A Theory of Disclosure in Speculative Markets

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Abstract

This paper presents a theory of disclosure in a market where investors have heterogeneous beliefs and face short-sale constraints. Assets trade above fundamentals reflecting the value of the option to sell to more optimistic investors in the future. The initial seller has an incentive to commit to an imprecise disclosure policy, despite the negative effect this has on the fundamental value of the asset, in order to increase the potential for disagreement and hence the magnitude of the speculative premium. I show that there is a strategic complementarity between sellers in their disclosure decisions. This explains why financial misreporting and episodes of speculation occur together in waves and demonstrates that the endogenous choice of imprecise disclosure amplifies the extent to which assets are overpriced.

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1 Introduction

A prevalent feature of the housing and credit market boom that preceded the 2007-08 financial crisis was the imprecise disclosure of financial information. Extensive evidence has documented that borrowers, loan originators, and RMBS underwriters systematically misrepresented: (1) the presence of a second lien, (2) owner-occupier status, (3) borrower household income, and (4) mortgage collateral value.\footnote{For evidence see: (1) the presence of a second lien (Piskorski et al. (2015), Griffin and Maturana (2016b)), (2) owner-occupier status (Haughwout et al. (2011), Piskorski et al. (2015), Griffin and Maturana (2016b)) (3) borrower household income (Jiang et al. (2014)), and (4) mortgage collateral value (Ben-David (2011), Carrillo (2013) Garmaise (2015), Griffin and Maturana (2016b)).}

To compound this, several papers have shown that the rating of the collateralized debt obligations that were backed by these loans was systematically imprecise during this period.\footnote{For evidence see Ashcraft et al. (2010), Griffin and Tang (2011), Griffin and Tang (2012), and Griffin et al. (2013).}

The first central question addressed in this paper is: how can the misrepresentation of information arise in equilibrium? At first consideration, it is not difficult to explain why a seller would exploit the opportunity to inflate the perceived value of the asset they are selling. The more challenging question is: why would the seller not elect to commit to avoid the possibility of misrepresenting the assets value in the first place? If investors are rational, then the average sale price should take into account the expected level of misrepresentation.\footnote{An alternate possibility is that investors are naive as per Hong et al. (2008).}

One possibility is that misrepresentation does not affect communication and therefore may have no cost, which could be the case if it is unraveled by investors on a case by case basis, as per the signal jamming theories of Fudenberg and Tirole (1986) and Stein (1989). Counter to this, considerable evidence suggests that investors were unable to unravel the misrepresentation they encountered. For example, Piskorski et al. (2015) show that RMBS securities prices did not vary across mortgage pools that contained more or less hidden second liens.\footnote{See also evidence in Coval et al. (2009), Ben-David (2011), Jiang et al. (2014), Garmaise (2015), and Griffin and Maturana (2016b).}

If misrepresentation is not unraveled on a case by case basis, rational investors will adjust their average assessment of a seller’s securities to recognize that the information they receive is distorted. In addition, not only will misrepresentation be factored into average sale prices, the anticipation of any loss in value due to the release of imprecise information will cause the ex ante price of any security to be further discounted. Examples of the costs of imprecise information include the discount from anticipated adverse selection, exacerbated moral hazard, or inefficient future decision-making.\footnote{Papers that demonstrate each cost are: 1) adverse selection (Akerlof (1970), Bernanke and Gertler (1989), and Kiyotaki and Moore (1997)), 2) moral hazard (Holmstrom (1979) and Holmstrom and Tirole (1993)), and 3) inefficient decision making (Fishman and Hagerty (1989), Boot and Thakor (1997), Dow and Gorton (1997), Subrahmanyam and Titman (1999), and Kedia and Philippon (2009)).} All of these theories therefore suggest that the seller of a security would optimally commit to precise disclosure practices to minimize this discount. This is an important reason why security issuers commit ex
ante to legally binding contractual warranties and external credit ratings, and why firms commit to scrutiny from external accountants and auditors. In many instances from the credit market boom, the cost of committing to more accurate information, such as the presence of a second lien with the same originator, would have been virtually costless and hence theory predicts that sellers would commit to practices that made such misrepresentation impossible so as to avoid the discount that would be priced into these securities in anticipation.

This paper presents a theory that reconsiders the decision of a seller who can decide to commit ex ante to limit the possibility of misrepresentation, recognizing that any misrepresentation will, on average, be anticipated by rational investors. In particular, I study the ex ante disclosure policy choice of a seller who must hire an agent to manage the asset that is being sold. In order to induce the manager to work, the seller will offer her an incentive contract that is based on the performance information that will be released about the asset. A byproduct of the incentive scheme is that it will also provide the manager with an incentive to overstate the information that she discloses to investors about the value of the asset in order to increase her own pay. The extent of misrepresentation that is possible is constrained by the seller’s initial choice of disclosure policy for the asset. The opportunity to misrepresent also has a random component that is privately observed by the manager ex post. This stochastic component is motivated by the evidence, cited above, that misrepresentation is generally not unraveled on a case-by-case basis. Intuitively, knowing that a manager is able to make an inaccurate report leads rational investors to unravel the average amount of misrepresentation and recognize that the disclosure is less precise. The more misrepresentation a disclosure policy allows, the less precise rational investors anticipate the disclosures to be.

The fundamental trade-off that determines the seller’s initial disclosure policy choice is driven by the impact the precision of the information that the manager will release will have. Less precision comes at the cost of less effective monitoring of the manager (as per Holmstrom (1979) and Holmström and Tirole (1993)). Counterbalancing this, the market in which the asset is sold is such that asset pricing bubbles are possible. I capture this by supposing that investors have heterogeneous prior beliefs about the value of the security and hence will potentially disagree about its value. This disagreement, combined with short sales constraints, produces a speculative premium to the asset’s price. Investors are willing to pay more than their own assessment of the asset’s fundamental value because it also contains the option to resell the asset to a more optimistic investor in some future state of the world. I show that this presents the seller of the asset with an incentive to allow misrepresentation as a way of committing to less precise disclosures. Releasing perfectly accurate information about a security’s value will end any disagreement over its value and thereby

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6 Many papers have established that differences of opinion and short sale constraints will lead to speculative pricing. See for example Miller (1977), Harris and Raviv (1993), Chen et al. (2002), Scheinkman and Xiong (2003), and Hong et al. (2008). Empirical work confirming this premise includes Diether et al. (2002), Lamont and Thaler (2003), and Oke and Richardson (2003).
eliminate the possibility of a speculative price premium. I show that the initial seller of the security commits to a disclosure policy that allows the manager of the asset to make more distorted and less informative disclosures so as to sponsor the speculative component of the asset’s price, despite the negative effect on the asset’s fundamental value.

The theory matches several key empirical facts. First, it is consistent with the finding that misrepresentation during the housing and credit boom was more prevalent among loans that were more likely to be securitized and sold to investors where trading and speculation over the asset value was possible (Keys et al. (2010)). Further, it is consistent with evidence that areas with a higher incidence of mortgage misrepresentation exhibited higher price growth and volatility during the housing market boom years of 2003 to 2006, and larger price decreases after the boom ended (Ben-David (2011) and Griffin and Maturana (2016a)). It is also consistent with the observation that the end of the housing and credit boom also coincided with a sharp drop in misrepresentation, both of which were precipitated by the growth of the credit default swap for mortgage-backed securities, which effectively made short-selling possible and thereby unwound the speculative component of pricing in this market (Stulz (2010)).

A direct implication of the model is that sellers will commit to increasingly less accurate disclosures that will promote higher speculative prices when the initial heterogeneity of beliefs among investors is higher. Differences of opinion are likely to be higher in industries and during episodes where new technologies are introduced. The model is therefore consistent with the fact that in the U.S. speculative episodes have coincided with the following major technological breakthroughs: (1) railroads, (2) electricity, (3) automobiles, (4) radio, (5) microelectronics, (6) personal computers, (7) biotechnology, and (8) the Internet. Further, it offers an explanation for why episodes of speculation and imprecise financial information have gone hand in hand. For example, practices to distort accounting information were particularly widespread among “New Economy” firms in the 1990s dot-com boom (see for example Kahn (2000), MacDonald (2000), Davis (2002), and Ball (2009)). The predictions of the model are also consistent with evidence that misreporting of financial information is more prevalent at firms when investors are more optimistic about industry growth (see Fernandes and Guedes (2010), Wang et al. (2010), and Filip and Raffournier (2014)) and matches the finding that earnings reports contain more information during recessions (Jenkins 2014).

7Further evidence is also contained in Keys et al. (2009), Nadauld and Sherlund (2013), Jiang et al. (2014), and Griffin and Maturana (2016b).
8For other evidence of this, Sloan (1996) and Teoh et al. (1998) both show finds that firms with imprecise disclosures (measured by high discretionary accruals) are on average overpriced (measured by having negative expected future abnormal stock returns).
10Other examples include the radio and electronics led boom of the 1920s (Galbraith John (1972) and Baskin and Miranti Jr (1999)), the “go-go” years of the 1960s (Brooks (1999)), and, the leveraged buy-out boom of the 1980s (Kaplan and Stein (1993)).
I extend the analysis to consider the case where investors learn about the value of the asset from its disclosure as well as from other external sources. I demonstrate that when investors have other reliable sources of information about the value of an asset, this eliminates the potential for disagreement among investors and thus there is no scope for the seller to sponsor speculation by choosing a disclosure policy that allows for misrepresentation. As a result, accurate external information “crowds in” the release precise information.

This complementarity suggests that speculative episodes and inaccurate disclosures will occur in waves. To demonstrate this, I consider an environment in which there are many sellers in the same industry each selling assets that are positively correlated in value. This is motivated by evidence that shows one firm’s financial disclosures are used by investors to alter their assessment of other firms in the same industry.\textsuperscript{11} I show that this generates a strategic complementarity in disclosure precision. When other sellers in the industry commit to release precise information, there is little scope for disagreement, and therefore no speculation. Thus no individual seller has an incentive to allow misrepresentation and imprecision in their own disclosures. Conversely, when other sellers adopt policies that result in the release of imprecise information, this opens the possibility for speculation and thus encourages each individual firm to follow such a policy. I show that this complementarity implies that the equilibrium disclosure response to an increase in the heterogeneity of beliefs is larger than in the case of a firm in isolation. Moreover, the endogenous equilibrium response of disclosure policy amplifies the degree of speculation and overvaluation in the industry that the presence of different prior beliefs allows.

This paper proceeds as follows. Section 2 describes the environment in which the seller of the asset is choosing the disclosure policy to put in place. Section 3 solves for the equilibrium price of the firm and optimal managerial incentives taking the precision of the firm’s disclosure as given. Section 4 analyzes the seller’s choice of disclosure policy when this is the only source of information about the asset’s value. Section 5 extends the model to allow for investors to learn about the value of the asset from the disclosures of other assets in the industry. Here I emphasize the strategic complementarity which exists between each seller’s initial disclosure choice. Section 6 discusses the related literature and provides a brief conclusion. All derivations are provided in the Appendix.

\textsuperscript{11}This has been documented for earnings announcements (see Foster (1981), Clinch and Sinclair (1987) and Freeman and Tse (1992)), for earnings forecasts (see Han et al. (1989)) and for sales announcements (see Olsen and Dietrich (1985)). See Dye (1990) and Admati and Pfleiderer (2000) for other theoretical studies of disclosure in the presence of information externalities across firms.
2 The Seller’s Problem

2.1 The Environment

I consider the problem of a seller who is choosing an ex ante disclosure policy. The seller can be thought to represent a financial intermediary who is putting in place the process that will be used to originate loans which will be sold to investors, or a founder, or a venture capitalist who is setting the governance and reporting structure of a firm she is about to sell in an IPO. The goal of the seller is to maximize the price she receives for selling the asset she owns. To fix terminology, I will refer to the seller as “the founder” and the asset being sold as “the firm”. There are three periods denoted by \( t = 0, 1, 2 \). For simplicity, I abstract from discounting. The timing of events, summarized in Figure 1, is as follows.

2.1.1 Events at \( t = 0 \)

At the start of period 0 the firm is owned by the founder. Prior to the sale, the founder makes several publicly observed decisions that determine how the firm will be run in the future and therefore how it will be valued by investors. First, the founder selects a disclosure policy that is captured by a choice of \( \tau_a \geq 0 \) which determines the accuracy of the information that will be released by the manager about the firm’s value in the future. This decision captures such disclosure and transparency arrangements as the type of auditor the firm hires, the strength and independence of the firm’s board, the composition of its audit committee, the incentives given to rating agencies to objectively assess the firm, the independence of underwriters and monitors, and the extent of legal protections against misrepresentation. The firm’s disclosure will serve two roles: it is used to provide incentives to the manager of the firm, and it is used by investors to revise their assessment of the firm’s value. Committing the firm to more accurate disclosure comes at a small marginal cost of \( c \geq 0 \) per unit of \( \tau_a \).\(^{12}\) Next, the founder hires a manager to operate the firm (or a loan officer to originate a loan) and sets her incentive contract. Management of the firm is subject to moral hazard: after being hired, the manager privately selects an uncontractible level of effort \( e \) that increases the expected output of the firm. Once these decisions are in place, the founder sells the firm to investors.

