

Competitive Poaching in Sponsored Search Advertising and Its Strategic Impact on Traditional Advertising

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Traditional advertising, such as TV and print advertising, primarily builds awareness of a firm's product among consumers, whereas sponsored search advertising on a search engine can target consumers closer to making a purchase because they reveal their interest by searching for a relevant keyword. Increased consumer targetability in sponsored search advertising induces a firm to "poach" a competing firm's consumers by directly advertising on the competing firm's keywords; in other words, the poaching firm tries to obtain more than its "fair share" of sales through sponsored search advertising by free riding on the market created by the firm being poached. Using a game theory model with firms of different advertising budgets, we study the phenomenon of poaching, its impact on how firms allocate their advertising budgets to traditional and sponsored search advertising, and the search engine's policy on poaching. We find that, as budget asymmetry increases, the smaller-budget firm poaches more on the keywords of the larger-budget firm. This may induce the larger-budget firm to allocate more of its budget to traditional advertising, which, in turn, hurts the search engine's advertising revenues. Therefore, paradoxically, even though poaching increases competition in sponsored search advertising, the search engine can benefit from limiting the extent of poaching. This explains why major search engines use "ad relevance" measures to handicap poaching on trademarked keywords.

Keywords: online advertising; paid search; poaching; keyword relevance score; competitive strategy; game theory

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1. Introduction

Online advertising is the fastest-growing channel of advertising, likely to exceed 30% of the total U.S. advertising expenditure by 2015 (eMarketer 2012a). This rapid growth in online advertising is impressive given that television advertising, which firms have been using for many decades, has a market share of about 35%. On aggregate, firms allocate nearly half of their online advertising spend to sponsored search advertising (eMarketer 2012b). There are several unique advantages of sponsored search advertising, including effective targetability, ease of setting up a campaign, and ease of measurement of return on investment. Given its unique characteristics and spectacular growth, sponsored search advertising is gaining increasing attention from researchers and practitioners. However, although firms are dedicating progressively larger fractions of their advertising budgets to sponsored search advertising at the expense of traditional channels of advertising, the strategic implications of the interactions between

these two types of advertising have not been carefully researched.

Consumers go through several stages of involvement before purchasing a product, and different types of advertising influence consumers differently in these stages. A widely employed marketing framework that captures the various sequential stages of a typical consumer's decision process before final purchase is the awareness-interest-desire-action (AIDA) model. Traditional channels of advertising, such as television, newspapers, radio, and billboards, generate awareness among consumers and are directed more toward the initial stages of the AIDA model. In traditional advertising, communication with potential consumers is initiated by the firm to make them aware of and interested in the firm's brand or product. Sponsored search, however, is directed more toward the later stages of the AIDA model, and it influences a consumer close to the purchase decision. Sponsored search effectively targets the consumers who are already aware of the product and

have shown some interest or desire in the product by searching for an associated keyword on a search engine. In the context of the AIDA model, traditional advertising can be interpreted as “upstream advertising” and sponsored search can be interpreted as “downstream advertising.” Thus, traditional awareness-generating advertising and sponsored search advertising are interrelated and play complementary roles in successfully consummating the sale of a product.

In a strategic market with competing firms, creating awareness has benefits as well as perils, especially when the awareness created through traditional advertising for one brand can be exploited by a competing firm through “free riding” in sponsored search advertising. Competitors, instead of allocating their advertising budget to create awareness about their own products, can advertise in sponsored search on the keywords of a competing firm in the same industry that is creating awareness by investing in traditional advertising, trying to steal the latter’s potential customers. We refer to this as “poaching” in sponsored search.

Such poaching is evident in the example of the shoe company Skechers, which advertised its “Shape Ups” model during Super Bowl XLV (held on February 6, 2011). The upper panel of Figure 1 shows the effect of the television ad on the search volumes of the keywords “Skechers” and “Shape Ups” on Google in

the days after the Super Bowl. It can be easily seen that the advertising created considerable awareness, resulting in heavy keyword search on the Internet. Interestingly, whereas Skechers spent millions of dollars for the Super Bowl commercials, its competitor, Reebok, poached on the keyword “Shape Ups” to advertise its competing model called “Easy Tone,” as shown in the screenshot of a Google search for the keyword “Shape Ups” in the lower panel of Figure 1. This is only one of many instances of poaching, which is happening with increasing frequency on the Internet. To further establish the phenomenon of poaching, we conduct the following empirical investigations.

