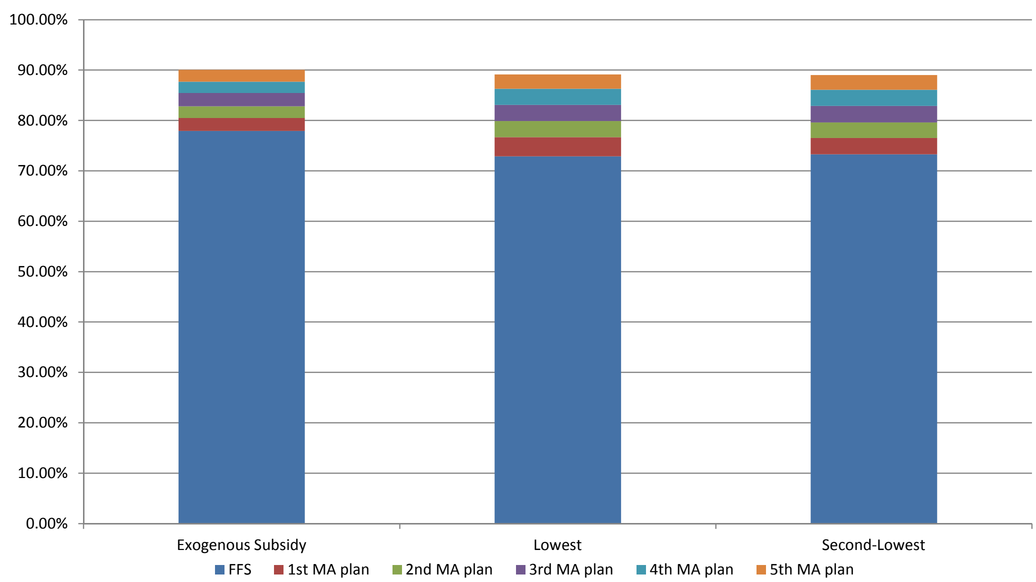


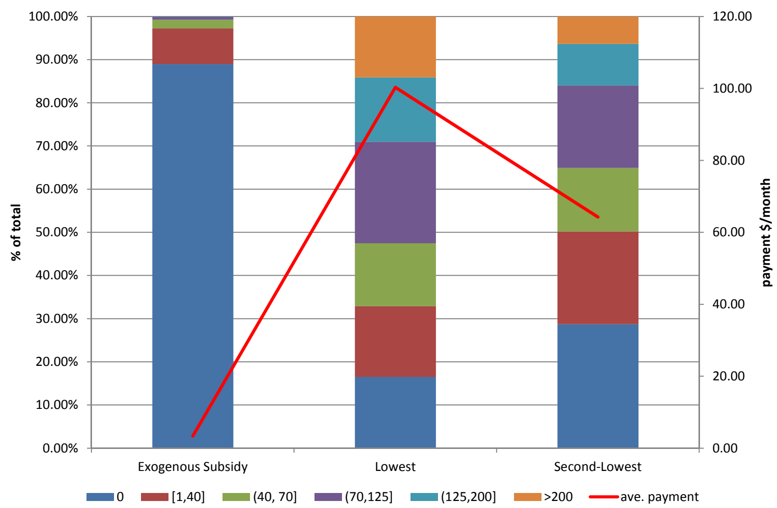
Table 2 shows that the intensified competition among the private insurers, under an endogenously specified capitation rate, translates into a reduction of the market share of the traditional Medicare (FFS) plans from 78% to 73%, but only in the absence of switching costs; under an inertia cost value of 4, the average FFS market share drops by one percentage point only. The lowest priced plans increase their market share by some 27%, from 2.57% to 3.27%, but each individual MA plan continues to have a relatively small market share. Thus, market concentration remains low, boding well for the continuation of a healthy competitive environment. The results also indicate that beneficiaries consider many non-price related attributes in their plan choices. The same results are shown graphically in Figure 2.

Table 3: Distribution of out-of-pocket payment (%) for FFS and MA enrollees

out-of-pocket payment ($/month)	No inertia cost $S_1 = 0$			With inertia cost $S_1 = 2$			With inertia cost $S_1 = 4$		
	Exog.	Lowest	Second	Exog.	Lowest	Second	Exog.	Lowest	Second
<= 0	88.96	16.50	28.73*	88.96	16.63	28.56	88.96	17.35	28.91
$(0, 40]$	8.23	16.36	21.48	8.23	16.14	21.66	8.23	16.04	21.22
$(40, 70]$	2.03	14.62	14.68	2.03	14.45	14.34	2.03	14.15	14.47
$(70, 125]$	0.68	23.50	19.12	0.68	23.10	19.00	0.68	22.51	17.55
$(125, 200]$	0.07	14.90	9.64	0.07	15.32	9.79	0.07	14.93	10.69
> 200	0.03	14.12	6.35	0.03	14.36	6.65	0.03	15.02	7.16
Ave. ($/month)	3.37	100.27	64.23	3.37	101.23	65.25	3.37	102.25	66.80

Note*: This number (28.73%) is much larger than the sum of market shares for the lowest MA plan (3.27%) and second-lowest MA plan (3.09%). The difference comes from the beneficiaries enrolled in FFS plan, whose price is less than the subsidy.

Figure 3: Out-of-pocket payment for FFS and MA enrollees (without inertia cost)



Finally, Table 3 shows what percentage of all beneficiaries incurs an out of pocket expense in five specific cost buckets. Under the second lowest bid scheme, a majority would continue to pay less than $40 per month, while 97% do under the current exogenous subsidy scheme. Twenty-nine percent of beneficiaries would continue to pay nothing or get a rebate. Figure 3 displays the cumulative percentage of beneficiaries who pay less than a given amount. For example, about 50% (or 65%) of beneficiaries pay less than $40 (or $75) per month under the payment scheme under which the capitation rate is determined by the second-lowest bid.

8 Multiple Customer Classes: Switching Cost

The MNL model assumes that the utility measures attributed to the different products, while random, are identically distributed. In many applications, this assumption is overly restrictive. Instead, the market needs to be partitioned into a set of $K \geq 2$ segments, each with its own population size h_k and its own specification of the random utility measures in (1) and (2), as follows:

$$u_{ijk} = a_{ik} - b_i f(p_i - g(p)) + \epsilon_{ijk}; \quad i = 1, 2, \ldots, N; \; k = 1, 2, \ldots, K; \; j = 1, \ldots \tag{28a}$$

$$u_{0jk} = a_{0k} - b_0 f(p_0 - g(p)) + \epsilon_{0jk}; \quad k = 1, 2, \ldots, K; \; j = 1, \ldots \tag{28b}$$

Here u_{ijk} denotes the utility attributed by the j-th potential customer in segment k to product i ($k = 1, 2, \ldots, K; \; i = 0, 1, \ldots, N$). $\{\epsilon_{ijk}\}$, again, represent a set of i.i.d. random variables with the Gumbel distribution. The MMNL model, without subsidization, i.e., with $g(p) \equiv 0$, has been employed in countless studies in industrial organization, marketing and operations management, among many other areas, see e.g. Caplin and Nalebuff (1991) and Allon et al. (2012). Even in the base model, without subsidization [$g(p) = 0$], a pure Nash equilibrium may fail to exist, see Allon et al. (2012). The latter have developed a set of sufficient conditions for the existence of a pure Nash equilibrium or its uniqueness. It remains an open question how these conditions can be generalized for our model with a general subsidy structure $g(\cdot)$, specified as one of the cases of Sections 3–5.

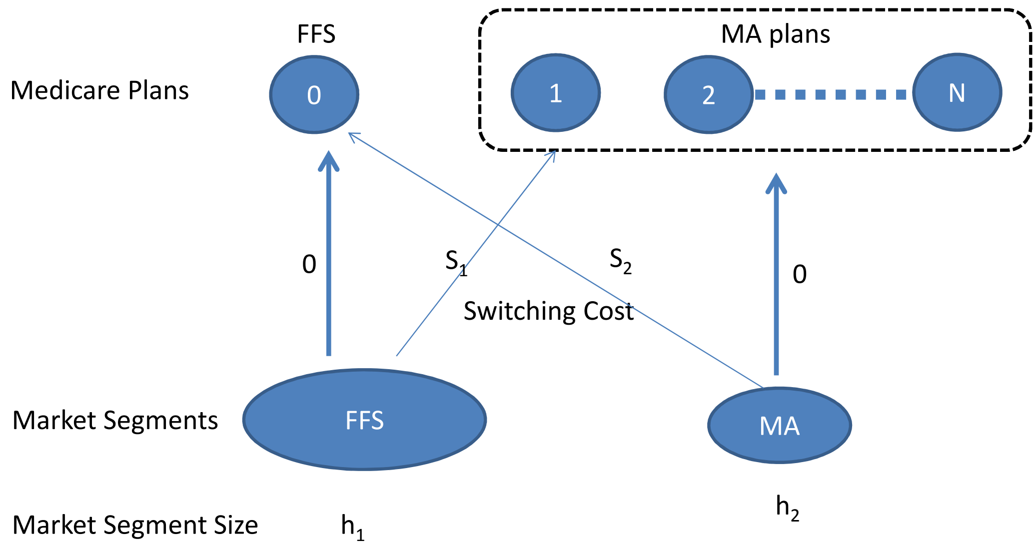
As mentioned in the previous section, we have applied this model to the Medicare market to incorporate the impact of switching costs. To this end, we partition the population of all beneficiaries in a given county into two segments: segment $k = 1$ represents beneficiaries enrolled in the traditional Medicare program, in 2009, the year preceding the study year; segment $k = 2$ represents the remainder of the market. A switching cost S_1 is incurred when a beneficiary switches from the traditional Medicare program to a MA plan, and S_2 when the switch is in the opposite direction, see Figure 4 for a pictorial representation of the market segmentation and switching costs.

