

HETEROGENEOUS TIME PREFERENCES WITHIN THE HOUSEHOLD

ANDREW HERTZBERG*

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Abstract

Substantial evidence suggests that discount factors vary significantly between individuals and that this variation exists between members of the same household. This paper introduces a model of consumption and savings in which household members discount the utility from their future consumption at different rates. Each period household members bargain efficiently over their consumption and saving choices. The ex-ante optimal consumption plan will prescribe a declining share of household consumption to the impatient household member and a savings rate later in life that is determined primarily by the time preferences of the patient member. However, as evidence suggests, the household lacks dynamic commitment and can renegotiate any consumption plan that was made in an earlier period. I show that if both members have constant bargaining power the ex-ante optimal consumption plan is time inconsistent despite both members being rational forward looking agents with time consistent preferences. Later in life the impatient member will bargain for both a higher share of consumption and a higher propensity to consume out of wealth than under the agreed ex-ante optimum. The household can achieve a optimal path of consumption and savings by increasing the bargaining power of the patient member over time. The household's ability to achieve the full commitment optimum is constrained by the altruism between household members. The framework provides an explanation of how control over household assets will evolve over time.

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*Columbia Business School. E-mail: ah2692@columbia.edu. I thank seminar participants at Columbia Business School, Southern Methodist University, and the Norwegian School of Economics. All remaining errors are my own.

I Introduction

The standard theory of household financial decision making is based upon the optimization of a single unified maximizing agent. However a considerable body of empirical work strongly rejects the unitary model as an adequate description of household decision making (see for example Lundberg, Pollak, and Wales 1997; Browning and Chiappori 1998; Phipps and Burton 1998; and, Ashraf 2009). Taking up the challenge posed by these empirical findings has led to considerable theoretical work that reconsiders the theory of household demand for goods and labor supply as the outcome of bargaining process between members with their own distinct objectives (see for example Chiappori 1988; Browning et al. 1994; Browning and Chiappori 1998; Chiappori Fortin and Lacroix 2002). Little attention has been directed to studying how bargaining among household members will affect financial decision making.¹ In this paper I focus on one important source for disagreement in the household: different time preferences among members. Considerable evidence suggests that time preferences vary substantially between individuals. For example, Warner and Pleeter (2001) show that military downsizing payout choices imply personal discount rates that vary between 0 and 30%. Schaner (2012) measures the difference in time preferences of Kenyan household members and finds that 50% of couples have weekly discount factors that vary by more than 0.193. Motivated by this evidence, the particular question I ask in this paper is: are households in which members have heterogeneous time preferences able to carry out optimal consumption and savings plans? As a follow on to this question, I ask how the household will choose to allocate power over these decisions over time to implement the optimal consumption and savings plan.

In the model I propose the household is comprised of two members who have different exponential discount factors. Each period they negotiate on how much to dedicate to consumption on private consumption goods for each member, how much to spend on shared

¹Recent exceptions include Mazzocco 2005; Mazzocco 2007; Schaner 2011.

public consumption and how much to save. The crucial assumption I make is that the household lacks commitment. At any period the household can renegotiate any plan they made in previous periods. Both members are rational forward looking agents who anticipate any future renegotiation. The ex-ante optimal lifetime consumption and savings plan has two distinct features. First, the impatient household member's share of consumption should fall over time. Second, the savings rate of the household should increase over time to increasingly reflect the preferences of the patient household member.

I start by studying the scenario in which both members have constant bargaining power over the life of the household. I show that in this case the optimal path of consumption and savings is time inconsistent. Later in life both members will renegotiate to give a higher share of consumption to the impatient member than under the optimal plan. In addition future savings rates will be renegotiated to give more weight to the time preferences of the impatient member than under the ex-ante optimal plan. This force is similar to the one studied by Jackson and Yariv (2012). This time inconsistency occurs despite the fact that both members of the household have time consistent time preferences and are rational forward looking agents who, on their own, would form time consistent savings plans. The household is willing to pay for a technology that allows them to achieve the ex-ante Pareto optimal consumption plan. I compute the value of commitment and show that most of the welfare cost from the sub-optimal consumption plan comes from the misallocation of resources between household members.

I then ask what dynamic pattern of household control would implement the optimal consumption and savings plan. I show that this may be achieved if the patient member receives increasing bargaining power over the life of the household.² I show that such a pattern of household control can be achieved without any commitment as long as members can choose each period the bargaining power they will carry into the next period. This may be imple-

²Aura (2005) provides evidence that changes in bargaining power within the household can affect savings rates.

mented in practice for example if the household decides what fraction of household savings are controlled by each member at the end of each period. Despite the fact that this arrangement can be renegotiated next period it is sufficient to implement the optimal consumption and savings plan. As such the model provides a theory for how control of the assets of the household evolve over time. This result is related to Gollier and Zeckhauser (2005) who study a decentralized economy in which agents have heterogenous time preferences. They show that the optimal dynamic allocation of consumption is achieved because patient agents share of aggregate wealth will increase over time and so the effective social discount factor will tend towards their rate of time preference.

In many cases however it is not feasible to implement the optimal consumption and savings plan if altruism between each member is too high. In this case, even though an optimal ex-ante plan may call for assigning very little consumption to member *A* later in life, *B* will not chose to do so when that point in time is reached, even if she has full control over all household consumption and savings decisions. This contrasts sharply with the result in Hertzberg (2012) where the dynamic commons problem that leads to over consumption is made strictly worse when altruism within the household is lower.

This paper is related to the literature that studies individuals having hyperbolic discount factors or self control problems that render optimal savings plans time inconsistent (for example: Thaler Shefrin 1981; Laibson 1997; Harris and Laibson 2001; Laibson, Repetto, and Tobacman 2003). That literature is based on considerable evidence at the individual level (see for example Ainslie 1992; Frederick Lowenstein and O'Donoghue 2002; Shapiro 2005). The goal of this paper is not to replace that explanation of time-inconsistency but rather to stress that heterogeneous time preferences within the household can lead to further savings distortions and will rationalize commitment technologies not previously considered by the literature focusing on individual time preferences or self control (see for example Thaler and Benartzi 2004; Ashraf, Karlan, and Yin 2006; Beshears et al. 2011).

This paper is closely related to the theoretical literature that incorporates misaligned

preferences within the household (see Lundberg and Pollack 2007; Browning, Chiappori, and Lechene 2006 for comprehensive surveys of the literature). In these papers, household decision making is typically modeled as the outcome of an efficient bargaining process and the focus is directed to determining what determines the threat points and bargaining weights of each household member.³ Evidence on the question of whether households are able to enforce Pareto efficient allocations is mixed. Chiappori and Donni (2009) point out that tests for static efficiency (see for example Bobonis 2009, Browning and Chiappori 1998, and Chiappori Fotin and Lacroix 2002) find in the affirmative whereas tests for dynamic efficiency find the opposite (see for example de Mel et al. 2009, Duflo and Udry 2004, Mazzocco 2007, Robinson 2011, Udry 1996). The framework presented in this paper adheres to this dichotomy by modeling allocations within each period as the outcome of Pareto efficient bargaining, but incorporates limited commitment over time which leads to the dynamic time inconsistency I study.

The paper proceeds as follows. Section II sets up the base model. Section III characterizes the equilibrium consumption choices of the household, compares them to the full commitment solution, and computes the value of commitment for the household. Section IV characterizes how bargaining power should evolve over time to implement the optimal consumption plan and shows how altruism will limit how closely the optimal plan can be achieved.

II Model of Household Consumption

The household has two members indexed by i labeled A and B . Time is discrete and indexed by t . The household is formed at the beginning of period $t = 1$. Both household members live for T years. I assume that the household remains together for their entire lives with certainty and abstract from endogenous household formation.

³As an exception non-cooperative decision making within the household has been considered by Lundberg and Pollack 1993; Chen Woolley 2001; and, Lundberg and Pollack 2003.

