

Revenue Management with Strategic Customers: Last-Minute Selling and Opaque Selling

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Companies in a variety of industries (e.g., airlines, hotels, theaters) often use last-minute sales to dispose of unsold capacity. Although this may generate incremental revenues in the short term, the long-term consequences of such a strategy are not immediately obvious: More discounted last-minute tickets may lead to more consumers anticipating the discount and delaying the purchase rather than buying at the regular (higher) prices, hence potentially reducing revenues for the company. To mitigate such behavior, many service providers have turned to opaque intermediaries, such as Hotwire.com, that hide many descriptive attributes of the service (e.g., departure times for airline tickets) so that the buyer cannot fully predict the ultimate service provider. Using a stylized economic model, this paper attempts to explain and compare the benefits of last-minute sales directly to consumers versus through an opaque intermediary. We utilize the notion of rational expectations to model consumer purchasing decisions: Consumers make early purchase decisions based on expectations regarding future availability, and these expectations are correct in equilibrium. We show that direct last-minute sales are preferred over selling through an opaque intermediary when consumer valuations for travel are high or there is little service differentiation between competing service providers, or both; otherwise, opaque selling dominates. Moreover, contrary to the usual belief that such sales are purely mechanisms for disposal of unused capacity, we show that opaque selling becomes more preferred over direct last-minute selling as the probability of having high demand increases. When firms randomize between opaque selling and last-minute selling strategies, they are increasingly likely to choose the opaque selling strategy as the probability of high demand increases. When firms with unequal capacities use the opaque selling strategy, consumers know more clearly where the opaque ticket is from and the efficacy of opaque selling decreases.

Key words: distribution channels; competition; revenue management; strategic consumer behavior; rational expectations

History: Received March 18, 2008; accepted November 6, 2009, by Ananth Iyer, operations and supply chain management. Published online in *Articles in Advance* February 12, 2010.

1. Introduction

Firms in the travel industry (e.g., airlines, hotels, and car rentals) face the problem of uncertain demand for their services. Although these firms typically begin by selling regularly priced products through direct channels as well as through a variety of intermediaries, later in the selling cycle for the product, they often utilize last-minute sales discounts to sell off their leftover capacity (in the airline industry, often termed “distressed inventory”). Because service capacity in these industries is hard to adjust in the short term and the marginal cost of providing service is negligible, such dynamic discounts are pervasive. One common practice that firms adopt is to sell last-minute tickets at low prices through their own websites. For instance, in the case of US Airways, consumers can visit <http://www.usairways.com/awa/faresale/eSaver.aspx> and find the current week’s

discounted fares. Unfortunately, last-minute sales directly to consumers are dangerous in that they condition potential consumers to expect that there might be a deal available at the last moment at a much lower price. As a result, some consumers may prefer to wait for last-minute sales rather than purchase earlier at a higher price. Many industry analysts, observers, and executives have questioned the last-minute sales approach altogether because it “... starts a cycle of price degradation that will eventually lead to ... destroying the airlines” (Sviokla 2004).

Another mechanism for selling distressed inventory appeared more recently under the name of *opaque* selling. Before purchasing the opaque product from an intermediary, a consumer does not know which company will ultimately provide the service, when exactly will the service commence, and how long will it take (for air tickets), etc. In the travel industry, firms

like Hotwire.com, CheapTickets.com, Priceline.com,¹ OneTravel.com, and many others have popularized this selling mechanism in which the intermediaries hide some attributes of the product from the consumers and reveal them only after the purchase has been made.

This is the opposite of *transparent* sales, discussed above, whereby all attributes are observable up front. One frequently cited reason for the existence of the opaque channel is that it "...enables airlines to generate incremental revenue by selling distressed inventory cheaply without disrupting existing distribution channels or retail pricing structures" (Smith et al. 2007, p. 75).

Using opaque sales can have two opposing effects: By hiding key attributes of the product, the firms may persuade some consumers to buy regularly priced service (and to some extent mitigate the "cycle of price degradation"), but there is also a chance that consumers indifferent among multiple service providers may be diverted to the opaque channel if "the price is right." Given that 60% of consumers shopping online buy the lowest fare available (PhoCusWright 2004) and that opaque selling of products has been widely prevalent in recent years among travel companies (Lambert 2006, Harrison 2006), it is important to understand how channel choice affects profitability, prices, and consumer segmentation.

The goal of this paper is to understand the dynamics of last-minute discounts and to shed some light on the relative merits of last-minute sales directly by the firm versus through an opaque intermediary. To analyze these issues, we propose a stylized economic model in which two firms sell horizontally differentiated products to consumers on a Hotelling line in two time periods. The firms have fixed capacities but can adjust prices from period to period. The firms use transparent sales in the first period and may use either transparent last-minute sales or opaque last-minute sales in the second period, if there is leftover inventory. Our model yields the following major findings:

1. When firms sell through opaque channels, consumers (who prefer a ticket from one firm over the other) form expectations about the availability of the tickets from either firm and factor this into their purchase decisions. When firms have symmetric capacities, we demonstrate that in equilibrium the consumers expect that the opaque product comes from either of the two firms with equal probability

and the two firms supply equal quantities of their products to the opaque channel; i.e., the firms achieve "perfect masking" of the product identity.

2. When there is no uncertainty in demand, selling through the opaque channel weakly increases the firms' profits compared to selling only through the direct channel. This is because using the opaque channel as a "clean-up" mechanism by charging a low opaque price but masking the product attributes leads to ex ante expected surplus of zero for all consumers—who, therefore, purchase the product. This does not distort the regular pricing structure (i.e., in the absence of the opaque channel), whereas consumers who would not have purchased otherwise, if any, now purchase the opaque product.

3. When demand is uncertain, both opaque last-minute sales and direct last-minute sales can lead to higher profits under different conditions. Under uncertain demand, consumers trade off the possibility of buying later at a lower price with the risk of not buying at all (if demand turns out to be high and inventory runs out). In opaque selling, the identity of the product is masked, which makes the ex ante expected surplus from an opaque product zero. Hence, consumers do not benefit from waiting, and the firms use this to charge high prices in the first period, even when consumer valuations for tickets are low. In direct last-minute sales, the firms price in the first period to extract the surplus from consumers who have a high preference for purchasing, whereas low-preference consumers choose to wait and buy at low prices through the last-minute sales if any products are left over. In this strategy, the firms make the bulk of their profits through first-period prices. However, if consumer valuations are low, these first-period prices are low; as valuations increase, these prices (and, correspondingly, firm profits) increase. Hence, opaque sales are preferred over transparent last-minute sales when consumer valuations are low, and vice versa.

4. We demonstrate that, as the probability of having high-demand realization increases, opaque selling becomes more and more preferred over direct last-minute selling. This finding is contrary to the traditional understanding of opaque selling as a mechanism to clear leftover inventories when demand is low. The intuition behind this result is again that masking the product identity leads to ex ante expected surplus of zero from an opaque product, so that consumers do not have a benefit from waiting and they prefer to purchase in the first period. As the probability of high demand (and therefore the possibility of not getting a product) increases, the competition (or "clamor") among consumers for products in the first period increases, which enables the firms to charge higher first-period prices and increase profits.

¹ In addition to opacity, "Name-Your-Own-Price[®]" is an interesting parallel concept that Priceline.com uses (i.e., consumers can haggle for ticket prices). In this paper, we focus on studying opacity and therefore abstract away from the bargaining/haggling process; see Terwiesch et al. (2005) for further discussion of online haggling.

5. We analyze a situation in which firms randomize between selling through opaque or through their own channels in the second period. We find that as probability of high demand increases, firms are more likely to adopt opaque selling strategies (i.e., they sell through opaque intermediaries with higher probability).

6. When firms are asymmetric, with one firm having larger capacity than the other, they will not be able to achieve perfect masking of the opaque product because consumers will rationally expect that the probability that it is from the larger firm is higher. Hence, because of the imperfect masking in the opaque channel, the pricing power of the opaque intermediary and the two firms will diminish. We find that the firm with a larger capacity is at a greater disadvantage. In spite of this finding, using opaque sales still helps asymmetric firms to increase their profits.

To summarize, our study is the first to shed light on the comparative advantages of direct selling versus opaque selling as last-minute sales strategies. We explicitly model the effect of consumers' strategic behavior regarding product availability on firms' sales strategies to dispose of inventories, and we characterize conditions under which firms prefer one strategy over the other. The rest of this paper is organized as follows. In §2, we review related literature from economics and operations and revenue management. In §3, we describe our model; in §4, we analyze the case of deterministic demand; and in §5, we analyze the case of stochastic demand. In §6, we extend our basic model to the case of asymmetric firms; in §7, we consider the consequences of relaxing several other assumptions. We conclude in §8 with a discussion.

2. Related Literature

The work on intertemporal sales started with the seminal Coase conjecture (Coase 1972), which postulates that given an infinite number of selling opportunities over time, a monopolist will eventually decrease a product's price to its marginal cost, and all consumers will anticipate this decrease and delay their purchases. Numerous subsequent papers modeled scenarios in which the Coase conjecture may not hold (Stokey 1979, Besanko and Winston 1990, DeGraba 1995). Specifically, DeGraba (1995) suggested that under uncertain demand, if product availability is limited, consumers might not have an incentive to wait for a lower price because of the threat of unavailability. We build on this trade-off between price and availability in the context of opaque sales when consumers form rational expectations about future availability.