The general utility measures in (28), thus, take the form,

$$u_{ij1} \;=\; a_i - S_1 \mathbb{I}(i \neq 0) - b_i \cdot f\left(p_i - g(\mathbf{p})\right) + \epsilon_{ij1}; \quad i = 0, 1, 2, \ldots, N; \; j = 1, \ldots \tag{29}$$

$$u_{ij2} \;=\; a_i - S_2 \mathbb{I}(i = 0) - b_i \cdot f\left(p_i - g(\mathbf{p})\right) + \epsilon_{ij2}, ; \quad i = 0, 1, 2, \ldots, N; \; j = 1, \ldots \tag{30}$$

Figure 4: Price competition with switching cost



For each MA plan $i = 1, \ldots, N$, its market share in segment $k = 1, 2$ is given by

$$\begin{aligned} d_{i1}(\mathbf{p}) &= \frac{\exp\left(a_i - b_i \cdot f\left(p_i - g(\mathbf{p})\right) - S_1\right)}{\exp\left(a_0 - b_0 \cdot f\left(p_0 - g(\mathbf{p})\right)\right) + \sum_{k=1}^{N} \exp\left(a_k - b_k \cdot f\left(p_k - g(\mathbf{p})\right) - S_1\right)}, \\ &= \frac{\exp\left(a_i - b_i \cdot f\left(p_i - g(\mathbf{p})\right)\right)}{\exp\left(a_0 - b_0 \cdot f\left(p_0 - g(\mathbf{p})\right) + S_1\right) + \sum_{k=1}^{N} \exp\left(a_k - b_k \cdot f\left(p_k - g(\mathbf{p})\right)\right)}, \quad (31) \\ d_{i2}(\mathbf{p}) &= \frac{\exp\left(a_i - b_i \cdot f\left(p_i - g(\mathbf{p})\right)\right)}{\exp\left(a_0 - b_0 \cdot f\left(p_0 - g(\mathbf{p})\right) - S_2\right) + \sum_{k=1}^{N} \exp\left(a_k - b_k \cdot f\left(p_k - g(\mathbf{p})\right)\right)}, \quad (32) \end{aligned}$$

Similarly, the market share of the FFS plan 0 in each segment is given by

$$\begin{aligned} d_{01}(\mathbf{p}) &= \frac{\exp\left(a_0 - b_0 \cdot f\left(p_0 - g(\mathbf{p})\right)\right)}{\exp\left(a_0 - b_0 \cdot f\left(p_0 - g(\mathbf{p})\right)\right) + \sum_{k=1}^{N} \exp\left(a_k - b_k \cdot f\left(p_k - g(\mathbf{p})\right) - S_1\right)}, \\ &= \frac{\exp\left(a_0 - b_0 \cdot f\left(p_0 - g(\mathbf{p})\right) + S_1\right)}{\exp\left(a_0 - b_0 \cdot f\left(p_0 - g(\mathbf{p})\right) + S_1\right) + \sum_{k=1}^{N} \exp\left(a_k - b_k \cdot f\left(p_k - g(\mathbf{p})\right)\right)}, \quad (33) \\ d_{02}(\mathbf{p}) &= \frac{\exp\left(a_0 - b_0 \cdot f\left(p_0 - g(\mathbf{p})\right) - S_2\right)}{\exp\left(a_0 - b_0 \cdot f\left(p_0 - g(\mathbf{p})\right) - S_2\right) + \sum_{k=1}^{N} \exp\left(a_k - b_k \cdot f\left(p_k - g(\mathbf{p})\right)\right)}, \quad (34) \end{aligned}$$

The total sales volume of plan i is given by $d_i(\mathbf{p}) = h_1 d_{i1}(\mathbf{p}) + h_2 d_{i2}(\mathbf{p})$, and its profit function by $\pi_i(p_i, p_{-i}) = (p_i - c_i) d_i(\mathbf{p})$.

Without loss of generality, we normalize $S_2 = 0$, otherwise, if $S_2 \neq 0$, (31)–(34) may be normalized via the following transformations

$$\bar{a}_1 = a_1 - S_2 \quad \text{and} \quad \bar{S}_1 = S_1 + S_2.$$

We analyze the model for a given switching cost value S_1 and a given estimate $\hat{b}$ for the price sensitivity coefficient $b_i = b$. In Appendix B, we show how the remaining model parameters, i.e., the intercepts $\{a_i\}$ and the marginal cost rates $\{c_i\}$ can be determined.

9 Conclusions and Extensions to Prospect-theoretical Models

In this paper, we have analyzed a general price competition model for settings where the (random) utility attributed to a given product depends on its price, via a non-linear response function $f(\cdot)$ of the *net* price, i.e., the *differential* between the nominal price and a reference value $g(p)$, see equations (1) and (2). Our model is motivated by settings where the product or service is subsidized by a third party and $g(p)$ represents the subsidy level. We have characterized the equilibrium behavior of the price competition model under various specifications of the reference subsidy level $g(p)$, and derived comparison results across different subsidy structures. We have applied our model to the Medicare market to estimate what impact, specification of the subsidy or capitation rate as the lowest or second lowest price, would have on the premia, government expenditures and out-of-pocket costs for individual beneficiaries. Such competitive bidding schemes have been proposed in the Wyden-Ryan and Domenici-Rivlin plans.

We have employed a non-standard representation of the "no-purchase option", as in (2). As discussed, the specification applies to the Medicare market, but not to applications where consumers may opt not to avail themselves of *any* of the product alternatives. In this case, the utility measure $u_{0j} = a_j + \epsilon_{0j}$, i.e., the second term in (2) is to be omitted, or $b_0 = 0$. It is easily verified that all of the results in Sections 3 and 4 for an exogenous subsidy or a subsidy specified by the lowest bid, continue to apply. The same therefore applies to the comparison results in Section 6. However, it is no longer clear when a pure Nash equilibrium can be guaranteed when the subsidy is given by the n-th lowest price for $n \geq 2$, see Section 5. The problem arises because Lemma 2 no longer applies.

Prospect-theoretical models in behavioral economics represent other settings, where a product's utility depends on the difference between the product's price and a *reference value* $g(p)$, via a non-linear response function $f(\cdot)$, as in (1) and (2), see Kahneman and Tversky (1979) and Tversky and Kahneman (1991). Such a MNL-based model was first proposed by Hardie et al. (1993) and applied to the market of orange juice brands. The authors considered a piecewise linear response function of the type specified in Assumption 3 and several specifications for the reference value $g(p)$, in particular, (i) $g(p) = p_1$, the price of the product with the dominant market share, or (ii) the last brand chosen. The results of Section 4, with $g(p) = p_{(1)}$, easily carry over to the first specification: $g(p) = p_1$. (Incidentally, the *lowest* price $g(p) = p_{(1)}$ is another reasonable choice for the reference value.) If a consumer's reference price is given by the price of the last purchased brand, a MMNL model, as in Section 8, is needed, with a similar segmentation of the market as in the case of switching costs.

To our knowledge, the results in this paper, provide the first characterizations of the equilibrium behavior of a price competition model based on a MNL consumer choice model with a prospect theoretical specification of price dependency. Heidhues and Kőszegi (2008) recently analyzed a Hotelling type price competition model in which the utility measure of each product depends on its price via a piecewise linear function of the *difference* between the price and an exogenous reference value. (This is similar to the structure considered in Section 3.) See Ellison (2006) and Spiegler (2011) for discussions of the importance of price competition models in which consumers are assumed to be loss averse relative to a reference value.

References

Aaron, H., A. Frakt. 2012. Why now is not the time for premium support. *New England J. Med.* **366** 877-879.

Allon, G., A. Federgruen, M. Pierson. 2012. Price competition under multinomial logit demand functions with random coefficients. *Management Sci.* Forthcoming.

Anderson, S., A. Palma, J. Thisse. 2001. Discrete choice theory of product differentiation. MIT Press, Cambridge MA.