2.1.2 Events at \( t = 1 \)

At \( t = 1 \) a publicly observable and contractible signal, denoted \( a_c \), of the firm’s profitability is released. The manager is able to distort this signal by misrepresenting the true expected profitability

\(^{12}\) For most the analysis I will assume that \( c > 0 \) to ensure the choice of \( \tau_a \) is finite.
of the firm. As a result, the signal that is released publicly is:

\[ a = \theta + e + m, \]  
(1)

where: \( \theta \) is the uncertain profitability of the firm (see below) and \( e \) is the manager’s effort. The first two terms in (1) represent information about the true underlying value of the firm that the signal contains. In contrast, \( m \) is the misrepresentation introduced into this signal by the manager.

The ability of the manager to misrepresent information is constrained by the initial disclosure policy chosen by the founder, as measured by \( \tau_a \), with higher values representing less ability for the manager to introduce misrepresentation into the disclosure. To capture this idea, the manager can choose to inflate the accounts of the firm up to some threshold \( \bar{m}(\tau_a) \) so that any choice of \( m \leq \bar{m}(\tau_a) \) is possible. Since misrepresentation is costless to the manager, she will always set \( m = \bar{m}(\tau_a) \) since this maximizes the ex post compensation of the manager. The manager’s capacity to misrepresent information is privately observed by the manager at \( t = 1 \) and is determined in the following way:

\[ \bar{m}(\tau_a) = f(\tau_a) + \varepsilon_a. \]  
(2)

The first term in (2), \( f(\tau_a) \), is a strictly decreasing deterministic function. This captures the idea that when the disclosure policy of the firm is more transparent, the manager has less capacity to misrepresent the information that the firm releases. The second term in (2), \( \varepsilon_a \), is an error term distributed \( N\left(0, \frac{1}{\tau_a}\right) \). This captures the idea that misrepresentation introduces imprecision into the information that is ultimately released. The manager privately learns how much misrepresentation is possible at \( t = 1 \) and exploits whatever opportunities for misrepresentation she finds. For this reason misrepresentation is not simply unraveled by investors on a case-by-case basis because they are unable to discern exactly how much was possible. Crucially, the lower is \( \tau_a \), the more capacity there is for misrepresentation on average and the more uncertainty there is regarding the extent to which this has been realized. Since \( f(\tau_a) \) is common knowledge, rational investors will unravel the expected amount of misrepresentation in equilibrium and hence will not be systematically misled by the manager’s exaggeration of the firm’s performance. However, if the firm adopts a less transparent disclosure policy, investors will recognize that the disclosure provides less precise information about the firm’s value. Similarly, the firm’s disclosure will also provide a less accurate measure of the effort of the manager.

In addition, a second external signal, \( s \), of the firm’s profitability is simultaneously released at

\(^{13}\)All error terms in the model are assumed to be independent. Technically, (2) implies that the realized value of \( \bar{m}(\tau_a) \) can be negative. Truncating the distribution of \( \bar{m}(\tau_a) \) at zero would not alter the analysis and would come at the cost of considerable tractability. If \( f(\tau_a) \) is arbitrarily high, this changes none of the results and reduces this possibility to an arbitrarily small probability event.
date 1. Specifically:
\[ s = \theta + \varepsilon_s, \]  
where \( \varepsilon_s \sim N\left( 0, \frac{1}{\tau_s} \right) \) is noise contained in the external signal. In Section 5 the accuracy of this signal will be determined by the endogenously chosen precision of other firm’s disclosures.

2.1.3 Events at \( t = 2 \)

In period 2 the output of the firm, \( y \), is realized and is publicly observed. The output is paid to whoever owns the firm at date 2 net of any costs incurred at time 0 and 1. The realized output, \( y \), is determined by the profitability of the firm and the effort of the manager,
\[ y = \theta + e + \varepsilon_y, \]  
where \( \varepsilon_y \sim N\left( 0, \frac{1}{\tau_y} \right) \) is a random component to the firm’s final output.

2.2 Moral Hazard and Managerial Incentives

The manager is risk averse. Her utility is:
\[ u(W, e) = -e^{-r(W - \phi(e))}, \]  
where \( W \) is the monetary payment the manager receives from the firm, \( r \) is the manager’s coefficient of absolute risk aversion, and \( \phi(e) \) is the manager’s private cost of exerting effort. The manager’s cost of effort is quadratic\(^1\) in \( e \):
\[ \phi(e) = \frac{e^2}{2\beta}. \]  
The manager has an outside opportunity which delivers utility of \( -e^{-rw} \). I assume \( c < \frac{1}{2\beta^2} \) to ensure that the signal is not prohibitively expensive, thus guaranteeing that the founder optimally selects some positive value for the precision of the firm’s disclosure.

For ease of exposition I place two restrictions upon the contract which the founder can offer to the manager. The first restriction is to assume that the manager must be paid at \( t = 1 \)\(^1\). This can be thought to reflect the fact that the manager may have private liquidity or consumption needs at \( t = 1 \) and is restricted in their ability to borrow against future earnings. As a result, the manager’s contract can only be made contingent upon information that is released at \( t = 1 \) (only \( a \) and \( s \)). This is reasonable if we interpret \( y \) as the stream of profits generated by the firm over its entire

\(^1\)The qualitative results of the model are unchanged if \( \phi(e) \) is simply assumed to be increasing and convex.

\(^1\)Since the firm does not generate any income at \( t = 1 \), I assume that the firm borrows \( W \) at \( t = 1 \) and repays the loan at \( t = 2 \).
The qualitative results of the model are unchanged if this assumption is relaxed as long as \( a \) is informative for the manager’s effort after controlling for \( y \). This is true whenever \( \tau_y \) is finite.\(^{16}\)

The second restriction I place upon the manager’s contract is that it must be linear. So the manager’s contract is of the form:

\[
W = w_0 + w_a a + w_s s,
\]

where \( w_0, w_a, \) and \( w_s \) are parameters to be chosen by the founder at \( t = 0 \) and are publicly observed. The assumption of linearity is made for two reasons. First, it avoids the standard Mirrlees (1974) problem where moral hazard can be solved costlessly by extreme punishments. Second, linear contracts can be motivated by supposing that this model is a static approximation to a dynamic moral hazard problem (Holmstrom and Milgrom (1987)). The qualitative results of the model are unchanged if linearity is relaxed, and instead, the manager is protected by limited liability. Assuming linearity has the advantage that it allows me to solve analytically for the manager’s contract. All that is required for the analysis is that the fundamental value of the firm is affected by the accuracy of its disclosures. Here this will be true because it is less expensive to solve the moral hazard problem with the manager if \( a \) is a more informative signal of the manager’s effort decision. This is a general feature of models of moral hazard (see for example Hölmstrom (1979) and Holmström and Tirole (1993)). The additional assumptions are made only for tractability.

2.3 Trading, Investor Beliefs, and Short-Sale Constraints

The potential for trade exists at two times in the life of the firm. First, at the end of period 0, the founder sells the stock of the firm to public investors. Second, subsequent to the release of information (signals \( a \) and \( s \)) at date 1, investors will reassess their value of the firm and can trade the firm’s shares accordingly.\(^{17}\) All investors and the founder are assumed to be risk neutral.

At time 0 the profitability of the firm, \( \theta \), is uncertain. The crucial assumption I make is that investors have heterogeneous prior beliefs about the distribution of \( \theta \).\(^{18}\) In particular, there is a continuum of investors indexed by \( i \). At time 0 investor \( i \) believes that \( \theta \) is distributed \( N \left( 0, \frac{1}{\tau_i^\theta} \right) \). Thus at \( t = 0 \) all investors agree about the expected value of \( \theta \) but disagree about the variance of the distribution.\(^{19}\) By assuming the investors have prior beliefs that differ in precision, Bayes’

\(^{16}\)In the limit if \( \frac{\tau_y}{\tau_a} \to 0 \) the optimal long term contract will only pay the manager based on \( a \).

\(^{17}\)I assume the manager is unable to trade the stock of the firm. This removes the possibility that investors can learn the manager’s private information about \( \bar{m} (\tau_a) \) by observing her trading behavior.

\(^{18}\)While I take the initial heterogeneity of prior beliefs as given, a growing set of papers in the behavioral finance literature endogenously derive disagreement among investors by assuming a particular pathology in the way that agents form beliefs (for a survey see Daniel et al. (2002) and Barberis and Thaler (2003)).

\(^{19}\)The model can be extended to the case where investors have heterogeneous beliefs about both the mean and variance of \( \theta \) at \( t = 0 \). It is straightforward to show that in equilibrium, for any given level \( \tau_i^\theta \), it is only the investor
rule implies that they will update their beliefs, based on any information that is disclosed about the firm, to arrive at a different assessment of its expected profitability. This is analogous to the assumption made in papers such as Daniel et al. (1998), Daniel et al. (2001), and Scheinkman and Xiong (2003) where investors place relatively more weight on their own source of information and therefore arrive at heterogeneous assessments of a stock’s value.

To describe the beliefs of investors, let $\tau^M_\theta$ be the median precision among all investors and let $\tau_\theta^+$ and $\tau_\theta^-$ denote the precision of the investor with the strongest and weakest prior respectively. There is a continuum of investors with deep pockets at each precision. The initial degree of heterogeneity among investors’ beliefs is measured by $\Delta$ so that:

$$\tau_\theta^+ = \tau^M_\theta + \frac{\Delta}{2} \quad \text{and} \quad \tau_\theta^- = \tau^M_\theta - \frac{\Delta}{2}.$$  

The manager’s prior belief at $t = 0$ is that $\theta \sim N(0, \frac{1}{\tau^\text{man}_\theta})$ where $\tau^\text{man}_\theta \in [\tau^-_\theta, \tau^+_\theta]$. All agents agree about the distribution of the other fundamental random variables ($\epsilon_a, \epsilon_s, \text{and} \epsilon_y$). Given their prior beliefs, each agent is fully rational in the sense that they use Bayes’ rule when updating their beliefs at $t = 1$ when the signal is released. Agents correctly anticipate how others will update their beliefs at $t = 1$ and recognize that they may agree to disagree in the way that they update their beliefs at that time. Note that investors do not have any private information in this framework.

Since investors agree to disagree over the value of the firm at $t = 1$, they will wish to take unlimited long or short positions in the stock. To determine the equilibrium price, I assume that investors are unable to take short positions. As a result, the equilibrium stock price of the firm at each point in time will be determined by the investor with the highest assessment of its value at that moment (as per Miller (1977) and Scheinkman and Xiong (2003)).

### 3 Equilibrium Price and Optimal Incentives

Before studying the founder’s optimal choice of the disclosure precision, I solve for the equilibrium price of the firm in each period and the optimal managerial incentive contract for any initial with the highest initial prior who will ever own the stock and hence influence its price (due to short-sale constraints). The current setup is therefore equivalent to allowing for heterogeneity in both the mean and variance of beliefs over $\theta$ at $t = 0$ and normalizing the initial expected value of the most optimistic investors to zero. If investors only disagreed about the mean of their beliefs at $t = 0$ then no speculative premium exists because the same initially optimistic investor would own the stock in every state of the world. Speculative premia arise from the option value of being able to sell to a more optimistic investor in some future states of the world (Harrison and Kreps (1978)).

20 Several papers provide evidence that investors are limited in their ability to short stocks and show that this leads to over pricing (see for example Jones and Lamont (2002), Lamont and Thaler (2003), Chen et al. (2002), and Ofek and Richardson (2003)). Of particular interest is D’Avolio (2002) who shows that proxies for the extent of disagreement among investors predict high costs of shorting.
choice of $\tau_a$. These form the two channels through which the disclosure decision impacts the price at which the firm can be sold at date 0.