A Preferences

Each period member i consumes a single consumption good and a shared non-rival public good. Let $C_{i,t}$ denote the amount of the private good consumed by member i in period t and let H_t denote the public good. The utility derived by member i from their own consumption in period t is

$$u_{i,t} = \mu \ln C_{i,t} + (1 - \mu) \ln H_t \quad (1)$$

where μ is the weight each member places on private consumption relative to public consumption. Note that member i does not directly derive utility from member j 's private consumption.

Both household members discount utility from future consumption using an exponential discount factor $\delta_i \in (0, 1)$. Household members have different time preferences so that in general $\delta_A \neq \delta_B$. Without loss of generality I will adopt the convention that B is the patient household member: $\delta_A < \delta_B$. The individual discounted utility of household member i in period t is

$$U_{i,t} = \sum_{x=0}^{T-t} \delta_i^x u_{i,t+x}. \quad (2)$$

Thus $U_{i,t}$ is the utility of household member i absent any concern for the other household member. Note that these are standard time preferences so that when considered on their own the optimal consumption plan for each household member will be time consistent.

One of the defining characteristics of the household is that its members are altruistic. I capture this by supposing that member i places weight $\gamma_i \in (0, 1)$ on their own utility and weight $1 - \gamma_i$ on the utility of the other member. In general I will assume that $\gamma_i \geq \frac{1}{2}$ since the evidence on household consumption decisions suggests that household members generally care more about their own consumption than the other members of the household. The framework can also be applied to the case where the reverse is true.

The objective function of member i at t is:

$$V_{i,t} = \gamma_i U_{i,t} + (1 - \gamma_i) U_{j,t}. \quad (3)$$

The goal of each household member in bargaining will be to maximize this objective.

B Decision Making and Budget Constraint

The present value of all combined household wealth at the beginning of $t = 1$ is W_1 . For simplicity I assume the household starts life with wealth W_1 which is taken as given and has no income.⁴ Household members make consumption decisions by bargaining cooperatively each period. Household members cannot commit to a path of future consumption. As a result any agreed upon consumption plan that was formed in the past can be renegotiated if both members choose to do so. Each period household members cooperatively simultaneously decide how much of the remaining household wealth W_t to spend on each of the three consumption goods $C_{A,t}, C_{B,t}, H_t$ and how much to save. The dynamic equilibrium path of consumption will be the subgame perfect solution to the bargaining game between these two members. Both members are forward looking and will anticipate any renegotiation that will occur in the future. Let $\eta_t \in [0, 1]$ be the bargaining power of member A in period t . For now I will take the sequence of bargaining power over the life of the household as given and then later study how this will evolve if the household is able to determine η_{t+1} in period t . Let a single “*” denote the cooperative renegotiation proof equilibrium quantities $C_{i,t}^*$ and H_t^* that will be achieved in period without commitment. Formally these will be the solution

⁴This is identical to assuming that household labor supply decisions are fixed in which case W_t is the present value of all future income plus (minus) any savings (debt) that the household has at period t .

in period t to

$$\max_{C_{A,t}, C_{B,t}, H_t} \eta_t V_{A,t} + (1 - \eta_t) V_{B,t} \quad (4)$$

subject to

$$C_{i,x} = C_{i,x}^*(W_x) \text{ for } x = t + 1, \dots, T \text{ and } i = A, B \quad (5)$$

$$H_x = H_x^*(W_x) \text{ for } x = t + 1, \dots, T \quad (6)$$

$$W_t \geq X_t \text{ where } X_t \equiv C_{A,t} + C_{B,t} + H_t \quad (7)$$

$$W_{t+1} = R(W_t - X_t) \quad (8)$$

$$H_t, C_{i,t} \geq 0 \text{ for } i = A, B. \quad (9)$$

In this problem (5) and (6) represent the constraints faced by each member recognizing that they will renegotiate consumption plans in the future. The household budget constraint is captured by (7) and (8) which ensure that the household cannot spend more than its entire wealth and that wealth evolves over time by any savings earning a fixed gross rate of interest R . The constraints in (9) ensure that all consumption choices are nonnegative, however with log utility this will never bind.

C Full Commitment Problem and the Value of Commitment

To evaluate the optimality of the equilibrium consumption path that the household obtains without commitment, I compare it to the consumption path that would be achieved if the household was able to fully commit to consumption choices at the start of $t = 1$. Consider the problem the household would face in setting a full commitment path. Whenever $\gamma_i \neq \frac{1}{2}$ household members will disagree over the optimal allocation. However any allocation that they would choose must be Pareto optimal and hence I characterize the solution to the

following full commitment Pareto problem:

$$\max_{\{C_{A,t}, C_{B,t}, H_t\}_{t=1}^{t=T}} \Pi = \eta_1 V_{A,1} + (1 - \eta_1) V_{B,1} \quad (10)$$

$$\text{subject to } W_1 - \sum_{x=1}^T R^{-(x-1)} [C_{A,x} + C_{B,x} + H_x] \geq 0 \text{ and} \quad (11)$$

$$\{C_{A,t}, C_{B,t}, H_t\}_{t=1}^{t=T} \geq 0. \quad (12)$$

where $\eta_1 \in [0, 1]$ is the Pareto weight placed on the objective of member A . Let a double “**” denote the full commitment Pareto optimal consumption quantities $C_{i,t}^{**}$ that solve this problem.

To quantify the welfare loss incurred by the household under the non-cooperative equilibrium I calculate how much the household would be willing to pay at $t = 1$ for a technology that allowed them to commit to an optimal consumption path. Let $V_{i,1}^*(W_1)$ be the discounted lifetime utility that will be achieved by household member i absent commitment as a function of initial household wealth. Let $V_{i,1}^{**}(W_1(1 - \phi), \eta)$ be the counterpart for the case where the household has spent a fraction ϕ of their initial wealth W_1 to achieve the full commitment plan that places weight η on the preferences of member A . The value of commitment ϕ^{**} is defined as the most that the household will pay while ensuring that there exists a weight η so that the purchase is a Pareto improvement for both members. Formally ϕ^{**} solves:

$$\phi^{**} = \max_{\phi, \eta} \phi \quad (13)$$

$$\text{subject to } V_{i,1}^{**}(W_1(1 - \phi), \eta) \geq V_{i,1}^*(W_1) \text{ for } i \in \{A, B\}, \text{ and } \eta \in [0, 1].$$

An analytical solution for ϕ^{**} is intractable in most cases so this will be solved for numerically.

III Consumption Choices and Value of Commitment

A Equilibrium Consumption Choices

The equilibrium consumption path is solved in the Appendix. The equilibrium level of total household consumption in period t is

$$X_t^* = \frac{1}{1 + (1 - \theta_t) \sum_{x=1}^{T-t} \delta_A^x + \theta_t \sum_{x=1}^{T-t} \delta_B^x} W_t. \quad (14)$$

where

$$\theta_t = \gamma_B + \eta_t (1 - \gamma_A - \gamma_B)$$

is the weight placed on B 's individual discounted utility, $U_{B,t}$, in the bargaining problem in period t . Conversely $1 - \theta_t$ is the weight placed on $U_{A,t}$. Observe that θ_t is determined by both the bargaining power η_t and the altruism of each member γ_A and γ_B so that even if member B has all the bargaining power at t ($\eta_t = 0$) she will still place weight $1 - \gamma_B$ on A 's individual discounted utility. The propensity to consume in period t is determined by the bargaining power that each member has in that period and the weight that is placed on their individual time preferences. Since the household lacks commitment the propensity to consume does not depend on the time preferences of either member earlier in their life (i.e. before t). Note that the propensity to consume is not affected by the relative importance that public consumption plays in the household.

In equilibrium total household expenditure is divided between the private and public consumption goods in the following way:

$$C_{A,t}^* = (1 - \theta_t) \mu X_t^* \quad (15)$$

$$C_{B,t}^* = \theta_t \mu X_t^* \quad (16)$$

$$H_t^* = (1 - \mu) X_t^* \quad (17)$$

In words, the relative bargaining power and the altruism of each member in t determines how consumption is allocated in that period. As with the propensity to consume, it is unaffected by the time preferences of the members prior to period t .