Our paper contributes to the small but growing literature in operations management that models strategic consumer behavior. Recent papers in this stream that explicitly incorporate product availability considerations and demand uncertainty into last-minute sales models include Cachon and Swinney (2009), Lai et al. (2010), Su (2007), Su and Zhang (2008), and Yin et al. (2009). For instance, Cachon and Swinney (2009) demonstrate that the value of quick response strategies is higher in the presence of consumers who strategically wait for sales. However, none of the aforementioned papers considers competition and opacity in product attributes. For a rich compilation of recent work in operations management that incorporates strategic consumer behavior, the reader is referred to Netessine and Tang (2009).

There is a rich literature studying revenue management practices in the travel industry (Talluri and van Ryzin 2004), which is the primary, but not the only, adopter of opaque selling. This literature usually assumes that availability of products or competition does not affect consumer demand (see Liu and van Ryzin 2008 and Netessine and Shumsky 2005 for exceptions). Koenigsberg et al. (2008) consider the impact of airline capacity and the number of customer segments (differing in price sensitivity) on the pattern of sales by airlines. However, strategic waiting by consumers for low prices, an extremely important issue that airlines face every day, has not been considered in the revenue management literature. This paper is an attempt to unravel the impact of strategic consumers on airline firms' selling strategies.²

To our knowledge, only a few very recent papers explicitly consider opaque selling mechanisms. Jiang (2007) considers opaque selling for two flights owned by the same monopolist firm but scheduled at different times throughout the day. Fay and Xie (2008) study "probabilistic selling," under which a monopolist creates a probabilistic good by clubbing several distinct goods together. Fay (2008) considers a model of competition with deterministic demand and shows that the opaque channel increases the degree of price rivalry and reduces industry profits unless firms have very loyal customers. Shapiro and Shi (2008) model competing firms selling opaque products through a passive intermediary that posts prices

² Xie and Shugan (2001) consider the strategy of advance selling of tickets to strategic consumers. Here consumers' valuations for the product are uncertain in advance and are realized after the purchase decision has been made. In our model, both consumers and firms are certain about consumers' valuations for the product; the uncertainty is on the realization of demand and correspondingly on future product availability. Su (2009) considers a two-period model where firms face inertial and rational customers, assuming that inertial customers have a tendency to refrain from purchasing. Levin et al. (2009) consider dynamic pricing of limited capacity sold over multiple periods to strategic customers.

dictated by firms while hiding the identity of the products. There are two segments of consumers with different strengths of brand preference. They find that opaque sales intensifies competition for customers with low brand preferences and enables firms to commit to high prices for customers with high brand preference so that the total profit can increase.

All of these papers utilize single-period models, in which opaque sales occur simultaneously with transparent sales. Our model is very different in many ways. First, we explicitly recognize that the opaque sales strategy is a last-minute sales mechanism and we study it using the resulting dynamic model. The dynamic aspect is an important practical consideration for opaque selling (Elkind 1999, Harrison 2006): in the travel industry, opaque products are sold only a few days before service delivery,³ whereas transparent products are sold at regular prices up to a year in advance. Second, we also allow for demand uncertainty, which is only resolved very late in the selling horizon. By virtue of the above two aspects, consumers face a trade-off between buying at a higher price early versus at a lower price (under the threat of stockout) later, which is the key trade-off of our model. Third, we compare opaque selling with another commonly observed last-minute sales mechanism, direct last-minute sales, and characterize the conditions under which one is better than the other for firms involved.

Versioning (Varian 2000) and “damaged goods” (Deneckere and McAfee 1996) are related price discrimination strategies in which a high-quality product is sold in its original form and also in an inferior version with some of its features disabled. Opaque selling is related to these strategies because (i) the same product is sold both through a transparent channel and through an opaque channel with some attributes hidden and (ii) airline reservations made through opaque channels cannot typically be modified or exchanged without an extra fee. However, the key difference for opaque selling is because it requires the availability of an alternative competing product. Furthermore, all consumers who purchase the inferior product in the form of versioning or damaged goods receive lower utility from consuming it (relative to consuming the original product), whereas a fraction of consumers that purchases the opaque product end up receiving the same utility (and higher net surplus)

³To be precise, opaque products are sometimes available in advance, but they are priced at the same level as transparent products, which makes them unattractive to any consumer. This can be observed through a simple experiment on Hotwire.com, which starts selling deeply discounted opaque tickets only a few days before the service date.

relative to what they would have received by purchasing a transparent product. Finally, rational expectations regarding product availability (because of limited capacity and demand uncertainty) never come into consideration in either the versioning or the damaged goods literature.

3. The Model

Two competing firms, A and B, each hold a quantity $K/2$ of inventory (later, we consider firms with asymmetric capacities). The inventories can be service capacities, e.g., seats on flights operated by the firms on a particular date or rooms for a particular day in similar hotels run by the firms. The products are perishable in the sense that they have to be sold before a certain time and have no value if they remain unsold. For travel services, this is a reasonable assumption (e.g., products have no value after the flights take off, and hotel rooms have no value if they remain unfilled by the day under consideration).

Consumers have heterogeneous preferences between firms. The reason might be loyalty to the firm, preference for a brand, or simply an established relationship with the company (e.g., through rewards programs). We capture this consideration by invoking a horizontal differentiation model similar to that of Fay (2008) and other papers on opaque selling.⁴ We assume that the two competing firms A and B are located at each end of a Hotelling line of length 1, and a continuum of consumers is spread on the horizontal line over the interval $[0, 1]$ with uniform density. A population of J consumers is spread uniformly over the entire line. We consider cases of deterministic low demand ($J < K$), deterministic high demand ($J > K$), and random demand ($J = H > K$ with probability α , and $J = L < K$ with probability $1 - \alpha$).

Each consumer has a valuation V for the product and purchases at most one unit. The brand preference of every consumer is completely characterized by his location $x \in [0, 1]$ on the line, which influences the utility a consumer derives when he purchases a product from a firm. The parameter t denotes the strength of brand preference in the market. A consumer at x incurs a disutility tx when buying a product from firm A and a disutility $t(1 - x)$ when buying a product from firm B. Thus, the customers have varying preferences toward the competing firms. If firms A and B charge prices p_A and p_B , respectively, then a consumer located at x receives net utility $V - tx - p_A$ when purchasing a product from firm A and receives net utility $V - t(1 - x) - p_B$ when purchasing a product from firm B. We assume that $V/t \geq 1/2$ so that every

⁴We assume that there is no vertical differentiation between products of the two firms; i.e., one product is not inherently superior to the other for all customers.

consumer would receive nonnegative utility from the product from at least one of the firms if it were offered for free by both firms.

We will encounter the ratio V/t frequently in the analysis to follow. This ratio can be interpreted as a “brand preference-adjusted valuation” for a product and it reflects the degree of competition between the firms. If V is large, the valuation for a product in the market is high and the market will be competitive, and vice versa. Further, if t is small, the consumers do not care about the firm from which they buy and competition will be high, and vice versa. Overall, as V/t increases, the market becomes more competitive.

We divide the selling horizon into two periods so that each firm has two pricing opportunities. We assume no discounting between the two periods. The firms choose one of the following strategies for selling products:

1. The firms can sell the products through their own channels in both periods and offer different prices in each period of sale.

2. The firms can sell in the “transparent” channel in the first period and sell opaque products through an intermediary (such as Hotwire.com) in the second period. The intermediary, denoted by I , makes its own pricing decision p_I . It keeps a fraction $1 - \delta$ of the revenue it makes. The remaining fraction of the revenue δ from each product goes to the airline whose product the intermediary sells (see Elkind 1999, PriceLine.com 2006, and Phillips 2005 for a description of such arrangements in the travel industry).

Because the firms sell products over two periods, possibly at different prices, the consumers strategically time their purchases based on their valuations, expectations of future inventory availability, and the firms’ pricing strategies. In other words, a consumer might decide to purchase products in the second period rather than in the first period. A consumer purchasing from the opaque intermediary does not know from which firm the product is coming, but develops expectations about the service provider based on expectation of inventory supplied to the intermediary by the two competitors. All consumers have the same beliefs. To differentiate from the actual equilibrium outcomes, we represent beliefs (or expectations) by using the superscript e .

We assume consumers are forward looking and have rational expectations; i.e., the availability of tickets from each firm in every period matches consumers’ expectations on the availability.⁵ Amaldoss

⁵ The rational expectations concept assumes strong rationality on the part of the agent (Muth 1961); i.e., an agent can correctly expect the future equilibrium path in a multiperiod game and acts on these expectations, so that this equilibrium indeed arises. There is a debate in the literature around this strong rationality assumption.

and Jain (2008) have a similar model with two time periods, where consumers in the first time period form expectations about the future popularity of a product and make their purchase decisions accordingly. Diamond and Fudenberg (1989), Stokey (1981), and several others solve similar multiperiod games using the rational expectations concept.

We first consider the case of deterministic demand, with both low demand (demand is lower than capacity) and high demand (demand is higher than capacity), and then consider the case of uncertain demand. In all cases, the firms and consumers know the values of all parameters. The above is a general description of the model. We provide specific details for each case as appropriate.

4. Deterministic Demand

In this section, we explore the two strategies of the firms when demand is deterministic: (i) the firms can sell through their own channels and have the option of offering different prices in the two periods of sale and (ii) the firms can sell opaque products in the second period after sales in the first period have concluded. We consider two possible scenarios for each strategy: (i) low-demand scenario ($J < K$) and (ii) high-demand scenario ($J > K$). The deterministic demand model helps us gain insights into the players’ decisions when demand is lower or higher than capacity and it serves as a logical building block for the more complex model with demand uncertainty (§5).