Antos, J. 2012. The Wyden-Ryan proposal–A foundation for realistic Medicare reform. *New England J. Med.* **366** 879-881.

Antos, J., L. Bilheimer. 1999. Medicare reform: Obstacles and options. *Amer. Econ. Rev.* **89** 217-221.

Bernstein, F, A. Federgruen. 2004. A general equilibrium model for industries with price and service competition. *Oper. Res.* **52** 868-886.

Cabral, L., M. Villas-Boas. 2005. Bertrand supertraps. *Management Sci.* **51** 599-613.

Caplin, A., B. Nalebuff. 1991. Aggregation and imperfect competition: On the existence of equilibrium. *Econometrica* **59** 25 -59.

Dowd, B., R. Feldman, R. Coulam. 2003. The effect of health plan characteristics on medicare+choice enrollment. *Health Services Research* **38** 113-135.

Dube, J., G. Hitsch, P. Rossi. 2009. Do switching costs make markets less competitive? *J. Marketing Res.* **46** 435-445.

Ellison, G. 2006. Bounded rationality in industrial organization, in *Advances in Economics and Econometrics: Theory and Applications*, eds Blundell R., Newey W., and Persson T. (Cambridge, UK: Cambridge University Press).

Feldman R., R. Coulam, B. Dowd. 2012. Competitive bidding can help solve Medicares fiscal crisis. http://www.aei.org/outlook/health/healthcare-reform/competitive-bidding-can-help-solve-medicares-fiscal-crisis/. *American Enterprise Institute.* February, 2012.

Gallego, G., T. Huh, W. Kang, R. Phillips. 2006. Price competition with the attraction demand model: Existence of unique equilibrium and its stability. *Manufacturing & Service Operations Management* **8** 359-375.

Hall, A. 2007. The value of Medicare managed care plans and their prescription drug benefits. *Finance and Economics Discussion Series*, Federal Reserve Board of Governers, Washington, D.C.

Hardie, B., E. Johnson, P. Fader. 1993. Modeling loss aversion and reference effects on brand choice. *Marketing Sci.* **12** 378 - 394.

Heidhues, P., B. Kőszegi. 2008. Competition and price variation when consumers are loss averse. *Amer. Econ. Rev.* **98** 1245 - 1268.

House Budget Committee. Wyden and Ryan advance bipartisan plan to strengthen Medicare and expand health care choices for all [news release]. http://www.budget.house .gov/bipartisanhealthoptions. Published December 15, 2011.

House Budget Committee. Fiscal Year 2013 Budget Resolution: The Path to Prosperity a Blueprint for American Renewal. http://budget.house.gov/UploadedFiles /Pathtoprosperity2013.pdf. Published March, 2012.

Kahneman, D., A. Tversky. 1979. Prospect theory: An analysis of decision under risk. *Econometrica* **47** 263 - 291.

Lustig, J. 2008. The welfare effects of adverse selection in privatized medicare. Working Paper, Boston University.

Nosal, K. 2012. Estimating switching costs for medicare advantage plans. Working paper, University of Mannheim, Germany.

Song, Z., D. Cutler, M. Chernew. 2012a. Potential Consequences of Reforming Medicare Into a Competitive Bidding System. *The J. of the Amer. Med. Association* **308** 596-560.

Song, Z., M. Landrum, M. Chernew. 2012b. Competitive bidding in Medicare: Who benefits from competition? *The Amer. J. of Managed Care* **18** 546-a555.

Spiegler, R. 2011. *Bounded Rationality and Industrial Organization.* Oxford University Press, New York.

Topkis, D. 1998. *Supermodularity and Complementarity.* Princeton University Press, Princeton, NJ.

Tversky A., D. Kahneman. 1991. Loss aversion in riskless choice: A reference-dependent model. *The Quarterly J. of Econ.* **106** 1039 - 1061.

Vives, X. 2001. *Oligopoly pricing: Old ideas and new tools.* The MIT Press.

Appendix A: Proofs

Proof of Proposition 1. Analogous to the proof of Theorem 1, we first establish the Proposition when f is piecewise linear; we then extend the proof for a general convex and increasing function f, by approximating this function by a sequence of piecewise linear functions,

Part 1: Assume, the function $f(\cdot)$ is piecewise linear, as specified in Part 1 of the proof of Theorem 1, with $0 \le \beta_1 \le \cdots \le \beta_{M+1}$ in view of convexity. For any firm $i = 1, 2, \ldots, N$, we show that the logarithms of the profit function, $\log(\pi_i)$, is quasi-concave. In what follows, we show this property by considering two cases: (1) $\Delta p_i \notin \mathbb{P}$; (2) $\Delta p_i = x_m$ for some $m = 1, 2, \ldots, M$.

(i) $\underline{\Delta p_i \notin \mathbb{P}}$, say, $\Delta p_i \in (x_{m-1}, x_m)$ for some $m = 1, 2, \ldots, M+1$. Note that, the function $f(\Delta p_i)$ is differentiable and, by (7),

$$\frac{\partial \log(\pi_i)}{\partial p_i} = \frac{1}{p_i - c_i} + \frac{\partial d_i}{\partial p_i}/d_i = \frac{1}{p_i - c_i} - b_i f'\left(\Delta p_i\right)(1 - d_i).$$

Since $f'\left(\Delta p_i\right) = \beta_m$ when $\Delta p_i \in (x_{m-1}, x_m)$ and d_i is non-increasing in p_i, by (7), it follows that $\frac{\partial \log(\pi_i)}{\partial p_i}$ is non-increasing in p_i.

(ii) $\underline{\Delta p_i = x_m \text{ for some } m = 1, 2, \ldots, M.}$ We show that $\frac{\partial_+ \log \pi_i(C+x_m, p_{-i})}{\partial p_i} \le \frac{\partial_- \log \pi_i(C+x_m, p_{-i})}{\partial p_i}$, which is equivalent to $\frac{\partial_+ \log d_i(C+x_m, p_{-i})}{\partial p_i} \le \frac{\partial_- \log d_i(C+x_m, p_{-i})}{\partial p_i}$, or, by (7),

$$\begin{aligned}
\frac{\partial_+ \log d_i(C + x_m, p_{-i})}{\partial p_i} &\equiv \lim_{p_i \searrow C + x_m} \frac{\partial \log d_i(p_i, p_{-i})}{\partial p_i} = -b_i f'_+(x_m)\left(1 - d_i(C + x_m, p_{-i})\right) \\
&= -b_i \beta_{m+1}\left(1 - d_i(C + x_m, p_{-i})\right) \\
&\le -b_i \beta_m \left(1 - d_i(C + x_m, p_{-i})\right) = -b_i f'_-(x_m)\left(1 - d_i(C + x_m, p_{-i})\right) \\
&= \lim_{p_i \nearrow C + x_m} \frac{\partial \log d_i(p_i, p_{-i})}{\partial p_i} \equiv \frac{\partial_- \log d_i(C + x_m, p_{-i})}{\partial p_i}.
\end{aligned}$$

Here the inequality follows from the fact that $0 \le \beta_m \le \beta_{m+1}$ and $1 - d_i(C + x_m, p_{-i}) \ge 0$.

We have thus shown that $\log \pi_i(p_i, p_{-i})$ is quasi-concave in $p_i \geq c_i$ for any p_{-i} when the response function f is increasing and convex piecewise linear.

Part 2: Assume now that f is a general increasing and convex function. It is well-known that there exists a sequence of increasing and convex piece-wise linear functions $\{f^{(k)}(\cdot)\}$ such that $\lim_{k\to\infty} f^{(k)}(x) = f(x)$ for any x. We prove the result by contradiction. Assume that there exists some firm $i = 1, 2, \ldots, N$ and price vector p_{-i}^0 such that $\log(\pi_i(\cdot, p_{-i}^0))$ is not quasi-concave, that is,

$$\log \pi_i\left(\lambda_0 p_i^1 + (1-\lambda_0)p_i^2, p_{-i}^0\right) < \min\left\{\log \pi_i\left(p_i^1, p_{-i}^0\right), \log \pi_i\left(p_i^2, p_{-i}^0\right)\right\} \quad \text{for some } p_i^1, p_i^2 \text{ and } \lambda_0 \in (0,1) \tag{A-1}$$

Let $\pi_i^{(k)}$ denote firm i's profit function associated with the function $f^{(k)}$. By a simple continuity argument, we have, for any price vector $\hat{p}$, $\lim_{k\longrightarrow\infty} \log \pi_i^{(k)}(\hat{p}) = \log \pi_i(\hat{p})$. Hence, by (A-1), there exists $k_0 \geq 1$ such that

$$\log \pi_i^{(k)}\left(\lambda_0 p_i^1 + (1-\lambda_0)p_i^2, p_{-i}^0\right) < \min\left\{\log \pi_i^{(k)}\left(p_i^1, p_{-i}^0\right),\ \log \pi_i^{(k)}\left(p_i^2, p_{-i}^0\right)\right\} \quad \text{for any } k \geq k_0.$$