B Comparison to Full Commitment Consumption Path

To assess the optimality of the equilibrium consumption path that the household will achieve without commitment, I compare it to the set of Pareto optimal consumption path that would be chosen if the household had access to perfect commitment at $t = 1$. The total level of consumption that the household would commit to in any period is

$$X_t^{**} = \frac{1}{1 + \frac{(1-\theta_1)\delta_A^{t-1} \sum_{x=1}^{T-t} \delta_A^x + \theta_1 \delta_B^{t-1} \sum_{x=1}^{T-t} \delta_B^x}{(1-\theta_1)\delta_A^{t-1} + \theta_1 \delta_B^{t-1}}} W_t. \quad (18)$$

Suppose that the bargaining power of each household member η_t is constant over time. This might be the case if it was determined primarily by institutional forces outside of the marriage such a family law, the labor market, and the market for spouses if these factors are fairly static over time. In this case, comparing the full commitment solution (18) to the level of consumption without commitment (14) leads directly to the following proposition:

Proposition 1: *Suppose that bargaining power is constant over time. If $\delta_A \neq \delta_B$ then the propensity to consume out of wealth is strictly higher without commitment, $\frac{X_t^*}{W_t^*} > \frac{X_t^{**}}{W_t^{**}}$, for all $t = 2, 3, \dots, T - 1$.⁵*

Proposition 1 is proved in the Appendix. The intuition for this result is as follows. In the full commitment solution the marginal utility of each member in each period is evaluated from the perspective of $t = 1$. As a result when the household would like a propensity to consume that becomes increasingly determined by the patient member over time since it is her marginal utility that is most important in an ex-ante sense. However, without commitment

⁵ W_t^* and W_t^{**} refer to the equilibrium level of household wealth in period t under the no commitment and full commitment consumption path.

the relative importance of the impatient member's time preferences does not diminish over time. As a result the consumption rate remains consistently above the rate that the household would like to have committed to at the start of its life. Panel A of Figure 1 compares the path of total household consumption absent commitment to the path of consumption the household would like to commit to. For the early part of the household's life the level of consumption without commitment is above the level the household would like to commit to due to the inefficiently high propensity to consume. Later in the life of the household the intertemporal budget constraint reverses this and the household consumes far less in the late years of its life than under the efficient plan.

A similar intuition applies to the allocation of consumption within each period. Under the full commitment consumption plan total consumption is allocated in the following way:

$$C_{A,t}^{**} = \frac{(1 - \theta_1) \delta_A^{t-1}}{(1 - \theta_1) \delta_A^{t-1} + \theta_1 \delta_B^{t-1}} \mu X_t^{**} \quad (19)$$

$$C_{B,t}^{**} = \frac{\theta_1 \delta_B^{t-1}}{(1 - \theta_1) \delta_A^{t-1} + \theta_1 \delta_B^{t-1}} \mu X_t^{**} \quad (20)$$

$$H_t^{**} = (1 - \mu) X_t^{**} \quad (21)$$

The optimal full commitment allocation directs an increasing share of private consumption to the patient member over time. This is not achieved in the household without commitment if both members maintain constant bargaining power. This is summarized in the following Proposition.

Proposition 2: *Suppose that bargaining power is constant over time. If $\delta_A < \delta_B$ then*

$$\frac{C_{A,t}^*}{C_{B,t}^*} > \frac{C_{A,t}^{**}}{C_{B,t}^{**}} \text{ for all } t = 2, 3, \dots, T.$$

The intuition for Proposition 2 is that with constant bargaining power each member will obtain a constant share of consumption each period. Panel B of Figure 1 compares the allocation of private consumption under the full commitment solution to the one that will be achieved without commitment if bargaining power is constant. The figure is drawn for the

case where bargaining power is equal over the life of the household. In this scenario in each period the marginal utility of consumption will be identical in each period. As a result, the private consumption of each member will be equal throughout the life of the household. In the efficient full commitment solution more private consumption should be directed to the patient member (member B) later in life.

Note that this allocative inefficiency only exists for the way private consumption is shared between each member. The share of consumption that is directed to public consumption is optimal in each period. By this argument we can use the parameter μ to vary the extent of each source of inefficiency. When $\mu = 0$ then only the intertemporal inefficiency characterized in Proposition 1 exists. Conversely, when $\mu = 1$ then both problems exist with the allocative inefficiency being the most severe from the perspective of ex-ante household welfare.

C The Value of Commitment

Having shown that the allocation without commitment is inefficient I now turn to quantifying this inefficiency. To do this I ask what fraction ϕ^{**} of the household's initial wealth would both household members agree to spend at $t = 1$ in order to achieve the full commitment allocation. I defer the discussion of the particular commitment technologies that the household may employ to achieve efficiency to later in the paper. I make the comparison to the case where the bargaining power of each household member is constant over the life of the household. Due to the assumption of log utility ϕ^{**} will be independent of the level of initial household wealth. Despite this, an analytical solution is in most cases intractable. Instead I solve for this fraction numerically. Figure 2 shows the value of commitment for the household for varying levels of weight on private consumption μ . When $\mu = 0$ the household only consumes the public consumption good. The only source of inefficiency in this case comes to the intertemporal misallocation documented in Proposition 1. For the parameters shown, the household is willing to pay up to 2.03% of household wealth at $t = 1$ to achieve full commitment in this case. When μ is positive the allocative inefficiency also is present.

Figure 2 shows that the cost of this allocative inefficiency can be substantial. In the figure, if household members place equal weight on public and private consumption ($\mu = 0.5$) the value of commitment rises to 11.04% and if household members consume only private goods ($\mu = 1$) then the household is willing to pay 19.23% of total wealth to achieve commitment. Thus the most substantial problem the household faces is that a large share of private consumption will be directed to the impatient member later in life.

IV Optimal Bargaining Power

In the previous section the ex-ante optimal consumption path was compared to the path that will be achieved without commitment if the bargaining power of each member is constant over time. The direct conclusion was that a household in which members have different discount rates cannot achieve the optimal consumption path if their relative bargaining power is unchanged over time. However the household may be able to change the relative power of each member over time. For example assets may be saved in the name of one member giving them increased control over future household decisions. Similarly, investments in the human capital of a member will also affect the future relative bargaining power within the household. I set aside the specific mechanism through which household bargaining power might be achieved and ask the question: if the household were able to choose a path of bargaining power what would it chose? What would be the optimal choice and would this be sufficient to implement the optimal consumption plan (18), (19), (20) and (21)? As before, I continue to assume that the household is unable to commit to the path of consumption itself and is able to renegotiate any plan in each period. As a result, for a chosen path of bargaining power the consumption and savings path achieved by the household will be given by (14), (15), (16), and (17).

To begin, suppose that the household is able to commit to an entire lifetime sequence

of bargaining power at $t = 1$.⁶ It is somewhat counter intuitive that the household will be able to commit to a path of bargaining power while not possessing the ability to commit to a consumption plan. As such, this problem should be thought of as a constrained optimum benchmark against which the analysis below with far more limited commitment can be evaluated. The problem solved by the household is as follows:

$$\max_{\{\theta_t\}_{t=2}^T} \Pi_1 = \eta_1 V_{A,1} + (1 - \eta_1) V_{B,1} \quad (22)$$

subject to (14), (15), (16), (17) and

$$\theta_t \in [1 - \gamma_A, \gamma_B] \quad \forall t \in [2, 3, \dots, T]. \quad (23)$$

The constraint (23) comes directly from the fact that the bargaining power of each member must lie on the unit interval. This program is solved in the appendix.