4.1. Selling Through Firms’ Direct Channels

When firms sell only through their direct channels and demand is deterministic, we find that each firm charges the same price in the two periods.⁶ Intuitively, if the firms were to try to charge a higher price in the first period and a lower price in the second period, the consumers, being strategic and having full information about demand, would wait to buy products until the prices were lowered. (In case of uncertain demand, we will see that this result changes.) We provide the detailed analysis in §A1.1 in the technical

Experimentally, Amaldoss and Jain (2005), Sunder (1995), and others show that markets can converge to predictions of rational expectations equilibria, even though Garner (1982), Smith et al. (1988), and others show that expectations at the individual level may not always correspond to theoretically predicted individual-level rational expectations. Liu and van Ryzin (2007) show that adaptive learning mechanisms can also lead to an equilibrium identical to the rational expectations equilibrium.

⁶ Prices would not be identical across periods if consumers discounted their second-period utility. However, the *discount-adjusted* prices would be identical across periods. Introducing discounting makes the analysis more tedious, although all our insights continue to hold.

appendix (provided in the e-companion)⁷ and provide only the main insights here.

The prices charged and market covered depend on whether demand is low or high and on the value of the quantity V/t . In the case of low demand ($J < K$), when V/t is small ($1/2 \leq V/t < 1$), the firms act as local monopolies, and each firm prices at $V/2$ to cover the length $V/(2t)$ on its side of the Hotelling line, which is less than half. As V/t increases, the market covered increases, until the firms start to compete—specifically, when $1 \leq V/t < 3/2$, each firm prices at $V - t/2$, covers half the market and sells $J/2 < K/2$ tickets; and when $V/t \geq 3/2$, each firm prices at t , covers half the market, and sells $J/2 < K/2$ tickets.

In the case of high demand ($J > K$), the same pattern of coverage that increases with V/t holds. However, in this case, the firms never get into competition and always act as local monopolies because each firm can sell a maximum of $K/2$ tickets. Hence, if V/t is large enough (specifically, $V/t \geq K/J$), the firms charge a price of $V - [K/(2J)]t$ and cover exactly $K/(2J) < 1/2$ on their side of the market.

4.2. Opaque Selling

As we described in the introduction, firms often sell products and services through opaque intermediaries very close to the terminal time, i.e., after consumers have bought in the transparent channel, but the firms still have some inventory of products left over. In this case, we assume that the firms declare that they might sell through an opaque channel late in the selling horizon (e.g., airlines selling opaque tickets through Hotwire declare this by having their names listed on the website Hotwire.com). Firms will engage in opaque selling only if there are products that are left unsold through their own direct channels. More formally, the game proceeds in the following manner:

1. Every consumer is endowed with expectations, given by γ_A^e and γ_B^e , about the probabilities that the product he will obtain from the opaque seller will be from firm A or firm B, respectively. In the first period, firms A and B set prices p_A and p_B in the direct-to-consumers channel (transparent channel).

2. Given prices p_A and p_B and his expectations about availability in the opaque channel from both firms, every consumer makes a purchase decision in the transparent channel.

3. After the transparent channel sales are over, the leftover products are made available to the opaque intermediary I by both firms. The opaque intermediary sets a price p_I for the opaque product. Consumers who did not purchase in the transparent channel now make their purchase decisions in the opaque channel.

A consumer may not obtain an opaque product if the number of leftover products is less than the number of consumers who are willing to purchase at price p_I . We denote the probability that the consumer can obtain an opaque product by β so that each consumer desiring a product is equally likely to obtain it.

4. The opaque intermediary keeps a fraction $1 - \delta$ of the revenues from the opaque channel. The remaining fraction δ is distributed between firms A and B in proportion to the products sold for each firm.⁸

Consequently, for the consumer at x_A who is indifferent between buying from firm A and buying in the opaque channel, the following condition holds in equilibrium:

$$V - p_A - tx_A = \beta(V - p_I - \gamma_A^e tx_A - \gamma_B^e t(1 - x_A)).$$

We now characterize the equilibria under low- and high-demand cases. Without loss of generality, we focus on $\delta = 1$; any $\delta \in [0, 1]$ yields the same insights. We provide the detailed analysis in §A1.2 in the technical appendix and provide the main insights here.

First, consider the case of low demand ($J < K$). To solve for the equilibrium, we use backward induction. We start with the opaque intermediary's problem in the second period—the intermediary has access to consumers on the line segment $[x_A, x_B]$ (x_A and x_B are arbitrary, and the intermediary can choose to cover this market fully or partially). This determines the actual number of leftover tickets from each firm and therefore the realized probability of obtaining tickets from firm A in the opaque channel through the expectation function as $\gamma_A = (K/2 - x_A J)/(K/2 - x_A J + K/2 - (1 - x_B)J)$. We solve for the equilibrium through the following procedure.

The opaque intermediary sets the price p_I it will charge to sell the leftover tickets from the two firms. Because this is the case of low demand, there is no shortage of tickets, so that $\beta = 1$. Based on the above, the intermediary sets the price p_I as function of location of the indifferent customers x_A , x_B , and first-period prices p_A^1 and p_B^1 . In the first period, both firms set their prices, based on the prices set by the opaque intermediary under all second-period possibilities.

As we demonstrate in §A1 in the technical appendix, in equilibrium, the rational expectations of the fraction of opaque products from each firm are $\gamma_A^e = \gamma_B^e = 1/2$. This implies that if the firms have equal capacities, then it is rational for consumers to expect that in the opaque channel half of the products come from one firm and the other half come from the other.

⁷ An electronic companion to this paper is available as part of the online version that can be found at <http://mansci.journal.informs.org/>.

⁸ This revenue sharing contract with opaque intermediaries is consistent with observations and industry practice (see Elkind 1999, Priceline.com 2006, Phillips 2005).

This result further implies that, in equilibrium, at price p_I , the ex ante utility of each consumer from purchasing in the opaque channel is $V - p_I - t/2$, so the intermediary charges a price of $p_I = V - t/2$ to make every consumer's ex ante surplus equal to zero. The benefit from the opaque channel lies in the fact that by masking the identity of the product, the ex ante surplus from purchasing an opaque ticket is zero for all consumers in $[x_A, x_B]$, and hence they purchase tickets. Recall that in the transparent channel, the surplus from purchasing a ticket is zero only for the customers at x_A and x_B who are indifferent between purchasing from firms A and B, respectively. For every other customer in $[x_A, x_B]$, the surplus is negative and they do not purchase tickets in the transparent channel.

The price charged by the opaque intermediary, $p_I = V - t/2$, is always lower than the price charged in the transparent channel in the first period, which is equal to $p_A = p_B = V/2$ for $1/2 \leq V/t \leq 1$. Hence, the opaque channel serves as a clean-up mechanism to increase profits by selling leftover tickets at a low price to consumers who would otherwise not have purchased at all, and masking the identity of the tickets enables this. When $V/t \geq 1$, there are no opaque sales because each firm covers half the market directly with the transparent sales. Hence, firms use opaque selling for the range $(1/2 \leq V/t < 1)$.

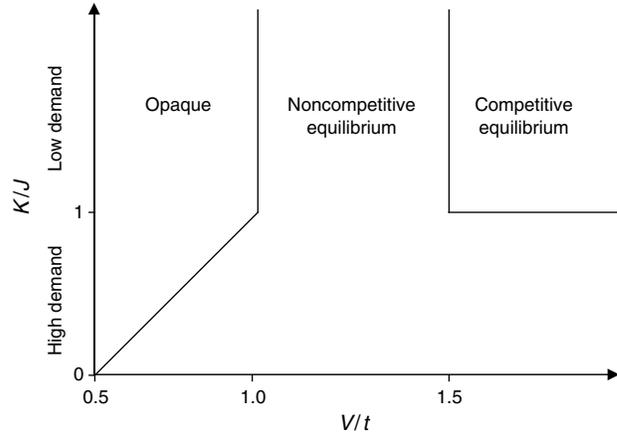
In the case of high demand ($J > K$), the same insights hold in equilibrium, $\gamma_A = \gamma_B = 1/2$, $p_I = V - t/2$, and the ex ante expected surplus of consumers in the opaque channel is zero because of masking the product identity. However, in this case, one difference is that as V/t increases and firms increase their market coverage, they stock out when they cover a length of $K/(2J)$, which occurs for $V/t \geq K/J$. Hence, firms use opaque selling for a smaller range $(1/2 \leq V/t < K/J)$. The second difference is that when firms do use opaque sales, all the consumers in $[x_A, x_B]$ are willing to purchase opaque tickets, but only a fraction $\beta = (K - (V/t)J)/(J - (V/t)J) < 1$ actually obtain opaque tickets.

4.3. Comparison of Strategies Under Deterministic Demand

In both high- and low-demand scenarios, the opaque channel acts as a clean-up mechanism to dispose of unsold products by selling them to consumers who would otherwise not have purchased at all. Hence, if opaque selling is used (when the market is not fully covered by the transparent channels), it will strictly improve firm profits.

Figure 1 depicts the optimal strategies for the firms given different values of consumer valuations (the ratio V/t) and inventory availability relative to demand (the ratio K/J). Under both high and low demand, firms sell products through the opaque

Figure 1 Equilibrium Outcomes Under Deterministic Demand for Different Valuations and Capacity and Demand Scenarios



channel only if V/t is small enough because in this case the firms do not cover the full market in the transparent channels and they use opacity as a mechanism to dispose of unsold products. As the ratio V/t increases above a threshold, the firms have the option of using an opaque channel, but they price in the transparent channels to cover the market anyway and do not need to resort to selling cheaper opaque products. Figure 1 also shows that if demand is high, opaque sales will be seen less frequently (for a smaller range of V/t) than if demand is low. This is consistent with the notion that the opaque channel is used to dispose of distressed inventory (Harrison 2006).