This contradicts the quasi-concavity of $\pi_i^{(k)}$ in part 1. □

Proof of Theorem 2. It suffices to show that the function $\pi_i(p_i, p_{-i})$ is quasi-concave in its own price for each firm $i = 1, 2, \ldots, N$. By (4), for firm $i = 1, 2, \ldots, N$, one has

$$\frac{\partial \pi_i}{\partial p_i} = d_i + (p_i - c_i)\frac{\partial d_i}{\partial p_i} = \begin{cases} d_i\left(1 - (p_i - c_i)\sum_{k\neq i} d_k b_k f'(p_k - p_i)\right), & p_i < p_{(1)}^{-i} \\ d_i\left(1 - (p_i - c_i) b_i f'\left(p_i - p_{(1)}^{-i}\right)(1 - d_i)\right), & p_i > p_{(1)}^{-i} \end{cases} \tag{A-2}$$

Also,

$$\frac{\partial_+ \pi_i(p_i, p_{-i})}{\partial p_i} = d_i + (p_i - c_i)\frac{\partial_+ d_i(p_i, p_{-i})}{\partial p_i}, \tag{A-3}$$

$$\frac{\partial_- \pi_i(p_i, p_{-i})}{\partial p_i} = d_i + (p_i - c_i)\frac{\partial_- d_i(p_i, p_{-i})}{\partial p_i}. \tag{A-4}$$

Thus, one has

$$\frac{\partial \pi_i}{\partial p_i} \geq 0 \quad \Leftrightarrow \quad \begin{cases} (p_i - c_i)\sum_{k\neq i} d_k b_k f'(p_k - p_i) \leq 1, & p_i < p_{(1)}^{-i} \\ (p_i - c_i) b_i f'\left(p_i - p_{(1)}^{-i}\right)(1 - d_i) \leq 1, & p_i > p_{(1)}^{-i} \end{cases} \tag{A-5}$$

Next, we will show quasi-concavity of $\pi_i(p_i, p_{-i})$ by showing that $\frac{\partial \pi_i}{\partial p_i} \geq 0$ if and only if $p_i \leq \hat{p}_i(p_{-i})$ for some threshold value $\hat{p}_i(p_{-i})$. In what follows, we show this property by considering three cases: (1) $p_i > p_{(1)}^{-i}$; (2) $p_i < p_{(1)}^{-i}$; and (3) $p_i = p_{(1)}^{-i}$.

(1) If $\underline{p_i > p_{(1)}^{-i}}$, $f'\left(p_i - p_{(1)}^{-i}\right)$ increases in p_i since f is convex on $[0, +\infty)$, $1 - d_i$ is increasing in p_i since $\frac{\partial d_i}{\partial p_i} \leq 0$, therefore, $(p_i - c_i) b_i f'\left(p_i - p_{(1)}^{-i}\right)(1 - d_i)$ is increasing in p_i. Thus, once the function

reaches a value ≥ 1, i.e., once $\frac{\partial \pi_i(p_i,p_{-i})}{\partial p_i}$ is decreasing in a certain point, the same applies to any larger price value. This implies that the function $\pi_i(p_i, p_{-i})$ is quasi-concave on the interval $(p_{(1)}^{-i}, \infty)$.

(2) If $p_i < p_{(1)}^{-i}$, the cross price derivatives of the demand functions are given by

$$\frac{\partial d_i}{\partial p_j} = b_j f'(p_j - p_i)\, d_i d_j, \text{ for any } j \neq i. \tag{A-6}$$

Let

$$H_i(p_i) = (p_i - c_i)\sum_{k\neq i} d_k b_k f'(p_k - p_i) - 1 = (p_i - c_i)\sum_{k\neq i}\frac{\partial d_i}{\partial p_k}/d_i - 1 = (p_i - c_i)\sum_{k\neq i}\frac{\partial \ln(d_i)}{\partial p_k} - 1.$$

Then, $\frac{\partial \pi_i}{\partial p_i} = -d_i H_i(p_i)$ by (A-2). Obviously, $H_i(c_i) = -1 < 0$ and we show quasi-concavity of π_i by considering two cases: (2a) $H_i(p_{(1)}^{-i}-) \leq 0$; (2b) $H_i(p_{(1)}^{-i}-) > 0$.

Case (2a) $H_i(p_{(1)}^{-i}-) \leq 0$: then $H_i(p_i) \leq 0$ for all $p_i \in [c_i, p_{(1)}^{-i})$ since $H_i(c_i) < 0$ and $H_i(p_i)$ is quasi-convex by condition (M). Thus, $\frac{\partial \pi_i}{\partial p_i} = -d_i H_i(p_i) \geq 0$ for all $p_i \in [c_i, p_{(1)}^{-i})$.

Case (2b) $H_i(p_{(1)}^{-i}-) > 0$: by the quasi-convexity of $H_i(p_i)$ on $[c_i, p_{(1)}^{-i})$ and $H_i(c_i) < 0$, there exists $\hat{p}_i(p_{-i}) \in (c_i, p_{(1)}^{-i})$ such that $H_i(p_i) \leq 0$, thus, $\frac{\partial \pi_i}{\partial p_i} = -d_i H_i(p_i) \geq 0$, if and only if $p_i \leq \hat{p}_i(p_{-i})$.

Therefore, we have shown π_i is quasi-concave in p_i on the interval $[c_i, p_{(1)}^{-i})$ as well.

Thus, to complete the proof that the function $\pi_i(\cdot, p_{-i})$ is quasi-concave on the complete interval $[c_i, \infty)$, it suffices to show that $\frac{\partial_+ \pi_i}{\partial p_i}(p_{(1)}^{-i}, p_{-i}) \leq \frac{\partial_- \pi_i}{\partial p_i}(p_{(1)}^{-i}, p_{-i})$. By (A-3) and (A-4), it suffices to show $\frac{\partial_+ d_i(p_{(1)}^{-i}, p_{-i})}{\partial p_i} \leq \frac{\partial_- d_i(p_{(1)}^{-i}, p_{-i})}{\partial p_i}$, or

$$\begin{aligned}
\lim_{p_i \searrow p_{(1)}^{-i}} \frac{\partial d_i(p_i, p_{-i})}{\partial p_i} \equiv \frac{\partial_+ d_i(p_{(1)}^{-i}, p_{-i})}{\partial p_i} &\leq \frac{\partial_- d_i(p_{(1)}^{-i}, p_{-i})}{\partial p_i} = \lim_{p_i \nearrow p_{(1)}^{-i}} \frac{\partial d_i(p_i, p_{-i})}{\partial p_i} \\
&= \lim_{p_i \nearrow p_{(1)}^{-i}} \left\{ -d_i \sum_{k\neq i} d_k b_k f'(p_k - p_i) \right\} = \lim_{p_i \nearrow p_{(1)}^{-i}} -\sum_{k\neq i}\frac{\partial d_i}{\partial p_k},
\end{aligned}$$

which is equivalent to (D). □

Proof of Lemma 1.

(a) <u>Condition (D)</u>: For any $i = 1, 2, \ldots, N$, and any given p_{-i}, we have

$$\begin{aligned}
& \lim_{p_i \nearrow p_{(1)}^{-i}} \sum_{j \neq i} \frac{\partial d_i}{\partial p_j}(p_i, p_{-i}) \leq \lim_{p_i \searrow p_{(1)}^{-i}} |\frac{\partial d_i}{\partial p_i}(p_i, p_{-i})| \\
\iff & \lim_{p_i \nearrow p_{(1)}^{-i}} \left\{ \left(\sum_{j \neq i} d_i d_j b_j f'\left(p_j - p_i\right) \right) \right\} \leq \lim_{p_i \searrow p_{(1)}^{-i}} \left\{ b_i f' \left(p_i - p_{(1)}^{-i} \right) (1 - d_i)\, d_i \right\} \\
\overset{(a1)}{\iff} & d_i(p_{(1)}^{-i}, p_{-i}) \cdot \left(\sum_{j \neq i} d_j(p_{(1)}^{-i}, p_{-i}) b\alpha \right) \leq d_i(p_{(1)}^{-i}, p_{-i}) b\alpha \left(1 - d_i(p_{(1)}^{-i}, p_{-i}) \right) \\
\iff & \sum_{j \neq i} d_j(p_{(1)}^{-i}, p_{-i}) \leq 1 - d_i(p_{(1)}^{-i}, p_{-i}),
\end{aligned}$$

which is trivially true. (a1) holds because $b_j = b$, $f'\left(p_j - p_i\right) = \alpha$ for any $p_i < p_{(1)}^{-i} = \min\{p_k, k \neq i\}$ and for any $j \neq i$.