### 3.1 Equilibrium Prices

I start by taking the manager’s contract ($w_o$, $w_a$, and $w_s$) as given and solve for the resulting equilibrium price of the firm each period. Let $p_t$ denote the market clearing price of the firm in period $t$. The price of the firm in period 2 is equal to its realized value so that:

$$p_2 = y - W - c \tau_a.$$  \hfill (9)

At date 1 investors observe both $a$ and $s$ and use these to update their belief about the expected value of the firm in date 2. Investor $i$’s updated expectation of $p_2$ is:

$$E_1(p_2|a,s; \tau^i_\theta) = \hat{e} - w_0 - w_a a - w_s s - c \tau_a + \left(\frac{\tau_a (a - \hat{e} - f(\tau_a)) + \tau_s s}{\tau^i_\theta + \tau_a + \tau_s}\right).$$  \hfill (10)

where $\hat{e}$ is the conjectured level of managerial effort$^{21}$

Competition among investors with heterogeneous beliefs for an asset that is in finite supply and faces a short-sale constraint, implies that at any point in time the price of the firm is determined by the investor with the highest expected value of the firm:

$$p_1 = \hat{e} - w_0 - w_a a - w_s s - c \tau_a + \max_i \left\{ \frac{\tau_a (a - \hat{e} - f(\tau_a)) + \tau_s s}{\tau^i_\theta + \tau_a + \tau_s} \right\}.$$  \hfill (11)

Hence only the investor with the strongest or weakest prior will determine the date 1 market value of the firm. If good news is released at date 1, so that investors revise upwards their assessment of the firm’s value ($\tau_a (a - \hat{e} - f(\tau_a)) + \tau_s s > 0$), then investors with the weakest prior ($\tau^i_\theta$) will be the most optimistic about the firm and thus determine its price. Conversely, if the news is unfavorable so that it causes investors to lower their assessment of the firm’s value ($\tau_a (a - \hat{e} - f(\tau_a)) + \tau_s s < 0$), then the investor with the strongest prior ($\tau^+_\theta$) will be least swayed by the news and thus will hold the highest valuation of the firm. The equilibrium value for the price of the firm at date 1 can thus be written as:

$$p_1 = \hat{e} - w_0 - w_a a - w_s s - c \tau_a + \max \left\{ \frac{\tau_a (a - \hat{e} - f(\tau_a))}{\tau^+_\theta + \tau_a + \tau_s}, \frac{\tau_a (a - \hat{e} - f(\tau_a)) + \tau_s s}{\tau^-_\theta + \tau_a + \tau_s} \right\}.$$  \hfill (12)

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$^{21}$In equilibrium this conjecture will be correct so that $\hat{e} = e$. 

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Returning to date 0, investor $i$’s expectation of $p_1$ is:

$$E_0(p_1; \tau^i_\theta) = (1 - w_a) \bar{e} - w_a f(\tau_a) - w_0 - c \tau_a + \frac{C}{\sqrt{2\pi}} \sqrt{\frac{\tau_a + \tau_s}{\tau^i_\theta}} \left[ \frac{\tau_a + \tau_s}{\tau^i_\theta} + 1 \right],$$  \hspace{1cm} (13)

where:

$$C \equiv \frac{\Delta}{(\tau^-_\theta + \tau_a + \tau_s)(\tau^+_\theta + \tau_a + \tau_s)} > 0.$$

$E_0(p_1; \tau^i_\theta)$ is strictly decreasing in $\tau^i_\theta$ and hence the date 0 value of the firm will be determined by the belief of of investors with the weakest prior. These investors buy the firm at date 0 because they believe that $\theta$ is the most volatile and hence expect the biggest divergence in beliefs at date 1. As a result, these investors place the most value on the option to resell at date 1. Therefore the equilibrium price of the firm at date zero is

$$p_0 = [(1 - w_a) \bar{e} - w_a f(\tau_a) - w_0 - c \tau_a] + V^{Spec}(\tau_a)$$  \hspace{1cm} (14)

where

$$V^{Spec}(\tau_a) \equiv \frac{\Delta}{\sqrt{2\pi \tau^-_\theta}} \tau^{-\frac{1}{2}} (\tau^-_\theta + \tau_a + \tau_s)^{-\frac{1}{2}} \tau^+ (\tau_a + \tau_s)^{-1}. \hspace{1cm} (15)$$

Using the dichotomy of Harrison and Kreps (1978) there are two components to the firm’s price at date 0. The first of these represents the “fundamental” expected value of the firm, its expected net output. Absent the ability to resell the firm at $t = 1$ this would be the date 0 price of the firm. At $t = 0$ all agents agree upon the fundamental value of the firm. The second term, $V^{Spec}$, is the speculative component of the firm’s price. This represents the option value of the firm’s stock which derives from the fact that with some probability the investor will be able to sell their shares to another investor with more optimistic beliefs at $t = 1$. Observe that this speculative component disappears if all investors have the same prior belief ($\Delta = 0$) and consequently update beliefs based on information released at $t = 1$ in the same way.

### 3.2 Optimal Managerial Incentives

For any given level of precision for the firm’s disclosure ($\tau_a$), the founder sets the incentive contract of the manager ($w_0, w_a, w_s, $ and $e$) so as to maximize the price at which they can sell the firm in date 0. Formally, her problem is:

$$\max_{w_0, w_a, w_s, e} (1 - w_a) e - w_a f(\tau_a) - w_0 - c \tau_a + V^{Spec}(\tau_a), \hspace{1cm} (16)$$
subject to:

\[ c'(e) = w_a, \quad \text{and} \]
\[ w_0 + w_a f(\tau_a) + w_a e - c(e) - \frac{r}{2} \left( \frac{w_a + w_s}{\tau_{q_{Man}}} \right)^2 + \frac{w_a^2}{\tau_a} + \frac{w_s^2}{\tau_s} \geq w. \]  

The first constraint (17) ensures that effort level \( e \) is chosen by the manager. The second constraint (18) is the manager’s participation constraint. For the remainder of the analysis I normalize \( w = 0 \) which is sufficient to ensure that the founder will always choose to employ the manager.

Let “\( * \)” denote the optimal managerial contract \( \{ w_0^*, w_a^*, w_s^*, e^* \} \) as a function of \( \tau_a \). These are solved for in the appendix and are:

\[
\begin{align*}
    w_0^* &= -\frac{(w_a^*)^2}{2\beta} \left( 1 - r\beta \left( \frac{1}{\tau_s + \tau_{q_{Man}}} + \frac{1}{\tau_a} \right) \right) - w_a^* f(\tau_a), \\
    w_a^* &= \left[ 1 + r\beta \left( \frac{1}{\tau_s + \tau_{q_{Man}}} + \frac{1}{\tau_a} \right) \right]^{-1}, \\
    w_s^* &= -w_a^* + (\tau_{q_{Man}} \tau_a)^{-1} w_a^*, \quad \text{and} \\
    e^* &= \frac{w_a^*}{\beta}.
\end{align*}
\]

In words, the optimal managerial contract induces managerial effort by paying more when the firm’s disclosure is high: \( w_a^* > 0 \). The contract is more sensitive to the disclosure when the precision of the firm’s disclosure is higher \( (\frac{\partial w_a^*}{\partial \tau_a} > 0) \). It is less costly to induce effort when the disclosure is more accurate and thus the optimal level of effort is also increasing in the precision of the firm’s disclosure \( (\frac{\partial e^*}{\partial \tau_s} = \frac{\partial e^*}{\partial w_a^*} \frac{\partial w_a^*}{\partial \tau_s} > 0) \). Notice that although the manager distorts the signal released by the firm at \( t = 1 \) to maximize her compensation, this is fully anticipated at \( t = 0 \) and does not alter the expected level of her compensation. The optimal contract offers reward for performance measured relative to the outside signal: \( w_s^* < 0 \). This enables the firm to filter out part of the noise contained in the firm’s disclosure coming from the uncertainty about the firm’s productivity, \( \theta \), that is unrelated to effort. As the external signal grows in precision, this increases the strength of the optimal incentives offered to the manager \( (\frac{\partial w_a^*}{\partial \tau_a} > 0 \quad \text{and} \quad \frac{\partial w_s^*}{\partial \tau_s} < 0) \) and raises the effort supplied by the manager \( (\frac{\partial e^*}{\partial \tau_s} = \frac{\partial e^*}{\partial w_a^*} \frac{\partial w_a^*}{\partial \tau_s} > 0) \).

Using the solution for the optimal contract, we can write the founder’s objective as a function of the precision of the firm’s disclosure:

\[ V(\tau_a) = V^{\text{Fund}}(\tau_a) + V^{\text{Spec}}(\tau_a), \]  

\(^{22}\) The manager’s objective function is strictly concave in \( e \) so the first order condition is sufficient to characterize the manager’s effort choice.
where:
\[
V^{\text{Fund}}(\tau_a) \equiv \left[ 2\beta \left( 1 + r\beta \left( \frac{1}{\tau_M^{\text{Man}}} + \frac{1}{\tau_s} + \frac{1}{\tau_a} \right) \right) \right]^{-1} - c\tau_a. \tag{21}
\]

\(V^{\text{Fund}}(\tau_a)\) represents the fundamental value of the firm. This is the price at which any investor would be willing to buy the firm at \(t = 0\) if they were unable to trade at \(t = 1\). It is also the price which investors would be willing to buy the firm at \(t = 0\) if there was no heterogeneity of beliefs (i.e. \(\Delta = 0\)).

4 Optimal Disclosure Precision: No External Information

Having fully characterized equilibrium prices and optimal managerial incentives for any chosen level of disclosure precision, I now turn to the central question of the paper: what precision will the founder select for the firm’s disclosure at the start of date 0? I start the analysis by focusing on the case where there is no external signal about the profitability of the firm (\(\tau_s = 0\)) and then consider the role the external signal plays in Section 5. As (20) makes clear, in order to maximize the price at which the founder can sell the firm at \(t = 0\), this decision is determined by the impact of the disclosure precision on the fundamental value of the firm as well as the speculative component of the firm’s price. In order to understand this trade-off, I start by characterizing the properties of the two components of the firm’s price.

4.1 Disclosure Precision and the Fundamental Component of the Firm’s Price

Increasing the precision of the firm’s disclosure reduces the cost of inducing managerial effort. The fundamental value of the firm is maximized by setting \(\tau_a\) so that the marginal value of its contribution to reducing the moral hazard problem is equal to the marginal cost of information production, \(c\). The fundamental value of the firm, \(V^{\text{Fund}}(\tau_a)\), is concave in \(\tau_a\) and is maximized at a unique \(\tilde{\tau}_a > 0\) where:
\[
\tilde{\tau}_a = \sqrt{\frac{\tau_s - r\beta}{1 + \frac{r\beta}{\tau^{\text{Man}}}}}. \tag{22}
\]

If there was no heterogeneity in investors’ prior beliefs (\(\Delta = 0\)), then there would be no speculative component to the firm’s price and the founder would set \(\tau_a = \tilde{\tau}_a\).

For the remainder of this section I focus on the case where \(c > 0\) but arbitrarily small which ensures that \(\tilde{\tau}_a\) is finite but arbitrarily large. In the limit, if monitoring is costless (\(c = 0\)), then the fundamental value of the firm is maximized by committing to a disclosure that contains no noise (\(\tilde{\tau}_a^{-1} = 0\)).
4.2 Disclosure Precision and the Speculative Component of the Firm’s Price

The speculative component of the firm’s price, as given by (15), varies non-monotonically with the precision of the firm’s disclosure. The form of this relationship is illustrated in Figure 2 and is established formally in the Appendix. Specifically, there exists a unique:

$$\hat{\tau}_a = \frac{1}{4} \left[ -\tau^0 + \sqrt{(\tau^0)^2 + 8\tau^0 \tau^0} \right] > 0 \quad (23)$$

that maximizes the speculative component of the firm’s price. Moreover, \( \frac{\partial V^{\text{Spec}}(\tau_a)}{\partial \tau_a} \geq 0 \) for all \( \tau_a \leq \hat{\tau}_a \) and \( \frac{\partial V^{\text{Spec}}(\tau_a)}{\partial \tau_a} \leq 0 \) for all \( \tau_a \geq \hat{\tau}_a \).

The intuition is as follows. There is no speculative premium when the firm’s disclosure is pure noise (i.e. \( V^{\text{Spec}}(\tau_a = 0) = 0 \)). This occurs because disagreement at \( t = 1 \) comes from the fact that investors differ in the way that they interpret the firm’s disclosure. When the disclosure is so garbled that it contains no information (\( \tau_a = 0 \)), there is nothing for investors to disagree over at \( t = 1 \), and hence no speculative premium. When \( \tau_a \leq \hat{\tau}_a \), this force dominates so that increasing \( \tau_a \) increases disagreement at \( t = 1 \) and raises the speculative component of the firm’s price. Conversely, when the firm’s disclosure contains no noise (\( \tau_a^{-1} = 0 \)), this fully reveals the expected value of the firm at \( t = 1 \). This prevents any disagreement among investors at \( t = 1 \) and thus also results in no speculative premium at \( t = 0 \) (i.e. \( V^{\text{Spec}}(\tau_a = \infty) = 0 \)). When \( \tau_a \geq \hat{\tau}_a \), this force dominates so that increasing \( \tau_a \) lowers disagreement at \( t = 1 \) and reduces the speculative component of the firm’s price.