First, we show that if a firm runs an effective traditional advertising campaign providing a short-term impetus to search activity for its keywords, competitors respond with increased poaching on this firm’s keywords in sponsored search advertising. To show this, we consider the time periods before and after Super Bowl XLVI, held on February 5, 2012. Based on reports in the popular press, we collected keywords related to the names of companies and their specific products across multiple industries for which ads were expected to be shown during the Super Bowl telecast. We also collected keywords related to the names of companies and products that are close competitors of the advertisers but were known to *not* be advertising during the Super Bowl telecast. For advertisers, we obtain the following: cars

Figure 1 Poaching of Skechers by Reebok for the Keyword “Shape Ups”



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("Camry," "Toyota," "CR-V," "Honda," "Chrysler," "GS 350," "Lexus," "Audi," "Acura," "Volt," "Chevy," "BMW"), yogurt ("Dannon"), tax software ("Taxact"), Internet domain name registration ("Go Daddy"), and online trading ("Etrade"). For nonadvertisers, we obtain the following: cars ("Ford," "Infiniti," "Nissan," "Mercedes-Benz"), yogurt ("Yoplait"), tax software ("TurboTax," "H&R Block"), Internet domain name registration ("Network Solutions"), and online trading ("TD Ameritrade," "Scottrade," "Fidelity"). For each of these keywords, for three days before and after the Super Bowl telecast, we collected two types of data.¹ First, we collected data on the keyword's search volume on Google using Google Insights. Second, we queried Google with the relevant keyword, roughly once every hour, to crawl all the sponsored search results.

We find that, for the keywords in the advertised category, the average search volume was higher by 44.8% in the three days after the Super Bowl compared with the three days before the Super Bowl, whereas for the keywords in the nonadvertised category, the average search volume showed only a modest increase of 3.7% (the increase is possibly due to spillovers). In terms of poaching activity, poaching on keywords in the advertised category increased by 55.6% on average in the days after the Super Bowl, whereas poaching on keywords in the nonadvertised category decreased by 39.4% on average. We also find that roughly between three days to a week after the Super Bowl, search traffic and poaching activity by competitors returned to pre-Super Bowl levels.

Second, we show that over a long time span, among competing firms in an industry, firms with lesser keyword traffic poach more and are poached upon less, whereas firms with greater keyword traffic poach less and are poached upon more. We consider the following industries and firms: insurance (Geico, Progressive, Allstate, State Farm), online trading (E*TRADE, Scottrade, TD Ameritrade, ShareBuilder), e-readers (Kindle, Nook, Playbook), and pornography (Playboy, Penthouse, Hustler, Fling). For the keywords associated with each firm, for the time period from August 2010 to November 2011, we collected two types of data. First, using Google Insights, we collect monthly search traffic on Google for every company's keywords. Second, using the website Spyfu.com, we collected monthly poaching data for every company's keyword. (Spyfu periodically queries Google with millions of keywords and crawls the sponsored search results.) From these data, for each industry, we can

reconstruct who poached on whom and how frequently, and we can construct monthly indices for the same.

Using the monthly data from August 2010 to November 2011, we find the correlations between (i) keyword traffic for a firm and the frequency with which this firm's keyword was poached by competitors in its industry and (ii) keyword traffic for a firm and the frequency with which it poached the keywords of competitors in its industry. The resulting numbers are provided in Table 1. The first row of the table shows that, for every industry, the correlation between keyword traffic and the frequency of being poached is positive. The second row of the table shows that, for every industry, the correlation between keyword traffic and the frequency of poaching others is negative.

The above analyses indicate that, in both the short term and the long term, higher keyword traffic on search engines induces competitors to poach in sponsored search advertising. In summary, poaching happens when a firm creates awareness that results in pertinent keyword search by consumers on the Internet, and competing firms aggressively bid on these keywords (typically sold through position auctions run by the search engine) to display their ads. In fact, since the competitors are spending less to create awareness, they can bid more aggressively on sponsored search keywords and thus can even enjoy an advantage over the firm that has attracted the customers in the first place. Poaching has implications not only for the competing firms' strategies on the sponsored search and traditional advertising channels, it also strategically affects the search engine's auction strategy. In this paper, we examine these issues using a game theory framework. We address three broad questions. First, under what conditions will poaching arise? Second, what are the effects of poaching on competing firms' decisions for budget allocation among traditional and sponsored search advertising? Third, what are the consequences of poaching for the search engine, and how should it adapt its auction mechanism anticipating poaching by competing advertisers?

We consider a scenario with two competing firms with different advertising budgets. Each firm decides how to allocate its budget between traditional advertising and sponsored search advertising, and from the budget allocated to sponsored search advertising,

Table 1 Correlation Table for Long-Term Poaching Behavior

	Insurance	Online trading	E-readers	Pornography
Traffic vs. being poached	0.58	0.72	0.95	0.94
Traffic vs. poaching others	-0.14	-0.22	-0.33	-0.66

¹ We consider alternative ways of spelling these keywords, e.g., for the company E*TRADE, we use the keywords "Etrade," "E-Trade," and "E Trade."

each firm also decides the proportion to allocate to advertising on its own keyword versus on the competitor's keyword. Traditional advertising spending by a firm generates direct sales for the firm, as well as a proportional flow of potential customers for the firm into the sponsored search market. These customers in the sponsored search market are, however, targetable by the competitor; therefore, poaching by a firm implies a strategy of obtaining more than its "fair share" of sales through stealing customers in the sponsored search market.