5. Uncertain Demand

Uncertainty in demand volume is a pervasive feature in many industries. In the travel industry, for example, firms usually can estimate the demand distribution for a given airline route or hotel using historical records, but the precision of such estimates is quite limited (see Talluri and van Ryzin 2004). As the departure date approaches, the firms can improve the forecast and therefore project with a higher degree of confidence whether the demand for the route is higher or lower than the available capacity. Building on the analysis in previous sections, this section extends our model to incorporate demand uncertainty.

Due to the presence of demand uncertainty, consumers cannot always credibly adopt a strategy of waiting in the early stages of the game because market demand could be high and products could be unavailable later. However, a consumer can form expectations about future availability and buy early if the expected utility from doing so is higher than the expected utility from waiting. These dynamics capture the practical consideration that not all consumers wait for last-minute discounts, and they allow us to

derive several insights beyond the model with deterministic demand.

The specifications of the model remain the same, except that the level of demand is now variable. We assume that with probability α , the total number of consumers in the market is $H (>K)$, and with probability $1 - \alpha$, the total number of consumers in the market is $L (<K)$. As before, each firm has capacity $K/2$. The parameters α , L , H , and K are common knowledge. The selling horizon is divided into two periods. In the first period, the firms and the consumers know the distribution of demand but do not know its realization (whether demand is H or L). At the end of the first period, but before the second period begins, the realization of demand is observed by the firms and the consumers.⁹

We assume that in any selling period, if the number of consumers who are willing to buy a product is higher than the capacity available, products are allocated randomly to the consumers. In other words, if a certain number of consumers desires products at the announced price, but the number of products available is lower than the number of products demanded (which can be the case if demand is high), it is possible that consumers with a lower expected (but positive) surplus obtain products at the expense of consumers with a higher expected surplus. In the following sections, we analyze the two strategies of selling through the firms' direct channels ("last-minute sales strategy," or LMSS) and opaque selling ("opaque sales strategy," or OpSS).

5.1. Selling Through Firms' Direct Channels

The following is the order of events in the game when firms adopt an LMSS.

1. In the first period, firm A prices its products at p_A^1 , firm B prices its products at p_B^1 , and both firms declare that there might be last-minute sales.
2. All consumers form expectations about the number of consumers purchasing in each period (and therefore the corresponding future availability) and strategically make or postpone their purchase.
3. At the end of period 1 and before period 2 begins, demand uncertainty is fully resolved. The level of demand is determined as H or L and is observed by both the firms and the consumers. The firms then set their prices (firm A sets price p_A^{2L} if demand is low and p_A^{2H} if demand is high, and similarly for firm B).
4. The consumers who postponed their purchase in the first period decide to purchase or not in the second period at the announced prices.

The equilibrium solution LMSS is provided in the following proposition.

PROPOSITION 5.1. *When the firms sell products through their own channels, the following equilibrium always exists: In the first period both firms set prices to cover $x_A = 1 - x_B = K/(2H)$ of the market. If demand is high, no products are sold in the second period because the firms stock out in the first period. If demand is low, consumers located between $x_A = K/(2H)$ and $x_B = 1 - K/(2H)$ buy in the second period. The first- and second-period prices are as follows:*

$\frac{V}{t}$	First-period prices ($p_A^1 = p_B^1$)	Second-period prices when demand is low ($p_A^{2L} = p_B^{2L}$)
$\frac{1}{2} \leq \frac{V}{t} < 1 - \frac{K}{2H}$	$\frac{1+\alpha}{2} \left(V - \frac{K}{2H}t \right)$	$\frac{1}{2} \left(V - \frac{K}{2H}t \right)$
$1 - \frac{K}{2H} \leq \frac{V}{t} < \frac{3}{2} - \frac{K}{H}$	$\alpha \left(V - \frac{K}{2H}t \right) + (1-\alpha) \left(V - \frac{t}{2} \right)$	$V - \frac{t}{2}$
$\frac{V}{t} \geq \frac{3}{2} - \frac{K}{H}$	$\alpha \left(V - \frac{K}{2H}t \right) + (1-\alpha)t \left(1 - \frac{K}{H} \right)$	$t \left(1 - \frac{K}{H} \right)$

In the equilibrium, all consumers who attempt to buy a product in the first period obtain a product but pay the high price p_A^1 or p_B^1 as in the proposition above. Note that the first-period prices are such that the customer who is indifferent between purchasing in the first period from firm A and waiting for the second period to purchase from firm A has a positive (and of course equal) surplus from purchasing the ticket in either period. If demand is high, firm A sells to $H(K/(2H)) = K/2$ consumers in the first period and thus exhausts its capacity, so there are no products sold in the second period through last-minute sales. If demand is low, firm A sells to $K/(2H) \cdot L (< K/2)$ consumers in the first period and will have some products left over to sell in the second period. Moreover, there are more of these leftover products than the number of unserved consumers in the market in the second period. Therefore, the consumers who waited for the "last-minute" products obtain them at lower prices only if demand is lower than capacity. The situation is symmetric for firm B.

To summarize, in the first period all consumers with "high brand preference" (located in the interval $[0, K/(2H)]$) purchase at a high price from firm A. If there are any leftover products, the consumers with "low brand preference" (located in the interval $[K/(2H), 1/2]$) purchase from firm A during the last-minute sales at lower prices. If there are no leftover products, there are no sales in the second period. In effect, the firms are separating out consumers who are ready to pay a higher price under the threat of stock-out and making most of their profits from the higher

⁹ In practice, some residual uncertainty in demand would remain.

prices charged to the high-preference consumers in the first period.

5.2. Opaque Selling

The following is the order of events in the game when the firms adopt an opaque sales strategy.

1. Every consumer is endowed with expectations about the probabilities that the product he will obtain from the opaque seller in the second period will be from firm A or firm B for both high- and low-demand realizations. In the first period, firm A prices its products at p_A^1 , firm B prices its products at p_B^1 , and both firms declare intention of sales through an opaque channel.

2. Consumers strategically purchase or postpone purchasing based on expectations about availability in the second period (β) and about the probability with which they will obtain the opaque product from each firm ($\gamma_A^{H,e}, \gamma_B^{H,e}, \gamma_A^{L,e}, \gamma_B^{L,e}$).

3. At the end of period 1 and before period 2 begins, demand uncertainty is resolved; the level of demand is determined as H or L and is observed by the firm and the consumers. The leftover products, if any, are made available to the opaque intermediary I , who then sets a price p_I^H if the demand realization is H or a price p_I^L if the demand realization is L .

4. Consumers who have not purchased in the transparent channel now make their purchasing decision in the opaque channel. The intermediary commits to a credible opaque strategy and sells products from both firms at price p_I . For every product sold, it keeps a fraction $1 - \delta$ of the revenue accrued from the opaque channel and distributes the remaining fraction δ to firm A or B whose product it sold.

Let the locations of consumers indifferent between buying in the first period and buying opaque tickets in the second period be x_A and x_B . Then we have the following two cases based on the realization of demand.

1. If the level of demand is low, then leftover products for firm A must be $I_A^L = \max\{K/2 - x_A L, 0\}$ and leftover products for firm B must be $I_B^L = \max\{K/2 - (1 - x_B)L, 0\}$. In equilibrium, the customers will obtain a product from firm A with probability $\gamma_A^L = I_A^L / (I_A^L + I_B^L)$ and from firm B with probability $\gamma_B^L = I_B^L / (I_A^L + I_B^L)$. (If there are no leftover products, the probabilities are set to zero.)

2. If the level of demand is high, then the leftover for firm A is $I_A^H = \max\{K/2 - x_A H, 0\}$ and the leftover for firm B is $I_B^H = \max\{K/2 - (1 - x_B)H, 0\}$. In equilibrium, if the opaque channel exists, the customers will obtain a product from firm A with probability $\gamma_A^H = I_A^H / (I_A^H + I_B^H)$ and from firm B with probability $\gamma_B^H = I_B^H / (I_A^H + I_B^H)$, if defined.

Based on the demand realization and availability, the opaque intermediary fixes opaque ticket prices.

Based on the second-period decision of the opaque intermediary, the firms optimally fix the first-period prices. The equilibrium of the above game is characterized in Proposition 5.2. (To keep results simple, we present the case with $\delta = 1$. The analysis for any $\delta \in [0, 1]$ yields the same insights.)

PROPOSITION 5.2. *When the firms sell products through the opaque intermediary, the following rational expectations equilibrium exists:*

$\frac{V}{t}$	First-period prices ($p_A = p_B$)	Opaque prices (p_I^L, p_I^H)
$\frac{1}{2} \leq \frac{V}{t} < \frac{K}{H}$	$\frac{V}{2}$	$V - \frac{t}{2}, V - \frac{t}{2}$
$\frac{K}{H} \leq \frac{V}{t} < \frac{K}{H} + \frac{\alpha}{1-\alpha} \frac{K}{2L}$	$V - \frac{K}{2H}t$	$V - \frac{t}{2}, -$
$\frac{K}{H} + \frac{\alpha}{1-\alpha} \frac{K}{2L}$	$\frac{V}{2} + \frac{\alpha}{1-\alpha} \frac{K}{4L}t$	$V - \frac{t}{2}, -$
$\leq \frac{V}{t} < 1 + \frac{\alpha}{1-\alpha} \frac{K}{2L}$		
$1 + \frac{\alpha}{1-\alpha} \frac{K}{2L}$	$V - \frac{t}{2}$	$-, -$
$\leq \frac{V}{t} < \frac{3}{2} + \frac{\alpha}{1-\alpha} \frac{K}{L}$		
$\frac{V}{t} \geq \frac{3}{2} + \frac{\alpha}{1-\alpha} \frac{K}{L}$	$\left(1 + \frac{\alpha}{1-\alpha} \frac{K}{L}\right)t$	$-, -$

Note that in equilibrium, consumers have the same probability of obtaining a ticket from either firm in the opaque market (if it exists), irrespective of whether the demand realization is high or low. This also implies that the consumer who is indifferent between purchasing a transparent ticket in the first period and an opaque ticket in the second period has the ex ante expected surplus of zero.