Comparing (22) and (23) yields a unique \( \tilde{c} > 0 \) for which \( \tilde{\tau}_a = \tilde{\tau}_a \). I focus solely on the case where the marginal cost of information is small, so that \( c < \tilde{c} \), which is necessary and sufficient to ensure that disclosure precision that maximizes the fundamental value of the firm is strictly larger than that which maximizes its speculative component: \( \tilde{\tau}_a > \hat{\tau}_a \).

4.3 Optimal Disclosure Precision

Having studied how \( \tau_a \) affects the speculative and fundamental component of the firm’s price independently, I now characterize how the founder balances these two forces to set \( \tau_a \) to maximize the sale price of the firm at \( t = 0 \). The resolution of this trade-off is described in the following proposition.

**Proposition 1.** If \( \Delta > 0 \), the founder’s optimal choice for the precision of the firm’s disclosure, \( \tau_a^* \), is such that \( \tilde{\tau}_a < \tau_a^* < \hat{\tau}_a \). This results in \( V^{\text{Fund}}(\tau_a^*) < V^{\text{Fund}}(\tilde{\tau}_a) \) and \( V^{\text{Spec}}(\tau_a^*) > V^{\text{Spec}}(\hat{\tau}_a) \).

---

\( \tilde{c} \) Formal expression given in (44) in the appendix.
Proposition 1 highlights the initial two key results of this paper. The first key result is that the founder will lower the precision of the firm’s disclosures below the value which maximizes the fundamental value of the firm when investors have heterogeneous prior beliefs. A small increase in $\tau_a$ above $\tau^*_a$ improves the fundamental value of the firm by making it less expensive to induce the manager to exert effort. However, the founder elects not to do this because increasing the informativeness of the firm’s disclosure reduces the extent of disagreement among investors at $t = 1$ and thereby lowers the speculative component of the firm’s price.

The second key result highlighted in Proposition 1 is that the endogenous change in the firm’s disclosure precision amplifies the overvaluation of the firm. Several papers have stressed that the combination of heterogeneous beliefs and short-sale constraints will cause a firm’s stock to be overpriced. This is equivalent to observing that $V^{\text{Spec}}(\tilde{\tau}_a) > 0$. Proposition 1 highlights that the overvaluation which arises from the difference of opinion is magnified by the endogenous response in the firm’s disclosure policy relative to the standard case where $\tau_a$ is chosen to maximize the fundamental value of the firm’s cashflows ($V^{\text{Spec}}(\tau^*_a) > V^{\text{Spec}}(\tilde{\tau}_a)$).

### 4.4 Comparative Statics

I now ask how changes in the heterogeneity of investors’ prior beliefs, as parameterized by $\Delta$, alter the founder’s choice of disclosure precision. Figure 3 Panel A shows how the founder’s choice of $\tau^*_a$ varies with $\Delta$ holding all else constant. For parameters used in Figure 3 the fundamental value of the firm is maximized at $\tilde{\tau}_a = 9.21$. The founder chooses this precision only when there is no heterogeneity among the prior beliefs of investors ($\Delta = 0$). As the heterogeneity of investors’ prior beliefs increases, the founder now has the opportunity to increase the sale price by committing to a less informative disclosure policy and thereby increasing the speculative component of the firm’s price. This produces a monotonic negative relationship between $\tau^*_a$ and $\Delta$.

The endogenous response of the founder to the increase in the heterogeneity of investor beliefs amplifies its effect upon the overvaluation of the firm. To illustrate this I calculate the proportion of the firm’s price at $t = 0$ that is accounted for by the speculative premium. The solid line in Figure 3 Panel B shows how this varies with the initial heterogeneity in investors’ beliefs (as parameterized by $\Delta$). There are two forces which drive this increase. The first is that for any given level of $\tau_a$, as $\Delta$ rises there will mechanically be more disagreement among investors at $t = 1$. This effect is captured by the dashed line in Figure 3 Panel B which shows the proportion of the firm’s price accounted for by the speculative premium in the hypothetical case where $\tau_a$ is fixed at $\tilde{\tau}_a$. The difference between this dashed line and the solid line represents the additional overvaluation that arises endogenously due to the founder’s disclosure decision.

---

See for example Harrison and Kreps (1978), Morris (1996), and Scheinkman and Xiong (2003).
I establish the following comparative static results analytically.

**Lemma 1.** \( \tau^*_a \) is weakly increasing in \( \tau_\theta^- \) (holding \( \tau_\theta^+ \) and \( \tau_{Man} \) constant). \( V_{Spec}(\tau^*_a) \) is weakly decreasing in \( \tau_\theta^- \). \( V_{Fund}(\tau^*_a) \) is weakly increasing in \( \tau_\theta^- \).

Lemma 1 examines what happens when the pool of investors expands to include agents with weaker prior beliefs. The increased heterogeneity in investors’ beliefs increases the founder’s incentive to sponsor speculation by reducing the precision of the firm’s disclosure. A fall in \( \tau_\theta^- \) (holding all else equal) lowers \( \tau^*_a \) and as a result increases the speculative component of the firm’s price while lowering its fundamental value.

The addition of investors with stronger prior beliefs (an increase in \( \tau_\theta^+ \) holding \( \tau_\theta^- \) and \( \tau_{Man} \) constant) may increase or decrease the founder’s choice of \( \tau^*_a \) (depending on parameters). This ambiguity stems from the fact that investors with stronger prior beliefs are influenced less by the firm’s disclosure. If \( \tau_\theta^+ \) increases, founders can, in some cases, increase disagreement by releasing more information. Consequently, investors with weak priors (\( \tau_\theta^- \)) will react strongly and those with strong priors (\( \tau_\theta^+ \)) will update their beliefs less. This can be seen by noting that \( \hat{\tau}_a \), the disclosure policy that maximizes the speculative component of the firm’s price, is increasing in \( \tau_\theta^+ \).

Despite the ambiguous effect of \( \tau_\theta^+ \) on \( \tau^*_a \) an increase in the divergence of prior beliefs will always increase the price at which the founder sells the firm at \( t = 0 \). This is summarized in the following lemma.

**Lemma 2.** \( V(\tau^*_a) \) is weakly increasing in \( \tau_\theta^+ \) and weakly decreasing in \( \tau_\theta^- \).

### 4.5 Welfare and Policy

#### 4.5.1 Welfare

I now ask whether the founder’s choices at \( t = 0 \) produce a Pareto optimal allocation. In order to assess the welfare of any choice made by the founder, I denote the prior beliefs of the social planner as \( \theta \sim N \left(0, \frac{1}{\tau_{Pol}} \right) \). In the Appendix I show that, with an appropriate set of transfers, the following social welfare function can be used to evaluate any allocation for a given \( \tau_{Pol}^\theta \):

\[
U^{SWF} = \frac{w_a}{\beta} - \frac{w_a^2}{2\beta} - \frac{r}{2} \left[ \frac{1}{\tau_{Pol}^\theta} + \frac{1}{\tau_a} \right] w_a^2 - c \tau_a . \tag{24}
\]

Any change in \( w_a \) or \( \tau_a \) that produces an increase in (24) results in a strictly Pareto superior allocation for a given belief precision \( \tau_{Pol}^\theta \). I adopt the belief-neutral welfare criterion of [Brunnermeier et al. (2014)](http://example.com) which contends with the possibility that the social planner herself is agnostic between all possible reasonable prior beliefs. Specifically, following [Brunnermeier et al. (2014)](http://example.com), welfare
rankings between two allocations can only be made if the ordering of the social welfare function is the same for all possible $\tau^\text{Pol}_{\theta} \in [\tau^-_{\theta}, \tau^+_{\theta}]$.

The effect of applying the welfare concept of Brunnermeier et al. (2014) is to fully remove the speculative premium from (24). The speculative component of the firm’s price is eliminated from the social welfare function because this arises purely due to different investors assessing states of the world with different probabilities. The social welfare function instead contains only the expected output of the firm and the manager’s expected utility that is assessed using the policy-maker’s belief precision $\tau^\text{Pol}_{\theta}$. $U^{SWF}$ is strictly concave in $\tau_a$, and the smallest value of $\tau_a$ that maximizes (24) for any $\tau^\text{Pol}_{\theta} \in [\tau^-_{\theta}, \tau^+_{\theta}]$ is:

$$\tau^\text{Wel}_{-a} = \sqrt{\frac{r^2}{c} - r\beta \tau^-_{\theta}} - r\beta \tau^+_{\theta}.$$ (25)

It follows that if the equilibrium choice of disclosure precision falls below this lower bound, then there must exist a Pareto superior allocation, regardless of what belief precision the policy-maker holds. This is stated formally in the following proposition.

**Proposition 2.** If the founder has selected a disclosure precision $\tau^*_a < \tau^\text{Wel}_{-a}$ and managerial contract $\{w^*_o, w^*_a\}$ this is strictly belief-neutral inferior to an allocation in which the disclosure precision is increased to $\tau^\text{Wel}_{-a}$ and the manager’s contract remains $\{w^*_o, w^*_a\}$.

Whenever $\Delta > 0$ the founder sets the disclosure precision below $\tau^\text{Wel}_{-a}$ so that, no matter what the social planner’s prior belief precision $\tau^\text{Pol}_{\theta}$, a strictly Pareto superior allocation exists in which higher disclosure is chosen. This is shown numerically in Figure 4 which plots both $\tau^*_a$ and $\tau^\text{Wel}_{-a}$ for different degrees of belief heterogeneity $\Delta$. Note that the lower bound, $\tau^\text{Wel}_{-a}$, is itself decreasing in $\Delta$. This occurs because the fundamental value of the disclosure precision derives from mitigating the moral hazard problem between the firm and the manager. This value is diminished when $\theta$ is believed to be more volatile because any incentive contract exposes the risk averse manager to more risk. An increase in $\Delta$ lowers $\tau^-_{\theta}$, and the policy-maker must evaluate social welfare giving consideration to the possibility that more such volatility exists.

### 4.5.2 Disclosure Policy

The welfare analysis so far has focused on the question of whether the founder’s choice of $\tau^*_a$ is Pareto efficient or not. The finding that choices below $\tau^\text{Wel}_{-a}$ are Pareto inferior, suggests the possibility of a welfare improving policy that mandates a minimum disclosure precision. However,

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25 A corresponding upper bound for disclosure precision also exists but this is above $\tau_a$ and hence the founder will never elect to set disclosure precision too high relative to this standard.
any disclosure policy regulation also needs to take into account the impact that mandating a level of \( \tau_a \) will have on the founder’s choice of \( w_a \). Specifically, I assume that the planner is unable to mandate \( w_a \) and therefore is constrained by the fact that any regulation mandating a minimum level of \( \tau_a \) will alter the manager’s contract according to (19). Note that \( w_a^* \) uses the prior beliefs of the manager, regardless of what value of \( \tau_{\theta Pol} \) the planner adopts.

The social welfare function to assess any mandated disclosure policy that takes this effect into account, is given by:

\[
U^{SWF*}(\tau_a; \tau_{\theta Pol}) = \left( 2 \beta \left( 1 + r \beta \left( \frac{1}{\tau_{Man}} + \frac{1}{\tau_a} \right) \right) \right)^{-1} \left( 2 - \frac{1 + r \beta \left( \frac{1}{\tau_{Pol}} + \frac{1}{\tau_a} \right)}{1 + r \beta \left( \frac{1}{\tau_{Man}} + \frac{1}{\tau_a} \right)} \right) - c \tau_a. \tag{26}
\]

To start the analysis, suppose that the social planner assesses welfare using a single belief that is the same as the prior of the manager so that \( \tau_{\theta Pol} = \tau_{\theta Man} \). In this case (26) reduces to \( V^{Fund}(\tau_a) \) and it follows immediately that the unique disclosure policy that maximizes social welfare is to mandate that \( \tau_a = \bar{\tau}_a \) so as to maximize the fundamental value of the firm.