Our first key contribution is that we characterize the budget allocation and poaching strategies of firms. Broadly speaking, we find that for small budget asymmetry, only the larger-budget firm poaches; for medium budget asymmetry, either firm may poach on the other; and for larger budget asymmetry, only the smaller-budget firm poaches while the larger-budget firm focuses on more traditional advertising. Regarding the impact of poaching on different players, when budget asymmetry is small and only the larger-budget firm poaches, poaching can benefit firms as well as the search engine. However, when budget asymmetry is larger, the smaller-budget firm does not do any traditional advertising and uses all of its budget on poaching. This hurts the larger-budget firm because the smaller-budget firm is free riding in the sponsored search market created by the larger-budget firm and not creating any sponsored search prospects on its own. This can induce the larger-budget firm to allocate more of its advertising budget to traditional advertising. This motivates our second key contribution, whereby we find that the search engine may penalize poaching to prevent excessive free riding by firms in order to prevent other competitor firms from moving their money away from sponsored search advertising. Specifically, the search engine benefits from allowing poaching but controlling its extent; i.e., the search engine benefits from *decreasing* competition in its own auctions in a measured way. This result offers a possible explanation for why search engines are in support of allowing bidding on trademarked keywords by competitors (Parker 2011) yet still employ "ad relevance" measures to underweight bids of firms bidding on competitors' keywords.

A growing theoretical and empirical literature on sponsored search advertising has enhanced our understanding of its different aspects; this includes Athey and Ellison (2009), Chan and Park (2010), Desai et al. (2014), Ghose and Yang (2009), Jerath et al. (2011), Katona and Sarvary (2010), Rutz and Bucklin (2011), Yang and Ghose (2010), Yao and Mela (2011), and Zhu and Wilbur (2011). Our work is distinctly different from the above work because these

papers consider sponsored search advertising in isolation; we model it in a multichannel advertising setting. Goldfarb and Tucker (2011a, b) study substitution between online and off-line advertising induced by better targeting in online advertising and advertising bans on off-line advertising for certain products. Joo et al. (2014) and Zigmund and Stipp (2010) empirically show that television advertising increases Internet search volume; we use their finding as a building block in our model. Kim and Balachander (2010) model sponsored search in a multichannel setting. However, they do not consider poaching behavior of competing firms, and the resulting strategic response of the search engine (in terms of auction design). In our research, the analysis of these two aspects leads to a rich set of novel results and insights. Since poaching involves elements of free riding under competition, our work is also related to the literature on strategic targeting and free riding (Carlton and Chevalier 2001, Shaffer and Zhang 1995, Shin 2007, Singley and Williams 1995, Telser 1960).

Before we proceed further, we note that downstream advertising, where the aim is to reach customers expected to have a high likelihood of purchase, is also possible in certain channels other than sponsored search. For instance, firms may advertise in yellow pages to reach customers when they are specifically looking for the provider of a product or service before making a purchase. However, targetability is weak in yellow pages, which makes it difficult to poach a competitor's customers; for instance, among the customers who are consulting yellow pages, firms cannot distinguish between those who are interested in a competitor versus those who are already interested in the firm itself. Similarly, "checkout coupons" used in retail stores, powered by technology from providers such as Catalina Marketing, target customers based on their data-based profiles (purchase history, gender, location, etc.). This allows firms to target consumers who purchased a competitor's product in a category (Pancras and Sudhir 2007). However, in this case, the identification of the customer and subsequent targeting is done after the current purchase is made (with the idea of making the customer switch at the next purchase occasion), which makes poaching less effective. Sponsored search, on the other hand, makes for a unique combination of features that make it an effective channel for poaching—based on the keyword searched, consumers self-identify whether they are interested in the competitor, the firm itself, or the category; consumers can be targeted after they have revealed interest in the product but before making their purchase; and, based on the keyword searched, different consumers can be targeted differently by showing them different ad copies and landing pages upon clicking.

The rest of the paper is structured as follows. In §2, we describe the model. In §3, we describe our analysis and develop key insights regarding the budget allocation and poaching strategies of firms. After developing this basic understanding of poaching, in §4, we analyze ad relevance scores as a device used by a search engine to strategically control the degree of poaching for its maximum benefit. In §5, we consider two extensions of the basic model and show that the key insights remain unchanged. In §6, we conclude with a discussion.

