What if Marketers Put Customers Ahead of Profits?*

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Abstract

We examine a duopoly where one of the firms does not maximize profit, but instead maximizes consumer surplus subject to a minimum profit constraint. Competition between the firms is modeled as a two-stage game, which is solved by backward induction. In the first stage, firms choose product quality levels sequentially, anticipating (simultaneous move) Bertrand price competition in the second stage of the game. The identity of the first moving firm is taken as exogenous, and thus we separately consider the cases of a surplus-maximizing and a profit-maximizing first mover. We consider both cases of convex fixed costs and convex variable costs of producing quality. Our primary goal in conducting the analysis is to evaluate changes in key market outcomes as the surplus-maximizing firm becomes more altruistic in its orientation, as measured by the extent to which it sacrifices profits. As a limiting case, our framework considers competition between a profit-maximizing firm and a non-profit firm. Interim cases may be viewed as models of competition involving a form of “social enterprise,” i.e., a firm organized as a for-profit entity but whose primary objective is to advance consumer welfare. From the analysis, we conclude that the surplus-maximizing firm can deliver significant additional value to consumers by forgoing some of the profit it would have earned as a profit maximizer. However, the effectiveness of this strategy depends upon the cost structure of the industry and which firm is the first mover. With fixed costs of quality, there is a significant first mover advantage. When the surplus-maximizing firm moves first, the tradeoff between sacrificing profits and generating consumer surplus is quite favorable, with surplus approaching its fully efficient level as the surplus-maximizing firm approaches being a non-profit. With variable costs of quality, higher levels of consumer surplus are achieved when the surplus-maximizing firm is the follower in quality choice, although differences in outcomes diminish as the surplus-maximizing firm sacrifices more profit. A key implication of these results is that in order to achieve highly favorable outcomes, the surplus-maximizing firm must move first when production costs are fixed, whereas the order of quality moves is less restrictive when costs are variable.

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1 Introduction

Models of competition in marketing and economics typically assume that profit maximization is the objective guiding the product design and pricing decisions of every firm in the market. However, many markets contain firms that, to varying degrees, are concerned with consumer welfare-related goals. At the extreme of this continuum, profit-maximizing firms compete against non-profit organizations. For example, in healthcare markets for-profit hospitals frequently compete with non-profit hospitals for patients, while setting prices (service fees) and choosing product quality (which doctors to hire). Similarly, in education markets for-profit universities (e.g., University of Phoenix) compete with non-profit universities (e.g., Yale) for students, while setting competitive prices (tuition) and choosing product quality (which faculty to hire). Less extreme cases involve competition between strict profit maximizers and “consumer-centric” firms, for-profit entities that are willing to sacrifice some potential profit in order to benefit consumers. These firms, sometimes referred to as for-profit “social enterprises,” manage to a “double bottom line” – delivering financial returns to shareholders and economic value to society through its consumers. A celebrated class of “social enterprises” are micro-finance institutions (MFI’s) such as Grameen Bank, which competes with profit-maximizing banks for consumer loans when setting prices (interest rates) and product quality (loan characteristics).

Motivated by these examples, in this paper we consider the following thought experiment: What if marketers put customers ahead of profits? How would the nature of competition in the industry change? To analyze the problem, we construct a duopoly model of strategic competition between a profit-maximizing firm and a firm that maximizes total consumer surplus. We assume the surplus-maximizing firm must attain some minimum level of profit, bounded below by zero and bounded above by the amount of profit it would have obtained under a profit maximization objective. Products are vertically differentiated and thus have two attributes: quality and price. We model consumer demand for these products as a discrete choice, including the option to purchase neither firm’s product. Competition is modeled as a two stage game in which firms sequentially choose product quality, followed by the simultaneous choice of prices in the second stage. We compare solutions of this model to the optimally efficient market outcome and the outcome when both firms operate as profit-maximizers. These comparisons allow us to infer both the absolute and relative effectiveness
of sacrificing profit in order to increase consumer surplus.

From the analysis, we conclude that the surplus-maximizing firm can deliver significant additional value to consumers by forgoing some of the profit it would have earned as a profit maximizer. However, the effectiveness of this strategy depends upon the cost structure of the industry and which firm is the first mover in choosing quality. When production costs are fixed (independent of demand), it is advantageous for the surplus-maximizing firm to move first. In this case, the surplus efficiency (the percentage of consumer surplus attained in the optimally efficient market outcome) ranges from 45% when the surplus-maximizing firm must generate competitive-level profits to 99% when it is a non-profit. That is, a 1% reduction in the required level of profit on average generates a 0.54% increase in surplus efficiency. By contrast, when moving second, the surplus efficiency ranges from 40% (competitive-level profits) to 52% (non-profit) – thus on average, a 1% reduction in profits only generates a 0.12% increase in surplus efficiency. When production costs are variable (proportional to demand), it is advantageous for the surplus-maximizing firm to be the follower in choosing quality, although this advantage diminishes as the surplus-maximizing firm sacrifices more profit. When the surplus-maximizing firm moves second, the surplus efficiency ranges from 70% (competitive-level profits) to 94% (non-profit), a 0.24% increase in consumer surplus for every 1% sacrifice in profits. When the surplus-maximizing firm moves first, the surplus efficiency ranges from 57% (competitive-level profits) to 94% (non-profit), a 0.37% increase in consumer surplus for every 1% sacrifice in profits. A key implication of these results is that in order to achieve highly favorable outcomes, the surplus-maximizing firm must move first when production costs are fixed, whereas the order of quality moves is less restrictive when costs are variable.

The paper adds to the literature on models of vertical product competition (e.g., Shaked and Sutton, 1982; Moorthy, 1988; Ronnen, 1991; Motta, 1993; Lehmann-Grube, 1997; Kuhn, 2007). These papers consider competition among profit-maximizing firms under different assumptions about production costs (variable or fixed), the timing of quality choices (simultaneous or sequential), and the nature of price competition (Bertrand or Cournot). We believe our paper is the first in this stream to document the sequential equilibrium solution with two profit maximizing firms competing in quality/price and convex (quadratic) fixed costs (the competitive benchmark of Section 3.1). The paper also adds to the literature on firms that have an objective other than profit maximization. Non-profit organizations, which by definition operate under an alternative objective function, have
been the most frequent subject of these studies.\(^1\) Common assumptions for non-profit objectives include maximizing consumer surplus (Steinberg and Weisbrod, 2005), output (Hansmann, 1981; Liu and Weinberg, 2004), product quality (Hansmann, 1981), budget (Hansmann, 1981), or a weighted combination of these factors (Newhouse, 1970; Steinberg, 1986; Dranove, 1988; Ansari et al., 1996). However, the nature of equilibrium when alternative-objective firms compete with for-profit entities has been largely unresearched. One exception is Liu and Weinberg (2004), who develop an analytical model of duopoly competition between an output-maximizing non-profit and a profit-maximizing firm. In addition, Park (2013) conducts an empirical study in which a purely profit-maximizing firm competes with a firm whose objective is to maximize a convex combination of profit and consumer surplus. Using data from the Singapore milk market, she finds the “alternative objective” firm weighs consumer surplus to profit in a 1 to 7 ratio. A final related stream of literature considers corporate social responsibility (CSR) and business ethics (Sen and Bhattacharya, 2001; Bagnoli and Watts, 2003; Besley and Ghatak, 2007; Iyer and Soberman, 2013). This literature emphasizes the role of “altruistic” consumer preferences (as expressed by the demand for products) in driving “socially responsible” behavior on the part of firms. Recognizing that corporate managers may possess similar preferences, in this paper we consider the implications of firms proactively choosing a consumer-centric or “altruistic” objective.

The paper proceeds as follows. In Section 2 we develop our model of strategic competition between a surplus-maximizing firm and a profit-maximizing firm. In Section 3 solve the model for fixed costs of quality production, while in Section 4, we present the model solution for variable costs of quality production. Section 5 concludes with a summary and a discussion of potential extensions.

2 Model

We consider product and price competition in a duopoly consisting of a profit-maximizing firm (\(PM\)) and a firm that maximizes total consumer surplus (\(CS\)), but which is required to achieve a minimum level of profit (\(\Pi_0\)). Later on, we express \(\Pi_0\) as a fraction of the profit the \(CS\) firm could have made had it instead pursued a profit maximization objective, assuming firms choose product quality in the same order.

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\(^1\)See Hansmann (1987) for a survey of economic theory on non-profit organizations.
We begin by developing the demand model and then turn to supply considerations.

2.1 Demand

We consider demand for vertically differentiated products, which are fully characterized by their quality \((s \in \mathbb{R}^+)\) and price \((p \in \mathbb{R}^+)\). Consumer willingness to pay (WTP) for quality \((t)\) is heterogeneous and uniformly distributed on the interval \([0, 1]\). Consumers consider the CS firm’s product \((s_{CS}, p_{CS})\), the PM firm’s product \((s_{PM}, p_{PM})\), and the outside alternative (neither firm’s product). The decision rule is to choose the product that maximizes consumer surplus (i.e., willingness to pay for the product minus its price), where the outside alternative is normalized to have zero surplus. For example, a consumer willing to pay \(t\) dollars per unit of quality will choose the CS firm’s product if \(ts_{CS} - p_{CS} = \max\{ts_{CS} - p_{CS}, ts_{PM} - p_{PM}, 0\}\).