Proposition 3: *Suppose $\delta_B > \delta_A$.⁷ The optimal bargaining power in each period $t \geq 2$ is*

$$\theta_t^{**} = \min \left\{ \frac{\delta_B^{t-1} \theta_1}{\delta_A^{t-1} (1 - \theta_1) + \delta_B^{t-1} \theta_1}, \gamma_B \right\}. \quad (24)$$

Under this solution the weight given to the patient member's time preferences weakly increases over time. Specifically, there is a cutoff period

$$\bar{\tau} \equiv \left\lfloor \frac{\ln \left(\frac{\gamma_B}{1 - \gamma_B} \right) - \ln \left(\frac{\theta_1}{1 - \theta_1} \right)}{\ln \left(\frac{\delta_B}{\delta_A} \right)} \right\rfloor \quad (25)$$

such that for all $t \leq \bar{\tau}$ the relative weight given to the patient member's time preferences

⁶Since the choice of bargaining power is itself made through bargaining the initial bargaining power η_1 must be taken as given.

⁷A symmetric solution characterizes the case where $\delta_B < \delta_A$.

strictly increases over time in the following way:

$$\frac{\theta_t^{**}}{1 - \theta_t^{**}} = \left(\frac{\delta_B}{\delta_A} \right) \frac{\theta_t^{**}}{1 - \theta_t^{**}}. \quad (26)$$

For $t > \bar{\tau}$ the patient member, in this case B , will already have complete control of the household's consumption and savings decisions and hence $\theta_t = \gamma_B$ from there on. This leads to the following result.

Corollary 1: *If the household is able to commit to a path of bargain power θ_t^{**} then the level and allocation of consumption will be the same as under the full commitment consumption plan in (18) , (19) and (20) for periods $t \leq \bar{\tau}$. For $t > \bar{\tau}$ bargaining power in the household will be constant and will be suboptimal as per Proposition 2.*

Corollary 1 follows directly by substituting (24) into (14) , (15) and (16) for $t \leq \bar{\tau}$. If $\bar{\tau} \geq T$ then the path of bargaining power prescribed by θ_t^{**} is sufficient to ensure the optimal consumption and savings path for the household over its entire life. The comparative statics on $\bar{\tau}$ illustrate what factors will limit the effectiveness of committing to the path of household bargaining power. First, $\bar{\tau}$ is decreasing in the altruism of the patient member. If B cares only about herself ($\gamma_B = 1$) then $\bar{\tau}$ will be infinite. However altruism from member B to member A places a lower limit on how much B will disregard A later in life. Put differently, even with full bargaining power, if B is altruistic she will relent later in life and direct more consumption to A even though this was not optimal from the perspective of $t = 1$. Moreover because she will continue to care about the utility of member A later in life she will also consider his time preferences. Next, $\bar{\tau}$ is decreasing in relative difference of the time preferences between both members. If this difference is large then to implement the optimal consumption plan requires a rapid increase in B 's bargaining power and will therefore reach limit under which she has full control sooner. Finally, $\bar{\tau}$ is decreasing in the initial bargaining power of the patient member. If B already has most of the control over household decisions then there is little scope for increasing her say over time.

A Implementing Optimal Bargaining Power

So far I have said nothing about how the household could commit to a sequence of future bargaining power. Given that the household is unable to commit to a consumption path ex ante it seems equally likely that they are unable to commit to a lifetime path of bargaining power either. They may however be able to set the bargaining power that each member carries into the next period each period. As one example the household could decide at the end of each period who has control over household assets. As an example, this might come from adjusting whose name household savings are held under as a way of controlling who has say over consumption and savings decisions in the future. Of course this choice can also be renegotiated at each point in time. As a result I consider a household in which the household starts period t taking W_t and θ_t as given and decides how much to consume $(C_{A,t}, C_{B,t}, H_t)$ and how much control to give each member in the following period (θ_{t+1}) . As before the household recognizes that all consumption plans and allocations of control can be renegotiated at any point in the future.

The problem of the household in any period now is:

$$\max_{C_{A,t}, C_{B,t}, H_t, \theta_{t+1}} \eta_t V_{A,t} + (1 - \eta_t) V_{B,t} \quad (27)$$

subject to

$$C_{i,k} = C_{i,k}^*(W_k, \theta_k) \text{ for } k = t + 1, \dots, T \text{ and } i = A, B \quad (28)$$

$$H_k = H_x^*(W_k, \theta_k) \text{ for } k = t + 1, \dots, T \quad (29)$$

$$\theta_{k+1} = \theta_{k+1}^* \text{ for } k = t + 1, \dots, T \quad (30)$$

$$W_t \geq X_t \text{ where } X_t \equiv C_{A,t} + C_{B,t} + H_t \quad (31)$$

$$W_{t+1} = R(W_t - X_t) \quad (32)$$

$$H_t, C_{i,t} \geq 0 \text{ for } i = A, B \quad (33)$$

$$\theta_{t+1} \in [1 - \gamma_A, \gamma_B] \quad (34)$$

As before the household decides on the current level of consumption and bargaining power subject to the constraint that the value of these will be renegotiated according to the same program in the future. For future consumption this is captured by (28) and (29) and for future bargaining power by (30). Note that the single “*” indicates the solution this program with limited commitment in period t . This problem is solved in the appendix.

Proposition 4: *A household that can set θ_{t+1} in t will achieve the same dynamic path of bargaining power $\theta_t^* = \theta_t^{**}$ and consumption path as in the case where the household can commit to the entire series of bargaining power.*

The direct implication of Proposition 4 is that being able to set bargaining power one period in advance is sufficient to achieve the same constrained optimal consumption path that the household could achieve if a full commitment to bargaining power were possible. Combining this with Corollary 1, if there was no altruism in the household then the renegotiated path of bargaining power would implement the full commitment consumption path. However, in general this will be limited by the altruism of the patient member for the impatient member. As before, this means that after $\bar{\tau}$ the patient member will have been assigned all the households bargaining power but will still taking into consideration the preferences of her partner despite this being sub-optimal from an ex-ante perspective.

V Discussion and Empirical Implications

The primary lesson of this paper is that a household comprised of agents who have different time preferences will be time inconsistent despite both members, on their own, have standard time preferences. The optimal full commitment household plan will direct an increasing fraction of consumption to the patient member over time and a savings rate that evolves to reflect the discount rate of the patient member. However, if both members do not have a commitment technology and have equal bargaining power over their life then ex-post renegotiation between both members will undo this plan. An inefficiently high fraction of

consumption will be directed to the patient member later in life and the savings rate will always remain above that under the optimal plan.

The household can solve this problem if in each period both members are able to set the bargaining power they will have in the following period. Both members will agree to give an increasing degree of bargaining power to the patient member which will increase her share of consumption and give her time preferences greater weight in setting the household saving rate. One possible way that the household might implement this dynamic path of bargaining power is by deciding who has control over household wealth. If the fraction of wealth held in each member's name determines their bargaining power then this would suggest that household savings should be increasingly held in the name of the patient member. The effectiveness of this solution will, somewhat paradoxically, be limited by the degree of altruism between each member. If altruism is high then even allocating all the bargaining power to the patient member will still, over represent the interests of the impatient member later in life.

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VI Appendix

A Household Consumption without Commitment

Combining (3) and (4), the objective which household bargaining maximizes in any period can be written as

$$\Pi_t = (1 - \theta_t) U_{A,t} + \theta_t U_{B,t} \tag{35}$$

where

$$\theta_t \equiv \gamma_B + \eta_t (1 - \gamma_A - \gamma_B). \tag{36}$$

A-1 Consumption at $t = T$

In the final period the household enters the period with wealth W_t and solves:

$$\max_{C_{A,T}, C_{B,T}, H_T} (1 - \theta_T) \mu \ln C_{A,T} + \theta_T \mu \ln C_{B,T} + (1 - \mu) \ln H_T \quad (37)$$

subject to

$$W_T \geq X_t \text{ where } X_T \equiv C_{A,T} + C_{B,T} + H_T \quad (38)$$

$$H_T, C_{i,T} \geq 0 \text{ for } i = A, B. \quad (39)$$

Begin by ignoring the non-negativity conditions in (39). Writing the Lagrangian for the remaining problem with $\Gamma_T \geq 0$ being the multiplier on the resource constraint (38) we have

$$\max_{C_{A,T}, C_{B,T}, H_T} (1 - \theta_T) \mu \ln C_{A,T} + \theta_T \mu \ln C_{B,T} + (1 - \mu) \ln H_T + \Gamma_T [W_T - C_{A,T} - C_{B,T} - H_T].$$