2. Model

Our model consists of three entities: the firms, the users, and the search engine. Two firms, Firm 1 and Firm 2, produce identical products. Firm i , $i \in \{1, 2\}$, has an exogenously specified total budget B_i allocated for advertising,² and has to decide how to allocate its advertising budget to traditional advertising and sponsored search advertising to maximize total sales. For Firm i , we denote the money spent on traditional advertising by T_i , which implies that the money spent on sponsored search is given by $B_i - T_i$. Note that we bundle all nonsponsored search channels of advertising together into traditional advertising.

We assume that if Firm 1 spends T_1 and Firm 2 spends T_2 on traditional advertising, $(1 + \alpha)\sqrt{T_1 + T_2}$ customers become aware of the product. We will clarify the meaning of the $\alpha > 0$ parameter shortly. The concave functional form, $\sqrt{\cdot}$, captures the fact that as more money is spent on traditional advertising, its effectiveness in generating awareness in the population decreases. We also assume that, out of the total customers made aware, a share $T_i/(T_1 + T_2)$ of customers become aware of Firm i .

The customers who become aware through a traditional ad either purchase the product directly or search for the product at a search engine. Each firm is associated with a specific keyword that consumers use to search for it on the search engine. For instance, if Apple sells the iPad tablet and Samsung sells the Galaxy Tab tablet, then the keywords associated with Apple and Samsung are “iPad” and “Galaxy Tab,” respectively. The transaction of a customer who searches the product on a search engine is either influenced by the sponsored links or not influenced. In our model, a customer who purchases directly from the firm after being exposed to a traditional ad is equivalent to a customer who searches before purchasing but is not influenced by the sponsored search results (e.g., is influenced only by the organic results). Without loss of generality, we assume that all the customers who

search on the search engine are influenced by sponsored search results. Specifically, we assume that out of the $(1 + \alpha)\sqrt{T_1 + T_2}$ customers made aware by traditional advertising, $\alpha\sqrt{T_1 + T_2}$ customers purchase the product independent of what they see in sponsored search, and the remaining $\sqrt{T_1 + T_2}$ customers search for the product on a search engine and purchase the product that they see advertised in the sponsored section of the search results (which may or may not be the product of the firm whose traditional ad they first saw and which motivated their online search).

To summarize, the above assumptions imply that if Firm 1 spends T_1 and Firm 2 spends T_2 , then $(1 + \alpha) \cdot (T_i/(T_1 + T_2))\sqrt{T_1 + T_2}$ customers get aware of the product of Firm i . Out of this, $\alpha(T_i/(T_1 + T_2))\sqrt{T_1 + T_2}$ customers directly purchase the product of Firm i through the traditional channel, whereas $(T_i/(T_1 + T_2))\sqrt{T_1 + T_2}$ customers search its keyword on the search engine. Each of these $(T_i/(T_1 + T_2))\sqrt{T_1 + T_2}$ customers will purchase from the firm whose product she sees advertised in the sponsored section of the search results (which may not be Firm i , even though the keyword is of Firm i) and is therefore susceptible to being poached by the other firm.

For simplicity, we assume that there is only one advertising slot available for each keyword; i.e., only one firm is shown in response to a keyword search.³ When a customer searches a keyword, the search engine uses a pay-per-click second-price auction to sell the advertising slot. In this auction, the slot is sold to the firm that bids higher for the keyword. However, when a consumer clicks on the sponsored link, the winner has to pay the loser’s bid. We assume that a third passive bidder with bid R is always present in the auction. Therefore, Firms 1 and 2 have to bid at least R to win the advertising slot. We assume that if one of the firms bids R , the tie is resolved in the favor of the firm (i.e., against the passive bidder).

We assume that the marginal profit from selling one unit of the product is 1 for both firms. By fixing the margin exogenously, we are able to focus solely on advertising competition between the firms and not confound it with other aspects of competition.

The order of moves in the model is as follows. In stage I, the search engine announces the rules of the auction (that it is a second-price, pay-per-click auction). We note that, in the current model, this is a “dummy” stage as there is no decision to make in this stage. In §4, we enable the search engine to decide and announce in this stage the relevance score multiplier for a poaching bid, i.e., a bid by a firm on a competitor’s keyword. In stage II, the two firms simultaneously decide how to allocate their budgets to the

² In §5.1, we make the budget endogenous and confirm that the results of our basic model are robust.

³ In §5.2, we consider the extension in which multiple advertising slots are available for each keyword.

traditional channel, their own keyword in sponsored search, and their competitor’s keyword in sponsored search.⁴ In stage III, consumers see traditional ads and a fraction $\alpha/(1 + \alpha)$ of them purchase directly from the firm whose traditional ad they saw. The remaining consumers go to the search engine sequentially and search the keyword of the firm whose traditional ad they saw, and the sequential second-price auction is played out. Each consumer who searches purchases from the firm that is shown to her in the sponsored search results.