<u>Condition (M)</u>: For any $i = 1, 2, \ldots, N$, any given p_{-i}, and any $p_i \in [c_i, p_{(1)}^{-i})$, let

$$H_i(p_i) = (p_i - c_i) \sum_{j \neq i} \frac{\partial \ln(d_i)}{\partial p_j} = (p_i - c_i) \sum_{j \neq i} d_j b_j f'\left(p_j - p_i\right) = (p_i - c_i) \sum_{j \neq i} d_j b\alpha.$$

Taking derivatives with respect to p_i, we get

$$\begin{aligned}
H_i'(p_i) &= \left[\sum_{j \neq i} d_j + (p_i - c_i) \sum_{j \neq i} \frac{\partial d_j}{\partial p_i} \right] b\alpha \\
&\overset{(a2)}{=} \left[\sum_{j \neq i} d_j + (p_i - c_i) \sum_{j \neq i} d_j \left(b\alpha - \sum_{k \neq i} d_k b\alpha \right) \right] b\alpha \\
&= \left[1 + b\alpha(p_i - c_i) \left(1 - \sum_{j \neq i} d_j \right) \right] \cdot \left(\sum_{j \neq i} d_j \right) b\alpha \\
&\geq 0,
\end{aligned}$$

where (a2) holds because $d_j = \frac{\exp(a_j - bf(p_j - p_i))}{\exp(a_i) + \sum_{k \neq i} \exp(a_k - bf(p_k - p_i))} = \frac{\exp(a_j - b\alpha(p_j - p_i))}{\exp(a_i) + \sum_{k \neq i} \exp(a_k - b\alpha(p_k - p_i))}$ for any $p_i < p_{(1)}^{-i}$ and any $j \neq i$ by (16). Taking derivative w.r.t p_i yields $\frac{\partial d_j}{\partial p_i} = d_j \left(b\alpha - \sum_{k \neq i} d_k b\alpha \right)$ for any $j \neq i$ and $p_i < p_{(1)}^{-i}$. Thus, $(p_i - c_i) \sum_{j \neq i} \frac{\partial \ln(d_i)}{\partial p_j}$ is non-decreasing, which is quasi-convex, in p_i on the interval $[c_i, p_{(1)}^{-i})$.

(b) We distinguish between two cases: (i) <u>$p_i \leq p_{(1)}^{-i}$</u>: the monotone property is immediate from (16). (ii) <u>$p_i \geq p_{(1)}^{-i}$</u>: By (18), the monotonicity of d_i with respect to p_j is immediate when $p_j > p_{(1)}^{-i}$. When $p_j = p_{(1)}^{-i}$,

$$d_i(p_i, p_{-i}) = \frac{\exp\left(a_i - bf\left(p_i - p_j\right)\right)}{\exp\left(a_i - bf\left(p_i - p_j\right)\right) + \sum_{k \neq i} \exp\left(a_k - bf\left(p_k - p_j\right)\right)}, \tag{A-7}$$

and the expression remains valid when p_j is decreased downward from $p_{(1)}^{-i}$. Thus,

$$\begin{aligned} \frac{\partial_- d_i}{\partial p_j} &= bf'(p_i - p_j)d_i - bd_i \sum_{k \neq j} f'_+(p_k - p_j)d_k \\ &= b\alpha d_i - b\alpha d_i \sum_{k \neq j} d_k = b\alpha d_i \left(1 - \sum_{k \neq j} d_k\right) \geq 0. \quad \text{whenever } p_j = p_{(1)}^{-i}. \end{aligned} \tag{A-8}$$

Similarly, when p_j is the *unique* lowest price, i.e., $p_j < p_k$ for all $k \neq j$, (A-7) continues to apply even when p_j is increased *upward* from $p_{(1)}^{-i}$, so that $\frac{\partial_+ d_i}{\partial p_j} = \frac{\partial_- d_i}{\partial p_j} = b\alpha d_i \left(1 - \sum_{k \neq j} d_k\right) \geq 0$. The remaining case has:

$$p_j = p_{(1)}^{-i} = p_l \quad \text{for some } l \neq j, i.$$

In view of (A-8), it suffices to show that $\frac{\partial_+ d_i}{\partial p_j} \geq 0$, which is immediate from the representation of d_i in (18).

□

Proof of Theorem 3. As in the proof of Theorem 1, for $p_{i1} > p_{i2}$, we show that the difference in the logarithms of firm i's profit function, under p_{i1} versus p_{i2}, is non-decreasing in $p_j, j \neq i$, for any p_{-i}. To this end, we distinguish among the following three cases: (i) $p_{i2} < p_{i1} \leq p_{(1)}^{-i}$; (ii) $p_{(1)}^{-i} \leq p_{i2} < p_{i1}$; (iii) $p_{i2} < p_{(1)}^{-i} < p_{i1}$.

(i) $\underline{p_{i2} < p_{i1} \leq p_{(1)}^{-i}}$ or (ii) $\underline{p_{(1)}^{-i} \leq p_{i2} < p_{i1}}$: Note that

$$\log \pi_i(p_{i1}, p_{-i}) - \log \pi_i(p_{i2}, p_{-i}) = \int_{p_{i2}}^{p_{i1}} \frac{\partial \log \pi_i(p_i, p_{-i})}{\partial p_i} dp_i,$$

since $\log \pi_i(p_i, p_{-i})$ is differentiable everywhere on the interval (p_{i2}, p_{i1}). Hence, it is sufficient to show that $\frac{\partial \log \pi_i(p_i, p_{-i})}{\partial p_i}$ is non-decreasing in p_j. By (17) and (19), we have

$$\frac{\partial \log \pi_i}{\partial p_i} = \begin{cases} \frac{1}{p_i - c_i} + \frac{\partial d_i}{\partial p_i}/d_i = \frac{1}{p_i - c_i} - b\sum_{k \neq i} f'\left(p_k - p_i\right) d_k = \frac{1}{p_i - c_i} - b\alpha(1 - d_i), & \text{if (i) } p_i \leq p_{(1)}^{-i} \\ \frac{1}{p_i - c_i} - bf'\left(p_i - p_{(1)}^{-i}\right)(1 - d_i) = \frac{1}{p_i - c_i} - b\alpha(1 - d_i), & \text{if (ii) } p_i > p_{(1)}^{-i} \end{cases}$$

which is non-decreasing in p_j for any $j \neq i$, by Lemma 1 (b).

(iii) $\underline{p_{i2} < p_{(1)}^{-i} < p_{i1}}$: Similar to the proof of Theorem 1, fix $\delta \leq \min\left\{p_{i1} - p_{(1)}^{-i}, p_{(1)}^{-i} - p_{i2}\right\}$,

$$\begin{aligned} \log \pi_i(p_{i1}, p_{-i}) - \log \pi_i(p_{i2}, p_{-i}) &= \overbrace{\log \pi_i(p_{i1}, p_{-i}) - \log \pi_i(p_{(1)}^{-i} + \delta, p_{-i})} \\ &\quad + \underline{\log \pi_i(p_{(1)}^{-i} + \delta, p_{-i}) - \log \pi_i(p_{(1)}^{-i} - \delta, p_{-i})} \\ &\quad + \underbrace{\log \pi_i(p_{(1)}^{-i} - \delta, p_{-i}) - \log \pi_i(p_{i2}, p_{-i})}. \end{aligned}$$

By cases (i) and (ii), both the first term and the third term are non-decreasing in p_j. It thus suffices to show that the second term, $\log \pi_i(p_{(1)}^{-i} + \delta, p_{-i}) - \pi_i(p_{(1)}^{-i} - \delta, p_{-i})$, is non-decreasing in $p_j, j \neq i$ as well. We show, in fact, that

$$\Delta_j(\delta) = \frac{\partial \left[\log \pi_i(p_{(1)}^{-i} + \delta, p_{-i}) - \log \pi_i(p_{(1)}^{-i} - \delta, p_{-i})\right]}{\partial p_j} \geq 0, \text{ for all } p_j,\ j \neq i. \tag{A-9}$$

Unless p_j is the unique lowest price among firm i's alternatives, so that the increase of p_j is accompanied by an increase of $p_{(1)}^{-i}$, the proof of (A-9) is identical to the proof of case (ii.a) in Theorem 1. The remaining case for (A-9) has $p_j = p_{(1)}^{-i} < p_k$ for all $k \neq i, j$.

$$\begin{aligned}
\Delta_j(\delta) &= \left.\frac{\partial \left[\log \pi_i(p_{i1}, p_{-i}) - \log \pi_i(p_{i2}, p_{-i})\right]}{\partial p_j}\right|_{p_{i1}=p_{(1)}^{-i}+\delta,\ p_{i2}=p_{(1)}^{-i}-\delta} \\
&= \left.\frac{\partial \left[\log d_i(p_{i1}, p_{-i}) - \log d_i(p_{i2}, p_{-i})\right]}{\partial p_j}\right|_{p_{i1}=p_{(1)}^{-i}+\delta,\ p_{i2}=p_{(1)}^{-i}-\delta} \\
&= b\alpha d_j \left(p_{(1)}^{-i} + \delta, p_{-i}\right) - b\alpha d_j \left(p_{(1)}^{-i} - \delta, p_{-i}\right) \geq 0. \quad \text{by Lemma 1 (b).}
\end{aligned}$$

(The last equality follows from the fact that

$$\frac{\partial d_i(p_{i1}, p_{-i})}{\partial p_j} = b\alpha d_i \left(1 - \sum_{k \neq j} d_k\right) = b\alpha d_i d_j, \quad \text{for any } p_{i1} > p_{(1)}^{-i} = p_j \text{ by (A-8) in Appendix,}$$

$$\frac{\partial d_i(p_{i2}, p_{-i})}{\partial p_j} = bf'(p_j - p_{i2}) d_i d_j = b\alpha d_i d_j, \quad \text{for any } p_{i2} < p_{(1)}^{-i} = p_j \text{ by (16).)}$$

Thus, we have shown that the price competition game is log-supermodular.