The first order conditions are

$$C_{A,T} : \frac{(1 - \theta_T) \mu}{C_{A,T}} - \Gamma_T = 0$$

$$C_{B,T} : \frac{\theta_T \mu}{C_{B,T}} - \Gamma_T = 0$$

$$H_T : \frac{1 - \mu}{H_T} - \Gamma_T = 0.$$

Solving these along with the condition that (38) binds with equality gives

$$C_{A,T}^* = (1 - \theta_T) \mu W_T \quad (40)$$

$$C_{B,T}^* = \theta_T \mu W_T \quad (41)$$

$$H_T^* = (1 - \mu) W_T. \quad (42)$$

Notice that at the bargained solution that the individual utility of each member is

$$U_{A,T}(C_{A,T}^*, H_T^*) = \mu \ln [(1 - \theta_T) \mu] + (1 - \mu) \ln [(1 - \mu)] + \ln W_T. \quad (43)$$

$$U_{B,T}(C_{B,T}^*, H_T^*) = \mu \ln [\theta_T \mu] + (1 - \mu) \ln [(1 - \mu)] + \ln W_T. \quad (44)$$

A-2 Solve for Subgame Perfect Consumption Path by Induction

Conjecture 1: *The subgame perfect equilibrium household allocation without commitment t until T is proportional to W_t . As a result at the subgame perfect equilibrium consumption path $\{C_{A,x}^*, C_{B,x}^*, H_x^*\}_{x=t}^T$ the discounted individual utility of member i can be written as $U_{i,t} = \left(\sum_{x=0}^{T-t} \delta_i^x\right) \ln W_t + g_{i,t}$ where $\varphi_{i,t}$ and $g_{i,t}$ are constants independent of W_t .*

Observe that Conjecture 1 is verified for $t = T$. It will now be established by induction

$\forall t < T$. Using Conjecture 1 to characterize the bargaining outcome for $t+1$ and onwards, the bargaining problem solved by the household in period t can be characterized as a solution to the following

$$\begin{aligned} & \max_{C_{A,t}, C_{B,t}, H_t} (1 - \theta_t) \mu \ln C_{A,t} + \theta_t \mu \ln C_{B,t} + (1 - \mu) \ln H_t \\ & + \left[(1 - \theta_t) \delta_A \sum_{x=0}^{T-(t+1)} \delta_A^x + \theta_t \delta_B \sum_{x=0}^{T-(t+1)} \delta_B^x \right] \ln (W_t - C_{A,t} - C_{B,t} - H_t) \end{aligned}$$

where the intertemporal budget constraint (8) has been substituted into the objective and the non-negativity conditions (9) and the static household budget constraint (7) have been set aside since these will not be at the solution (this will be verified below). The first order conditions to this problem are:

$$\begin{aligned} C_{A,t} & : \frac{(1 - \theta_t) \mu}{C_{A,t}} - \frac{(1 - \theta_t) \sum_{x=1}^{T-t} \delta_A^x + \theta_t \delta_B \sum_{x=1}^{T-t} \delta_B^x}{W_t - C_{A,t} - C_{B,t} - H_t} = 0 \\ C_{B,t} & : \frac{\theta_t \mu}{C_{B,t}} - \frac{(1 - \theta_t) \sum_{x=1}^{T-t} \delta_A^x + \theta_t \delta_B \sum_{x=1}^{T-t} \delta_B^x}{W_t - C_{A,t} - C_{B,t} - H_t} = 0 \\ H_t & : \frac{1 - \mu}{H_t} - \frac{(1 - \theta_t) \sum_{x=1}^{T-t} \delta_A^x + \theta_t \delta_B \sum_{x=1}^{T-t} \delta_B^x}{W_t - C_{A,t} - C_{B,t} - H_t} = 0. \end{aligned}$$

Solving simultaneously gives (14), (15), (16), and (17). Note that these verify that (9) and (7) are indeed satisfied. All that remains is to establish Conjecture 1. Using (14) the wealth of the household next period will be

$$W_{t+1} = R \left(\frac{(1 - \theta_t) \sum_{x=1}^{T-t} \delta_A^x + \theta_t \delta_B \sum_{x=1}^{T-t} \delta_B^x}{1 + (1 - \theta_t) \sum_{x=1}^{T-t} \delta_A^x + \theta_t \delta_B \sum_{x=1}^{T-t} \delta_B^x} \right) W_t.$$

and so using Conjecture 1 for $U_{A,t+1}$ and (15) and (17) to find the discounted individual utility of member A in period t gives

$$\begin{aligned} U_{A,t} & = \mu \ln \left(\frac{(1 - \theta_t) \mu W_t}{(1 - \theta_t) \sum_{x=1}^{T-t} \delta_A^x + \theta_t \delta_B \sum_{x=1}^{T-t} \delta_B^x} \right) \\ & + (1 - \mu) \ln \left(\frac{(1 - \mu) W_t}{(1 - \theta_t) \sum_{x=1}^{T-t} \delta_A^x + \theta_t \delta_B \sum_{x=1}^{T-t} \delta_B^x} \right) \\ & + \delta_A \left(\sum_{x=0}^{T-(t+1)} \delta_A^x \right) \ln \left(R \left(\frac{(1 - \theta_t) \sum_{x=1}^{T-t} \delta_A^x + \theta_t \delta_B \sum_{x=1}^{T-t} \delta_B^x}{1 + (1 - \theta_t) \sum_{x=1}^{T-t} \delta_A^x + \theta_t \delta_B \sum_{x=1}^{T-t} \delta_B^x} \right) W_t \right) + \delta_A g_{A,t+1} \end{aligned}$$

which simplifies to

$$U_{A,t} = \left(\sum_{x=0}^{T-t} \delta_A^x \right) \ln W_t + g_{A,t}.$$

A symmetric argument applies for $U_{B,t}$. This establishes Conjecture 1 by an argument of induction.

B Full Commitment Consumption Plan

With full commitment the household will set a Pareto optimal consumption plan at $t = 1$ that solves

$$\begin{aligned} & \max_{\{C_{A,t}, C_{B,t}, H_t\}_{t=1}^T} (1 - \theta_1) \mu \sum_{t=1}^T \delta_A^{t-1} \ln C_{A,t} + \theta_1 \mu \sum_{t=1}^T \delta_B^{t-1} \ln C_{B,t} & (45) \\ & + (1 - \mu) \left[\left((1 - \theta_1) \sum_{t=1}^T \delta_A^{t-1} \ln H_t + \theta_1 \sum_{t=1}^T \delta_B^{t-1} \ln H_t \right) \right] \\ & + \Gamma \left[W_1 - \sum_{t=1}^T R^{-(t-1)} [C_{A,t} + C_{B,t} + H_t] \right] \end{aligned}$$

where $\Gamma \geq 0$ is the Lagrangian multiplier on the resource constraint (11). I have ignored the non-negativity conditions in (12) and it is easy to verify they are satisfied at the solution found below. The first order conditions imply that $\forall t$:

$$C_{A,t} : \frac{(1 - \theta_1) \mu \delta_A^{t-1}}{C_{A,t}} - \Gamma R^{-(t-1)} = 0 \quad (46)$$

$$C_{B,t} : \frac{\theta_1 \mu \delta_B^{t-1}}{C_{B,t}} - \Gamma R^{-(t-1)} = 0 \quad (47)$$

$$H_t : (1 - \mu) \left[\frac{(1 - \theta_1) \delta_A^{t-1} + \theta_1 \delta_B^{t-1}}{H_t} \right] - \Gamma R^{-(t-1)} = 0 \quad (48)$$

Using (46), (47), and (48) to solve for $C_{A,t}$, $C_{B,t}$, and H_t in terms of Γ and adding these up gives

$$X_t = \frac{(1 - \theta_1) \delta_A^{t-1} + \theta_1 \delta_B^{t-1}}{\Gamma R^{-(t-1)}}. \quad (49)$$