2.1. Key Intermediate Result

Suppose that τ_i customers search keyword i on the search engine. These customers arrive sequentially at the search engine, and it runs a separate auction for each customer. In other words, if τ_i customers search keyword i , the search engine sequentially runs τ_i auctions, one for each customer. In each auction, the firms submit their bids simultaneously. Each firm decides its bid in an auction based on how much of its allocated budget is remaining at that time. Using subgame-perfect equilibrium, we show in Theorem A1 in the appendix that the unique outcome of this sequential second-price auction coincides with the outcome of a *market-clearing-price mechanism*. We state this result in the lemma below.

LEMMA 1. *Assume that Firm 1 spends L_1 and Firm 2 spends L_2 on keyword i searched by τ_i consumers. If $L_1 + L_2 \geq \tau_i R$, then $L_1/(L_1 + L_2) \cdot \tau_i$ customers purchase from Firm 1 and $L_2/(L_1 + L_2) \cdot \tau_i$ customers purchase from Firm 2. If $L_1 + L_2 < \tau_i R$, then L_1/R customers purchase from Firm 1 and L_2/R customers purchase from Firm 2.*

This result is interesting in itself and, to the best of our knowledge, is new to the auctions literature. This is also a very useful result because, for the analysis in the rest of the paper, it allows us to reduce bidding in a complicated sequential auction to a much simpler form that abstracts away from the auction and, in fact, represents a simple market-clearing allocation. In the analysis to follow, rather than modeling bidding between competitors in every scenario, we will simply use this lemma repeatedly. Note that we make the assumption in the model that each firm, given its budget exogenously, maximizes sales (i.e., it has no value from leftover budget), which is important for us to be able to invoke this lemma. In the appendix, we show that in the case of a tie in bids in any round of the sequential second-price auction, if the search engine breaks the tie in favor of the firm with the

larger leftover budget at the time, then neither firm has any leftover budget at the end of the sequential auction, and the market-clearing-price mechanism can indeed be invoked. Furthermore, in §5.1, we allow the budget of each firm to be endogenously determined by the firm and show that, upon incorporating this “budget elasticity,” our insights remain unchanged.

3. Analysis and Results

We first describe our analysis procedure. We denote firm strategies by the tuple (T_1, T_2) , where $T_1, T_2 \geq 0$. This denotes that Firm i spends T_i on traditional advertising and $B_i - T_i$ on sponsored search advertising. If Firm i spends T_i on traditional advertising, $\alpha(T_i/(T_1 + T_2))\sqrt{T_1 + T_2}$ customers purchase the product from Firm i without being influenced by search advertising, and using Lemma 1, $\min((B_i - T_i)/R, ((B_i - T_i)/(B_1 + B_2 - T_1 - T_2))\sqrt{T_1 + T_2})$ purchase Firm i ’s product by being influenced through search advertising. Note that as there is no reduced conversion rate if a firm poaches (and no poaching penalty), mathematically, each firm is indifferent between being displayed in response to a search for its own keyword and a search for the competitor’s keyword. We make the assumption that a firm prefers being displayed against its own keyword;⁵ i.e., it exhausts its own keyword’s supply before it poaches. Our insights are robust to these simplifications and assumptions. However, these assumptions make the analysis simpler.⁶ We note that our assumptions imply that each firm is effectively deciding how to split its budget between traditional and sponsored search advertising, and poaching is a strategy for a firm to obtain more than its “fair share” of sales through sponsored search.

The profit of Firm i is

$$\pi_i = \alpha \frac{T_i}{T_1 + T_2} \sqrt{T_1 + T_2} + \min\left(\frac{B_i - T_i}{R}, \frac{B_i - T_i}{B_1 + B_2 - T_1 - T_2} \sqrt{T_1 + T_2}\right).$$

⁵ This can be justified by assuming that a firm has an infinitesimally higher conversion rate when it is displayed in response to a search for its own keyword compared with when it is displayed in response to a search for the competitor’s keyword.

⁶ The analysis becomes simpler because we can treat both keywords as being equivalent from the points of view of both firms. Specifically, both keywords will have the same price in equilibrium (otherwise, firms would have the incentive to move their budgets to the cheaper keyword). As a consequence, instead of differentiating between the two keywords and their search volumes—say, x and y , respectively—we can treat the sponsored search advertising channel as offering search volume $x + y$. Then, depending on how much each firm spends on traditional advertising and search advertising, and the assumption that a firm exhausts its own keyword’s supply before it poaches, we can determine whether or not a firm is poaching.