Evaluating policy choices using the welfare criteria of Brunnermeier et al. (2014) where the policy-maker is agnostic over all possible prior beliefs complicates the analysis. In this situation the policy-maker must also contend with the fact that changes in the mandated level of disclosure precision will impact the incentive contract which the founder offers the manager. For values of \( \tau_{\theta Pol} < \tau_{\theta Man} \) the policy-maker judges the incentive contract to set \( w_a \) too high and expose the manager to an inefficiently high level of risk. Increasing \( \tau_a \) has the effect of exacerbating this inefficiency. This complication also renders (26) such that it does not produce a tractable analytical solution. Instead I show the belief-neutral minimum disclosure precision policy numerically, where this is defined as:

\[
\tau_{\theta Pol} - \equiv \arg\min_{\tau_{\theta Pol}} \left[ \arg\max_{\tau_a} \left[ U^{SWF*}(\tau_a; \tau_{\theta Pol}) \right] \right]. \tag{27}
\]

In words, \( \tau_{\theta Pol}^- \) is the smallest social welfare maximizing disclosure precision for any value of \( \tau_{\theta Pol} \) that the planner may adopt (it is determined by the least precise possible prior: \( \tau_{\theta Pol} = \tau_{\theta}^- \)). If the founder chose to set \( \tau_a^* < \tau_{\theta Pol}^- \), then mandating an increase in disclosure precision to \( \tau_{\theta Pol}^- \) will lead to a strict Pareto improvement, no matter which \( \tau_{\theta Pol} \in \left[ \tau_{\theta}^-, \tau_{\theta}^+ \right] \) the social planner uses to assess welfare. Unlike Proposition 2 this incorporates the impact this policy will have on the managerial contract.

\[26\]Similarly, if the social planner disregards the welfare of the firm’s manager, and focuses only on the welfare of investors and the founder of the firm, mandating that the founder set \( \tau_a = \bar{\tau}_a \) is also the unique optimal disclosure policy regardless of what belief the policy-maker adopts. Recall that at any equilibrium allocation the manger’s contract will be set so that her expected utility is \( w_a^* \) according to her own prior belief that \( \theta \sim N \left( 0, \frac{1}{\tau_{Man}} \right) \).
The belief-neutral minimum disclosure standard is shown as the dotted line in Figure 4. The figure highlights two features of the optimal belief-neutral disclosure policy. When belief heterogeneity is positive but takes on low to intermediate values, the minimum disclosure policy will be binding: $\tau_{Pol}^a > \tau^a_a$. In this case an agnostic social planner will be able to limit speculation and be certain welfare is improved.

However this is no longer true when belief heterogeneity is large. This stems from the fact that the social-planner recognizes that the manager’s contract will be set according to the manager’s perceived prior belief $\tau_{Man}^\theta$ and for $\tau_{Pol}^\theta = \tau^\theta$ the planner believes that this forces an inefficiently high degree of risk on the planner - more than she or the founder recognizes. Increasing the disclosure precision makes the performance contract even more sensitive and further exacerbates this problem. This is why $\tau_{Pol}^a < \tau_{Wel}^a$: the welfare standard holds constant the contract offered to the manager while the optimal disclosure policy must take this into account. For this reason, when the heterogeneity of beliefs is high, the social planner cannot be sure that raising the low level of disclosure set by the founder will lead to a belief-neutral welfare improvement and hence the minimum disclosure standard is not binding $\tau_{Pol}^a < \tau^a_a$. This suggests that an agnostic social planner is unable to use disclosure policy to limit speculation, and be sure that welfare is improved, exactly when prior belief heterogeneity is high and the potential problem is the most pronounced.

5 Disclosure Policy in the Presence of External Information

The analysis in the previous section focused on the case where the firm’s disclosure is the only source of information that investors receive ($\tau_s = 0$). I now extend the analysis of the model to incorporate the fact that investors also learn about the profitability of an asset from other sources including their own private research, the business press, rating agencies, and the financial disclosures of other sellers in the same industry. In this section I show how these other sources of information impact upon the founder’s choice of disclosure precision. To begin with, I allow there to be an exogenous source of information. I then consider the case where that information is generated endogenously from the disclosures of other firms in the same industry. The goal is to demonstrate that there exists a complementarity between the founder’s choice and the precision of this external information. This complementarity suggests that speculative episodes and inaccurate disclosures will occur in waves.

5.1 Exogenous External Information

Start by supposing that the founder sets the disclosure precision for her firm taking $\tau_s > 0$ as given. To commence the analysis, consider how external information impacts the fundamental and
speculative components of the price at $t = 0$.

5.1.1 Disclosure Precision and the Fundamental Component of the Firm’s Price ($\tau_s > 0$)

The fundamental value of the firm, $V^{Fund} (\tau_a)$ as given by (21), is strictly increasing in the precision of the external signal ($\tau_s$) because it provides the founder with a more accurate benchmark against which to judge the manager’s performance. The fundamental value of the firm remains strictly concave in $\tau_a$ and is now maximized at:

$$\tilde{\tau}_a' = \sqrt{\frac{\tau_s}{\tau_a^2 - \tau_s}},$$

where the prime is used to denote the case where $\tau_s > 0$. A more accurate external signal, by providing a more accurate benchmark against which to compare the firm’s own disclosure, increases the marginal benefit to the fundamental value of the firm from raising the precision of the firm’s own disclosure and hence $\tilde{\tau}_a'$ is increasing in $\tau_s$.

5.1.2 Disclosure Precision and the Speculative Component of the Firm’s Price ($\tau_s > 0$)

The speculative component of the firm’s price, as given by (15), is determined solely by the total precision of all the information released at $t = 1$: $\tau_a + \tau_s$. The relationship between the speculative component of the firm’s price and the precision of its disclosure is the same as shown in Figure 2 but shifted to the left by $\tau_s$. As a result, the value of $\tau_a$ which maximizes $V^{Spec} (\tau_a)$ is $\tilde{\tau}_a' = \max \{0, \tilde{\tau}_a - \tau_s\}$. The disclosure precision which maximizes the speculative component of the firm’s price is unambiguously decreasing in the amount of outside information available to investors. In this way, external information can “crowd out” the informativeness of the firm’s own disclosure for the purposes of maximizing the speculative component of the firm’s price. Conversely, when the external signal is highly informative ($\tau_s$ is large) this lowers the possible disagreement among investors at $t = 1$ and thereby lowers the size of the speculative component of the firm’s price for any level of $\tau_a$. By reducing the importance of the speculative component of the firm’s price, relative to the fundamental component, external information can “crowd in” precision of the firm’s own disclosure. I now study this trade-off.

5.1.3 Optimal Disclosure Precision ($\tau_s > 0$)

As before, the founder’s optimal choice of disclosure precision will maximize the sum of the fundamental and the speculative component of the firm’s price. This will lead the founder to choose
a level of disclosure that is strictly lower than that which maximizes the fundamental value of the firm. Formally:

**Proposition 3.** If $\Delta > 0$, the founder’s optimal choice for the precision of the firm’s disclosure, $\tau^*_a$, is such that $\tilde{\tau}^*_a < \tau^*_a < \tilde{\tau}'_a$.

Figure 5 shows how the founder’s choice of disclosure precision $\tau^*_a$ varies with the amount of external information, $\tau_s$. There is a non-monotonic relationship between $\tau_s$ and the founder’s choice. When $\tau_s$ is small, $\tau^*_a$ is decreasing in $\tau_s$. This is because, for the purposes of generating speculation, the outside information substitutes for the firm’s disclosure. In order to sponsor speculation, the founder must reduce the precision of their firm’s disclosure so as to maintain the potential level of disagreement among investors at $t = 1$. However, when $\tau_s$ becomes sufficiently large, this eliminates most of the potential disagreement among investors. When this occurs, the founder sets $\tau^*_a$ primarily to maximize the fundamental value of the firm. To do this, the founder raises the precision of the firm’s disclosure. Taking this argument to the extreme, if the external signal contains no noise ($\tau_s^{-1} = 0$) there will be no disagreement among investors and no speculative premium. In this case the founder of the firm will set $\tau^*_a = \tilde{\tau}'_a$ and simply maximize the fundamental value of the firm. This is summarized in the following corollary.

**Corollary 1.** If the outside signal contains no noise (so that $\tau_s^{-1} = 0$) then the founder will set $\tau^*_a = \tilde{\tau}'_a$.

### 5.2 Learning from the Disclosures of Other Firms

Until now I have treated the external signal which is released at $t = 1$ as being exogenously determined. However, if $\theta$ captures the profitability of assets being sold in the same industry, then one of the most important external sources of information are the disclosures of other sellers or firms.

#### 5.2.1 Setup with Multiple Sellers

To formalize this idea, suppose that there are $N$ firms in the industry indexed by $j \in \{1, 2, \ldots, N\}$. Each firm has the same structure that I studied in the previous section and shares a common underlying level of profitability $\theta$. Suppose also that the only information released at $t = 1$ is the

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27 Note that $\tilde{\tau}'_a = \sqrt{\frac{\tau}{\tau_s^2} - r\beta}$ in the case where the external signal is perfectly informative.

28 Here I am assuming the profitability of each firm is perfectly correlated. The analysis can easily be extended to the case where there is any non-zero correlation between each firm’s profitability. This does not change the qualitative results of the model.
disclosure of each firm. Assume that the noise in each firm’s disclosure \( \varepsilon_{a,j} \) is independent across firms. From the perspective of firm \( j \) the external signal is now determined by the disclosure policy of all other firms. By Bayes’ rule, this signal is a weighted sum of the disclosure of every other firm:

\[
s_j = \sum_{k \neq j} \tau_{a,k} \left( a_k - e_k - f \left( \tau_{a,k} \right) \right),
\]

so that:

\[
s_j \sim N \left( 0, \frac{1}{\tau_{s,j}} \right) \text{ with } \tau_{s,j} = \sum_{k \neq j} \tau_{a,k}.
\]

At \( t = 0 \) each founder simultaneously and non-cooperatively selects \( \tau_{a,j} \) and the incentive contract of their manager simultaneously. As per (19), the incentive scheme each founder offers the manager is now, in effect, a function of the disclosure of every firm through relative performance evaluation. At \( t = 1 \) the disclosure of each firm is released simultaneously. Investors observe all \( N \) signals and trade based upon their updated information. Date 2 is unaffected.

The equilibrium market prices and the pattern of trading remains as described in the analysis of the single firm. An equilibrium is now defined as Nash vector of disclosure precisions \( \{ \tau_{a,1}^{**}, \tau_{a,2}^{**}, \ldots, \tau_{a,N}^{**} \} \) where each founder’s choice of \( \tau_{a,j}^{**} \) is optimal given the choices of all other founder’s \( \tau_{a,-j}^{**} \). I focus on locally stable symmetric equilibria.

### 5.2.2 Equilibria with Costless Disclosure Precision

The strategic complementarity between each firm’s disclosure decision can be seen most clearly in the case where each firm’s disclosure precision is costless (\( c = 0 \)). In this case there is always an equilibrium in which each founder selects perfectly accurate disclosure.

**Proposition 4.** If \( c = 0 \) then \( \left( \tau_{a,j}^* \right)^{-1} = 0 \forall j \in \{1, \ldots, N\} \) is a Nash equilibrium. In this equilibrium \( V_j^{Spec} \left( \tau_{a,j}^*, \tau_{a,-j}^* \right) = 0 \forall j \in \{1, \ldots, N\} \).

Proposition 4 follows directly from Corollary 1. If every other firm chooses a perfectly accurate disclosure then the true value of \( \theta \) will be known with certainty at \( t = 1 \). This eliminates the potential for any disagreement among investors at \( t = 1 \) and the speculative component of the firm’s price will therefore be zero. In this case the founder’s only incentive then is to set \( \left( \tau_{a,j}^* \right)^{-1} = 0 \) so as to maximize the fundamental value of the firm. Observe that this equilibrium always exists no matter how diverse the beliefs of investors are. Extending the logic of Proposition 4 produces the following corollary.

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29 The analysis could easily be extended to also allow for an additional exogenous source of information. This would not qualitatively alter the results obtained below.
Corollary 2. Assume $c = 0$. If there exists a Nash equilibrium in which any firm sets $\tau_{a,j}^{**}$ finite then it must be that $\tau_{a,k}^{**}$ is finite $\forall k \neq j$.

This highlights the strategic complementarity between each founder’s decision. In order for a founder to sponsor the speculative component of their firm’s price, it is required that all other firms release noisy information so as not to destroy speculation in the industry.