As product quality is ordered, there will be one low-quality firm \(L \in \{CS, PM\}\) and one high-quality firm \(H \in \{PM, CS\}\).\(^2\) The quality ordering of firms in quality is endogenously determined in the model, as described in Sections 3 and 4. For now, it is convenient to express demand in terms of the high \((s_H, p_H)\) and low \((s_L, p_L)\) quality products. In formulating demand, we impose the condition that the low-quality firm offers the lower price per quality (greater quality value) product, i.e., \(0 < \frac{p_L}{s_L} < \frac{p_H}{s_H}\).\(^3\) Under this assumption, it is straightforward to compute the demanded quantities, which, by nature of having a unit mass of consumers, are equivalent to market shares:

\[
q_0 = \int_0^1 I(0 = \max\{ts_H - p_H, ts_L - p_L, 0\}) \, dt = \int_0^1 I\left(t < \frac{p_L}{s_L} < \frac{p_H}{s_H}\right) \, dt = \frac{p_L}{s_L} (1a)
\]

\[
q_L = \int_0^1 I\left(ts_L - p_L = \max\{ts_H - p_H, ts_L - p_L, 0\}\right) \, dt = \int_0^1 I\left(\frac{p_L}{s_L} < t < \frac{p_H - p_L}{s_H - s_L}\right) \, dt = \frac{p_H - p_L}{s_H - s_L} - \frac{p_L}{s_L} (1b)
\]

\[
q_H = 1 - q_0 - q_L = 1 - \frac{p_H - p_L}{s_H - s_L} (1c)
\]

\(^2\)In principle, firms can choose the same quality levels, but doing so leads to a situation where both firms make zero profits, which is clearly off the equilibrium path. With equal qualities, the price equilibrium is for the firms to charge the same price. Above the perfectly competitive price, there is an incentive for the profit-maximizing firm to deviate and undercut the existing price by a small amount in order to “steal the market.” Price is thus bid down to perfectly competitive levels.

\(^3\)If this condition is violated, the low-quality product is strictly dominated by the high-quality product, resulting in zero market share for the low-quality product. This outcome could not result in equilibrium, assuming rational firms and perfect information (as in our setup).
Note that the assumption of strictly positive prices and qualities ensures the outside good has non-zero market share (i.e., the market is “not covered”). In the expressions above, the quantity $t_L = \frac{p_L}{s_L}$ represents the willingness to pay of the marginal consumer who is indifferent between the outside alternative and low quality good, while $t_H = \frac{p_H - p_L}{s_H - s_L}$ is the willingness to pay of the marginal consumer who is indifferent between the low and high quality goods. As these WTP values lie within $[0,1]$, we have the relations $0 < \frac{p_L}{s_L} < \frac{p_H - p_L}{s_H - s_L} < 1$. These inequalities may be rearranged and combined to provide a set of strict ordering relations that must be maintained in equilibrium outcomes:

**Definition 1.** Let $R$ denote the feasible set of quality and price values, where:

$$R \equiv \{s_L, s_H, p_L, p_H : 0 < p_H - s_H + s_L < p_L < p_H(s_L/s_H) < s_L < p_H < s_H\}.$$

### 2.2 Supply

**Timing of moves and equilibrium concept**

We require that firms first commit to product quality choices before setting prices.\(^4\) Thus competition ensues in two stages: quality choices are taken first, and then prices are determined in a sub-game in which the quality choices are held fixed. In the first stage, firms sequentially chose product quality levels (Stackelberg competition). Sequential quality choice is realistic in the sense that, from the firm’s perspective, product quality choices are typically more permanent than price choices. The identify of the first-moving firm is taken as exogenous, and thus solutions for each case will be sought. In the second game stage, firms simultaneously set their product prices (Bertrand competition) conditional upon product quality choices. Under the assumption of complete information, pure strategy solutions to the game may be found via backward induction.

### Costs of production

Cost structures differ greatly across industries, with an important distinction being whether production costs are primarily fixed or variable. In our analysis, we consider both cases and compare

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\(^4\)We further note that firm objective functions are taken as exogenous and common knowledge. In the case of the surplus-maximizing firm, the objective function incorporates the required minimum profit level of profit. Firms are assumed to commit to these objectives in advance of the first stage quality competition and that rivals consider these commitments credible.
the welfare implications of having a surplus-maximizing firm participating in the market. Our baseline assumption is that the cost function for firm \( i \) (\( C_i \)) is convex (quadratic) in quality, i.e.:\(^5\)

\[
C_i = \begin{cases} 
  s_i^2 & \text{fixed costs of quality production} \\
  q_i s_i^2 & \text{variable costs of quality production}
\end{cases}
\]

The model therefore assumes no technological differences between the firms. Note that the assumption of quadratic costs ensures that qualities remain bounded. Indeed, the highest quality product which can be produced without incurring a loss is \( s = 1 \).\(^6\) Thus, we may constrain our search for equilibrium outcomes to the (four dimensional) unit simplex.

**Objective functions**

Here we formulate general expressions for firm profit and consumer surplus that take the firm decision variables as fixed. In subsequent sections, we describe how the model is solved for the values which obtain in equilibrium.

Firm’s profits are given by \( \Pi_i = q_i p_i - C_i \), where the appropriate form depends on the quality ordering of firms:

\[
\Pi_L = \left( \frac{p_H - p_L}{s_H - s_L} - \frac{p_L}{s_L} \right) p_L - C_L \quad (2a)
\]

\[
\Pi_H = \left( 1 - \frac{p_H - p_L}{s_H - s_L} \right) p_H - C_H \quad (2b)
\]

Firm \( i \)'s surplus is defined by summing consumer surplus over the firm’s customers: \( \Psi_i = \int_{t_{i1}}^{t_{i2}} (t s_i - p_i) \, dt = (t_{i2} - t_{i1}) \left( \frac{q_i}{2} (t_{i2} + t_{i1}) - p_i \right) \), where the values of \( \{t_{i1}, t_{i2}\} \) depend on the quality position of the firm. For low \( (L) \) quality firm the integration range is \( \{t_L, t_H\} \) while for the high \( (H) \) quality firm it is

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\(^5\) Some readers may wonder about the robustness of our results to changes in the scale of production costs, e.g., if we were to replace \( s_i^2 \) with \( \alpha s_i^2 \). For our model, it is straightforward to show that, in cases that may be solved analytically, shifting the scale of production costs simply scales the equilibrium (quality, price) solution by a factor of \( \alpha \). For cases requiring numerical solutions, we solve the model with \( \alpha \in \{1, 5, 2, 100\} \) and verify the same scaling property applies.

\(^6\) To see this, note that for fixed costs, profits are given by \( qp - s^2 \), where \( q < 1 \) and \( p < s \). Thus, to avoid a loss we must have \( 0 \leq qp - s^2 < s - s^2 = s(1 - s) \), and \( s(1 - s) > 0 \) clearly holds only for \( s < 1 \) given that by definition \( s > 0 \). Similarly for variable costs, profits are given by \( q(p - s^2) < s - s^2 \), and we obtain the same result that \( s < 1 \).
\{t_H, 1\}, yielding the following expressions for firm and total consumer surplus:

\[
\begin{align*}
\Psi_L &= \left( \frac{p_H - p_L}{s_H - s_L} - \frac{p_L}{s_L} \right) \left( \frac{1}{2} \left( \frac{p_H - p_L}{s_H - s_L} + \frac{p_L}{s_L} \right) - p_L \right) \\
\Psi_H &= \left( 1 - \frac{p_H - p_L}{s_H - s_L} \right) \left( \frac{1}{2} \left( 1 + \frac{p_H - p_L}{s_H - s_L} \right) - p_H \right) \\
\Psi_T &= \Psi_L + \Psi_H = \frac{p_H^2 s_H - 2p_H p_L s_L + s_L (s_H - p_H)^2 + s_L^2 (2p_H - s_H)}{2(s_H - s_L)s_L}
\end{align*}
\]

Note that consumer surplus for a firm is given by the product of the firm's market share and the surplus of the median consumer in its market segment, emphasizing the roles of market coverage and product value in delivering surplus.

In our model, when CS is the high quality firm, its objective is to maximize \(\Psi_T\) with respect to \((s_H, p_H)\) and subject to the constraint that \(\Pi_H \geq \Pi_0\). Similarly, CS maximizes \(\Psi_T\) with respect to \((s_L, p_L)\) and subject to the constraint that \(\Pi_L \geq \Pi_0\) when it is the low quality firm. Firm PM maximizes \(\Pi_L\) with respect to \((s_L, p_L)\) when it is the low quality firm, and similarly when it is the high quality firm.