⁴ Our results do not change qualitatively if firms decide traditional and sponsored search allocations sequentially and can observe each others’ traditional advertising allocations when deciding on sponsored search advertising actions.

If $(B_i - T_i)/R < ((B_i - T_i)/(B_1 + B_2 - T_1 - T_2))\sqrt{T_1 + T_2}$, this function reduces to

$$\pi_i^1 = \alpha \frac{T_i}{T_1 + T_2} \sqrt{T_1 + T_2} + \frac{B_i - T_i}{R}$$

If $(B_i - T_i)/R > ((B_i - T_i)/(B_1 + B_2 - T_1 - T_2))\sqrt{T_1 + T_2}$, this function reduces to

$$\pi_i^2 = \alpha \frac{T_i}{T_1 + T_2} \sqrt{T_1 + T_2} + \frac{B_i - T_i}{B_1 + B_2 - T_1 - T_2} \sqrt{T_1 + T_2}$$

For given T_j and B_j ($j \neq i$), Firm i decides how much to spend on search advertising to maximize profit π_i . Let T_i^1 and T_i^2 be the optimal values of traditional advertising for π_i^1 and π_i^2 , respectively. In other words,

$$T_i^1 = \arg \max \pi_i^1 \quad \text{and} \quad T_i^2 = \arg \max \pi_i^2$$

Furthermore, let T_i^3 be the solution to the following equation:

$$\frac{B_i - T_i^3}{R} = \frac{B_i - T_i^3}{B_1 + B_2 - T_i^3 - T_j} \sqrt{T_i^3 + T_j}$$

In other words, if Firm i spends T_i^3 on traditional advertising, we have $\pi_i^1 = \pi_i^2$. Note that T_i^1 , T_i^2 , or T_i^3 are functions of the other player’s decision and, essentially, potential segments of the best-response functions, which we define shortly.

Depending on the values of T_j , R , B_i , and B_j , the optimum value of traditional advertising spend, T_i^* , is T_i^1 , T_i^2 , or T_i^3 . A complete analysis of the conditions under which each of these values is the solution T_i^* is provided in the appendix. The values of T_i^1 , T_i^2 , and T_i^3 have the following intuitive interpretations. The value T_i^1 is relevant when the competition in search advertising channel is weak. In other words, the firm pays only price R for each unit. Therefore, for T_i^* to be T_i^1 , we must have large enough T_j (i.e., the competing firm should be spending a large fraction of its budget on traditional advertising) or large enough α (i.e., most of the customers are not affected by search advertising). On the other hand, we have $T_i^* = T_i^2$ only when competition in the search advertising channel is strong. In particular, the per-unit price of search advertising has to be more than R . Therefore, if T_j is small enough or α is small enough, we have $T_i^* = T_i^2$. Finally, T_i^3 captures the situation in which the per-unit price of search advertising is R , and any additional spending on search advertising increases the price. Firm i faces a trade-off between spending more on search advertising to increase its conversion and avoiding more competition in search advertising to keep the price low. We have $T_i^* = T_i^3$ for medium values of α .

For given B_i , B_j , α , and R , the best response of Firm i to Firm j , defined as $BR_i(T_j)$, is the profit-maximizing value T_i^* that Firm i spends on traditional advertising if Firm j spends T_j on traditional advertising. To calculate the equilibria of the game, we have to find values T_1^e and T_2^e (where the superscript e stands for “equilibrium”) such that

$$BR_1(T_2^e) = T_1^e \quad \text{and} \quad BR_2(T_1^e) = T_2^e$$

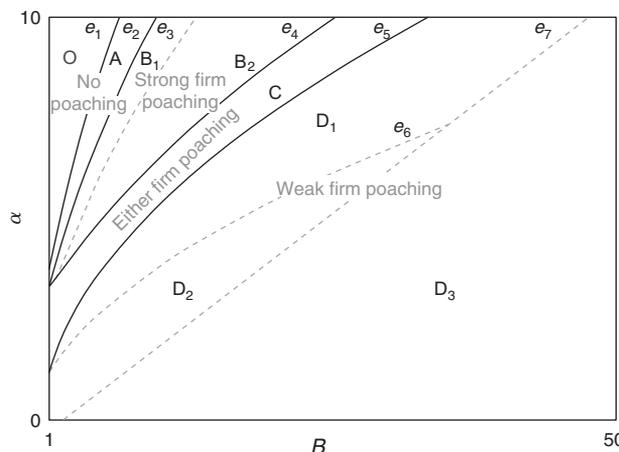
Note that equilibrium strategy T_i^e is T_i^1 , T_i^2 , or T_i^3 . We provide the details of the analysis in §A.2 in the appendix, where we obtain closed-form solutions for the firms’ strategies in terms of B_1 , B_2 , and R . Here, we focus on the insights obtained from the analysis.