Providing an analytical characterization of these equilibria is made difficult by the fact that the best response correspondence for $\tau_{a,j}^*$ is not well behaved. To avoid these problems I investigate these Nash equilibria numerically. Figure 6 draws the best response function of founder $j$ for different degrees of heterogeneity in prior beliefs (as parameterized by $\Delta$). In the example there are two firms ($N = 2$) and hence any point at which the best response function crosses the dashed forty five degree line is a Nash equilibrium. Among these, an equilibrium is stable if the best response function crosses the the forty five degree line from above on the left and below on the right. A locally stable symmetric Nash equilibria in which both firms choose disclosures of limited precision exist when the initial heterogeneity of beliefs is sufficiently large (for $\Delta = 2, 3$). When $\Delta = 1$ the speculative motive becomes so weak that no Nash equilibrium exists in which any founder chooses a finite disclosure precision. This example indicates that an equilibrium in which all founders choose to sponsor speculation only exists if the heterogeneity of beliefs among investors is sufficiently strong.\(^{30}\)

5.2.3 Equilibria with Costly Disclosure Precision

The multiplicity of equilibria which exist when $c = 0$ and $\Delta$ is large is a stark way to demonstrate the strategic complementarity between each founder’s disclosure decision. When we return to the case where the firm’s disclosure carries a small cost then the same forces are at work. When $c > 0$ each founder will only select a finite precision for their firm’s disclosure in equilibrium. In most cases this eliminates the multiplicity of equilibria. Figure 7 shows the best response functions of founder $j$ in the presence of a small marginal cost of disclosure. As before, for $N = 2$ a Nash equilibrium exists whenever the best response crosses the dashed forty five degree line. When the initial heterogeneity among investors is low ($\Delta = 1$) each founder commits their firm to highly accurate disclosure in equilibrium (in the example $\tau_{a,j}^{**} = 19.42$). The result is that in equilibrium the speculative premium in the industry is essentially eliminated.

The equilibrium disclosure precision is markedly lower when the heterogeneity of investors prior beliefs is sufficiently large. In the example $\tau_{a,j}^{**} = 0.815$ when $\Delta = 2$ and $\tau_{a,j}^{**} = 0.501$ when $\Delta = 3$. Each founder elects to sacrifice the fundamental value of their firm by lowering $\tau_{a,j}$ in order

\(^{30}\)In this example if founders could collectively choose between these multiple equilibria, they would select the equilibrium with low disclosure precision because this involves a higher sale price at $t = 0$.  

24
to sponsor the potential for investor disagreement. In combination this creates a large speculative premium for each firm in the industry.

Figure 8 shows how the symmetric Nash equilibrium choice of each founder \((\tau^{**}_{a,j})\) varies with the degree of heterogeneity in investors’ prior beliefs \((\Delta)\). When \(\Delta\) is sufficiently large, the industry equilibrium is one in which each founder commits to limit the precision of their firm’s disclosure at the cost of its fundamental value in order to sponsor speculation. Figure 3 showed that an individual founder would optimally choose to lower the precision of their firm’s disclosure when the heterogeneity of beliefs increased. By comparison Figure 8 shows a much stronger reaction of the equilibrium Nash disclosure choice. This comes from the strategic complementarity which exists in the industry equilibrium. Figure 9 shows the effect this has upon the speculative component of the firm’s total market value at \(t = 0\). The solid line shows the proportion of each firm’s price that comes from its speculative premium in equilibrium. To provide a comparison, the dashed line shows the speculative component of each firm’s price if each founder maximized only the fundamental value of their firm.\(^{31}\) Moving along the dashed line, as the heterogeneity of investors’ beliefs increases, the speculative component of each firm’s price grows. The solid line shows how strongly this effect is amplified when we take into account the endogenous equilibrium response of the founder’s disclosure decision. Comparing Figure 9 to the single firm case in Figure 8 shows that in a multi-seller setting, most of the speculative component of prices is due to the decision of sellers to commit to imprecise disclosures.

5.3 Implications

The results in this section highlight the important strategic complementarity that exists in each seller’s disclosure policy. It is only when other firms in the industry are releasing inaccurate information that the speculative motive is strong. The results of the single firm model predict that inaccurate disclosure and overvaluation will occur together. The strategic complementarity that I have highlighted in this section suggests that these phenomenon will occur in waves. It is consistent with the evidence (cited in the Introduction) that misrepresentation and overpricing both occur together on a widespread basis, such as during the housing and credit boom that preceded the 2007-08 financial crisis.

6 Discussion and Conclusion

In this section I discuss how this paper relates to literature and provide a brief conclusion.

\(^{31}\)In this example each founder would choose \(\tau_{a,j} = 23.93\) in a Nash equilibrium where they only maximized the fundamental value of the firm.
6.1 Relation to Literature

This paper relates to a broad literature that has studied the way firms respond when their market price varies from the fundamental value of their firm. Several papers argue that rational managers will respond to overpricing by increasing investment (Stein (1996), Baker et al. (2003), Panageas (2005), and Polk and Sapienza (2009)) and altering the capital structure of their firm (Baker and Wurgler (2002) and Shleifer and Vishny (2003)). Closer to the central message of this paper, Jensen (2004) argues that managers will take actions that reduce the fundamental value of their firm in order to prolong overvaluation. Bolton et al. (2006) show that the owners of a firm will distort the incentives they offer their firm’s manager in order to encourage her to undertake risky projects which have higher speculative premia. The theory presented in the current paper is the first to consider what impact the potential for overvaluation will have on the disclosure policy of a firm. This contribution seems particularly important since the equilibrium asset pricing account of overvaluation relies upon disagreement between investors stemming from how they interpret information that is released about the profitability of a firm.

This paper is also related to the literature on disclosure. For comprehensive surveys, see Verricchia (2001) and Dye (2001). The model in this paper falls into Verricchia’s category of “efficiency based theories of disclosure” that study a seller’s ex ante optimal choice of disclosure policy prior to having any private information to disclose. Earlier papers in this literature include Diamond (1985) and Fishman and Hagerty (1989). Within this literature, this paper is most closely related to papers that study disclosure decisions in environments where agents learn in non-standard ways. For example, Fishman and Hagerty (2003) examine the impact of mandatory disclosure rules when some buyers are incapable of understanding the disclosure. This paper also relates to other theories that suggest reasons for why it might be optimal for firms to reveal imprecise information (for a more comprehensive survey see Goldstein and Sapra (2013)). For example accurate disclosure may distort managerial investment decisions (Edmans et al. (2016)), aggravate principal agent problems (Hermalin and Weisbach (2012)), make it difficult to hire highly-skilled managers (Subrahmanyam (2005)), or depress the incentives for traders to gather information that will be embedded in prices (Goldstein and Yang (2016)).

The model presented in this paper predicts that an increase in disclosure regulation (forcing founders to raise $\tau_a$) should reduce the overpricing of securities by limiting the potential for disagreement among investors. Simon (1989) studies the impact of the 1933 Securities Act and provides evidence consistent with this hypothesis. She shows that the 1933 Securities Act marked a substantial increase in disclosure requirements only for firms on regional exchanges (as opposed to the NYSE). Exploiting this difference-in-difference, she shows that stock issues on regional

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32 See also Hirshleifer and Teoh (2003) and Hirshleifer et al. (2009).
exchanges were systematically overpriced prior to the passage of the Act (exhibiting negative expected cumulative risk adjusted returns of 24% in the first two years after issuance) and that mispricing ended after the adoption of the Act.

Finally, the paper is related to Povel et al. (2007) who also show that fraud is more likely to happen in booms. Their explanation focuses on changes in the ex post incentive of investors to investigate fraud in good and bad times. By a similar argument, the theory of Fernandes and Guedes (2010) suggests that managers are more likely to engage in fraud when the market’s expectation of a firm’s performance is high. Both papers focus on the ex post incentive to commit fraud rather than the ex ante incentives of the initial owner of the firm to determine how much misrepresentation is possible. Further, neither paper addresses the link between fraud and systematic overpricing since investors rationally anticipate the average amount of misrepresentation that occurs in equilibrium.

### 6.2 Conclusion

This paper provides an explanation for why a seller would ex ante commit to a policy of imprecise financial disclosure. A commitment to accurate disclosures has the benefit of reducing the degree of moral hazard between the firm and its manager, and thereby increases its fundamental value. However, if investors have heterogeneous prior beliefs and face short-sale constraints, prices will have a speculative component: the option value of selling the asset to a more optimistic investor in the future. Accurate disclosure reduces the potential for disagreement and hence can lower this speculative premium. This paper shows that in order to maximize the initial sale price of the asset, the seller will commit to an inaccurate disclosure policy, despite the negative impact this will have on the fundamental value of the firm. The choice of disclosure policy amplifies the overpricing caused by the underlying heterogeneity of beliefs.

This paper also offers an explanation for why episodes of financial misrepresentation and speculative overpricing occur together in waves. To show this I demonstrate that there is a strategic complementarity in disclosure decision across sellers of assets with correlated values. The potential for speculation is only present if other sellers commit to inaccurate disclosure policies that permit substantial disagreement. The complementarity between each seller’s disclosure policy further amplifies the contribution that endogenous disclosure choice makes to the overpricing of each asset.
References


7 Appendix - Mathematical Derivations

7.1 Derivations for Section 3.2

To begin, I solve for the optimal managerial contract \((w_0, w_a, w_s, e)\) for a given level of accounting precision \(\tau_a\). Let \(\lambda^{IC}, \lambda^{IR} \geq 0\) be the Lagrange multipliers on incentive compatibility and individual rationality constraints. The problem is

\[
\max_{w_0, w_a, w_s, e} \ (1 - w_a) e - w_a f (\tau_a) - w_0 + \lambda^{IC} (w_a - e\beta) - c\tau_a \\
+ \lambda^{IR} \left( w_0 + w_a f (\tau_a) + w_a e - \frac{e^2\beta}{2} - \frac{r}{2} \left[ \frac{(w_a + w_s)^2}{\tau_{Man}} + \frac{w_a^2}{\tau_a} + \frac{w_s^2}{\tau_s} \right] - w \right)
\]  

(31)

The first order conditions for this problem yield:

\[
w_0 : \lambda^{IR} = 1
\]  

(32)

\[
w_a : \lambda^{IC} - r \left[ \frac{(w_a + w_s)}{\tau_{Man}} + \frac{w_a}{\tau_a} \right] = 0
\]  

(33)

\[
w_s : \frac{(w_a + w_s)}{\tau_{Man}} + \frac{w_s}{\tau_s} = 0
\]  

(34)

\[
e : \lambda^{IC} = \frac{1 - w_a}{\beta}
\]  

(35)

Solving these simultaneously we get

\[
w_a^* = \left[ 1 + r\beta \left( \frac{1}{\tau_s} + \frac{1}{\tau_{Man}} + \frac{1}{\tau_a} \right) \right]^{-1}
\]  

(36)

\[
w_s^* = \left( \frac{\tau_s}{\tau_s + \tau_{Man}} \right) w_a^*
\]  

(37)

\[e^* = \frac{w_a^*}{\beta}
\]  

(38)

\[w_0^* = w - \frac{(w_a^*)^2}{2\beta} \left[ 1 - r\beta \left( \frac{1}{\tau_s + \tau_{Man}} + \frac{1}{\tau_a} \right) \right] - w_a^* f (\tau_a)
\]  

(39)

Substituting these into the objective function (and normalizing \(w = 0\)) gives the expression for \(V^{Fund} (\tau_a)\). Solutions for the single firm problem can be found by setting \(\tau_s = 0\).
7.2 Derivations for Section 4.1

Differentiating $V^\text{Fund}(\tau_a)$ with respect to $\tau_a$ yields

$$\frac{\partial V^\text{Fund}(\tau_a)}{\partial \tau_a} = r \left[ \tau_a \left( 1 + \frac{r \beta}{\tau_{Man}} \right) + r \beta \right]^{-2} - c \quad \text{and}$$

(40)

$$\frac{\partial^2 V^\text{Fund}(\tau_a)}{\partial \tau_a^2} = -r \left( 1 + \frac{r \beta}{\tau_{Man}} \right) \left[ \tau_a \left( 1 + \frac{r \beta}{\tau_{Man}} \right) + r \beta \right]^{-3} < 0. \quad (41)$$

This establishes that $V^\text{Fund}(\tau_a)$ is strictly concave and thus has a unique maximum. $\frac{\partial V^\text{Fund}(\tau_a)}{\partial \tau_a}$ is zero at

$$\tilde{\tau}_a = \sqrt{\frac{r}{2c} - r \beta} \quad \frac{1}{1 + \frac{r \beta}{\tau_{Man}}}.$$  

(42)

Observe that the assumption that $c < \frac{1}{2r \beta^2}$ ensures that $\tilde{\tau}_a > 0$. This establishes the Proposition.