□

Proof of Theorem 4. The proof is similar to that of Theorem 2; we show that each product i's profit function is quasi-concave in its own price, i.e., the function has no local minimum. Note first from (22) that, if $p_i \neq p_{(n-1)}^{-i}, p_{(n)}^{-i}$, each firm i's sales volume is differentiable in its own price, where

$$0 \geq \frac{\partial d_i}{\partial p_i} = \begin{cases} -b_i f'\left(p_i - p_{(n-1)}^{-i}\right) d_i(1 - d_i), & p_i < p_{(n-1)}^{-i} \\ -d_i \sum_{k \neq i} d_k b_k f'\left(p_k - p_i\right), & p_i \in (p_{(n-1)}^{-i}, p_{(n)}^{-i}) \\ -b_i f'\left(p_i - p_{(n)}^{-i}\right) d_i(1 - d_i), & p_i > p_{(n)}^{-i} \end{cases}, \tag{A-10}$$

Similar to (A-2), we therefore have

$$\frac{\partial \pi_i}{\partial p_i} = d_i + (p_i - c_i)\frac{\partial d_i}{\partial p_i} = \begin{cases} d_i \left(1 - (p_i - c_i) b_i f'\left(p_i - p_{(n-1)}^{-i}\right)(1 - d_i)\right), & p_i < p_{(n-1)}^{-i} \\ d_i \left(1 - (p_i - c_i) \sum_{k \neq i} d_k b_k f'\left(p_k - p_i\right)\right), & p_i \in (p_{(n-1)}^{-i}, p_{(n)}^{-i}) \\ d_i \left(1 - (p_i - c_i) b_i f'\left(p_i - p_{(n)}^{-i}\right)(1 - d_i)\right), & p_i > p_{(n)}^{-i} \end{cases}. \tag{A-11}$$

- If $\underline{p_i < p^{-i}_{(n-1)}}$, $(p_i - c_i)f'\left(p_i - p^{-i}_{(n-1)}\right)$ is non-decreasing since $f(\cdot)$ is convex. Moreover, $1 - d_i$ is non-decreasing in p_i since $\frac{\partial d_i}{\partial p_i} \leq 0$, therefore, $(p_i - c_i)b_i f'\left(p_i - p^{-i}_{(n-1)}\right)(1 - d_i)$, the product of two non-negative non-decreasing functions, is non-decreasing in p_i. Quasi-concavity of π_i in p_i on the interval of $[c_i, p^{-i}_{(n-1)})$ follows as in the proof of Theorem 2, see (A-5).

- If $\underline{p_i = p^{-i}_{(n-1)}}$: Clearly, $p^{-i}_{(n-1)} = p_i \geq c_i$. By (A-11),

$$\begin{aligned}
& \frac{\partial_- \pi_i}{\partial p_i}(p^{-i}_{(n-1)}, p_{-i}) - \frac{\partial_+ \pi_i}{\partial p_i}(p^{-i}_{(n-1)}, p_{-i}) \\
= \;& (p^{-i}_{(n-1)} - c_i)\left(\lim_{p_i \nearrow p^{-i}_{(n-1)}} \frac{\partial d_i}{\partial p_i}(p_i, p_{-i}) - \lim_{p_i \searrow p^{-i}_{(n-1)}} \frac{\partial d_i}{\partial p_i}(p_i, p_{-i})\right) \\
= \;& (p^{-i}_{(n-1)} - c_i)\left(\lim_{p_i \nearrow p^{-i}_{(n-1)}} \frac{\partial d_i}{\partial p_i}(p_i, p_{-i}) + \lim_{p_i \searrow p^{-i}_{(n-1)}} \sum_{j \neq i} \frac{\partial d_i}{\partial p_j}\right) \\
\geq \;& 0.
\end{aligned}$$

  The last equality follows from the fact that, by (22), $\frac{\partial d_i}{\partial p_i} = -d_i \sum_{j \neq i} d_j b_j f'(p_j - p_i)$ and $\frac{\partial d_i}{\partial p_j} = d_i d_j b_j f'(p_j - p_i)$ when $p_i \in (p^{-i}_{(n-1)}, p^{-i}_{(n)})$. The inequality follows from condition (D'). This shows that $p^{-i}_{(n-1)}$ fails to be a local minimum of the profit function π_i.

- If $\underline{p_i \in (p^{-i}_{(n-1)}, p^{-i}_{(n)})}$, by (22) and (A-11), $\frac{\partial \pi_i}{\partial p_i} = d_i\left(1 - (p_i - c_i)\sum_{j \neq i} \frac{\partial \ln(d_i)}{\partial p_j}\right)$. Note that $(p_i - c_i)\sum_{j \neq i} \frac{\partial \ln(d_i)}{\partial p_j}$ is non-decreasing by (M). Quasi-concavity of π_i on the interval $(p^{-i}_{(n-1)}, p^{-i}_{(n)})$ follows, as in the proof of Theorem 2, see (A-5).

- If $\underline{p_i = p^{-i}_{(n)}}$, similar to the second case $(p_i = p^{-i}_{(n-1)})$, one shows $\frac{\partial \pi_i}{\partial p_i}(p^{-i}_{(n)}-, p_{-i}) \geq \frac{\partial \pi_i}{\partial p_i}(p^{-i}_{(n)}+, p_{-i})$ by (D') so that $p^{-i}_{(n)}$ fails to be a local minimum.

- If $\underline{p_i > p^{-i}_{(n)}}$. The proof of quasi-concavity of π_i on $(p^{-i}_{(n)}, p^{max}]$ is identical to the proof of first case, merely replacing similar $p^{-i}_{(n-1)}$ by $p^{-i}_{(n)}$.

Therefore, $\pi_i(p_i, p_{-i})$ is quasi-concave in $p_i \geq c_i$ for any p_{-i}. □

Proof of Lemma 2. Conditions (D'): By (22), we have

$$\frac{\partial d_i}{\partial p_j} = bf'(p_j - p_i)\, d_i d_j \text{ for any } p_i \in (p^{-i}_{(n-1)}, p^{-i}_{(n)}).$$

Involving the first part of (A-10), we thus obtain

$$
\begin{aligned}
& \lim_{p_i \searrow p^{-i}_{(n-1)}} \sum_{j\neq i} \frac{\partial d_i}{\partial p_j}(p_i, p_{-i}) \geq | \lim_{p_i \nearrow p^{-i}_{(n-1)}} \frac{\partial d_i}{\partial p_i}(p_i, p_{-i})| \\
\Longleftrightarrow \quad & \lim_{p_i \searrow p^{-i}_{(n-1)}} \sum_{j\neq i} bf'\left(p_j - p_i\right) d_i d_j \geq \lim_{p_i \nearrow p^{-i}_{(n-1)}} bf'\left(p_i - p^{-i}_{(n-1)}\right) d_i(1-d_i) \\
\overset{\text{(e1)}}{\Longleftrightarrow} \quad & d_i(p^{-i}_{(n-1)}, p_{-i}) \cdot \left(\sum_{j\neq i:\ p_j > p^{-i}_{(n-1)}} \alpha d_j(p^{-i}_{(n-1)}, p_{-i}) + \beta \sum_{j\neq i:\ p_j \leq p^{-i}_{(n-1)}} d_j(p^{-i}_{(n-1)}, p_{-i}) \right) \\
& \geq \beta d_i(p^{-i}_{(n-1)}, p_{-i}) \left(1 - d_i(p^{-i}_{(n-1)}, p_{-i})\right) \\
\Longleftrightarrow \quad & \sum_{j\neq i:\ p_j > p^{-i}_{(n-1)}} \alpha d_j(p^{-i}_{(1)}, p_{-i}) + \beta \sum_{j\neq i:\ p_j \leq p^{-i}_{(n-1)}} d_j(p^{-i}_{(n-1)}, p_{-i}) \geq \beta \left(1 - d_i(p^{-i}_{(1)}, p_{-i})\right), \text{(A-12)}
\end{aligned}
$$

where (e1) holds from the facts that d_i and d_j are continuous in p_i, $f'(x) = \alpha$ if $x > 0$ and $f'(x) = \beta$ if $x < 0$. Since $\alpha \geq \beta \geq 0$, therefore the inequality to the right of the last implication in (A-12) holds

$$
\sum_{j\neq i:\ p_j > p^{-i}_{(n-1)}} \alpha d_j(p^{-i}_{(n-1)}, p_{-i}) + \beta \sum_{j\neq i:\ p_j \leq p^{-i}_{(n-1)}} d_j(p^{-i}_{(n-1)}, p_{-i}) \geq \beta \sum_{j\neq i} d_j(p^{-i}_{(n-1)}, p_{-i}) = \beta \left(1 - d_i(p^{-i}_{(n-1)}, p_{-i})\right).
$$

Hence, the first inequality of (D') is true. The proof of the second inequality in (D') is identical to the Lemma 1.