Since the objective function is strictly increasing, the optimal allocation will fully exhaust the household's resources and hence $W_1 = \sum_{t=1}^T R^{-(t-1)} X_t$. Combining this with (49) gives

$$\Gamma = \frac{(1 - \theta_1) \sum_{t=1}^T \delta_A^{t-1} + \theta_1 \sum_{t=1}^T \delta_B^{t-1}}{W_1} \quad (50)$$

and therefore provides an expression for the full commitment level of consumption as a fraction of W_1 :

$$X_t^{**} = \frac{(1 - \theta_1) \delta_A^{t-1} + \theta_1 \delta_B^{t-1}}{R^{-(t-1)} \left[(1 - \theta_1) \sum_{x=1}^T \delta_A^{x-1} + \theta_1 \sum_{x=1}^T \delta_B^{x-1} \right]} W_1. \quad (51)$$

Under the full commitment plan household wealth at t will be

$$W_t^{**} = R^{t-1} W_1 - \sum_{x=1}^{t-1} R^{t-x} X_x^{**}. \quad (52)$$

Combining (51) and (52) gives the relationship between W_t^{**} and W_1 to be

$$W_1 = \frac{W_t^{**}}{R^{t-1} \left[\frac{(1-\theta_1) \sum_{x=t}^T \delta_A^{x-1} + \theta_1 \sum_{x=t}^T \delta_B^{x-1}}{(1-\theta_1) \sum_{x=1}^T \delta_A^{x-1} + \theta_1 \sum_{x=1}^T \delta_B^{x-1}} \right]}. \quad (53)$$

Combining (53) with (51) gives (18).

To solve for the allocation of consumption within each period: combining (46) with (49) gives (19); combining (47) with (49) gives (20); and, combining (48) with (49) gives (21).

B-1 Proof of Proposition 1

Comparing (14) and (18) gives that $\frac{X_t^*}{W_t^*} > \frac{X_t^{**}}{W_t^{**}}$ if and only if

$$(1 - \theta_t) \sum_{x=1}^{T-t} \delta_A^x + \theta_t \sum_{x=1}^{T-t} \delta_B^x < \frac{(1 - \theta_1) \delta_A^{t-1} \sum_{x=1}^{T-t} \delta_A^x + \theta_1 \delta_B^{t-1} \sum_{x=1}^{T-t} \delta_B^x}{(1 - \theta_1) \delta_A^{t-1} + \theta_1 \delta_B^{t-1}}. \quad (54)$$

If bargaining power is constant so that $\theta_t = \theta_1 \forall t$, then (54) can be re-written as

$$(1 - \theta_1) \theta_1 (\delta_B^{t-1} - \delta_A^{t-1}) \left[\sum_{x=1}^{T-t} \delta_A^x - \sum_{x=1}^{T-t} \delta_B^x \right] < 0 \quad (55)$$

which holds whenever $t \in \{2, 3, \dots, T-1\}$ and $\delta_A \neq \delta_B$. This establishes Proposition 1.

B-2 Proof of Proposition 2

Comparing the ratio of private consumption with and without commitment using (19), (20), (15) and (16) gives that $\frac{C_{A,t}^*}{C_{B,t}^*} > \frac{C_{A,t}^{**}}{C_{B,t}^{**}}$ if and only if

$$\frac{1 - \theta_t}{\theta_t} > \frac{(1 - \theta_1) \delta_A^{t-1}}{\theta_1 \delta_B^{t-1}}. \quad (56)$$

If bargaining power is constant so that $\theta_t = \theta_1 \forall t$, then (56) can be re-written as

$$\delta_B^{t-1} > \delta_A^{t-1} \quad (57)$$

which holds whenever $t \in \{2, 3, \dots, T\}$ and $\delta_A < \delta_B$. This establishes Proposition 2.

C Optimal Bargaining Power

In any period t the objective of the household is to maximize (35) anticipating the renegotiated decisions of the household in the future. Conditional on the level of wealth W_k carried into any period k the consumption decisions given by the household will be given by (14), (15), (16), and (17) and depend only on the bargaining power of each member in that period θ_t . The individual objective of member A in period k can be written as

$$U_{A,k} = \mu \ln(C_{A,k}^*) + (1 - \mu) \ln(H_k^*) + \delta_A U_{A,k+1} \quad (58)$$

Applying (58) recursively and using the expressions for $C_{A,k}^*$ and H_k^* in (15) (17) this can be re-written as

$$U_{A,k} = \sum_{j=0}^{T-k} \delta_A^j (\mu \ln(1 - \theta_{k+j}) + \ln(X_{k+j}^*)) + d_{A,k} \quad (59)$$

where

$$d_{A,k} \equiv \sum_{j=0}^{T-k} \delta_A^j (\mu \ln \mu + (1 - \mu) \ln(1 - \mu)) \quad (60)$$

is a constant. Using the expression for X_t^* in (14) and recursively applying the intertemporal budget constraint (8) gives X_{t+n}^* as a function of W_t and the future path of bargaining power:

$$X_{t+n}^* = \left(\frac{\prod_{j=1}^n \left((1 - \theta_{t+n-j}) \sum_{x=1}^{T-(t+n-j)} \delta_A^x + \theta_{t+n-j} \sum_{x=1}^{T-(t+n-j)} \delta_B^x \right)}{\prod_{j=0}^n \left(1 + (1 - \theta_{t+n-j}) \sum_{x=1}^{T-(t+n-j)} \delta_A^x + \theta_{t+n-j} \sum_{x=1}^{T-(t+n-j)} \delta_B^x \right)} \right) R^n W_t \quad (61)$$

Substituting (61) into (59) gives

$$\begin{aligned} U_{A,k} = & \sum_{n=0}^{T-k} \left[\delta_A^n \left[\left(\sum_{j=1}^{T-(k+n)} \delta_A^j \right) \ln \left((1 - \theta_{k+n}) \left(\sum_{j=1}^{T-(k+n)} \delta_A^j \right) + \theta_{k+n} \left(\sum_{j=1}^{T-(k+n)} \delta_B^j \right) \right) \right. \right. \\ & \left. \left. - \left(\sum_{j=0}^{T-(k+n)} \delta_A^j \right) \ln \left(1 + (1 - \theta_{k+n}) \left(\sum_{j=1}^{T-(k+n)} \delta_A^j \right) + \theta_{k+n} \left(\sum_{j=1}^{T-(k+n)} \delta_B^j \right) \right) \right] \right] \\ & + \mu \sum_{n=0}^{T-k} [\delta_A^n \ln(1 - \theta_{k+n})] + d_{A,k} + \left(\sum_{j=0}^{T-k} \delta_A^j \right) \ln W_t + \left(\sum_{j=0}^{T-k} j \delta_A^j \right) \ln R. \end{aligned} \quad (62)$$

A symmetric argument for B 's objective gives

$$\begin{aligned}
U_{B,k} = & \sum_{n=0}^{T-k} \left[\delta_B^n \left[\left(\sum_{j=1}^{T-(k+n)} \delta_B^j \right) \ln \left((1 - \theta_{k+n}) \left(\sum_{j=1}^{T-(k+n)} \delta_A^j \right) + \theta_{k+n} \left(\sum_{j=1}^{T-(k+n)} \delta_B^j \right) \right) \right. \right. \\
& \left. \left. - \left(\sum_{j=0}^{T-(k+n)} \delta_B^j \right) \ln \left(1 + (1 - \theta_{k+n}) \left(\sum_{j=1}^{T-(k+n)} \delta_A^j \right) + \theta_{k+n} \left(\sum_{j=1}^{T-(k+n)} \delta_B^j \right) \right) \right] \right] \\
& + \mu \sum_{n=0}^{T-k} [\delta_B^n \ln(\theta_{k+n})] + d_{B,k} + \left(\sum_{j=0}^{T-k} \delta_B^j \right) \ln W_t + \left(\sum_{j=0}^{T-k} j \delta_B^j \right) \ln R.
\end{aligned} \tag{63}$$

where

$$d_{B,k} \equiv \sum_{j=0}^{T-k} \delta_B^j (\mu \ln \mu + (1 - \mu) \ln(1 - \mu)). \tag{64}$$