7.3 Derivations for Section 4.2

Differentiating $V^\text{Spec}(\tau_a)$ with respect to $\tau_a$ and collecting terms yields

$$\frac{\partial V^\text{Spec}(\tau_a)}{\partial \tau_a} = V^\text{Spec}(\tau_a) \left[ -2 \left( \tau_a^2 - \tau_{\theta}^+ \tau_a + \tau_{\theta}^- \tau_a \right) \right] \left[ 2 \tau_a \left( \tau_{\theta}^- + \tau_a \right) \left( \tau_{\theta}^+ + \tau_a \right) \right]. \quad \text{for } (43)$$

Since $V^\text{Spec}(\tau_a) \left[ 2 \tau_a \left( \tau_{\theta}^- + \tau_a \right) \left( \tau_{\theta}^+ + \tau_a \right) \right] \geq 0$ the sign of this derivative is determined by the sign of $\left[ -2 \left( \tau_a^2 - \tau_{\theta}^+ \tau_a + \tau_{\theta}^- \tau_a \right) \right]$. This is a quadratic function of $\tau_a$ which only has a single positive root: $\hat{\tau}_a = \frac{1}{4} \left[ -\tau_{\theta}^- + \sqrt{\left( \tau_{\theta}^+ \right)^2 + 8 \tau_{\theta}^- \tau_{\theta}^+} \right].$

This quadratic function is unambiguously concave which establishes that $\frac{\partial V^\text{Spec}(\tau_a)}{\partial \tau_a} \geq 0$ for $\tau_a \leq \hat{\tau}_a$ and $\frac{\partial V^\text{Spec}(\tau_a)}{\partial \tau_a} \leq 0$ for $\tau_a \geq \hat{\tau}_a$.

Note that comparing (22) and (23) and setting both expressions equal gives that $\tilde{c}$ is defined as:

$$\tilde{c} \equiv r \left[ \frac{1}{2} \left( r \beta + \frac{1}{4} \left( 1 + \frac{r \beta}{\tau_{Man}} \right) \left[ -\tau_{\theta}^- + \sqrt{\left( \tau_{\theta}^+ \right)^2 + 8 \tau_{\theta}^- \tau_{\theta}^+} \right] \right)^2 \right]^{-2} \quad \text{for } (44)$$

and makes clear that $\tilde{c} > 0$.

7.4 Proof of Proposition

Since I assume that $c < \tilde{c}$ it is assured that $\tilde{\tau}_a > \hat{\tau}_a$. As shown above, $V^\text{Spec}$ and $V^\text{Fund}$ are both strictly increasing for $\tau_a < \hat{\tau}_a$ and decreasing for $\tau_a > \hat{\tau}_a$. It follows that $\tau_a^* \in [\tilde{\tau}_a, \hat{\tau}_a]$. Since $c > 0$ ensures that $\tilde{\tau}_a$ is finite, this is equivalent to

$$\tau_a^* = \alpha \tilde{\tau}_a + (1 - \alpha) \hat{\tau}_a \quad \text{for } (45)$$
for some $\alpha \geq [0, 1]$. I now need to prove this is only true for $\alpha \in (0, 1)$. Since $\tilde{\tau}_{a} > \tilde{\tau}_{a}$ then

\[
\frac{\partial V^{\text{Spec}}(\tilde{\tau}_{a})}{\partial \tau_{a}} < 0 \quad \text{and} \quad \frac{\partial V^{\text{Fund}}(\tilde{\tau}_{a})}{\partial \tau_{a}} = 0 \tag{46}
\]

\[
\frac{\partial V^{\text{Spec}}(\tilde{\tau}_{a})}{\partial \tau_{a}} = 0 \quad \text{and} \quad \frac{\partial V^{\text{Fund}}(\tilde{\tau}_{a})}{\partial \tau_{a}} > 0 \tag{47}
\]

and hence $\alpha = 0$ and $\alpha = 1$ cannot be optimal. From above, we know that $V^{\text{Fund}}(\tau_{a})$ is strictly concave and so it must be that $V^{\text{Fund}}(\tau_{a}^{*}) < V^{\text{Fund}}(\tilde{\tau}_{a})$. Since $\tau_{a}^{*}$ is optimal it must be that $V(\tau_{a}^{*}) > V(\tilde{\tau}_{a})$ which implies that $V^{\text{Spec}}(\tau_{a}^{*}) > V^{\text{Spec}}(\tilde{\tau}_{a})$.

7.5 Proof of Lemma 1

First observe that

\[
\frac{\partial V^{\text{Spec}}(\tau_{a})}{\partial \tau_{\theta}} = -V^{\text{Spec}}(\tau_{a}) \left[ (\tau_{\theta}^{-} - \tau_{\theta}^{+})^{-1} + \frac{(\tau_{\theta}^{-})^{-1}}{2} + \frac{(\tau_{\theta}^{-} + \tau_{\theta}^{+})^{-1}}{2} \right] \leq 0. \tag{48}
\]

Next observe that

\[
\frac{\partial^{2} V^{\text{Spec}}(\tau_{a})}{\partial \tau_{a} \partial \tau_{\theta}} \bigg|_{\tau_{a} \geq \tilde{\tau}_{a}, \tau_{\theta} \geq \tilde{\tau}_{\theta}} = V^{\text{Spec}}(\tau_{a}) \frac{(\tau_{\theta}^{-} + \tau_{\theta}^{+})^{-2}}{2} + \frac{\partial V^{\text{Spec}}(\tau_{a})}{\partial \tau_{\theta}} \frac{\partial V^{\text{Spec}}(\tau_{a})}{\partial \tau_{a}} \left( \frac{1}{V^{\text{Spec}}(\tau_{a})} \right)^{2} \geq 0 \tag{49}
\]

where the inequality follows from combining (48) with the fact that

\[
\frac{\partial V^{\text{Spec}}(\tau_{a})}{\partial \tau_{a}} \leq 0 \quad \text{for} \quad \tau_{a} \geq \tilde{\tau}_{a}. \tag{50}
\]

Consider two values of $\tau_{\theta}$: $\tau_{\theta}^{+}$ and $\tau_{\theta}^{-}$ with $\tau_{\theta}^{-} > \tau_{\theta}^{-}$. Let $\tau_{a}^{+}$ and $\tau_{a}^{-}$ be the founder’s optimal choice of $\tau_{a}$ associated with $\tau_{\theta}^{+}$ and $\tau_{\theta}^{-}$ respectively. Since $\tau_{a}^{+}$ and $\tau_{a}^{-}$ are optimal choices it must be that

\[
V^{\text{Fund}}(\tau_{a}^{+}) + V^{\text{Spec}}(\tau_{a}^{+}, \tau_{\theta}^{-}) \geq V^{\text{Fund}}(\tau_{a}^{+}) + V^{\text{Spec}}(\tau_{a}^{+}, \tau_{\theta}^{-}) \tag{51}
\]

and

\[
V^{\text{Fund}}(\tau_{a}^{-}) + V^{\text{Spec}}(\tau_{a}^{-}, \tau_{\theta}^{-}) \geq V^{\text{Fund}}(\tau_{a}^{-}) + V^{\text{Spec}}(\tau_{a}^{-}, \tau_{\theta}^{-}) \tag{52}
\]

Subtracting (52) from (51) gives us that

\[
V^{\text{Spec}}(\tau_{a}^{+}, \tau_{\theta}^{-}) - V^{\text{Spec}}(\tau_{a}^{-}, \tau_{\theta}^{-}) \geq V^{\text{Spec}}(\tau_{a}^{+}, \tau_{\theta}^{-}) - V^{\text{Spec}}(\tau_{a}^{+}, \tau_{\theta}^{-}) \tag{53}
\]

Since $\tau_{a} > \tilde{\tau}_{a}$, we can apply (49) to (53) and conclude that $\tau_{a}^{+} \geq \tau_{a}^{-}$. This establishes that $\tau_{a}^{*}$ is weakly increasing in $\tau_{\theta}$. Since $V^{\text{Spec}}(\tau_{a})$ is decreasing in $\tau_{a}$ for $\tau_{a} \geq \tilde{\tau}_{a}$ this implies that $V^{\text{Spec}}(\tau_{a})$
is weakly decreasing in $\tau_\theta^-$. By a parallel argument, since $V^{\text{Fund}}(\tau_a)$ is increasing in $\tau_a$ for $\tau_a \leq \tilde{\tau}_a$. This implies that $V^{\text{Fund}}(\tau_a^*)$ is weakly increasing in $\tau_\theta^-$. This establishes the Lemma.

### 7.6 Proof of Lemma 2

First I prove that $V(\tau_a^*)$ is weakly decreasing in $\tau_\theta^-$. Consider two values of $\tau_\theta^-: \tau_\theta^{-h}$ and $\tau_\theta^{-l}$ with $\tau_\theta^{-h} > \tau_\theta^{-l}$. Let $\tau_a^{+,h}$ and $\tau_a^{+,l}$ be the founder’s optimal choice of $\tau_a$ associated with $\tau_\theta^{-h}$ and $\tau_\theta^{-l}$ respectively. Observe that

$$V^{\text{Fund}}(\tau_a^{+,h}) + V^{\text{Spec}}(\tau_a^{+,h}, \tau_\theta^{-l}) \geq V^{\text{Fund}}(\tau_a^{+,h}) + V^{\text{Spec}}(\tau_a^{+,h}, \tau_\theta^{-h})$$

which follows directly from (48). Combining (54) with (52) yields

$$V^{\text{Fund}}(\tau_a^{+,l}) + V^{\text{Spec}}(\tau_a^{+,l}, \tau_\theta^{-l}) \geq V^{\text{Fund}}(\tau_a^{+,h}) + V^{\text{Spec}}(\tau_a^{+,h}, \tau_\theta^{-h})$$

which establishes that $V(\tau_a^*)$ is weakly decreasing in $\tau_\theta^-$. 

Now I prove that $V(\tau_a^*)$ is weakly increasing in $\tau_\theta^+$. Consider two values of $\tau_\theta^+: \tau_\theta^{+,h}$ and $\tau_\theta^{+,l}$ with $\tau_\theta^{+,h} > \tau_\theta^{+,l}$. Let $\tau_a^{+,h}$ and $\tau_a^{+,l}$ be the founder’s optimal choice of $\tau_a$ associated with $\tau_\theta^{+,h}$ and $\tau_\theta^{+,l}$ respectively. First observe that

$$\frac{\partial V^{\text{Spec}}(\tau_a)}{\partial \tau_\theta^+} = V^{\text{Spec}}(\tau_a) \left[ (\tau_\theta^+ - \tau_a)^{-1} - (\tau_\theta^+ + \tau_a)^{-1} \right] \geq 0.\quad (56)$$

where the inequality follows from the fact that $\tau_\theta^-, \tau_a \geq 0$. Since $\tau_a^{+,h}$ is optimal then it must be that

$$V^{\text{Fund}}(\tau_a^{+,h}) + V^{\text{Spec}}(\tau_a^{+,h}, \tau_\theta^{+,l}) \geq V^{\text{Fund}}(\tau_a^{+,l}) + V^{\text{Spec}}(\tau_a^{+,l}, \tau_\theta^{+,h}).\quad (57)$$

Using (56) we have that

$$V^{\text{Fund}}(\tau_a^{+,l}) + V^{\text{Spec}}(\tau_a^{+,l}, \tau_\theta^{+,h}) \geq V^{\text{Fund}}(\tau_a^{+,l}) + V^{\text{Spec}}(\tau_a^{+,l}, \tau_\theta^{+,h}).\quad (58)$$

Combining (57) and (58) yields

$$V^{\text{Fund}}(\tau_a^{+,h}) + V^{\text{Spec}}(\tau_a^{+,h}, \tau_\theta^{+,h}) \geq V^{\text{Fund}}(\tau_a^{+,l}) + V^{\text{Spec}}(\tau_a^{+,l}, \tau_\theta^{+,h})\quad (59)$$

which establishes that $V(\tau_a^*)$ is weakly increasing in $\tau_\theta^+$. This establishes the Lemma.

### 7.7 Derivations for Section 4.5

#### 7.7.1 Welfare

The social planner assesses the expected welfare of all agents using a single prior belief $\theta \sim N\left(0, \frac{1}{\iota_b} \right)$ where $\tau_b$ can be any value on the interval $[\tau_b^-, \tau_b^+]$. The agents to be considered are:
1) investors with prior belief precision $\tau_\theta^+$ who will buy the stock of the firm at $t = 1$ if $a \leq \hat{e}$, 2) investors with prior belief precision $\tau_\theta^-$ who buy the stock at $t = 0$ and sell it at $t = 1$ if $a \geq \hat{e}$ (and hold the stock until $t = 2$ otherwise), 3) the manager of the firm, and 4) the founder of the firm. To establish a Pareto ranking between possible allocations, consider a set of transfers that the planner can make to each of these agents. Label these as $\{\Gamma^+, \Gamma^-, \Gamma^M, \Gamma^F\}$ respectively. The planner can select this set of transfers based on the belief $\tau_{\theta Pol}$ subject to the constraint that they must sum to zero.