Condition (M): The proof is identical to the proof in Lemma 1. □

Proof of Lemma 3. Note by (18) that the sales volume in the lowest price subsidy model satisfies

$$
d_i^{LOW}(\mathbf{p}) = \frac{\exp\left(a_i - b\alpha p_i\right)}{\exp\left(a_i - b\alpha p_i\right) + \sum_{k\neq i} \exp\left(a_k - b\alpha p_k\right)}.
$$

We distinguish between the following two cases: (i) $p^{-i}_{(1)} \geq C$; (ii) $p^{-i}_{(1)} < C$.

(i) $p^{-i}_{(1)} \geq C$. By (6), since $p_k \geq p^{-i}_{(1)} \geq C$ for any $k \neq i$, one has

$$
\begin{aligned}
d_i^{EXO}(\mathbf{p}) &= \begin{cases} \frac{\exp(a_i - b\alpha(p_i - C))}{\exp(a_i - b\alpha(p_i - C)) + \sum_{k\neq i} \exp(a_k - b\alpha(p_k - C))}, & p_i \geq C \\ \frac{\exp(a_i - b\beta(p_i - C))}{\exp(a_i - b\beta(p_i - C)) + \sum_{k\neq i} \exp(a_k - b\alpha(p_k - C))}, & p_i < C \end{cases} \\
&= \begin{cases} \frac{\exp(a_i - b\alpha p_i)}{\exp(a_i - b\alpha p_i) + \sum_{k\neq i} \exp(a_k - b\alpha p_k)}, & p_i \geq C \\ \frac{\exp(a_i - b(\beta p_i + (\alpha - \beta)C))}{\exp(a_i - b(\beta p_i + (\alpha - \beta)C)) + \sum_{k\neq i} \exp(a_k - b\alpha p_k)}, & p_i < C \end{cases}.
\end{aligned}
$$

It is obvious that $d_i^{EXO}(\mathbf{p}) = d_i^{LOW}(\mathbf{p})$ when $p_i \geq C$, thus, (23b) holds. When $p_i < C$, $p_{(1)} = p_i$, and

$$\begin{aligned}
& \alpha\left(1-d_i^{LOW}(\mathbf{p})\right)-\beta\left(1-d_i^{EXO}(\mathbf{p})\right) \\
= \;& \alpha\frac{\sum_{k\neq i}\exp\left(a_k-b\alpha p_k\right)}{\exp\left(a_i-b\alpha p_i\right)+\sum_{k\neq i}\exp\left(a_k-b\alpha p_k\right)}-\beta\frac{\sum_{k\neq i}\exp\left(a_k-b\alpha p_k\right)}{\exp\left(a_i-b\left(\beta p_i+(\alpha-\beta)C\right)\right)+\sum_{k\neq i}\exp\left(a_k-b\alpha p_k\right)} \\
= \;& \frac{\left(\sum_{k\neq i}e^{a_k-b\alpha p_k}\right)\cdot\left[(\alpha-\beta)\cdot\left(\sum_{k\neq i}e^{a_k-b\alpha p_k}\right)+\alpha e^{a_i-b(\beta p_i+(\alpha-\beta)C)}-\beta e^{a_i-b\alpha p_i}\right]}{\left[e^{a_i-b\alpha p_i}+\sum_{k\neq i}e^{a_k-b\alpha p_k}\right]\cdot\left[e^{a_i-b(\beta p_i+(\alpha-\beta)C)}+\sum_{k\neq i}e^{a_k-b\alpha p_k}\right]} \\
= \;& \frac{\left(\sum_{k\neq i}e^{a_k-b\alpha p_k}\right)\cdot\left[(\alpha-\beta)\cdot\left(\sum_{k\neq i}e^{a_k-b\alpha p_k}\right)+\alpha e^{a_i-b\alpha p_i}\left(e^{-b(\alpha-\beta)(C-p_i)}-\frac{\beta}{\alpha}\right)\right]}{\left[e^{a_i-b\alpha p_i}+\sum_{k\neq i}e^{a_k-b\alpha p_k}\right]\cdot\left[e^{a_i-b(\beta p_i+(\alpha-\beta)C)}+\sum_{k\neq i}e^{a_k-b\alpha p_k}\right]} \qquad \text{(A-13)} \\
\geq \;& \frac{\left(\sum_{k\neq i}e^{a_k-b\alpha p_k}\right)\cdot\left[(\alpha-\beta)\cdot\left(\sum_{k\neq i}e^{a_k-b\alpha p_k}\right)+\alpha e^{a_i-b\alpha p_i}\left(e^{-b(\alpha-\beta)(C-c_i)}-\frac{\beta}{\alpha}\right)\right]}{\left[e^{a_i-b\alpha p_i}+\sum_{k\neq i}e^{a_k-b\alpha p_k}\right]\cdot\left[e^{a_i-b(\beta p_i+(\alpha-\beta)C)}+\sum_{k\neq i}e^{a_k-b\alpha p_k}\right]} \\
\geq \;& 0,
\end{aligned}$$

where the first inequality follows from $p_i \geq c_i$, and the second inequality from $\alpha \geq \beta$ and $e^{-b(\alpha-\beta)(C-c_i)} \geq \frac{\beta}{\alpha}$ by $\mathbf{P}(\beta)$.

(ii) $\underline{p_{(1)}^{-i} < C}$. By (6), one has

$$\begin{aligned}
d_i^{EXO}(\mathbf{p}) \;&=\; \begin{cases} \frac{\exp(a_i-b\alpha(p_i-C))}{\exp(a_i-b\alpha(p_i-C))+\sum_{k\neq i:\; p_k<C}\exp(a_k-b\beta(p_k-C))+\sum_{k\neq i:\; p_k\geq C}\exp(a_k-b\alpha(p_k-C))}, & p_i\geq C \\ \frac{\exp(a_i-b\beta(p_i-C))}{\exp(a_i-b\beta(p_i-C))+\sum_{k\neq i:\; p_k<C}\exp(a_k-b\beta(p_k-C))+\sum_{k\neq i:\; p_k\geq C}\exp(a_k-b\alpha(p_k-C))}, & p_i<C \end{cases} \\
&\geq\; \begin{cases} \frac{\exp(a_i-b\alpha(p_i-C))}{\exp(a_i-b\alpha(p_i-C))+\sum_{k\neq i:\; p_k<C}\exp(a_k-b\alpha(p_k-C))+\sum_{k\neq i:\; p_k\geq C}\exp(a_k-b\alpha(p_k-C))}, & p_i\geq C \\ \frac{\exp(a_i-b\beta(p_i-C))}{\exp(a_i-b\beta(p_i-C))+\sum_{k\neq i:\; p_k<C}\exp(a_k-b\alpha(p_k-C))+\sum_{k\neq i:\; p_k\geq C}\exp(a_k-b\alpha(p_k-C))}, & p_i<C \end{cases} \\
&=\; \begin{cases} \frac{\exp(a_i-b\alpha p_i)}{\exp(a_i-b\alpha p_i)+\sum_{k\neq i}\exp(a_k-b\alpha p_k)} = d_i^{LOW}(\mathbf{p}), & p_i\geq C \\ \frac{\exp(a_i-b(\beta p_i+(\alpha-\beta)C))}{\exp(a_i-b(\beta p_i+(\alpha-\beta)C))+\sum_{k\neq i}\exp(a_k-b\alpha p_k)}, & p_i<C \end{cases},
\end{aligned}$$

where the inequality follows by replacing, in the denominator, each of the term in the index set $\{k \neq i, p_k < C\}$ by a larger value, since $0 < \beta \leq \alpha$. This proves (23b) when $p_i \geq C$. When $p_i < C$, one has

$$\begin{aligned}
& \alpha\left(1-d_i^{LOW}(\mathbf{p})\right)-\beta\left(1-d_i^{EXO}(\mathbf{p})\right) \\
\geq \;& \alpha\frac{\sum_{k\neq i}\exp\left(a_k-b\alpha p_k\right)}{\exp\left(a_i-b\alpha p_i\right)+\sum_{k\neq i}\exp\left(a_k-b\alpha p_k\right)}-\beta\frac{\sum_{k\neq i}\exp\left(a_k-b\alpha p_k\right)}{\exp\left(a_i-b\left(\beta p_i+(\alpha-\beta)C\right)\right)+\sum_{k\neq i}\exp\left(a_k-b\alpha p_k\right)} \\
\geq \;& 0. \quad \text{by (A-13), (Note that the left hand side coincides with the second expression in (A-13).)}
\end{aligned}$$