There is, however, a critical difference from the deterministic demand case. Under deterministic demand consumers know the state of demand, and the opaque channel is primarily a clearance mechanism used when the entire market could not be covered by the firms through transparent prices. In contrast, when demand is uncertain, there is an additional factor—because the consumers do not know the state of demand in the first period, they face the possibility of the firms stocking out if demand is high. In other words, if a consumer has nonnegative utility in the first period at the price offered by a firm, then he will purchase the product, inferring that he might not obtain it at all later if the demand turns out to be high. This consideration allows the firms to charge higher prices in the first period as compared to the prices in the deterministic case. Consequently, if demand is low, only a few products will be sold in the first period. However, in this eventuality, the

firms can use the opaque channel to “clean up” the leftover products if any. Selling to a smaller population at higher prices in the first period helps the firms to increase the expected profit across two periods.

The above argument naturally leads to the interesting insight that as the probability of high demand increases, the firm will rely more and more on the opaque channel (the formal proof is straightforward and is therefore omitted): If there is a greater chance that demand is high, the “competition for products” among consumers in the first period will be higher, which means that the firms will be able to raise the first-period prices. If demand turns out to be high, the firms will exhaust their capacities. On the other hand, even if demand turns out to be low, there will still be some consumers left in the market because of high first-period prices. Consequently, there will be some leftover products, and firms will sell them through the opaque channel.

5.3. Opaque Sales vs. Last-Minute Sales

We saw in the previous two sections that both LMSS and OpSS can increase firms’ profits. In this section, we can use our analytical results to answer the following question: When should firms employ LMSS versus OpSS? To illustrate our insights better, we compare the profits of the firms for these two strategies for a representative set of parameter values ($\alpha = 1/2$, $K = 1$, $L = 1/2$, $H = 3/2$, $t = 1$) in Figure 2. We use numerical and graphical illustrations for expositional simplicity. Qualitatively, the results do not change for other values of the parameters. From Figure 2, we obtain the insight that if V is low, the profits from OpSS are higher than the profits from LMSS which is consistent with the intuition stated above. As V increases, the profits from OpSS flatten out, whereas the profits from LMSS keep increasing.

Figure 2 Profits Accrued by a Firm (π_A, π_B) Under Uncertain Demand When the Firm Employs the Last-Minute Sales Strategy vs. the Opaque Sales Strategy



Note. For the figure, we use $\alpha = 1/2$, $K = 1$, $L = 1/2$, $H = 3/2$, $t = 1$, and $\delta = 1$.

Above a certain threshold for V , LMSS profits become higher than OpSS profits.

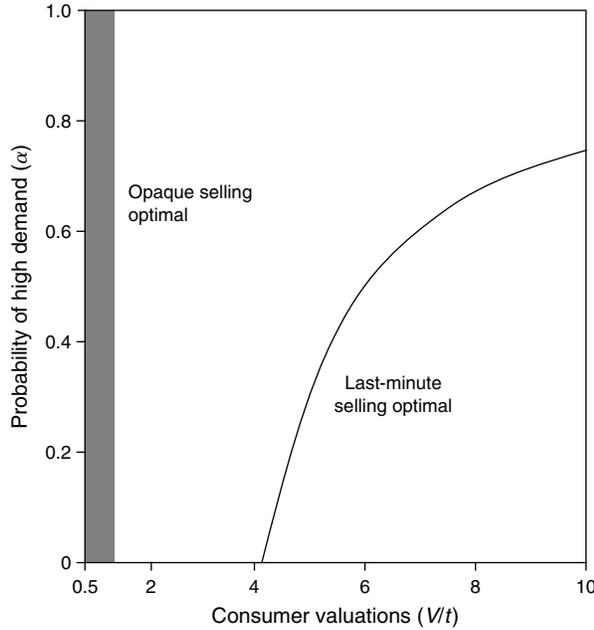
To see why the above result holds, note that under LMSS the bulk of a firm’s profits comes from products sold in the first period to the consumers that are closer to the firm on the Hotelling line. If the valuation for products in the market is high (i.e., V is high), the first-period prices are high. However, if the valuation for products is low, the first-period prices are low, the second-period prices are even lower, and hence profits from LMSS are low. In OpSS, on the other hand, the first-period prices are higher than in LMSS for low V because each firm is choosing to cover only a small portion of the market in the transparent channel by charging a price that makes the surplus of the marginal consumer equal to zero and covers the rest using the opaque channel. Moreover, note that the second-period prices in the opaque channel (if opaque sales are present) are equal to or higher than the second-period prices for LMSS (except when $V/t < 1 - K/(2H)$). As in the deterministic demand case, by masking the identity of the product, the opaque intermediary can sustain relatively higher second-period prices. Hence, for low V/t , OpSS yields higher profits.

As V increases, the revenue from LMSS increases faster, because the firms are able to separate out the consumers with a high preference for a particular firm and charge these consumers higher prices even if demand is low. In OpSS, on the other hand, prices are such that the firms cover a large portion of the market at lower prices if demand is low. In fact, if V is high enough, the firms are in a competitive equilibrium in the first period itself under OpSS when demand is low so that prices are very low. (In Figure 2, this is the region where the OpSS profits level off.) Hence, when V is high, LMSS yields higher profits because it allows the firms to “milk” the high-preference consumers in the first period, even if it has to charge lower prices in the second period when demand turns out to be low.

What happens as the probability of high-demand realization increases? As we discussed earlier for OpSS, as the probability of high-demand realization increases, consumers are under a higher threat of stockout in the first period. Thus, many more consumers prefer to buy in the first period, and therefore the firms increase prices. In other words, not only is there a higher chance that demand is high, the prices are also high. If demand turns out to be low, the first-period sales suffer, but the leftover capacity is cleared through the opaque channel. Over the two periods, expected profits increase.

In LMSS, however, the firms charge a first-period price that increases at a slower rate with an increase in the probability of high demand. Further, consumers with low firm preferences buy only if demand is low,

Figure 3 Strategies the Firms Should Adopt for Different Consumer Valuations (V/t) and Different Probabilities of High Demand (α)



Notes. The shaded area denotes the region where the opaque selling market exists for deterministic low demand (i.e., when $V/t \leq 1$). For the figure, we use $K = 1$, $L = 1/2$, $H = 3/2$, $t = 1$, and $\delta = 1$. Qualitatively, the results do not change for other values of the parameters.

which now happens with lower probability. Hence, even though expected profits increase (because there is a higher chance of high demand), the increase is slower than in OpSS. Figure 3 summarizes the comparison between the opaque strategy and the last-minute direct sales strategy for various probabilities of high demand ($\alpha \in [0, 1)$ on the y -axis) and consumer valuations (V on the x -axis).

5.3.1. Results for Subgame Perfect Nash Equilibrium.¹⁰ In the rational expectations equilibrium that we have derived above, consumers are assumed to rationally predict the full equilibrium path of the dynamic game; i.e., in the beginning of the first period, they develop correct *point* expectations regarding ticket availability (in equilibrium) from each firm in the second period. In contrast, in a subgame perfect Nash equilibrium, expectations on ticket availability from each firm are developed at the end of the first period as a *function* of the prices charged by the firms in the first period. Both rational expectations equilibrium and subgame perfect Nash equilibrium have been used as solution concepts for dynamic games in the extant literature. The formulation using the subgame perfect Nash equilibrium concept is

¹⁰ We thank an anonymous reviewer and the associate editor for suggesting this line of analysis.

algebraically intractable for our model, so we conduct a numerical study to analyze this equilibrium. We find that the results are qualitatively the same; i.e., as the probability of high demand (α) increases, opaque selling is preferred for larger values of consumer valuation of the product (V/t). Details on the numerical analysis are available in §A2.4 in the technical appendix.

5.4. Mixed-Strategy Equilibria¹¹

So far, we modeled a problem with two firms that announce their strategies in the first period (OpSS or LMSS), implicitly assuming that both firms select the same strategy. This is a natural assumption because if one firm does not select the opaque selling strategy, the other firm cannot implement it alone. One way to circumvent this difficulty and allow firms to pick different selling strategies is to consider mixed strategies—both firms randomize between the strategies under consideration and the relevant strategy is chosen in the second period, with some probability.