For any given disclosure precision $\tau_a$ and managerial contract $\{w_0, w_a\}$ which will induce effort level $\hat{e} = \frac{w_a}{\beta}$ (the planner must take the moral hazard problem within the firm as given) the planner’s assessment of the expected utility of each agent is as follows. Note that these expressions anticipate the equilibrium price at $t = 1$ given in (12) as given. The expected utility of investors with prior $\tau_\theta^+$ is

$$U^+ = \frac{1}{2} \left[ E \left( \tau_a (a - \hat{e}) \left( \frac{1}{\tau_{\theta Pol} + \tau_a} - \frac{1}{\tau_\theta^+ + \tau_a} \right) \right \vert a \leq \hat{e} \right) \right] + \Gamma^+. \quad (60)$$

The expected utility of the investors with prior $\tau_\theta^-$ is

$$U^- = -p_0 + \hat{e} - w_0 - w_a a - c \tau_a + \frac{1}{2} \left[ E \left( \tau_a (a - \hat{e}) \left( \frac{1}{\tau_{\theta Pol} + \tau_a} \right) \right \vert a \geq \hat{e} \right) + E \left( \tau_a (a - \hat{e}) \left( \frac{1}{\tau_\theta^+ + \tau_a} \right) \right \vert a \leq \hat{e} \right) \right] + \Gamma^- \quad (61)$$

The expected utility of the manager is

$$U^M = w_0 + w_a \hat{e} - \frac{(\hat{e})^2}{2} \beta - \frac{r}{2} \left[ \frac{1}{\tau_{\theta Pol} + \tau_a} \right] w_a^2 + \Gamma^M. \quad (62)$$

Suppose that $\{\Gamma^+, \Gamma^-, \Gamma^M\}$ are set to normalize (60) (61) (62) to zero. Using the restriction that $\Gamma^F = -(\Gamma^+ + \Gamma^- + \Gamma^M)$ the expected utility of the founder, net of transfers, is

$$U^F = \frac{w_a}{\beta} - \frac{w_a^2}{2\beta} - \frac{r}{2} \left[ \frac{1}{\tau_{\theta Pol} + \tau_a} \right] w_a - c \tau_a. \quad (63)$$

The expression for $U^F$ given in (63) therefore serves as a social welfare function (and hence is labeled $U^{SWF}$ in the body of the paper). Any change in $w_a$ or $\tau_a$ that increases (63) must, by construction, result in a strict Pareto improvement. The first order conditions which characterize the welfare maximizing choices of $w_a$ and $\tau_a$ are

$$w_a: \frac{1 - w_a}{\beta} - r \left[ \frac{1}{\tau_{\theta Pol} + \tau_a} \right] w_a = 0 \quad (64)$$

$$\tau_a: \frac{r}{2} \left( \frac{w_a}{\tau_a} \right)^2 - c = 0. \quad (65)$$

Note that $U^F$ is strictly concave in both choices. Solving (64) and (65) simultaneously gives the welfare maximizing choice of each, for any given $\tau_{\theta Pol}$. I use a superscript of “Wel” to denote these
welfare maximizing choices:

\[ w_{Wel}^a = \frac{1}{\beta - \sqrt{2c_r}} + \frac{1}{\beta} \]  \hspace{1cm} (66)

\[ \tau_{Wel}^a = \frac{\sqrt{\frac{r}{2c}} - r\beta}{1 + \frac{r\beta}{\tau_{Pol}^{\theta}}} \]  \hspace{1cm} (67)

Note that

\[ \frac{\partial \tau_{Wel}^a}{\partial \tau_{Pol}^{\theta}} = \frac{r\beta}{(\tau_{Pol}^{\theta})^2} \left( \frac{\sqrt{\frac{r}{2c}} - r\beta}{1 + \frac{r\beta}{\tau_{Pol}^{\theta}}} \right)^2 > 0 \]  \hspace{1cm} (68)

so the Pareto optimal choice of \( \tau_{Wel}^a \) must be on the interval

\[ \tau_{Wel}^a \in \left[ \frac{\sqrt{\frac{r}{2c}} - r\beta}{1 + \frac{r\beta}{\tau_{Pol}^{\theta}}}, \frac{\sqrt{\frac{r}{2c}} - r\beta}{1 + \frac{r\beta}{\tau_{Pol}^{\theta}}} \right] \]  \hspace{1cm} (69)

where (68) ensures that the first bracket term represents the lower bound of the interval. Note that \( \tau_{Wel}^a \) must lie on the interval in (69). Combining that fact with Proposition 1 ensures that the equilibrium choice for the firm’s disclosure precision must lie either within or below this interval. It follows immediately that if in equilibrium the founder selects \( \{ w_{a}^*, \tau_{a}^* \} \) with

\[ \tau_{a}^* < \tau_{Wel}^a - \equiv \frac{\sqrt{\frac{r}{2c}} - r\beta}{1 + \frac{r\beta}{\tau_{Pol}^{\theta}}} \]  \hspace{1cm} (70)

then a strictly Pareto superior allocation would be produced if the founder selected \( \{ w_{a}^*, \tau_{a}^{Wel-} \} \) instead. Note that since \( U^F \) is strictly concave and \( \tau_{Wel}^a \geq \tau_{a}^{Wel-} \) for all \( \tau_{Pol}^{\theta} \) then it is unambiguous that this increase in \( \tau_{a} \) must result in strictly higher welfare. This establishes Proposition 2.

### 7.7.2 Disclosure Policy

If the policy-maker contemplates setting limits on the disclosure precision and is unable to control the incentive contract that the founder sets for the firm then the planner must take into account that varying \( \tau_{a} \) will impact the manager’s incentive constraint as per (36). The resulting social welfare function of the policy-maker, as a function of \( \tau_{a} \), for any reasonable \( \tau_{Pol}^{\theta} \) is given by (26) in the paper.

### 7.8 Proof of Proposition 3

Since both \( \tilde{\tau}_{a} > \tilde{\tau}_{a} \) and \( \tilde{\tau}_{a} \equiv \max \{ 0, \tilde{\tau}_{a} - \bar{\tau}_{a} \} < \tilde{\tau}_{a} \) then the argument is identical to the proof for Proposition 1.
8 Appendix - Figures

Figure 1: Timeline
This figure shows the timeline of events in the model.

I. Founder publicly sets
   (i) Disclosure policy: \( \tau_s \)
   (ii) Manager’s contract: \( w_0, w_s, w_f \).

II. Founder sells firm to investor for \( p_0 \).

III. Manager privately chooses \( e \).

\[ t = 0 \]  \[ t = 1 \]  \[ t = 2 \]

I. Manager privately selects \( m \).

II. Firm’s financial disclosure is released: \( a \).

III. External information is released: \( s \).

III. Manager is paid \( W \).

IV. Investors trade stock of firm at price \( p_1 \).

I. Output is realized: \( y \).

II. Firm profit is paid to shareholders: \( p_2 = y - W - c\tau_s \).
Figure 2: The Speculative Component of the Firm’s Price: $V^{Spec}(\tau_a)$

This plot shows the speculative component of the firm’s price at $t = 0$, $V^{Spec}(\tau_a)$, as a function of the precision of the firm’s disclosure $\tau_a$. The following parameters are used: the manager’s belief precision is equal to the median investor precision and is $\tau_{\theta}^{Man} = \tau_{\theta}^M = 2$, the heterogeneity of investors’ beliefs is $\Delta = 2$, and there is no outside signal $\tau_s = 0$. 

![Plot showing the speculative component of the firm's price as a function of disclosure precision.]
Figure 3: Comparative Statics in the Heterogeneity of Investors’ Beliefs: Δ

Panel A shows how the optimal choice of the firm’s disclosure precision varies with the degree of heterogeneity of investors’ beliefs Δ. Panel B shows how the fraction of the firm’s price at \( t = 0 \) that is accounted for by the speculative premium, \( \frac{V^{\text{Spec}}(\tau_a)}{V(\tau_a)} \), varies with Δ. The solid line shows the case where the disclosure precision is set to maximize the price of the firm at \( t = 0 \): \( \tau_a = \tau_a^* \). For comparison, in order to show the impact of the endogenous adjustment of the firm’s disclosure policy, the dashed line shows the same fraction in the case where the precision of the firm’s disclosure is set purely to maximize the fundamental value of the firm: \( \tau_a = \tilde{\tau}_a \).

The following parameters are used: the manager’s belief precision is equal to the median investor precision and is \( \tau_{\theta, M} = \tau_{\theta}^M = 2 \), the marginal cost of information precision is \( c = 0.0005 \), \( \beta = 4 \), \( r = 1 \), and there is no outside signal \( \tau_s = 0 \).

(a) Optimal Disclosure Precision: \( \tau_a^* \)

(b) Speculative Premium as a Fraction of Firm’s Value
Figure 4: Welfare and Disclosure Policy
This plot compares the disclosure precision that the founder will choose, $\tau_a^*$ (shown in the solid line), to two belief-neutral disclosure benchmarks. The first comparison is to $\tau_a^{Wel-}$ (shown the dashed line): whenever $\tau_a^* < \tau_a^{Wel-}$ the equilibrium allocation is Pareto inefficient. The second comparison is to $\tau_a^{Pol-}$ (shown in the dotted line), the minimum belief-neutral disclosure precision standard. All three are shown for varying degrees of investor belief heterogeneity $\Delta$. The following parameters are used: the manager’s belief precision is equal to the median investor precision and is $\tau_{\theta}^{Man} = \tau_{\theta}^{M} = 2$, the marginal cost of information precision is $c = 0.0005$, $\beta = 4$, $r = 1$, and there is no outside signal $\tau_s = 0$. 
Figure 5: Founder’s Choice as Precision of External Information Varies

This figure shows how the optimal choice of the firm’s disclosure precision, \( \tau_a^* \), varies with the precision of the external signal \( \tau_s \). The following parameters are used: the manager’s belief precision is equal to the median investor precision and is \( \tau_{Man} = \tau_{Median} = 2 \), the marginal cost of information precision is \( c = 0.0005 \), the heterogeneity of investors’ beliefs is \( \Delta = 2 \), \( \beta = 4 \), and \( r = 1 \).
Figure 6: Nash Equilibria For Varying Levels of Heterogeneity in Beliefs ($c = 0$)
This figure shows firm $j$’s disclosure precision best response, $\tau_{a,j}^*$, (y-axis) varies with the precision of total amount of information released by all other firms $\tau_{a,-j}^*$. This best response is shown for three values of heterogeneity of investors’ beliefs is $\Delta = 1, 2, 3$. The following parameters are used: the manager’s belief precision is equal to the median investor precision and is $\tau_{\theta}^{Man} = \tau_{\theta}^{M} = 2$, the marginal cost of information precision is $c = 0$, $\beta = 4$, and $r = 1$. Where the best response function crosses the dotted 45 degree line, a symmetric Nash equilibrium exists.
Figure 7: Nash Equilibria With Costly Disclosure
This figure shows firm $j$’s disclosure precision best response, $\tau^{*}_{a,j}$, (y-axis) varies with the precision of total amount of information released by all other firms $\tau^{*}_{a,-j}$. This best response is shown for three values of heterogeneity of investors’ beliefs is $\Delta = 1, 2, 3$. The following parameters are used: the manager’s belief precision is equal to the median investor precision and is $\tau^{Man}_{\theta} = \tau^{M}_{\theta} = 2$, the marginal cost of information precision is $c = 0.0005$, $\beta = 4$, and $r = 1$. Where the best response function crosses the dotted 45 degree line, a symmetric Nash equilibrium exists.
Figure 8: Industry Nash Equilibria as $\Delta$ Changes

This figure shows how the symmetric Nash equilibrium choice of each firm’s disclosure precision, $\tau_{a,j} = \tau_{a,j}^*$, varies with the degree of heterogeneity of investors’ beliefs $\Delta$. The following parameters are used: the manager’s belief precision is equal to the median investor precision and is $\tau_{\theta}^{Man} = \tau_{\theta}^{M} = 2$, the marginal cost of information precision is $c = 0.0005$, $\beta = 4$, $r = 1$, and there are two firms in the industry $N = 2$. 
This figure shows how the fraction of the firm’s price at $t = 0$ that is accounted for by the speculative premium, $\frac{V^{\text{Spec}}(\tau_a)}{V(\tau_a)}$, varies with $\Delta$. The solid line shows the case where the disclosure precision of each firm is set according the symmetric Nash equilibrium disclosure precision at $t = 0$: $\tau_{a,j} = \tau_{a,j}^{**}$. For comparison, in order to show the impact of the endogenous adjustment of the firm’s disclosure policy, the dashed line shows the same fraction in the case where the precision of each firm’s disclosure is set purely to maximize the fundamental value of the firm: $\tau_a = \tilde{\tau}_a$. The following parameters are used: the manager’s belief precision is equal to the median investor precision and is $\tau^M_{\theta} = \tau^M_\theta = 2$, the marginal cost of information precision is $c = 0.0005$, $\beta = 4$, $r = 1$, and there are two firms in the industry $N = 2$. 

![Graph showing speculative premium as a proportion of firm value in Nash equilibrium](image)