3 Fixed costs of quality production

In this section, we consider the case of fixed costs of production, \(C_i = s_i^2\).

3.1 Benchmark cases

Interpretation of the model results is facilitated by developing relevant benchmarks. One benchmark is the fully efficient solution, which maximizes total surplus. The efficient solution gives the upper bound of total consumer surplus which can be realized by supplying two products to the market. The other benchmark is the competitive solution, i.e., the outcome when two profit-maximizing firms compete in quality and price according to the same game rules. We expect that our model will yield equilibrium objectives for firm CS that fall between these benchmark cases.

Efficient

In the efficient case, one firm offers the socially optimal products, i.e., the products which maximize total surplus (consumer surplus less costs). If a single product is offered, the firm objective
is to maximize $\Psi = \int_0^1 ts \, dt - s^2 = \frac{s^2 - p^2}{2s}$. However, as $\Psi$ is strictly decreasing in price and increasing in quality, the maximization is unbounded without further constraints. Thus, we require that profits are non-negative, implying the constraint $(1 - \frac{p}{s})p - s^2 \geq 0$. The constraint binds and therefore defines the optimal price $p = \frac{s}{2}(1 - \sqrt{1 - 4s})$. Substituting for $p$ into $\Psi$ and solving the first order condition in $s$ yields the optimal constrained solution of \( \{s = \frac{3}{16} = 0.1875, p = \frac{3}{64} = 0.0469\} \), demand (market share) of $\frac{3}{4}$, and a total surplus of $\frac{27}{512} (0.0527)$. The second order condition for $s$ indicates this solution is in fact a maximum. When the market is “uncovered,” it may be shown that adding a second product to the market does not increase total surplus, and thus $\frac{27}{512}$ also represents the upper bound for surplus in the market of interest, which contains two products.\(^7\)

**Competitive**

Our competitive benchmark is a sequential entry game played by two profit-maximizing firms. We refer to this game as \((PM_1, PM_2)\) to reflect that both firms use a profit maximization objective. Holding quality choices fixed, solving the system of first order conditions \(\frac{\partial \Pi_L}{\partial p_L} = 0, \frac{\partial \Pi_H}{\partial p_H} = 0\) yields equilibrium prices, which may be substituted to express profits entirely in terms of quality choices:\(^8\)

$$
\Pi_L(s_L, s_H) = \frac{s_L s_H (s_H - s_L)(1 + s_H - s_L)^2}{(4s_H - s_L)^2}
$$

$$
\Pi_H(s_L, s_H) = \frac{s_H^2 (s_H - s_L)(2 - 2s_H + s_L)^2}{(4s_H - s_L)^2}
$$

To solve the product equilibrium, note that the assumption of perfect information implies the first moving firm can fully anticipate its rival’s reaction to any choice of quality. With the first moving firm’s quality choice \((s_1)\) fixed, the follower can either produce a higher or lower quality product. Let \(s_L^*(s_1) = \arg\max_{s_L} \Pi_L(s_L, s_1)\) and \(s_H^*(s_1) = \arg\max_{s_H} \Pi_H(s_1, s_H)\) represent the best lower and upper quality responses for the follower. These reaction functions are given by solving the first order conditions \(\frac{\partial \Pi_L(s_L, s_1)}{\partial s_L} = 0\) and \(\frac{\partial \Pi_H(s_1, s_H)}{\partial s_H} = 0\) for \(s_L\) and \(s_H\), respectively. The follower will choose between the best lower and upper quality responses so that he gets the higher

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\(^7\)The firm’s problem is to solve $\max_{s_H, p_H \geq 0, p_L \in \mathbb{R}} \Psi_T$ subject to $\Pi_L + \Pi_H \geq 0$, which cannot be solved analytically. However, direct numerical optimization reveals that the limiting behavior of the problem is for \((s_H, p_H)\) to equal the single product case and for \((s_L, p_L)\) to be \(\lim_{\epsilon \to 0} (1, \frac{1}{4\epsilon})\). The surplus contribution from the second product thus approaches zero in the limit as its market share approaches zero.

\(^8\)Equilibrium prices are given by: $p_L = \frac{s_L (s_H - s_L)}{4s_H - s_L}$, $p_H = \frac{2s_H (s_H - s_L)}{4s_H - s_L}$.\
profit: \( s_2^*(s_1) = \arg\max_{s_L^*, s_H^*} \{ \Pi_L(s_L^*, s_1), \Pi_H(s_1, s_H^*) \} \). To determine his optimal quality choice, the leader evaluates his objective function, first assuming that the follower responds with the higher quality product and then assuming the follower responds with the lower quality product. Let \( s_{1H}^* = \arg\max_{s_1} \Pi_H(s_L^*(s_1), s_1) \) and \( s_{1L}^* = \arg\max_{s_1} \Pi_L(s_1, s_H^*(s_1)) \) represent the leader’s best “high” and “low” quality choices (where again these choices are given by solutions to the corresponding first order conditions \( \left. \frac{\partial \Pi_H(s_L^*(s_1), s_1)}{\partial s_1} \right|_{s_H=s_1} = 0 \) and \( \left. \frac{\partial \Pi_L(s_1, s_H^*(s_1))}{\partial s_L} \right|_{s_L=s_1} = 0 \)). The leader will then choose the quality level giving the greater profit, provided that choice of quality actually leads to the anticipated behavior of the rival. For example, \( s_1^* = s_{1H}^* \) if \( \Pi_H(s_L^*(s_{1H}^*), s_{1H}^*) \geq \Pi_L(s_{1L}^*, s_H^*(s_{1L}^*)) \) and \( s_{1H}^*(s_{1L}) = s_L^*(s_{1L}) \), where the first condition selects the global optimum for the leader and the second verifies that the follower’s best response is in fact to produce a lower quality product than the leader. Similar relations hold for \( s_1^* = s_{1L}^* \).

The first order conditions of this system yield complex polynomials that must be solved numerically. The reported solution in Table 1 below has second order conditions consistent with a maximum and is confirmed to be in the feasible set \( R \).

<table>
<thead>
<tr>
<th>Firm</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality</td>
<td>0.1226</td>
<td>0.0239</td>
</tr>
<tr>
<td>Price</td>
<td>0.0519</td>
<td>0.0051</td>
</tr>
<tr>
<td>Share</td>
<td>0.5256</td>
<td>0.2628</td>
</tr>
<tr>
<td>Profit</td>
<td>0.0122</td>
<td>0.0013</td>
</tr>
<tr>
<td>Surplus</td>
<td>0.0202</td>
<td>0.0008</td>
</tr>
<tr>
<td>Total surplus</td>
<td>0.0211</td>
<td></td>
</tr>
<tr>
<td>Total profit</td>
<td>0.0130</td>
<td></td>
</tr>
<tr>
<td>Welfare</td>
<td>0.0341</td>
<td></td>
</tr>
<tr>
<td>No purchase share</td>
<td>0.2116</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Fixed cost competitive equilibrium

Note that the first moving firm becomes the high quality firm but offers lower quality at a higher price than compared to the efficient solution, resulting in a considerable (72%) loss in total consumer welfare.
The competitive solution plays an important role in the specification of our model, since we wish to compare the equilibrium outcomes of our model to one in which both firms have a profit-maximizing objective. In particular, we will set the target level of profit $\Pi_0$ for firm $CS$ as a proportion of the profit it would have made if it had adopted a profit-maximization objective (the reference profit). Therefore, the appropriate reference level of profit depends on whether $CS$ moves first or second. That is, if $CS$ moves first, the reference profit is the profit of the first moving firm ($F_1$), $\Pi_{PM_1} = 0.0122$, whereas if it moves second, the appropriate reference profit is $\Pi_{PM_2} = 0.0013$. To summarize, for $\beta \in [0, 1]$ we define the target profit $\Pi_0$ as follows:

$$\Pi_0 = \begin{cases} \beta \Pi_{PM_1} = 0.0122 \beta & F_1 = CS \\ \beta \Pi_{PM_2} = 0.0013 \beta & F_1 = PM \end{cases}$$ (4)

### 3.2 Price equilibrium

With asymmetric firm objectives, the solution to the price equilibrium involving the surplus-maximizing ($CS$) and profit-maximizing ($PM$) firms will depend upon whether the $CS$ firm is the high or low quality firm. Before considering the separate cases, we derive a general result that informs the equilibrium solution.

**Proposition 2.** Regardless of the nature of firm costs, the minimum profit constraint is always binding for the surplus-maximizing firm.