Suppose firm A plays OpSS with probability $q_A \in [0, 1]$ and LMSS with probability $1 - q_A$, and firm B plays OpSS with probability $q_B \in [0, 1]$ and LMSS with probability $1 - q_B$. Let π_A^O denote firm A’s profit in the first period, $\pi_A^{O,2}$ denote firm A’s expected profit in the second period when opaque tickets are sold in the second period (so that total expected profit is $\pi_A^O = \pi_A^1 + \pi_A^{O,2}$), and $\pi_A^{T,2}$ denote firm A’s expected profit in the second period when transparent tickets are sold in the second period (so that total expected profit is $\pi_A^T = \pi_A^1 + \pi_A^{T,2}$). If firm A plays OpSS, with probability q_B opaque tickets will be sold in the second period (if firm B also plays OpSS) and with probability $1 - q_B$ transparent tickets will be sold in the second period (if firm B plays LMSS), firm A’s expected profit will be equal to $q_B \pi_A^{O,2} + (1 - q_B) \pi_A^{T,2}$. If firm A plays LMSS, only transparent tickets can be sold in the second period, and its expected profit will be $\pi_A^{T,2}$. Hence, for firm A, we obtain the condition for mixing between OpSS and LMSS as

$$\begin{aligned} &\text{Expected Profit from OpSS} \\ &= \text{Expected Profit from LMSS,} \\ &q_B \pi_A^{O,2} + (1 - q_B) \pi_A^{T,2} = \pi_A^{T,2} \\ &\Rightarrow \pi_A^{O,2} = \pi_A^{T,2} \Rightarrow \pi_A^1 + \pi_A^{O,2} = \pi_A^1 + \pi_A^{T,2} \\ &\Rightarrow \pi_A^{O,2} = \pi_A^{T,2}. \end{aligned} \tag{1}$$

Similarly, for firm B, we obtain $\pi_B^{O,2} = \pi_B^{T,2}$. The above result provides the interesting insight that each firm will adopt a mixed strategy when the profits from

¹¹ We thank the associate editor for suggesting this line of analysis.

opaque sales and transparent sales are equal in the second period (condition (1)). We obtain this condition because one firm alone cannot implement the opaque selling strategy. We provide the rest of the analysis in §A2.2 in the technical appendix and provide two salient insights here.

First, we obtain the mixing probabilities for both firms as $q_A = q_B = \sqrt{\alpha(H-K)/((1-\alpha)H + \alpha K)}$. This expression implies that as the probability of high demand (given by α) increases, the firms choose the opaque selling strategy with higher probability. This is in line with the insight from Figure 3, which shows that firms prefer opaque selling as α increases. Furthermore, as capacity becomes more constrained (H increases or, alternatively, K decreases), the firms choose the opaque selling strategy with higher probability. Note, however, that the mixing probabilities do not depend on L because $L < K$ and the market is always unconstrained in the low-demand state. Second, when the firms mix between opaque and transparent sales, the prices charged for transparent tickets in the second period are equal under high and low demand (and both are equal to $\delta(V - t/2)$). This is derived indirectly from condition (1), which states that in both high- and low-demand cases, the second-period profits from transparent sales and opaque sales are equal. Under opaque selling, in turn, prices are equal to $V - t/2$ in both the high- and low-demand cases (because for symmetric firms, tickets come from either firm with equal probability). This leads to the result that the prices charged for transparent tickets in the second period are equal in high- and low-demand cases.

6. Asymmetric Firms with Unequal Capacities¹²

So far, we assumed the two firms to be symmetric in all respects, which leads to “perfect masking” in the opaque channel; i.e., all consumers obtain a ticket from either firm with equal probability. If firms are asymmetric, with one firm having larger capacity than the other, then consumers will expect that the probability that an opaque ticket is from the larger firm is higher. In this section, we extend the basic model to investigate the implications of the inability to achieve perfect masking in the opaque channel. Broadly, we find that using opaque sales still helps firms to increase their profits. However, the pricing power of the opaque intermediary and of the two firms indeed diminishes, and the firm with a larger capacity is at a greater disadvantage.

6.1. Opaque Selling with Deterministic Demand

Consider first the case of deterministic low demand; i.e., the total capacity of the firms is more than the total demand in the market. Here we generalize the model in §4.2 by assuming that the capacities of firms A and B are given by K_A and K_B , respectively, and $K_B \geq K_A$. Our solution approach is also the same as in §4.2. We provide the details in §A3.1 in the technical appendix and discuss some salient insights here.

Because firm A has smaller capacity, consumers rationally expect that the probability that an opaque ticket is from firm A is less than half; i.e., $\gamma_A^e < 1/2$.¹³ This implies that in the opaque channel, the surplus for a consumer located at x , given by $V - p_I - \gamma_A^e tx - (1 - \gamma_A^e)t(1 - x)$, increases with x . In other words, the leftmost consumer on the Hotelling line who purchases an opaque ticket will have the lowest surplus, and all other consumers will have surplus greater than the surplus of this consumer. The opaque intermediary prices such that the consumer at x_A , who is indifferent between purchasing a transparent ticket from firm A and an opaque ticket, has zero surplus. (This gives us the equalities $V - p_A - tx_A = V - p_I - \gamma_A^e tx_A - (1 - \gamma_A^e)t(1 - x_A) = 0$, which also implies that firm A prices its transparent ticket to make the surplus of its marginal consumer equal to zero.)

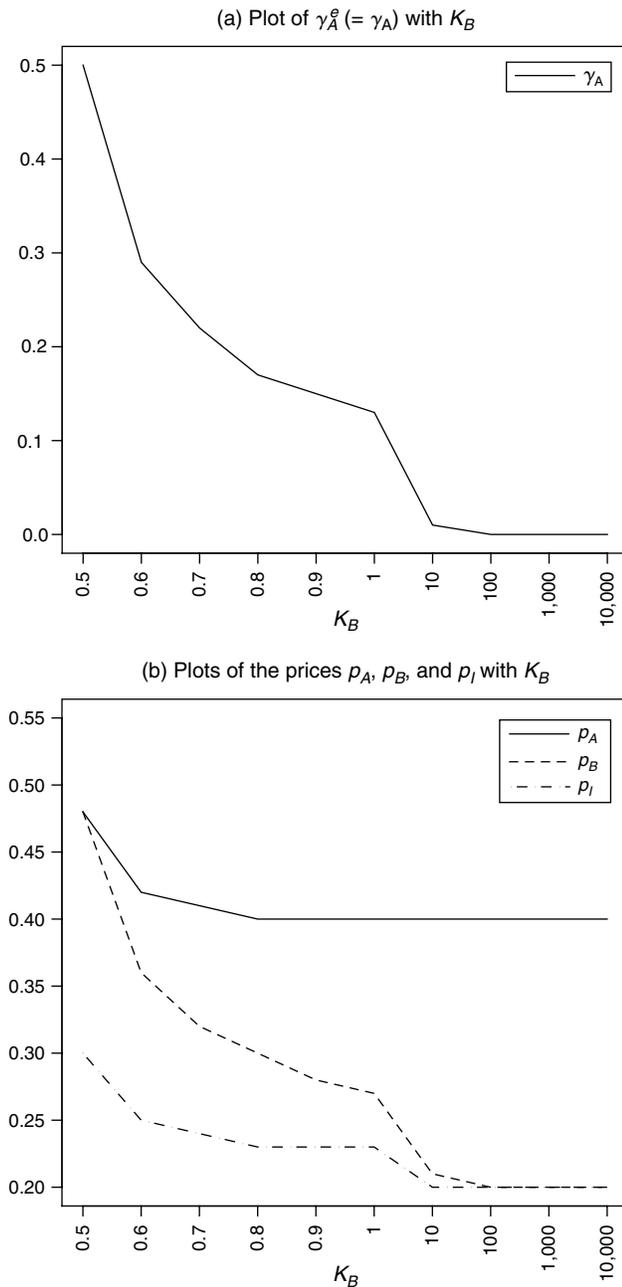
The above arguments also imply that all other consumers closer to firm B will have positive surplus in the opaque channel. Now consider the consumer at x_B who is indifferent between purchasing a transparent ticket from firm B and an opaque ticket. This consumer prefers a ticket from firm B, and there is a higher chance that the opaque ticket is from firm B. Hence, this consumer has a positive surplus in the opaque channel; i.e., $V - p_I - \gamma_A^e tx_B - (1 - \gamma_A^e) \cdot t(1 - x_B) > 0$ because $x_B > x_A$. Hence, firm B will charge a price p_B such that $V - p_B - t(1 - x_B) = V - p_I - \gamma_A^e tx_B - (1 - \gamma_A^e)t(1 - x_B) > 0$. This implies that, unlike firm A, firm B cannot price in the first period to extract full surplus from its marginal consumer and will therefore charge a lower price for its transparent ticket than firm A.

The above discussion also clarifies why “imperfect masking” in the asymmetric-capacities case reduces the profits in opaque selling as compared to the symmetric-capacities case. Under imperfect masking, the larger-capacity firm cannot extract full surplus from its indifferent consumer. In contrast, recall that with perfect masking, in the symmetric-capacities case, both firms were able to extract full surplus from their indifferent consumers. Hence, in opaque selling with asymmetric capacities, the larger firm has to

¹² We thank an anonymous reviewer for suggesting this analysis.

¹³ Note that we also check that the case when the firm with the larger capacity has fewer tickets in the opaque market (i.e., the probability of obtaining an opaque ticket from the larger-capacity firm is $< 1/2$) is off the equilibrium path.

Figure 4 Trends in Equilibrium Expectations of Product Availability and Prices in the Opaque Channel as the Capacity of Firm B Increases



Notes. Note the abrupt change in scale on the x-axis. For the plots above, the values of the other parameters are chosen as $V = 0.8$, $t = 1$, $J = 1$, $\delta = 1$, and $K_A = 0.5$.

leave some “surplus on the table” for its indifferent consumer; whereas when the firms have symmetric capacities, neither firm leaves any surplus on the table for its indifferent consumer.

Figure 4(a) shows how the probability of obtaining a ticket from firm A in the opaque channel varies with the capacity of firm B. (Other choices of the parameters yield qualitatively the same insights.

Notably, if V/t is higher, the prices are higher but follow the same patterns.) When firms have equal capacity, the probability is equal to half, and as firm B’s capacity increases, the probability goes down. At the extreme, as firm B’s capacity becomes much larger than firm A’s capacity, this probability tends to zero. Figure 4(b) shows the corresponding prices. When the capacities are equal, firms A and B charge equal prices. As firm B’s capacity increases and consumers become more certain about the identity of the product in the opaque channel, all prices decline. Firm B’s price is always lower than that of firm A, and as its capacity becomes much larger than the capacity of firm A, its price approaches the opaque channel price. Hence, in equilibrium, the firm with the larger capacity is forced to price lower than the firm with the lower capacity. Furthermore, even though the larger firm has a greater total sales volume, its profit is lower than that of the firm with the lower capacity due to lower prices.