So consider the problem of a household in period $k - 1$ that is negotiating on the optimal bargaining power for the future. For a given bargaining weight of θ_{k-1} and the objective of the household is

$$\Pi_{k-1} = \delta_A (1 - \theta_{k-1}) U_{A,k} + \delta_B \theta_{k-1} U_{B,k} \tag{65}$$

Using (62) and (63) we can rewrite (65) as

$$\begin{aligned}
\Pi_{k-1} = & \sum_{n=0}^{T-k} \Psi_{k+n} + \delta_A (1 - \theta_{k-1}) d_{A,k} + \delta_B \theta_{k-1} d_{B,k} \\
& + \left[\delta_A (1 - \theta_{k-1}) \left(\sum_{j=0}^{T-k} \delta_A^j \right) + \delta_B \theta_{k-1} \left(\sum_{j=0}^{T-k} \delta_B^j \right) + \right] \ln W_t \\
& + \left[\delta_A (1 - \theta_{k-1}) \left(\sum_{j=0}^{T-k} j \delta_A^j \right) + \delta_B \theta_{k-1} \left(\sum_{j=0}^{T-k} j \delta_B^j \right) + \right] \ln R
\end{aligned} \tag{66}$$

where

$$\begin{aligned}
\Psi_{k+n} &\equiv \mu \left[\delta_A^{n+1} (1 - \theta_{k-1}) \ln (1 - \theta_{k+n}) + \delta_A^{n+1} \theta_{k-1} \ln (\theta_{k+n}) \right] & (67) \\
&- \left[\delta_A^{n+1} (1 - \theta_{k-1}) \left(\sum_{j=0}^{T-(k+n)} \delta_A^j \right) + \delta_B^{n+1} \theta_{k-1} \left(\sum_{j=0}^{T-(k+n)} \delta_B^j \right) \right] \\
&\times \ln \left(1 + (1 - \theta_{k+n}) \left(\sum_{j=1}^{T-(k+n)} \delta_A^j \right) + \theta_{k+n} \left(\sum_{j=1}^{T-(k+n)} \delta_B^j \right) \right) \\
&+ \left[\delta_A^{n+1} (1 - \theta_{k-1}) \left(\sum_{j=1}^{T-(k+n)} \delta_A^j \right) + \delta_B^{n+1} \theta_{k-1} \left(\sum_{j=1}^{T-(k+n)} \delta_B^j \right) \right] \\
&\times \ln \left((1 - \theta_{k+n}) \left(\sum_{j=1}^{T-(k+n)} \delta_A^j \right) + \theta_{k+n} \left(\sum_{j=1}^{T-(k+n)} \delta_B^j \right) \right).
\end{aligned}$$

Observe that the only term in (66) that depends on future bargaining power (θ_{k+n} for $n \geq 0$) is $\sum_{n=0}^{T-k} \Psi_{k+n}$. Hence from the perspective of the household when setting future bargaining power in $t = k - 1$ all other terms in (66) can be ignored.

C-1 Committing to a Path of Bargaining Power

If at $t = k - 1$ the household is able to commit to the entire future path of bargaining power this will be set to solve the following problem

$$\max_{\{\theta_t\}_{t=k}^T} \sum_{n=0}^{T-k} \Psi_{k+n} \quad (68)$$

subject to

$$\theta_t \leq \gamma_B \quad (69)$$

$$\theta_t \geq 1 - \gamma_B \quad (70)$$

In the case where $\delta_B > \delta_A$ (70) can be ignored and verified at the end. Since (68) is additively separable in θ_t , the choice of each θ_t is simply the solution to

$$\max_{\theta_{k+n}} \Psi_{k+n} \quad (71)$$

subject to

$$\theta_t \leq \gamma_B. \quad (72)$$

The solution to this problem is characterized by a standard first order condition. Observe

that

$$\frac{\partial \Psi_{k+n}}{\partial \theta_{k+n}} = [\delta_B^{n+1} \theta_{k-1} - \theta_{k+n} [\delta_A^{n+1} (1 - \theta_{k-1}) + \delta_B^{n+1} \theta_{k-1}]] \times \Phi_{k+n} \quad (73)$$

where

$$\Phi_{k+n} \equiv \frac{\mu}{(1 - \theta_{k+n}) \theta_{k+n}} + \frac{\left[(1 - \theta_{k+n}) \left(\sum_{x=1}^{T-(k+n)} \delta_A^x \right) + \theta_{k+n} \left(\sum_{x=1}^{T-(k+n)} \delta_B^x \right) \right]^{-1} \left[\sum_{x=1}^{T-(k+n)} (\delta_B^x - \delta_A^x) \right]^2}{\left[1 + (1 - \theta_{k+n}) \left(\sum_{x=1}^{T-(k+n)} \delta_A^x \right) + \theta_{k+n} \left(\sum_{x=1}^{T-(k+n)} \delta_B^x \right) \right]}. \quad (74)$$

Note that $\Phi_{k+n} > 0$ and so the sign of $\frac{\partial \Psi_{k+n}}{\partial \theta_{k+n}}$ is determined by the sign of the first term in (73). Specifically,

$$\frac{\partial \Psi_{k+n}}{\partial \theta_{k+n}} = \begin{cases} > 0 & \text{if } \theta_{k+n} < \frac{\delta_B^{n+1} \theta_{k-1}}{\delta_A^{n+1} (1 - \theta_{k-1}) + \delta_B^{n+1} \theta_{k-1}} \\ = 0 & \text{if } \theta_{k+n} = \frac{\delta_B^{n+1} \theta_{k-1}}{\delta_A^{n+1} (1 - \theta_{k-1}) + \delta_B^{n+1} \theta_{k-1}} \\ < 0 & \text{if } \theta_{k+n} > \frac{\delta_B^{n+1} \theta_{k-1}}{\delta_A^{n+1} (1 - \theta_{k-1}) + \delta_B^{n+1} \theta_{k-1}} \end{cases}. \quad (75)$$

Since Ψ_{k+n} is concave in θ_{k+n} then it will be maximized at

$$\theta_{k+n}^{**} = \min \left\{ \frac{\delta_B^{n+1} \theta_{k-1}}{\delta_A^{n+1} (1 - \theta_{k-1}) + \delta_B^{n+1} \theta_{k-1}}, \gamma_B \right\} \quad (76)$$

which establishes Proposition 3 where $k = 2$.

C-2 Bargaining Power Renegotiated Each Period

To solve the problem where the household can only set θ_k at $k - 1$ we need to proceed by backward induction since the choice of θ_k will anticipate the effect on θ_{k+1} , and θ_{k+1} will have on θ_{k+2} and so on. I establish the unique subgame perfect path of bargaining power and consumption by induction. To begin, I make the following conjecture:

Conjecture 2: *The subgame perfect equilibrium household choice of bargaining power in $k + x$ for $x \geq 1$ will be*

$$\theta_{k+x}^{*Con} = \min \left\{ \frac{\delta_B \theta_{k+x-1}}{\delta_A (1 - \theta_{k+x-1}) + \delta_B \theta_{k+x-1}}, \gamma_B \right\}. \quad (77)$$

Note that the full commitment case establishes Conjecture 2 for $k = T - 1$. Applying (77) iteratively gives the following mapping between θ_k and θ_{k+n} .

$$\tilde{\theta}_{k+n}(\theta_k) = \begin{cases} \frac{\delta_B^n \theta_k}{\delta_A^n (1 - \theta_k) + \delta_B^n \theta_k} & \text{if } \theta_k < \bar{\theta}_n \\ \gamma_B & \text{if } \theta_k \geq \bar{\theta}_n \end{cases} \quad (78)$$

where

$$\bar{\theta}_n \equiv \frac{\gamma_B \delta_A^n}{\gamma_B \delta_A^n + (1 - \gamma_B) \delta_B^n}. \quad (79)$$

Notice that (79) defines a series of cutoffs that are ordered such that $\bar{\theta}_{t+1} < \bar{\theta}_t$ with $\bar{\theta}_0 = \gamma_B$.