**Proof.** Inspection of equation 3 reveals that total surplus is independent of the cost specification. To establish that the minimum profit constraint will bind, it suffices to show that $\Psi_T$ is strictly declining in prices over the feasible set $R$. Inspecting the partial derivative with respect to the lower price, $\frac{\partial \Psi_T}{\partial p_L} = \frac{p_L s_H - p_H s_L}{s_L(s_H - s_L)} < 0$ by nature of the conditions $p_L < p_H(s_L/s_H)$ and $s_L < s_H$ embedded in Definition 1. Similarly, $\frac{\partial \Psi_T}{\partial p_H} = \frac{1}{s_H - s_L} \left( p_H - p_L - (s_H - s_L) \right) < 0$ by nature of the condition $p_H - s_H + s_L < p_L$ in Definition 1. Thus, regardless of whether the $CS$ firm is the low or high quality firm, it will set its price to the lowest value that attains the required profit level $\Pi_0$.  

A corollary to this result is that the $CS$ firm’s best price response function when it is the high quality firm ($p_H^*$) will be the smaller-valued root that solves the quadratic equation $\Pi_H(s_L, s_H, p_L, p_H) = -$
Π₀ = 0 for \( p_H \) (and similarly when it is the low quality firm).

**Surplus-maximizing firm is the high quality firm**

When \( CS \) is the high quality firm, Proposition 2 implies the price equilibrium will be the unique vector \((p^*_L, p^*_H)\) that solves the system \( \frac{\partial \Pi_H}{\partial p_L} = 0 \) and the smaller root of \( \Pi_H = \Pi_0 \), which is given by: \(^9\)

\[
\begin{align*}
p^*_L(s_L, s_H) &= \frac{s_L}{2} \left( \frac{s_H - s_L}{2s_H - s_L} \right) \left( 1 - \sqrt{1 - \frac{2(\Pi_0 + s_H^2)(2s_H - s_L)}{s_H(s_H - s_L)}} \right) \\
p^*_H(s_L, s_H) &= s_H \left( \frac{s_H - s_L}{2s_H - s_L} \right) \left( 1 - \sqrt{1 - \frac{2(\Pi_0 + s_H^2)(2s_H - s_L)}{s_H(s_H - s_L)}} \right)
\end{align*}
\]

(5a) \( (5b) \)

The solution \((p^*_L, p^*_H)\) is a stable equilibrium. The second order condition \( \frac{\partial^2 \Pi_L}{\partial p^2 L} < 0 \), indicating the \( PM \) firm has no incentive to deviate. Similarly, \( CS \) will not deviate because it cannot further reduce price and increasing price lowers its objective.

It is informative to demonstrate the effect of varying the parameter \( \beta \), which scales the required level of profit from 0 (a non-profit) to 1 (competitive level). To do this, we first assume \( CS \) was the first mover and fix quality levels at the \( \{PM_1, PM_2\} \) values. That is, we assign \( s_{CS} = s_{PM_1} \), \( s_{PM} = s_{PM_2} \) and \( \Pi_0 = \beta \Pi_{PM_1} = 0.0122\beta \). Next, we plot the price reaction functions of the two firms in Figure 1 below. \(^10\) The \( PM \) firm’s reaction is independent of \( \beta \) and thus is represented with a single line. The \( CS \) firm’s reaction functions are plotted for \( \beta \in \{0, 0.25, 0.5, 0.75, 1\} \). Holding qualities fixed, the effect of lowering \( \beta \) is clearly to lower the prices of both firms. As expected, for \( \beta = 1 \), we recover the \( \{PM_1, PM_2\} \) prices.

\(^9\) The smaller valued root is the unique solution because total surplus is strictly declining in prices.

\(^10\) The price reaction functions are \( \frac{\partial \Pi_L}{\partial p_L} = 0 \) and the smaller root of \( \Pi_H = \Pi_0 \), which are: \( p_L = \frac{p_H + s_H - s_L}{2} \) and \( p_H = \frac{p_L + s_H - s_L}{2} \left( 1 - \sqrt{1 - \frac{4(s_H - s_L)(\Pi_0 + s_H^2)}{p_L^2 + \Pi + s_H - s_L}} \right) \).
Surplus-maximizing firm is the low quality firm

When CS is the low quality firm, Proposition 2 implies the price equilibrium will be the unique vector \((p^*_L, p^*_H)\) that solves the system \(\frac{\partial \Pi_H}{\partial p_H} = 0\) and the smaller root of \(\Pi_L = \Pi_0\), which is given by:

\[
p^*_L(s_L, s_H) = \frac{s_L(s_H - s_L)}{2(2s_H - s_L)} \left(1 - \sqrt{1 - \frac{8(\Pi_0 + s_L^2)(2s_H - s_L)}{s_L(s_H - s_L)}}\right) \tag{6a}
\]

\[
p^*_H(s_L, s_H) = \frac{(s_H - s_L)(4s_H - s_L)}{8s_H - 4s_L} - \frac{s_L(s_H - s_L)}{8s_H - 4s_L} \sqrt{1 - \frac{8(\Pi_0 + s_L^2)(2s_H - s_L)}{s_L(s_H - s_L)}}. \tag{6b}
\]

The second order optimality conditions are also easily verified for this case.

Again we demonstrate the effect of varying the parameter \(\beta\) in Figure 2 below, assuming PM was the first mover. We assign \(s_{CS} = s_{PM_2}, s_{PM} = s_{PM_1}\) and \(\Pi_0 = \beta \Pi_{PM_2} = 0.0013\beta\).\(^{11}\) Holding these qualities fixed, the effect of lowering \(\beta\) is again to lower the prices of both firms.

\(^{11}\)Here the price reaction functions are \(\frac{\partial \Pi_H}{\partial p_H} = 0\) and the smaller root of \(\Pi_L = \Pi_0\), which are: \(p_L = \frac{p_H + s_H - s_L}{2} \left(1 - \sqrt{1 - \frac{4s_H(s_H - s_L)(\Pi_0 + s_L^2)}{p_H^2s_L}}\right)\) and \(p_H = \frac{1}{2}(p_L + s_H - s_L)\).
3.3 Product equilibrium when the surplus-maximizing firm moves first

With the price equilibrium defined, the algorithm to derive the product equilibrium follows a similar process to the one described for the \{PM_1, PM_2\} equilibrium benchmark in Section 3.1.\footnote{One important difference is that, rather than solving first order conditions to infer the best quality response of the follower and the optimal quality choice of the first mover, we employ a global (direct) optimization approach. This strategy is taken as the first mover objective functions are not necessarily globally concave and thus the maxima may correspond to discontinuity points in the best quality response functions (as may be seen in the graphical solutions when the PM firm moves first – Figures 4 and 8). For these optimization procedures, we employ the simplex algorithm of Nelder and Mead (1965) using a convergence tolerance of $10^{-10}$ and a penalty barrier to ensure qualities and prices remain in the feasible set $R$.} When \(CS\) moves first, the follower is a profit maximizer, and its best quality reaction functions remain

\[
\begin{align*}
\hat{s}_L^* &= \argmax_{s_L} \Pi_L(s_L, s_1) \\
\hat{s}_H^* &= \argmax_{s_H} \Pi_H(s_1, s_H)
\end{align*}
\]

The leader \(CS\) first solves its best “high” and “low” quality choices as

\[
\begin{align*}
\hat{s}_1^H &= \argmax_{s_1} \Psi_T(s_L^*(s_1), s_1) \\
\hat{s}_1^L &= \argmax_{s_1} \Psi_T(s_1, s_H^*(s_1))
\end{align*}
\]

\(CS\) selects \(s_1^* = s_1^H \) if \(\Psi_T(s_L^*(s_1^H), s_1^H) \geq \Psi_T(s_1^L, s_H^*(s_1^L))\) and \(s_2^*(s_1^H) = s_1^L, s_1^* = s_1^L \) otherwise.

Once a value for \(\beta\) is set, the product equilibrium may be solved numerically. We demonstrate the solution process graphically for the case \(\beta = 0.75\) in Figure 3 below. The plot displays the \(PM\) firm’s best quality response function \(s_{PM}^*(s_{CS}) = s_2^*(s_{CS})\) and the \(CS\) firm’s objective function evaluated at \(PM\) firm’s best quality response \(\Psi_T(s_L^*(s_{CS}), s_{CS})\) if \(s_{PM}^*(s_{CS}) = s_1^*(s_{CS})\) and \(\Psi_T(s_{CS}, s_H^*(s_{CS}))\) if \(s_{PM}^*(s_{CS}) = s_2^*(s_{CS})\). The equilibrium values of \(\{s_{CS}, s_{PM}\}\) correspond to
the argument and value of the $PM$ firm’s best quality response function, at the point where the $CS$ firm’s objective is maximized, $\{s_{CS}, s_{PM}\} = \{0.1635, 0.0375\}$, which is indicated by the dashed vertical line.