Other specifications, such as when the total capacity in the market is held fixed but the percentage of capacity held by one firm is increased, yield similar insights. The case of asymmetric firms when demand is deterministic and high (i.e., the total demand in the market is more than the combined capacity of the two firms) also yields similar insights. Because of space considerations, we do not discuss these cases and proceed directly to the more interesting scenario when demand is uncertain.

6.2. Opaque Selling with Uncertain Demand

We now generalize the model in §5.2 by assuming that the capacities of firms A and B are given by K_A and K_B , respectively, and $K_B \geq K_A$. The details of the solution are in §A3.2 in the technical appendix.

As in the deterministic-demand case, the masking of the opaque ticket’s identity is imperfect in the uncertain-demand case; i.e., consumers know that there is a higher probability that the opaque ticket is from the larger-capacity firm (i.e., firm B), which hurts the profits in the opaque channel (as compared to profits with perfect masking in the symmetric-capacities case). Overall, this imperfect masking leads to lower prices charged by the intermediary in both the high- and low-demand opaque channels and, in turn, for both firms in the first period, with firm B charging a price lower than firm A’s. Furthermore, unlike in the symmetric case, consumers can have different expectations of product availability in the high- and low-demand opaque channels, and this can lead to different prices of the opaque product in the two cases.

We now develop some salient insights with the help of an illustrative example. Using other values for these parameters yields qualitatively similar insights.

Note from Figure 5(a) that as the capacity of firm B increases, the probability that an opaque ticket is from firm A decreases for both the high- and low-demand opaque channels. Moreover, this probability is always lower in the high-demand opaque channel: When demand is high both firms sell more tickets (compared to the case when demand is low) in the first period, and therefore the proportion of tickets in the opaque channel is larger (compared to the case when demand is low) for the firm with higher capacity. In other words, the probability that an opaque ticket (if available) is from firm B (the larger-capacity firm) is higher when demand is high than when demand is low, the masking of the opaque product is less perfect, and the consumers know more clearly where the opaque ticket is from. This implies that, ironically, the opaque intermediary will charge a lower price for the opaque product in the high-demand opaque channel than in the low-demand opaque channel. From Figure 5(b) we see that this is indeed the case. We also see that the first-period prices for both firms are higher than the second-period opaque channel prices, with firm B charging a lower price than firm A.

As the consumers' valuation for tickets (V) increases, keeping other parameters constant, all prices increase (as shown in Figure 5(d)) and firms A and B cover a larger part of the Hotelling line through transparent sales. Hence, both firms sell a larger number of tickets in the first period, the overflow of tickets into the second period decreases for both firms, and the proportion of tickets in the opaque channel increases for the larger firm with increasing V . Hence, as V increases, the probability that an opaque ticket is from firm A decreases, and this decrease is sharper in the case of high demand. This is shown in Figure 5(c). Finally, as the probability of high demand (α) increases, customers are under a larger threat of stockout in the second period. Hence, both firms charge higher transparent channel prices in the first period (Figure 5(f)) and cover a smaller market. As firms sell a smaller number of tickets in the first period, the overflow of tickets into the second period increases for both firms and the proportion of tickets in the opaque channel decreases for the larger firm with increasing α for both high- and low-demand scenarios. Hence, as α increases, masking of the opaque product improves (Figure 5(e)), which leads to increasing opaque prices (Figure 5(f)).

To summarize the results above, opaque selling always increases the profits of both firms in the asymmetric-capacities scenario as compared to not using opaque selling at all. However, imperfect masking of the opaque product reduces the efficacy of the opaque channel as compared to perfect masking in the symmetric-capacities scenario. Furthermore, this imperfect masking hurts the prices and profit of the

larger firm more than it hurts the prices and profit of the smaller firm, to the extent that the larger firm makes overall lower profit across the two periods than the smaller firm.

6.3. Comparison with LMSS

The above analysis leads to a natural question: Will asymmetric firms prefer OpSS or LMSS? To answer this question, we solve the LMSS game with asymmetric firms and obtain the following insights.¹⁴

The main difference between the asymmetric-capacities case and the symmetric-capacities case is that because firm B has larger capacity, it charges lower prices than firm A and sells more than firm A. Because of its higher sales, it makes a larger profit than firm A (though this is lesser than the symmetric-case profit due to reduced prices). However, even with asymmetric capacities, the nature of the LMSS equilibrium remains similar to that in the symmetric case (described in Proposition 5.1). Specifically, both firms price in the first period such that all consumers with "high brand preference" purchase at high prices from their preferred firms in the first period (because of the threat of stockout later), and if demand is high, both firms stock out in the first period itself. If demand turns out to be low, there is leftover capacity for both firms and they compete and price low in the second period. The bulk of the profits for each firm comes from the first period.

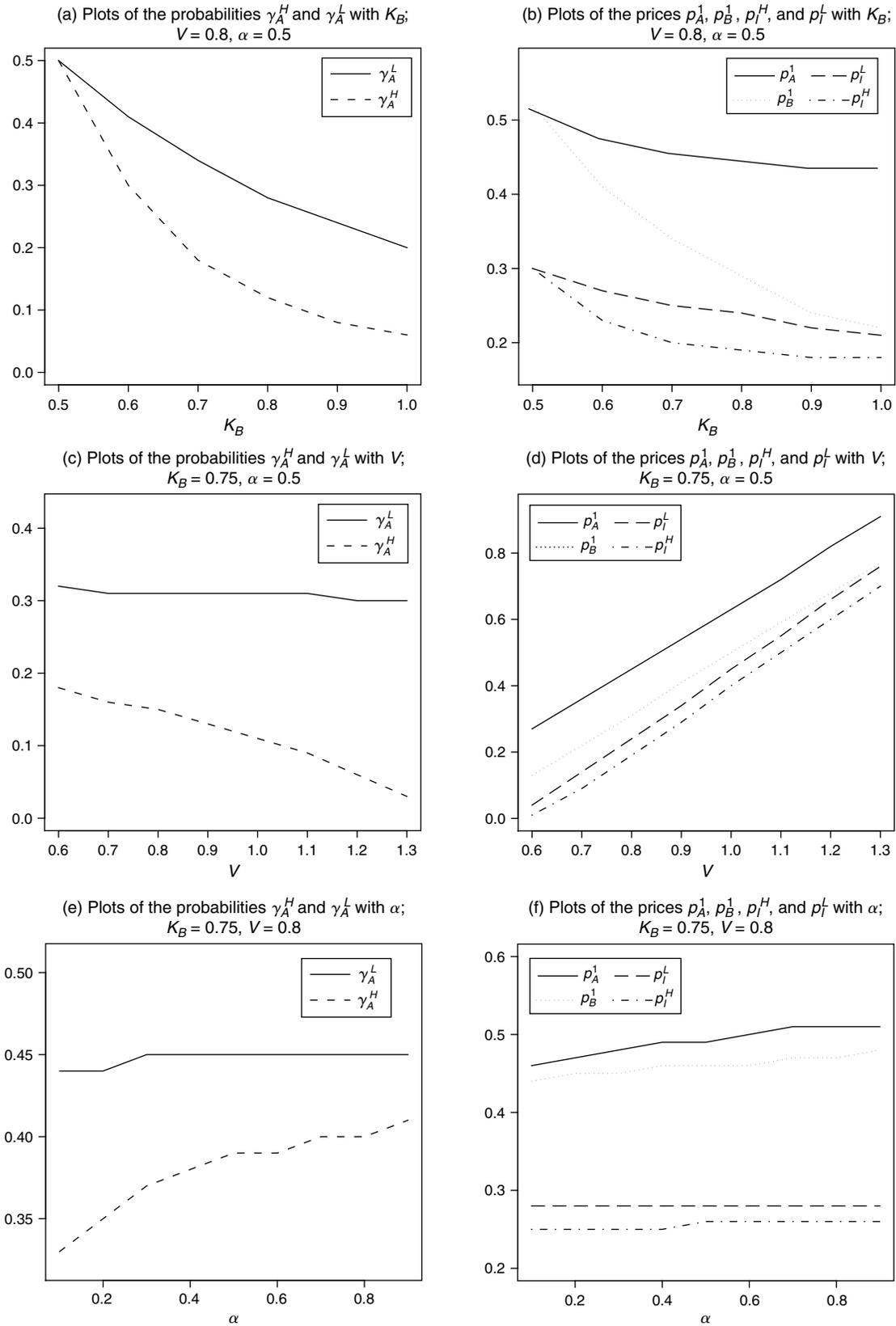
From a numerical comparison with OpSS, we find that asymmetric firms prefer LMSS over a larger region of the (V/t) - α space. (Specifically, the equal-profit contours in the (V/t) - α space in Figure 3 shift more and more outward in the top-left direction as K_B/K_A increases.) The reason is that in LMSS the nature of the equilibrium remains the same, as described above. In OpSS, however, asymmetric firms cannot achieve perfect masking in the opaque channel in the second period. Because the opaque channel profits hinge upon how well product identity can be masked, imperfect masking significantly reduces the efficacy of the opaque channel. As a consequence, there is reduced preference for opaque selling with asymmetric firms.

7. Other Modeling Considerations

Opaque sales and last-minute sales are encountered in a variety of practical situations, many of which are not fully reflected in our stylized model. Thus, apart from analyzing the core model above, we point out some modeling variations that that might form a good starting point for various future research considerations.