The problem the household faces when setting θ_k in $t = k - 1$ is

$$\max_{\theta_k} \sum_{n=0}^{T-k} \Psi_{k+n} \quad (80)$$

subject to

$$\theta_k \leq \gamma_B \quad (81)$$

$$\theta_k \geq 1 - \gamma_B \quad (82)$$

$$\theta_{k+n} = \tilde{\theta}_{k+n}(\theta_k) \text{ for all } n \geq 1. \quad (83)$$

As before, when $\delta_B > \delta_A$ the constraint (82) can be ignored and verified to hold at the end. Consider choosing θ_k to maximize Ψ_{k+n} for some $n \geq 0$:

$$\max_{\theta_k} \Psi_{k+n} \quad (84)$$

subject to

$$\theta_k \leq \gamma_B \quad (85)$$

$$\theta_{k+n} = \tilde{\theta}_{k+n}(\theta_k) \quad (86)$$

Notice that

$$\frac{\partial \Psi_{k+n}}{\partial \theta_k} = \frac{\partial \Psi_{k+n}}{\partial \theta_{k+n}} \times \frac{\partial \tilde{\theta}_{k+n}(\theta_k)}{\partial \theta_k} \quad (87)$$

and using (78) we have that

$$\frac{\partial \tilde{\theta}_{k+n}(\theta_k)}{\partial \theta_k} = \begin{cases} \frac{(\delta_A \delta_B)^n}{[\delta_A^n (1 - \theta_k) + \delta_B^n \theta_k]^2} > 0 & \text{if } \theta_k < \bar{\theta}_n \\ = 0 & \text{if } \theta_k \geq \bar{\theta}_n \end{cases}. \quad (88)$$

Combining (87), (88), (72) and (75) gives that the entire set of feasible values for θ_k that maximize Ψ_{k+n} is

$$\theta_k^{*n} = \begin{cases} \frac{\delta_B \theta_{k-1}}{\delta_A (1 - \theta_{k-1}) + \delta_B \theta_{k-1}} & \text{if } \bar{\theta}_n > \frac{\delta_B \theta_{k-1}}{\delta_A (1 - \theta_{k-1}) + \delta_B \theta_{k-1}} \\ [\bar{\theta}_n, \gamma_B] & \text{if } \bar{\theta}_n \leq \frac{\delta_B \theta_{k-1}}{\delta_A (1 - \theta_{k-1}) + \delta_B \theta_{k-1}} \end{cases}. \quad (89)$$

The only difference between θ_k^{*n} and $\theta_k^{*(n+1)}$ is that $\bar{\theta}_n > \bar{\theta}_{n+1}$. It follows that $\theta_k^{*n} \subseteq \theta_k^{*(n+1)}$ and hence θ_k^{*0} will maximize every Ψ_{k+n} for $n \geq 1$ as well. It follows that the solution to (80), (81), (82), and (83) is

$$\theta_k^* = \min \left\{ \frac{\delta_B \theta_{k-1}}{\delta_A (1 - \theta_{k-1}) + \delta_B \theta_{k-1}}, \gamma_B \right\}. \quad (90)$$

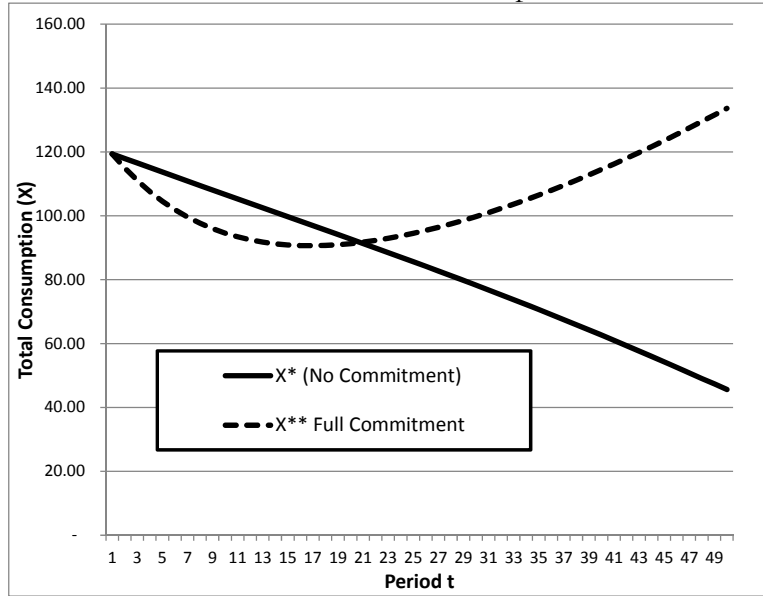
Note that this solution is unique. Finally observe that (90) establishes Conjecture 2 by an argument of induction and establishes Proposition 4.

Figure 1

Consumption Path with and Without Commitment – Constant Bargaining Power

Panel A of this plot shows the equilibrium level of total household expenditure in every period without commitment X_t^* , and the optimal full commitment consumption path X_t^{**} . Panel B shows the path of private consumption for both members without commitment $C_{A,t}^*$, $C_{B,t}^*$ and compares it to the full commitment allocation of private consumption $C_{A,t}^{**}$, $C_{B,t}^{**}$. It is drawn using the following parameters: discount factors are $\delta_A=0.85$ and $\delta_B=0.95$, the gross interest rate is $R=1.07$, both members place weight $\gamma_A=\gamma_B=0.6$ on their own utility, the bargaining power of both members is equal $\eta=0.5$, household members place equal weight on private and public consumption $\mu=0.5$, the household exists for $T=50$ periods, and starts life with wealth of $W_1=1,500$.

Panel A: Total Household Expenditure



Panel B: Allocation of Private Household Expenditure

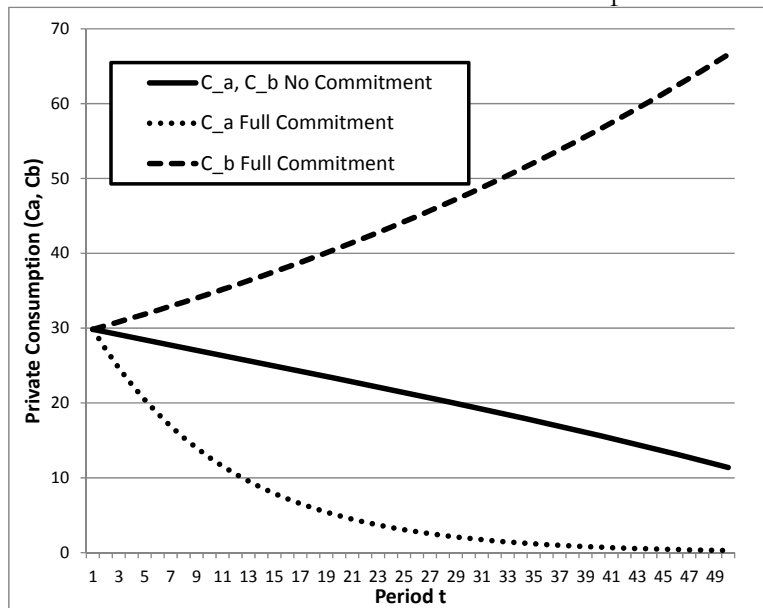


Figure 2
Comparative Statics: The Value of Commitment

This plot shows the fraction of W_t that the household would be willing to pay at $t=1$ to achieve the full commitment consumption path. Due to log additive utility functions this fraction is invariant to the choice of W_t . The figure shows how the value of commitment varies with the weight household members place on private consumption μ . The plot is drawn using the following parameters: discount factors are $\delta_A=0.85$ and $\delta_B=0.95$, the gross interest rate is $R=1.07$, both members place weight $\gamma_A=\gamma_B = 0.6$ on their own utility, the bargaining power of both members is constant and equal $\eta=0.5$, and the household exists for $T=50$ periods.

The Value of Commitment and the Weight on Private Consumption μ