Figure 3: Fixed cost quality equilibrium, $CS$ first mover ($\beta = 0.75$)

We present solutions for $\beta \in \{0, 0.25, 0.5, 0.75, 1\}$ in Table 5 of Appendix A. The table reports equilibrium values and comparison percentages to the analogous $\{PM_1, PM_2\}$ outcome. Additionally, we report the “surplus efficiency” of the equilibrium, a percentage comparison to the theoretical maximum surplus obtained in the efficient solution. Several aspects of the results are worth highlighting:

- As $\beta$ decreases, total surplus increases (as expected) as does total welfare, while the share of the outside good decreases. The surplus efficiency ranges from 45% when the surplus-maximizing firm must generate competitive-level profits to 99% when it is a non-profit. That is, a 1% reduction in the required level of profit on average generates a 0.54% increase in surplus efficiency. There is therefore a reasonable “return” on foregone profits for the $CS$ firm in terms of surplus production.

- $CS$ always becomes the high quality firm and offers a higher quality product than it would under a profit maximization objective (regardless of the value of $\beta$).
• As $\beta \to 0$, the CS firm’s quality increases, while its price decreases. CS’s market share and surplus increase.

• As $\beta \to 0$, the PM firm’s quality, price, share and surplus are increasing for $\beta \geq 0.75$, and decreasing for $\beta < 0.75$. That is, for small deviations from competitive level profits, total surplus is maximized by adding to the surplus contribution of both products, whereas for higher reductions in the target profit level it is more efficient for CS to maximize its own surplus at the expense of the PM firm. This change in regime corresponds to when the equilibrium products are minimally differentiated in quality over the values of $\beta$ ($\beta = 0.75$).

• When $\beta = 1$ (CS must make as much profit as in the competitive equilibrium), we do not recover the $\{PM_1, PM_2\}$ solution – rather, CS is able to moderately (13%) improve upon the total surplus by offering an equi-profit product at a higher quality and price (as compared to the PM1 firm). Thus, simply by committing to a surplus-maximizing objective, CS can improve its desired outcome without sacrificing profits. While overall welfare is increased (8%) relative to the $\{PM_1, PM_2\}$ equilibrium, the PM firm’s profits are reduced by 11%.

3.4 Product equilibrium when the profit-maximizing firm moves first

When PM moves first, the follower is the surplus-maximizer, and its best quality reaction functions become $s^*_L(s_1) = \arg\max_{s_L} \Psi_T(s_L, s_1)$ and $s^*_H(s_1) = \arg\max_{s_H} \Psi_T(s_1, s_H)$. The leader PM solves its best “high” and “low” quality choices as $s^*_L(s_1) = \arg\max_{s_1} \Pi_L(s^*_L(s_1), s_1)$ and $s^*_L(s_1) = \arg\max_{s_1} \Pi_L(s_1, s^*_H(s_1))$. PM selects $s^*_1 = s^*_1$ if $\Pi_H(s^*_L(s^*_1), s^*_1) \geq \Pi_L(s^*_1, s^*_H(s^*_1))$ and $s^*_2(s_1) = s^*_L(s^*_1)$, and $s^*_1 = s^*_1$ otherwise.

We demonstrate the solution process graphically for the case $\beta = 0.75$ in Figure 4 below. The plot displays the CS firm’s best quality response function $s^*_C(s_{PM}) = s^*_2(s_{PM})$ and the PM firm’s objective function evaluated at CS firm’s best quality response ($\Pi_L(s^*_L(s_{PM}), s_{PM})$ if $s^*_C(s_{PM}) = s^*_2(s_{PM})$ and $\Pi_L(s_{PM}, s^*_H(s_{PM}))$ if $s^*_C(s_{PM}) = s^*_H(s_{PM}))$. The equilibrium values of $\{s_{PM}, s_{CS}\}$ correspond to the argument and value of the CS firm’s best quality response function, at the point where the PM firm’s objective is maximized, $\{s_{PM}, s_{CS}\} = \{0.1170, 0.0345\}$, which is indicated by the dashed vertical line.
Figure 4: Fixed cost quality equilibrium, $PM$ first mover ($\beta = 0.75$)

We present solutions for $\beta \in \{0, 0.25, 0.5, 0.75, 1\}$ in Table 6 of Appendix A. Several aspects of the results are worth highlighting:

- Total surplus increases as $\beta$ decreases, while the share of the outside good decreases. The surplus efficiency ranges from 40% (competitive-level profits) to 52% (non-profit) – thus on average, a 1% reduction in profits only generates a 0.12% increase in surplus efficiency. Therefore, in this case, sacrificing profits is costly for the $CS$ firm in terms of surplus production.

- $PM$ always becomes the high quality firm and suffers relatively minor losses in profit as $\beta$ decreases. When $\beta = 1$ ($CS$ must make as much profit as in the competitive equilibrium), we recover the $\{PM_1, PM_2\}$ solution (no profit loss for $PM$). At $\beta = 0$, the $PM$ firm loses only 37% of its competitive-level profits. Thus, the first mover advantage is sufficiently great for $PM$ to largely mitigate the influence of a surplus-maximizing firm in the market.

- For $PM$, quality, market share, and surplus increase when $\beta$ decreases, while price and profits decline.

- For $CS$, quality, price, market share and surplus increase when $\beta$ decreases.

- Compared to when $CS$ is the first mover:
– Total surplus is lower (regardless of \( \beta \)). Thus, given the choice, CS will always prefer to be the first mover.

– The high quality is lower.

– Products are less quality differentiated (i.e., \( s_H - s_L \) is smaller).

4 Variable costs of quality production

In this section, we consider the case of variable costs of production, \( C_i = q_i s_i^2 \).

4.1 Benchmark cases

As in the case of fixed costs, we solve for the efficient and competitive benchmarks.

Efficient

If a single product is offered, the firm objective is to maximize

\[
\Psi = \frac{1}{T} \int (ts - s^2) \, dt = \frac{(s-p)(p+s-2s^2)}{2s}.
\]

Maximizing first with respect to price yields a marginal cost pricing rule, \( p = s^2 \). Substituting back in and maximizing with respect to \( s \) gives an optimal quality of \( s = \frac{1}{3} \), and thus a total surplus of \( \frac{2}{27} = 0.741 \) and market share of \( \frac{2}{3} \). The second order conditions for \( s \) and \( p \) indicate this solution is in fact a maximum. We are more concerned with the two product efficient solution, which is given by the qualities and prices \((s_1, s_2, p_1, p_2)\) which solve:

\[
\max_{s_1, s_2, p_1, p_2} \int_{s_1}^{p_2} (ts_1 - s_1^2) \, dt + \int_{s_1}^{p_2} (ts_2 - s_2^2) \, dt.
\]

Maximizing first with respect to prices \( \{p_L, p_H\} \) gives marginal pricing rules for both products. Substituting in and maximizing simultaneously with respect to \( \{s_L, s_H\} \) (and checking second order conditions and feasibility constraints) gives the solution summarized in Table 2 below:
<table>
<thead>
<tr>
<th>Product</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality</td>
<td>0.4000</td>
<td>0.2000</td>
</tr>
<tr>
<td>Price</td>
<td>0.1600</td>
<td>0.0400</td>
</tr>
<tr>
<td>Share</td>
<td>0.4444</td>
<td>0.4444</td>
</tr>
<tr>
<td>Profit</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Surplus</td>
<td>0.0640</td>
<td>0.0160</td>
</tr>
<tr>
<td>Total surplus</td>
<td>0.0800</td>
<td></td>
</tr>
<tr>
<td>Total profit</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Welfare</td>
<td>0.0800</td>
<td></td>
</tr>
<tr>
<td>No purchase share</td>
<td>0.1111</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Variable cost two product efficient solution

Allowing a second product results in products of higher and lower quality than the single product efficient solution, an increase in total consumer surplus of 0.006 (8%), and a 67% decrease in the share of the outside good.

**Competitive**

Our competitive benchmark is a sequential entry game played by two profit-maximizing firms, analogous to the model of Moorthy (1988) §6. With respect to variable costs, we again refer to this game as \((PM_1, PM_2)\). Holding quality choices fixed, solving the system of first order conditions \(\{\frac{\partial \Pi_L}{\partial p_L} = 0, \frac{\partial \Pi_H}{\partial p_H} = 0\}\) yields equilibrium prices, which may be substituted to express profits entirely in terms of quality choices.\(^{13}\)

\[
\Pi_L = \frac{SLSH(s_H - s_L)(1 + s_H - s_L)^2}{(4s_H - s_L)^2}
\]
\[
\Pi_H = \frac{s_H^2(s_H - s_L)(2 - 2s_H - s_L)^2}{(4s_H - s_L)^2}
\]

\(^{13}\)Equilibrium prices are: 
\(p_L = \frac{sl(sh-sl+s_h^2+2s_l^2)}{4s_H-s_L}\) and \(p_H = \frac{s_H(2(s_H-s_L+s_h^2)+s_L^2)}{4s_H-s_L}\).
As with the fixed cost \(\{PM_1, PM_2\}\) game, the product equilibrium must be solved using numerical methods.\(^{14}\) The solution is presented in Table 3 below.