We have thus shown that the inequalities (23) apply for any firm i and any $\mathbf{p}$. □

Appendix B: Identifying the parameters of the Medicare model with switching costs

For any $i = 1, 2, \ldots, N$, let

$$\Delta_i = a_i - b_i f(p_i - C) - a_0 + b_0 f(p_0 - C),$$

Thus, determining the intercepts $\{a_i\}$ is equivalent to computing the quantities $\{\Delta_i\}_{i=1}^{N}$. (Recall $a_0 = 0$, by normalization.) We have, by (31)–(34), for any MA plan $i = 1, 2, \ldots, N$, since we normalize $S_2 = 0$, that

$$\begin{aligned} d_i &= h_1 \frac{e^{a_i - b_i \cdot f(p_i - g(\mathbf{p}))}}{e^{S_1} \cdot e^{a_0 - b_0 \cdot f(p_0 - g(\mathbf{p}))} + \sum_{k=1}^{N} e^{a_k - b_k \cdot f(p_k - g(\mathbf{p}))}} + h_2 \frac{e^{a_i - b_i \cdot f(p_i - g(\mathbf{p}))}}{e^{a_0 - b_0 \cdot f(p_0 - g(\mathbf{p}))} + \sum_{k=1}^{N} e^{a_k - b_k \cdot f(p_k - g(\mathbf{p}))}} \\ &= h_1 \frac{e^{\Delta_i}}{e^{S_1} + \sum_{k=1}^{N} e^{\Delta_k}} + h_2 \frac{e^{\Delta_i}}{1 + \sum_{k=1}^{N} e^{\Delta_k}} \\ &= e^{\Delta_i} \cdot \left[\frac{h_1}{e^{S_1} + \sum_{k=1}^{N} e^{\Delta_k}} + \frac{h_2}{1 + \sum_{k=1}^{N} e^{\Delta_k}} \right]. \end{aligned}$$

Similarly, the sales volume of the FFS plan may be represented as

$$d_0 = \frac{h_1 e^{S_1}}{e^{S_1} + \sum_{k=1}^{N} e^{\Delta_k}} + \frac{h_2}{1 + \sum_{k=1}^{N} e^{\Delta_k}}.$$

Dividing the sales volume of MA plan $i, i \geq 2$ by the sales volume of MA plan 1, we obtain

$$\ln\left(\frac{d_i}{d_1}\right) = \Delta_i - \Delta_1 \quad \text{or} \quad \Delta_i = \Delta_1 + \ln(d_i) - \ln(d_1), \quad i = 1, 2, \ldots, N. \tag{B-1}$$

Thus, all $\{\Delta_i\}$ are specified once Δ_1 is computed. Dividing the market share of MA plan 1 by the market share of the FFS plan 0, we also have, by (31)–(34),

$$\begin{aligned} \frac{d_1}{d_0} &= e^{\Delta_1} \cdot \frac{h_1\left(1 + \sum_{k=1}^{N} e^{\Delta_k}\right) + h_2\left(e^{S_1} + \sum_{k=1}^{N} e^{\Delta_k}\right)}{h_1 e^{S_1}\left(1 + \sum_{k=1}^{N} e^{\Delta_k}\right) + h_2\left(e^{S_1} + \sum_{k=1}^{N} e^{\Delta_k}\right)} \\ &= e^{\Delta_1} \cdot \frac{(h_1 + h_2 e^{S_1}) + (h_1 + h_2)\sum_{k=1}^{N} e^{\Delta_k}}{(h_1 + h_2)e^{S_1} + (h_2 + h_1 e^{S_1})\sum_{k=1}^{N} e^{\Delta_k}} \\ &= e^{\Delta_1} \cdot \frac{(h_1 + h_2 e^{S_1}) + (h_1 + h_2)e^{\Delta_1}\sum_{k=1}^{N} e^{\Delta_k - \Delta_1}}{(h_1 + h_2)e^{S_1} + (h_2 + h_1 e^{S_1})e^{\Delta_1}\sum_{k=1}^{N} e^{\Delta_k - \Delta_1}} \\ &= e^{\Delta_1} \cdot \frac{(h_1 + h_2 e^{S_1}) + (h_1 + h_2)e^{\Delta_1}\sum_{k=1}^{N} d_k/d_1}{(h_1 + h_2)e^{S_1} + (h_2 + h_1 e^{S_1})e^{\Delta_1}\sum_{k=1}^{N} d_k/d_1}. \end{aligned}$$

Recall that the sizes of the two segments add up to 1, i.e., $h_1 + h_2 = 1$, thus, Δ_1 satisfies

$$\begin{aligned} 0 &= \frac{\sum_{k=1}^{N} d_k}{d_1}(e^{\Delta_1})^2 + \left(h_1 + h_2 e^{S_1} - (h_2 + h_1 e^{S_1})\frac{\sum_{k=1}^{N} d_k}{d_0}\right) e^{\Delta_1} - \frac{d_1}{d_0} e^{S_1} \\ &= \frac{1 - d_0}{d_1}(e^{\Delta_1})^2 + \left(h_1 + h_2 e^{S_1} - (h_2 + h_1 e^{S_1})\frac{1 - d_0}{d_0}\right) e^{\Delta_1} - \frac{d_1}{d_0} e^{S_1}. \end{aligned}$$

The unique positive root of this quadratic equation in e^{Δ_1} is:

$$
\begin{aligned}
e^{\Delta_1} &= \frac{-\left(h_1 + h_2 e^{S_1} - (h_2 + h_1 e^{S_1})\frac{1-d_0}{d_0}\right) + \sqrt{\left(h_1 + h_2 e^{S_1} - (h_2 + h_1 e^{S_1})\frac{1-d_0}{d_0}\right)^2 + 4\frac{1-d_0}{d_0} e^{S_1}}}{2(1-d_0)/d_1} \\
&= \frac{d_1}{d_0} \cdot \frac{-\hat{B} + \sqrt{\hat{B}^2 + 4d_0(1-d_0)e^{S_1}}}{2(1-d_0)}. \qquad \text{(B-2)}
\end{aligned}
$$

Here

$$
\hat{B} = (h_1 + h_2 e^{S_1})d_0 - (h_2 + h_1 e^{S_1})(1-d_0)
$$

Thus, by (B-1)–(B-2), for any $i = 1, 2, \ldots, N$, we have

$$
\Delta_i = \ln(d_i) - \ln(d_0) + \ln\left(\frac{-\hat{B} + \sqrt{\hat{B}^2 + 4d_0(1-d_0)e^{S_1}}}{2(1-d_0)}\right). \qquad \text{(B-3)}
$$

With all parameters of the demand functions fully specified, the remaining task is to identify the marginal cost rates $\{c_i\}$. As with the models in Sections 3-5, this is accomplished by observing that the observed price vector p^* is an interior point of the feasible price space and by assuming that it is a Nash equilibrium. It is easily verified, as in Section 3, that each profit function π_i is differentiable in its own price p_i, unless $p_i = C$, in which case the right and left derivatives $\frac{\partial_+ \pi_i}{\partial p_i}$ and $\frac{\partial_- \pi_i}{\partial p_i}$ exist. This implies that

$$
0 = \frac{\partial \pi_i(p^*)}{\partial p_i} = d_i(p^*) + (p_i^* - c_i)\frac{\partial d_i}{\partial p_i}; \quad \text{if } p_i^* \neq C. \qquad \text{(B-4)}
$$

$$
\begin{aligned}
0 &\geq \frac{\partial_+ \pi_i(p^*)}{\partial p_i} = d_i(p^*) + (C - c_i)\frac{\partial_+ d_i}{\partial p_i}; \quad \text{if } p_i^* = C, \qquad \text{(B-5)} \\
0 &\leq \frac{\partial_- \pi_i(p^*)}{\partial p_i} = d_i(p^*) + (C - c_i)\frac{\partial_- d_i}{\partial p_i}; \quad \text{if } p_i^* = C.
\end{aligned}
$$

Thus, as in the model of Section 3, if $p_i^* \neq C$, the marginal cost rate c_i is determined as the unique root of equation (B-4). If $p_i^* = C$, (B-5) determines an interval for the cost rate c_i.