¹⁴ We characterize the equilibrium in §A3.3 in the technical appendix but do not provide the details of the solution due to space constraints and because the derivation is almost exactly as in §A2.3 in the technical appendix.

Figure 5 Trends in Equilibrium Expectations of Product Availability in the Opaque Channel and Prices as the Capacity of Firm B Increases



Notes. For the plots above, we use $t = 1, L = 0.75, H = 1.25, \delta = 1,$ and $K_A = 0.5$. Values of the other parameters are specified next to the corresponding plots. Other choices of the parameter values yield qualitatively the same plots.

7.1. Damaged Opaque Goods

In our basic model, we assume that consumers derive the same utility from flying with a firm regardless of whether they buy in the direct channel or the opaque channel. However, firms sometimes force opaque sellers to sell “damaged” goods. For instance, firms might disallow re-booking or charge high cancellation fees for opaque goods. One way to make this consideration consistent with our model is to impose that “damaging” the goods affects valuations that consumers obtain on opaque purchases. In our model, this can be admitted by discounting the valuation in the opaque channel to $\delta_o V$, where $\delta_o \in (0, 1)$ is a factor that accounts for the possible disutility from the reduced flexibility in the opaque channel. As a consequence of reduced valuation, the intermediary charges a reduced price $p_I = \delta_o V - t/2$ in equilibrium. We note that in the symmetric model, this change affects the prices only on the opaque selling channel, whereas the LMSS prices remain unaffected. As a result, the revenues for the intermediary are reduced in the opaque channel, and more consumers purchase directly from the firms in the first period. Hence, firms are more likely to sell in their own channels. Thus, if opaque goods are damaged, LMSS may be preferred to OpSS over a larger range of problem parameters.

7.2. Concentrated vs. Monopolistic Markets

In this paper, we analyzed competing firms selling through an opaque intermediary. In some situations, however, the same service providers sell opaque products as well. For example, Norwegian Cruise Line offers both specific staterooms on their cruise ships as well as opaque staterooms that guarantee certain minimal amenities but not a specific location on the ship. Similarly, one airline might be selling opaque tickets with different departure times on the same route (e.g., morning versus evening flights). We considered a variation of our model in which both transparent products are managed by the same firm, which maximizes its total profit. We find that the monopoly firm is able to derive higher profit from LMSS (because second-period prices for transparent tickets are set in a monopolistic rather than a competitive scenario), whereas our other results remain qualitatively unchanged. Thus, without competition, LMSS may be preferred to OpSS over a larger range of problem parameters.

7.3. Heterogeneous Values for the Core Product

In the basic model, we assumed that consumers are homogeneous in their preference for the core product; i.e., valuation V does not vary by consumer. In practice, some companies (e.g., airlines) derive significant profits by discriminating between “business”

and “leisure” travelers, who typically have drastically different preferences for travel.

We believe that these considerations will not impact the main insights from the model because consumers with high utility for product consumption are likely to purchase the product at full price and would not participate in either opaque or last-minute sales channels. Thus, our model focuses exclusively on price-conscious consumers with relatively low value for the product itself. It is, however, possible to incorporate into our model heterogeneous consumers that differ in their core value for the product. For instance, we could introduce a second Hotelling line with a much larger core value \bar{V} representing consumers with high valuation for the product. (We observe that this approach introduces more complexity to the model.) Because these consumers have high willingness to pay, the firms will allocate capacity to satisfy these consumers first and then sell to consumers with lower V . Future research can carefully explore how this might affect the trade-off between OpSS and LMSS.

7.4. Multiple Hidden Product Attributes and Vertical Product Differentiation

In our model, we assume that products are characterized by a single attribute: the company that sells it. In practice, however, products may differ along multiple dimensions. Hotel rooms purchased on Hotwire.com differ in size, location, and amenities. Airline tickets differ in the number of stops, departure times, and trip lengths. All these different attributes can be hidden from (or revealed to) consumers in the opaque selling channel. Some opaque intermediaries allow consumers to select the level of opacity. For example, Priceline.com lets its consumers specify whether a “red eye” flight is acceptable and it also allows users to set the upper bound on the number of stops. Selecting the optimal level of opacity and the right attributes to hide provides potential for future research but is outside the scope of this study. We believe adding multiple dimensions of opacity might shift consumer preferences toward the direct last-minute sales channel. In our model, consumers are certain that the firms are selling products of identical valuations in both the channels. However, if there is additional uncertainty about exact features of the product purchased from the opaque channel, then the consumers will be more likely to purchase directly from the firms.

7.5. Channel Design and Intermediary Selection Decisions

Our paper models the decision of firms to choose between LMSS (selling directly) and OpSS (an opaque intermediary) to sell limited inventories to strategic

consumers. However, channel design and choosing between several available intermediaries remain as important decisions for a large number of firms. In a sequence of papers, Rangan (1986) and Rangan et al. (1986) discuss a prescriptive model for designing channels (with wholesalers and retailers) and choosing the right intermediary for the channel designed. Consumer demand is modeled exogenously in those papers. How strategic consumer behavior affects airline channel design (e.g., selling online through Expedia.com versus selling through travel agents) is a pertinent question that we leave for future research.

8. Discussion and Conclusions

Because of uncertain demand and (short-term) inflexible capacity, firms in travel industries often end up with one of the two extremes—a shortfall of capacity due to high demand or leftover unused (and expensive) capacity due to low demand. To deal with the mismatch between demand and supply, firms have implemented a variety of strategies; two of the most prominent strategies are direct last-minute sales at reduced prices and sales through an opaque intermediary. However, consumers are becoming more and more strategic—they have learned to anticipate this last-minute distress selling and might decide to postpone their purchase in expectation of future lower prices. The risk the consumers face while making this decision is not being able to obtain a product if demand turns out to be high.

In this paper, we model this strategic interaction between competing firms and consumers and shed light on the following question: When should firms offer last-minute sales directly to consumers versus through an opaque intermediary? We find that the answer depends on at least three factors: (1) the valuations that consumers have for the service, (2) the strength of brand preference that consumers have for competing firms (alternatively, the extent of service differentiation between competing firms), and (3) the probability that demand in the market exceeds capacity. If consumer valuation for a product is high or the strength of brand preference of the consumers in the market is low, or both, firms prefer direct last-minute sales over opaque sales. Furthermore, as the probability of high demand increases, firms start to prefer opaque sales over direct last-minute sales. At the extreme, if market demand is deterministic, direct last-minute sales are never offered, whereas opaque sales can be offered if consumer valuations for travel are very low. These findings immediately translate into empirically testable hypotheses.

We find that the dynamics underlying these two selling strategies are very different. By using direct last-minute sales, each firm prices in the first period

so that only consumers with high preference for the firm buy the product. Thus, each firm derives the bulk of its profits primarily by charging high prices to these consumers, whereas second-period prices are very low (however, these cheap products are available only if demand turns out to be lower than capacity). Quite differently, in the opaque selling strategy, if the consumer valuations are very low, the firms set first-period prices to extract maximum profits from consumers and then clear any remaining products through the opaque channel. When valuations are high, the firms price in the first period to ensure that the number of consumers that wants to buy products exceeds supply if demand is high (which, at this point in time, is unknown to everybody), introducing clamor for the limited number of products and leveraging the risk of product shortage to charge higher first-period prices. To summarize, the direct last-minute sales strategy can be construed as extracting profits from high-preference consumers, whereas the opaque sales strategy can be thought of as creating a frenzy for products to raise prices. Clearly, both strategies in our paper are far from simple “inventory clearance mechanisms”—they are indeed firms’ strategic responses to consumers’ strategic purchasing decisions.

We omitted several considerations from the model in order to obtain sharper insights, and these considerations pose several interesting questions for future research. First and foremost, how opaque should the opaque product be? Should consumers be able to specify at least some time intervals for the departure or not? Addressing this important question is beyond the scope of the current study. Second, we simplified the decision for the firms by allowing for two sales opportunities: one “regular” and one “sales.” In practice, for example, airlines offer many fares, and prices tend to increase until the very last moment when last-minute sales are announced.

In our model, all consumers fully know their brand preference-adjusted valuations. Those customers with higher valuations naturally buy earlier. However, customers may not always know their valuation for flying. For instance, some customers, despite having high valuation for flying for an occasion, may realize their need late in the selling season. This remains a challenging problem and is explored in a stream of research by Akan et al. (2008), Courty and Li (2000), and others, where factors such as product opacity and competition are not considered. It is fruitful to combine the research issues to explore the impact of learning behavior on opaque selling. Because of the complexities involved with dynamic learning models, we leave the solution of this problem for future research.

Incorporating the above considerations into modern decision support systems remains a challenge. Finally, although numerous studies have modeled airline revenue management decisions, there have been very few attempts to verify these findings empirically (see Koenigsberg et al. 2008 for an exception). Empirical studies tend to be limited by data availability—although airlines periodically share data with regulatory authorities, these data are not precise enough to distill specific pricing strategies employed by an airline. All these directions are promising areas of future research in years to come.

9. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at <http://mansci.journal.informs.org/>.

Acknowledgments

This research was supported by the Mack Center for Technological Innovation at the Wharton School. The authors thank the faculty of the OPIM and Marketing departments at the Wharton School and seminar participants at the University of British Columbia, New York University, Mini-Conference on Customer-Oriented Operations Models at Washington University in St. Louis, Marketing Science Conference 2008 Revenue Management and Pricing Conference 2008, and INFORMS 2009 for their helpful comments. They also thank Wilfred Amaldoss, Krishnan Anand, James Dana, Burton Hollifield, two anonymous reviewers, the associate editor, and the department editor for their extremely valuable suggestions, which improved this paper significantly.

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