<table>
<thead>
<tr>
<th>Firm</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality</td>
<td>0.2836</td>
<td>0.1444</td>
</tr>
<tr>
<td>Price</td>
<td>0.1318</td>
<td>0.0440</td>
</tr>
<tr>
<td>Share</td>
<td>0.3691</td>
<td>0.3263</td>
</tr>
<tr>
<td>Profit</td>
<td>0.0190</td>
<td>0.0075</td>
</tr>
<tr>
<td>Surplus</td>
<td>0.0367</td>
<td>0.0077</td>
</tr>
<tr>
<td>Total surplus</td>
<td>0.0444</td>
<td></td>
</tr>
<tr>
<td>Total profit</td>
<td>0.0265</td>
<td></td>
</tr>
<tr>
<td>Welfare</td>
<td>0.0709</td>
<td></td>
</tr>
<tr>
<td>No purchase share</td>
<td>0.3046</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Variable cost competitive equilibrium

Comparing the competitive solution to the efficient solution, we see that qualities are lower and that products are less quality differentiated. The first moving firm assumes the high quality position, which is close to the optimally efficient one product quality. Consumer surplus delivered by the products in the competitive case is about half that in the efficient case.

The relevant target profits \(\Pi_0\) for the CS firm with variable quality costs are as follows:

\[
\Pi_0 = \begin{cases} 
\beta \Pi_{PM_2} = 0.0190 \beta & F_1 = CS \\
\beta \Pi_{PM_2} = 0.0075 \beta & F_1 = PM 
\end{cases}
\]

\(^{14}\)In evaluating the model solution, we find small differences from the results reported by Moorthy (1988). Moorthy (1991) reports a revised solution to the simultaneous move equilibrium with two profit-maximizing firms, but does not explicitly reconsider the sequential move equilibrium.

19
4.2 Price equilibrium

Surplus-maximizing firm is the high quality firm

As with fixed quality costs, when $CS$ is the high quality firm, Proposition 2 implies the price equilibrium will be the unique vector $(p^*_L, p^*_H)$ that solves the system $\frac{\partial \Pi_L}{\partial p_L} = 0$ and the smaller root of $\Pi_H = \Pi_0$, which is given by:

$$p^*_L(s_L, s_H) = \frac{s_L}{4s_H(2s_H - s_L)} \left( 4s_H^2 s_L + s_H(s_H - s_L)(2 + 2s_H + s_L) - s_H(s_H - s_L)(2 - 2s_H + s_L) \sqrt{1 - \frac{8\Pi_0 s_H(2s_H - s_L)}{s_H^2(2 - 2s_H + s_L)^2(s_H - s_L)}} \right) \tag{8a}$$

$$p^*_H(s_L, s_H) = \frac{1}{4s_H - 2s_L} \left( 2s_H s_L^2 + s_H(s_H - s_L)(2 + 2s_H + s_L) - s_H(s_H - s_L)(2 - 2s_H + s_L) \sqrt{1 - \frac{8\Pi_0 s_H(2s_H - s_L)}{s_H^2(2 - 2s_H + s_L)^2(s_H - s_L)}} \right) \tag{8b}$$

The solution $(p^*_L, p^*_H)$ is a stable equilibrium. The second order condition $\frac{\partial^2 \Pi_L}{\partial^2 p_L} < 0$, indicating the $PM$ firm has no incentive to deviate. Similarly, $CS$ will not deviate because it cannot further reduce price and increasing price lowers its objective. In Figure 5 below we demonstrate the effect of varying the parameter $\beta$, assuming $PM$ was the first mover.\(^{15}\) We assign $s_{CS} = s_{PM1}$, $s_{PM} = s_{PM2}$ and $\Pi_0 = \beta \Pi_{PM1} = 0.0190\beta$. Holding these qualities fixed, the effect of lowering $\beta$ is to lower the prices of both firms.

---

\(^{15}\)The price reaction functions are: $p_L = \frac{4s_L}{2s_H}(p_H + s_H s)$, $p_H = \frac{1}{2}(p_L + s_H - s_L + s_H^2 - (p_L + s_H - s_L - s_H^2)\sqrt{1 - \frac{4\Pi_0(s_H - s_L)}{(p_L + s_H - s_L - s_H^2)^2}}}$.  

---

20
Surplus-maximizing firm is the low quality firm

When CS is the low quality firm, Proposition 2 implies the price equilibrium will be the unique vector \((p_L^*, p_H^*)\) that solves the system \(\frac{\partial \Pi_H}{\partial p_H} = 0\) and the smaller root of \(\Pi_L = \Pi_0\), which is given by:

\[
p_L^*(s_L, s_H) = \frac{1}{4s_H - 2s_L} \left(2s_H^2 s_L + s_L(s_H - s_L)(1 - s_H - s_L) - s_L(s_H - s_L)(1 + s_H - s_L)\right) \left(1 - \frac{8\Pi_0 s_L(2s_H - s_L)}{s_L^2(1 + s_H - s_L)^2(s_H - s_L)}\right)
\]

\[(9a)\]

\[
p_H^*(s_L, s_H) = \frac{1}{8s_H - 4s_L} \left(4s_H^2(1 + s_H) - 2s_L(3s_H - s_L) + s_L(s_H - s_L)(1 - s_H - s_L)ight) \left(1 - \frac{8\Pi_0 s_L(2s_H - s_L)}{s_L^2(1 + s_H - s_L)^2(s_H - s_L)}\right)
\]

\[(9b)\]

The second order optimality conditions apply. In Figure 6 below we demonstrate the effect of varying the parameter \(\beta\), assuming CS was the first mover.\(^\text{16}\) We assign \(s_{PM} = s_{PM_1}\), \(s_{CS} = s_{PM_2}\) and \(\Pi_0 = \beta \Pi_{PM_2} = 0.0075\beta\). Holding these qualities fixed, the effect of lowering \(\beta\) is to lower the

\(^\text{16}\)The price reaction functions are: \(p_L = \frac{1}{2s_H} \left(p_{HsL} + s_H s_L^2 - (p_{HsL} - s_H s_L^2)\sqrt{1 - \frac{4g_H s_H (s_H - s_L)}{s_L^2(p_H - s_H s_L)^2}}\right),\) \(p_H = \frac{1}{2}(p_L + s_H - s_L + s_H^2)\).
prices of both firms.

Figure 6: Variable cost price equilibrium for $s_{CS} < s_{PM}$ ($s_{CS} = s_{PM_2}, s_{PM} = s_{PM_1}$)

4.3 Product equilibrium when the surplus-maximizing firm moves first

The algorithm to solve the product equilibrium is exactly as described in Section 3.3 for fixed costs. We demonstrate the solution process graphically for the case $\beta = 0.75$ in Figure 7 below. The plot displays the $PM$ firm’s best quality response function and the $CS$ firm’s objective function evaluated at $PM$ firm’s best quality response. The equilibrium values of $\{s_{CS}, s_{PM}\}$ correspond to the argument and value of the $PM$ firm’s best quality response function, at the point where the $CS$ firm’s objective is maximized, $\{s_{CS}, s_{PM}\} = \{0.3333, 0.2054\}$, which is indicated by the dashed vertical line.
Equilibrium values are reported in Table 7 of Appendix B. Several aspects of the results are worth highlighting:

- Total surplus increases as $\beta$ decreases (as expected), as does welfare, while the share of the outside good (weakly) decreases. When the surplus-maximizing firm moves first, the surplus efficiency ranges from 57% (competitive-level profits) to 94% (non-profit), a 0.37% increase in consumer surplus for every 1% sacrifice in profits.

- $CS$ always becomes the high quality firm and both firms offer higher quality products than would obtain under dual profit maximization (regardless of the value of $\beta$).

- As with the variable cost $\{CS, PM\}$ equilibrium, the dependence of equilibrium values on $\beta$ is divided into two regimes at the point of minimal equilibrium quality differentiation ($\beta = 0.25$).

- For $\beta \geq 0.25$, as $\beta$ decreases:
  - $CS$ selects the one product optimally efficient quality of $\frac{1}{3}$, and reduces price.
  - $PM$ increases quality and price.
  - Quality and price are chosen so that market shares stay a constant $\frac{1}{3}$ for both firms (and the outside good).
The surplus contribution of both products increases. Thus, for most of the range of $\beta$, total surplus is maximized by adding to the surplus contribution of both products.

- For $\beta < 0.25$, as $\beta$ decreases:
  - CS increases quality and reduces price.
  - PM decreases quality and price.
  - CS's market share increases, while the outside good and PM shares decrease.
  - The surplus contribution of the CS product increases, while PM's declines.

- When $\beta = 1$, CS is able to slightly (3%) improve upon the total surplus by offering an equi-profit product at a higher quality and price (as compared to the PM$_1$ firm). The $\{CS, PM\}$ equilibrium here pareto-dominates the comparable $\{PM_1, PM_2\}$ equilibrium, as welfare is increased (4%) and the PM firm’s profits are increased by 23%.