To illustrate the results and insights, we assume that $B_1 = B \geq 1$ and $B_2 = 1$. We note that this scaling is without any loss of generality. We refer to Firm 2, the smaller-budget firm, as the “weak firm” and to Firm 1, the larger-budget firm, as the “strong firm.” We also assume that the passive bidder’s bid is $R = 1/2$. We note that the results are qualitatively the same for all other feasible values of $R > 0$. By plugging in the values $B_1 = B \geq 1$, $B_2 = 1$, and $R = 1/2$ in the relevant expressions in §A.2 in the appendix, we obtain the following proposition.

PROPOSITION 1. *For the case with $B_1 = B \geq 1$, $B_2 = 1$, and $R = 1/2$, the budget allocation strategies of the firms are summarized in Figure 2 and Table 2.*

Figure 2 shows different regions that demarcate the firms’ equilibrium strategies as functions of exogenous parameters $\alpha > 0$ (the relative measure of consumers who directly purchase after being exposed to a traditional ad) and $B \geq 1$ (the budget of the strong firm, the budget of the weak firm being fixed

Figure 2 Regions with Different Equilibrium Strategies as Functions of B and α



Note. The expressions for the loci e_1, \dots, e_7 are provided in Table A.1 in the appendix.

Table 2 Firms' Poaching Strategies and Budget Allocations in the Different Regions of Figure 2

Region	Poaching firm	Poaching characteristics	(T_1, T_2)
O	None	—	$(B, 1)$
A	None	—	$(\hat{B}, 1)$
B			
B ₁	Strong firm	Partial	$(\hat{B}, 1)$
B ₂	Strong firm	Partial	$(\frac{1}{8} + B - \frac{1}{8}\sqrt{17 + 16B}, 1)$
C	Either firm	Partial; multiple equilibria	$T_1 + T_2 = B - \frac{1}{8}\sqrt{16B + 17} + \frac{9}{8}$
D			
D ₁	Weak firm	Partial or full; multiple equilibria	$T_1 + T_2 = B - \frac{1}{8}\sqrt{16B + 17} + \frac{9}{8}$
D ₂	Weak firm	Partial	$(\frac{(1 + 3\alpha)((4 + 9\alpha)B - 2)}{6 + 33\alpha + 27\alpha^2}, \frac{(1 + 3\alpha)((4 + 9\alpha) - 2B)}{6 + 33\alpha + 27\alpha^2})$
D ₃	Weak firm	Full	$(\frac{2\alpha(B + 1) + 2B + 3 - \sqrt{8(\alpha + 1)B + 8\alpha + 9}}{2(\alpha + 1)}, 0)$

Note. \hat{B} is defined in (2) in the appendix.

at 1). In Regions O and A, neither firm poaches; in Region B, only the strong firm poaches; in Region C, either firm may poach; in Region D, only the weak firm poaches. All regions have unique equilibrium, except Regions C and D₁, which have multiple equilibria. If a firm uses a part of its budget for poaching, we call it *partial poaching*, and if a firm uses all of its budget for poaching, we call it *full poaching*. When discussing the firms' budget allocation strategies and their profits, we use the setting in which poaching is not possible (i.e., not allowed) as the benchmark scenario. The analysis for the benchmark scenario is also available in the appendix. We now discuss the results in more detail.

3.1. Region O

In Region O, both firms spend all their budget on the traditional channel. This is because the value of α is large (which implies that a relatively small number of customers are affected by the search advertising channel), whereas the budgets of both the firms are small. Because of small budgets, saturation is not reached in the traditional advertising channel; the price in the sponsored search channel, which would be at least R , is comparatively higher.

3.2. Region A

In Region A, since α is large, firms prefer the traditional channel. The weak firm spends all of its budget on traditional advertising in this region. The strong firm, which has a larger budget than in Region O, also spends at least as much on traditional advertising, but because of saturation effects in traditional advertising, it also spends part of its budget on search advertising of its own keyword. Note that the strong firm does not poach. (Since neither firm poaches, their strategies are not affected by whether or not poaching is allowed.)

3.3. Region B

Region B is similar to Region A except that, since the strong firm's budget B is larger than it is in Region A, the strong firm uses the search advertising supply of the weak firm in addition to its own search advertising supply. In other words, the weak firm still spends all of its budget on traditional advertising and the strong firm spends at least as much as the weak firm on traditional advertising as well. However, because of saturation in traditional advertising (which hurts the strong firm more than the weak firm), the strong firm spends part of its budget on search advertising of its own keyword and of the weak firm's keyword. We divide Region B into Regions B₁ and B₂, where the strong firm spends, respectively, \hat{B} (defined in (2) in the appendix) and $\frac{1}{8} + B - \frac{1}{8}\sqrt{17 + 16B}$ on traditional advertising. In Region B₁, the third passive player has positive allocation of sponsored search advertising; in Region B₂, this player has zero allocation.