- Relative to the $\{CS, PM\}$ equilibrium with fixed costs, qualities and surplus are higher. This result reflects the fact that total cost is lower when costs are variable.

4.4 Product equilibrium when the profit-maximizing firm moves first

The algorithm to solve the product equilibrium is exactly as described in Section 3.4 for fixed costs. We demonstrate the solution process graphically for the case $\beta = 0.75$ in Figure 8 below. The plot displays the CS firm’s best quality response function and the PM firm’s objective function evaluated at CS firm’s best quality response. The equilibrium values of $\{s_{PM}, s_{CS}\}$ correspond to the argument and value of the CS firm’s best quality response function, at the point where the PM firm’s objective is maximized, $\{s_{PM}, s_{CS}\} = \{0.3273, 0.2561\}$, which is indicated by the dashed vertical line.
Figure 8: Variable cost quality equilibrium, $PM$ first mover ($\beta = 0.75$)

Equilibrium values are reported in Table 8 of Appendix B. We note the following regarding the results:

- Total surplus increases as $\beta$ decreases (as expected), as does welfare, while the share of the outside good decreases. The surplus efficiency ranges from 70% (competitive-level profits) to 94% (non-profit), a 0.24% increase in consumer surplus for every 1% sacrifice in profits.

- Higher quality products are offered than in the $\{PM_1, PM_2\}$ equilibrium (regardless of the value of $\beta$), but the identity of the high quality firm depends on the value of $\beta$. For $\beta \geq 0.5$, we obtain the usual result that the first mover ($PM$) assumes the high quality position (high quality leader), while for $\beta < 0.5$ the $PM$ firm leads with low quality. This regime change again takes place at the point of minimal equilibrium quality differentiation ($\beta = 0.5$).

- For $\beta \geq 0.5$, as $\beta$ decreases:
  
  - $CS$ increases quality and reduces price.
  
  - $PM$ increases quality and price.
  
  - $CS'$s market share increases, while $PM'$s share and the outside good share decline.
- The surplus contribution of both products increases.

- For $\beta < 0.5$, as $\beta$ decreases:
  
  - $CS$ increases quality and reduces price.
  
  - $PM$ decreases quality and price.
  
  - $CS'$s market share increases, while the outside good and $PM$ shares decrease.
  
  - The surplus contribution of the $CS$ product increases, while $PM'$s declines. Thus, for most of the range of $\beta$, total surplus is maximized by adding to the surplus contribution of both products.

- When $\beta = 1$, $CS$ is still able to significantly (26%) improve upon the total surplus by offering an equi-profit product at a higher quality and price (as compared to the $PM_1$ firm). While overall welfare is increased (7%) relative to the $\{PM_1, PM_2\}$ equilibrium, the $PM$ firm’s profits are reduced by 37%. We conclude that the first mover advantage is insufficiently strong for $PM$ to counteract the effects of competing with a surplus-maximizing firm when the costs entering the binding profit constraint are variable.

- Compared to when $CS$ is the first mover, total surplus (hence surplus efficiency) is higher (at all levels of $\beta$). Thus, given the choice, $CS$ will prefer to be the second mover (follower) in quality.

- Relative to the $\{PM, CS\}$ equilibrium with fixed costs, qualities and surplus are higher (total cost is lower when costs are variable).
<table>
<thead>
<tr>
<th>Costs</th>
<th>Fixed</th>
<th>Variable</th>
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</thead>
<tbody>
<tr>
<td>First mover</td>
<td>CS</td>
<td>PM</td>
</tr>
<tr>
<td>min surplus efficiency ($\beta = 1$)</td>
<td>45%</td>
<td>40%</td>
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<tr>
<td>max surplus efficiency ($\beta = 0$)</td>
<td>99%</td>
<td>52%</td>
</tr>
<tr>
<td>$\Delta$ surplus efficiency from -1% CS profit</td>
<td>+0.54%</td>
<td>+0.12%</td>
</tr>
<tr>
<td>$CS$ is high quality firm</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>$CS$ quality higher than in ${PM_1, PM_2}$</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>$\beta \rightarrow 0 \implies CS$ quality increases</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>$\beta \rightarrow 0 \implies CS$ price decreases</td>
<td>Y</td>
<td>N</td>
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<tr>
<td>$\beta \rightarrow 0 \implies CS$ share increases</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>$\beta \rightarrow 0 \implies PM$ quality increases</td>
<td>$\beta \geq 0.75$</td>
<td>$\beta &lt; 1$</td>
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<tr>
<td>$\beta \rightarrow 0 \implies PM$ price decreases</td>
<td>$\beta &lt; 0.75$</td>
<td>Y</td>
</tr>
<tr>
<td>$\beta \rightarrow 0 \implies PM$ share increases</td>
<td>$\beta \geq 0.75$</td>
<td>Y</td>
</tr>
</tbody>
</table>

Table 4: Results summary

5 Conclusion

In this paper, we pose the question of what happens to competitive markets when one of the firms makes delivering value to consumers its primary objective. To analyze the problem, we construct a duopoly model of strategic competition between a profit-maximizing firm and a firm that maximizes the consumer surplus subject to a minimum profit constraint. Moves in quality are taken sequentially in the first stage of a two-stage game, while prices are set simultaneously in the second stage, conditional upon quality choices. Products are vertically differentiated and the cost to produce quality is convex. We separately consider the cases of quadratic fixed costs and quadratic variable costs. Under each assumption for costs, we compare the surplus-maximizing/profit-maximizing equilibrium at different levels of the minimum profit constraint with the equilibrium obtained when both firms operate as profit-maximizers, and to the optimally efficient market outcome. These comparisons allow us to infer both the absolute and relative effectiveness of sacrificing profit in order to increase consumer surplus.

Our analysis provides a number of key findings, which are summarized in Table 4 above. An intuitive result is that we find total consumer surplus increases as the minimum profit level for the surplus-maximizing firm declines. However, the efficiency of trading profit for surplus production depends upon both the cost structure and the order of quality choice. When costs are fixed, there is a significant first mover advantage associated with becoming the high quality firm. Simply
by committing to the surplus-maximizing objective (and not sacrificing any profits), the surplus-maximizing firm is able to increase total consumer surplus by 13%, relative to what would have been obtained if it were a profit-maximizing firm. The total consumer surplus in this case is 45% of that in the optimally efficient market outcome. Returns to sacrificing profits are also favorable: a 1% reduction in the required level of profit on average generates a 0.54% increase in surplus efficiency. By the surplus-maximizing firm forgoing all profit, consumer surplus reaches 99% of its theoretical limit with fully efficient products. By contrast, when the surplus-maximizing firm moves second, the best it can do without giving up profits is to recoup the competitive-level total consumer surplus. Further, returns to sacrificing profits are far smaller: on average, a 1% reduction in profits only generates a 0.12% increase in surplus efficiency. Even when the surplus-maximizing firm operates as a non-profit, total consumer surplus is only 52% of the fully efficient value. With variable costs, the order of move preference is reversed for the surplus-maximizing firm. When the surplus-maximizing firm chooses quality first, it becomes the high quality firm. With no sacrifice of profits, consumer surplus is 3% higher than with two profit maximizers, or 57% efficient. Decreasing the required profit by 1% increases surplus efficiency by 0.37%, to a maximum of 94% theoretical efficiency when the surplus-maximizing firm operates as a non-profit. When the surplus-maximizing firm chooses quality second, it becomes the high quality firm provided it sacrifices enough profit (>50% of the competitive level). However, even at more modest levels of “altruism,” surplus efficiency is higher than when the surplus-maximizing firm moves first. With no sacrifice of profits, consumer surplus is 26% higher than with two profit maximizers (and 70% efficient). Decreasing the required profit by 1% increases the surplus efficiency by 0.24%, to a maximum of 94% theoretical efficiency when the surplus-maximizing firm operates as a non-profit. A key implication of these results is that in order to achieve highly favorable outcomes, the surplus-maximizing firm must move first when production costs are fixed, whereas the order of quality moves is less restrictive when costs are variable.

There are several avenues by which the work may be extended. One possibility is to investigate the properties of a model in which the “alternative” firm maximizes an objective other than consumer surplus, such as market share or the surplus of only its own customer base. Another might be to relax the assumptions of a uniform distribution of consumer willingness to pay for quality or technologically equivalent firms. This exercise might shed light on how general our results are. Also of potential interest would be to explore the consequence of repeated interactions among the firms.
or increasing the number of market participants. A final extension might consider how outcomes would differ with horizontally differentiated products.
References


## A Results – Fixed cost of quality production

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<th>0.75</th>
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<td>0.1625 (33%)</td>
<td>0.1619 (32%)</td>
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<td>0.7228 (38%)</td>
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<td>0.0122 (0%)</td>
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<td>CS surplus</td>
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<td>0.0137 (-43%)</td>
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<td>0.0012 (-77%)</td>
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<tr>
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<td>0.0288 (37%)</td>
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<td>0.0031 (-76%)</td>
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<td>90%</td>
<td>79%</td>
<td>55%</td>
<td>49%</td>
<td>45%</td>
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Table 5: CS first mover (\{CS, PM\}) equilibrium at different levels of required CS profit (\(\Pi_0 = 0.0122,3\)). Numbers in parentheses denote percentage increases/decreases relative to analogous \(\{PM_1, PM_2\}\) values. Surplus efficiency is total consumer surplus divided by the maximum feasible two product surplus (0.0527).
Figure 9: Plot of CS first mover ($\{CS, PM\}$) equilibrium values. CS firm values in black, PM firm values in red. Dashed lines represent corresponding $\{PM_1, PM_2\}$ values. Dotted line in lower right panel represents the fully efficient surplus value.
<table>
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<tr>
<th>β</th>
<th>0.00</th>
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<td>51%</td>
<td>50%</td>
<td>48%</td>
<td>45%</td>
<td>41%</td>
<td>40%</td>
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Table 6: PM first mover (\{PM, CS\}) equilibrium at different levels of required CS profit (\(\Pi_0 = 0.0013\)). Numbers in parentheses denote percentage increases/decreases relative to analogous \{PM_1, PM_2\} values. Surplus efficiency is total consumer surplus divided by the maximum feasible two product surplus (0.0527).
Figure 10: Plot of CS first mover (\{CS, PM\}) equilibrium values. CS firm values in black, PM firm values in red. Dashed lines represent corresponding \{PM_1, PM_2\} values. Dotted line in lower right panel represents the fully efficient surplus value.
## Results – Variable cost of quality production

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<tr>
<th>β</th>
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<th>0.50 (18%)</th>
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<td>0.3333 (18%)</td>
<td>0.3333 (18%)</td>
<td>0.3333 (18%)</td>
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<td>CS price</td>
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<td>0.3333 (-10%)</td>
<td>0.3333 (-10%)</td>
<td>0.3333 (-10%)</td>
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<td>0.0685 (56%)</td>
<td>0.0599 (36%)</td>
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<tr>
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<td>0.3333 (9%)</td>
<td>0.3333 (9%)</td>
<td>0.3333 (9%)</td>
<td>0.3333 (9%)</td>
</tr>
<tr>
<td>surplus efficiency</td>
<td>94%</td>
<td>91%</td>
<td>84%</td>
<td>75%</td>
<td>66%</td>
<td>61%</td>
<td>57%</td>
</tr>
</tbody>
</table>

Table 7: CS first mover (\{CS, PM\}) equilibrium at different levels of required CS profit (\(\Pi_0 = 0.0190\beta\)). Numbers in parentheses denote percentage increases/decreases relative to analogous \{PM_1, PM_2\} values. Surplus efficiency is total consumer surplus divided by the maximum feasible two product surplus (0.08).
Figure 11: Plot of CS first mover ($\{CS, PM\}$) equilibrium values. CS firm values in black, PM firm values in red. Dashed lines represent corresponding $\{PM_1, PM_2\}$ values. Dotted line in lower right panel represents the fully efficient surplus value.
<table>
<thead>
<tr>
<th>$\beta$</th>
<th>0.00</th>
<th>0.10</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>0.90</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CS quality</strong></td>
<td>0.3452 (139%)</td>
<td>0.3450 (139%)</td>
<td>0.3453 (139%)</td>
<td>0.2806 (94%)</td>
<td>0.2561 (77%)</td>
<td>0.2371 (64%)</td>
<td>0.2180 (51%)</td>
</tr>
<tr>
<td><strong>CS price</strong></td>
<td>0.1192 (171%)</td>
<td>0.1204 (174%)</td>
<td>0.1227 (179%)</td>
<td>0.0883 (101%)</td>
<td>0.0810 (84%)</td>
<td>0.0757 (72%)</td>
<td>0.0702 (60%)</td>
</tr>
<tr>
<td><strong>CS share</strong></td>
<td>0.5648 (73%)</td>
<td>0.5572 (71%)</td>
<td>0.5425 (66%)</td>
<td>0.3956 (21%)</td>
<td>0.3672 (13%)</td>
<td>0.3489 (7%)</td>
<td>0.3327 (2%)</td>
</tr>
<tr>
<td><strong>CS profit</strong></td>
<td>0.0000 (-100%)</td>
<td>0.0008 (-90%)</td>
<td>0.0019 (-75%)</td>
<td>0.0038 (-50%)</td>
<td>0.0057 (-25%)</td>
<td>0.0068 (-10%)</td>
<td>0.0075 (0%)</td>
</tr>
<tr>
<td><strong>CS surplus</strong></td>
<td>0.0726 (844%)</td>
<td>0.0716 (831%)</td>
<td>0.0699 (809%)</td>
<td>0.0220 (185%)</td>
<td>0.0173 (125%)</td>
<td>0.0144 (88%)</td>
<td>0.0121 (57%)</td>
</tr>
<tr>
<td><strong>PM quality</strong></td>
<td>0.1800 (-37%)</td>
<td>0.1832 (-35%)</td>
<td>0.1916 (-32%)</td>
<td>0.3306 (17%)</td>
<td>0.3273 (15%)</td>
<td>0.3237 (14%)</td>
<td>0.3181 (12%)</td>
</tr>
<tr>
<td><strong>PM price</strong></td>
<td>0.0473 (-64%)</td>
<td>0.0487 (-63%)</td>
<td>0.0524 (-60%)</td>
<td>0.1238 (-6%)</td>
<td>0.1297 (-2%)</td>
<td>0.1336 (1%)</td>
<td>0.1357 (3%)</td>
</tr>
<tr>
<td><strong>PM share</strong></td>
<td>0.1726 (-53%)</td>
<td>0.1767 (-52%)</td>
<td>0.1840 (-50%)</td>
<td>0.2898 (-21%)</td>
<td>0.3165 (-14%)</td>
<td>0.3319 (-10%)</td>
<td>0.3453 (-6%)</td>
</tr>
<tr>
<td><strong>PM profit</strong></td>
<td>0.0026 (-86%)</td>
<td>0.0027 (-86%)</td>
<td>0.0029 (-85%)</td>
<td>0.0042 (-78%)</td>
<td>0.0071 (-62%)</td>
<td>0.0095 (-50%)</td>
<td>0.0119 (-37%)</td>
</tr>
<tr>
<td><strong>PM surplus</strong></td>
<td>0.0027 (-93%)</td>
<td>0.0029 (-92%)</td>
<td>0.0032 (-91%)</td>
<td>0.0461 (25%)</td>
<td>0.0462 (26%)</td>
<td>0.0453 (23%)</td>
<td>0.0440 (20%)</td>
</tr>
<tr>
<td><strong>total surplus</strong></td>
<td>0.0753 (70%)</td>
<td>0.0745 (68%)</td>
<td>0.0732 (65%)</td>
<td>0.0680 (53%)</td>
<td>0.0634 (43%)</td>
<td>0.0597 (34%)</td>
<td>0.0561 (26%)</td>
</tr>
<tr>
<td><strong>total profit</strong></td>
<td>0.0026 (-90%)</td>
<td>0.0034 (-87%)</td>
<td>0.0048 (-82%)</td>
<td>0.0080 (-70%)</td>
<td>0.0128 (-52%)</td>
<td>0.0163 (-38%)</td>
<td>0.0195 (-26%)</td>
</tr>
<tr>
<td><strong>welfare</strong></td>
<td>0.0779 (10%)</td>
<td>0.0779 (10%)</td>
<td>0.0780 (10%)</td>
<td>0.0760 (7%)</td>
<td>0.0762 (7%)</td>
<td>0.0761 (7%)</td>
<td>0.0755 (7%)</td>
</tr>
<tr>
<td><strong>no purchase share</strong></td>
<td>0.2626 (-14%)</td>
<td>0.2661 (-13%)</td>
<td>0.2735 (-10%)</td>
<td>0.3146 (3%)</td>
<td>0.3163 (4%)</td>
<td>0.3192 (5%)</td>
<td>0.3221 (6%)</td>
</tr>
<tr>
<td><strong>surplus efficiency</strong></td>
<td>94%</td>
<td>93%</td>
<td>91%</td>
<td>85%</td>
<td>79%</td>
<td>75%</td>
<td>70%</td>
</tr>
</tbody>
</table>

Table 8: $PM$ first mover ($\{PM,CS\}$) equilibrium at different levels of required $CS$ profit ($\Pi_0 = 0.0075\beta$). Numbers in parentheses denote percentage increases/decreases relative to analogous $\{PM_1,PM_2\}$ values. Surplus efficiency is total consumer surplus divided by the maximum feasible two product surplus (0.08).
Figure 12: Plot of CS first mover (\(\{CS, PM\}\)) equilibrium values. CS firm values in black, PM firm values in red. Dashed lines represent corresponding \(\{PM_1, PM_2\}\) values. Dotted line in lower right panel represents the fully efficient surplus value.