It is interesting to note that in Region B, both firms benefit from poaching compared with the case where poaching is not possible. The strong firm benefits because it can use the search advertising supply of the weak firm—in this way, the strong firm can avoid extra competition on traditional advertising that would happen if poaching was not possible. The weak firm also benefits when the strong firm can avoid spending more on the traditional channel. We state this in the following corollary to Proposition 1.⁷

COROLLARY 1. *In Region B, poaching increases both the weak firm's and the strong firm's profits.*

⁷The proofs of all corollaries are derived easily using the relevant expressions in §A.2 in the appendix, evaluated using $B_1 = B$, $B_2 = 1$, and $R = 1/2$.

3.4. Region C

As the strong firm's budget B increases even further (or if α decreases), we enter Region C. As B grows, the strong firm's spending on the traditional channel also grows, and the saturation of the traditional channel starts hurting the weak firm's profit. Therefore, the weak firm benefits from decreasing traditional advertising and increasing search advertising. Region C has multiple equilibria but they all share the following major common features. The price of search advertising is always exactly equal to the passive bidder's bid, R . Furthermore, in any equilibrium in Region C, if any firm increases the budget for search advertising (by any amount), the price of search advertising becomes larger than R . Therefore, no firm wants to increase its search advertising budget. On the other hand, decreasing the search advertising budget does not decrease the price of search advertising and only leaves part of the supply for the third passive player. For both firms, the marginal return on a dollar spent on the traditional channel is less than $1/R$, and therefore, they do not want to spend more on the traditional channel. In other words, both firms prefer search advertising at price R to traditional advertising; however, if they spend more on search advertising, the price would not be R anymore. In Region C, either firm may be poaching on the other firm's keyword. However, the degree of poaching cannot be large, because the firm that is being poached would benefit from spending more on search advertising even if it leads to a higher price in search advertising.

All of the equilibria in Region C have the same revenue for the search engine. From the firms' perspectives, the situation is similar to a "dove and hawk" game—if one firm poaches (playing "hawk"), the other firm has to spend more on traditional advertising (playing "dove"). The firm that poaches is better off and the firm that is being poached is worse off compared with the case where poaching is not possible. The slight difference from a classic dove and hawk game is that firms' strategies are *continuous* in this game. Therefore, a firm may play a "slightly hawk" and the best response from the other firm would be to play "slightly dove". Note that poaching is always partial in this region, even in the "hawkiest" strategy. Furthermore, it may happen that neither firm plays hawk or dove, which leads to a nonpoaching equilibrium, which is always one of the equilibria in Region C.

Finally, note that the case in which firms are symmetric (when $B = 1$) is part of Region C for medium values of α . As before, following the same dove and hawk pattern, either firm may partially poach on the other firm's keyword in equilibrium. We state the results above in the following corollary to Proposition 1.

COROLLARY 2. *In Region C, either firm may partially poach on the other firm's keyword. In response, the firm being poached spends more on traditional advertising than it would have if poaching was not possible. Furthermore, symmetric firms may follow asymmetric strategies in equilibrium, where only one firm poaches and the other spends more on traditional advertising.*

3.5. Region D

In Region D, only the weak firm poaches. Region D_1 is characterized by multiple equilibria in which only the weak firm poaches, and it may poach partially or fully. Region D_2 corresponds to the situation where α is not large. In this region, the firms' incentives for search advertising are large enough such that the equilibrium price of search advertising is greater than R . In Region D_3 , where B is large enough and α is small enough, the weak firm spends *all* of its budget for poaching on the strong firm's keyword. In Region D_2 , the weak firm partially poaches on the strong firm's keyword, in Region D_3 , the weak firm fully poaches on the strong firm's keyword, and the strong firm does not poach in either of these two regions. (Note, however, that the strong firm spends part of its budget on advertising on its *own* keyword in sponsored search.) In Region D_2 , the percentage of budget allocated to poaching increases as B increases or as α decreases, both of which lead to a greater flow of customers into sponsored search. In Regions D_2 and D_3 , the weak firm benefits from poaching while the strong firm's profit decreases compared with the situation where poaching is not possible.⁸

COROLLARY 3. *In Region D_2 , the weak firm spends part of its budget for poaching on the strong firm's keyword, and the percentage of budget allocated to poaching increases with B and decreases with α . In Region D_3 , the weak firm spends all of its budget for poaching on the strong firm's keyword. In both regions, the strong firm does not poach, and the percentage of its budget that the strong firm allocates to traditional advertising increases with B and α .*

Next, we study the equilibrium strategy of a firm when its competitor poaches. We first consider the strong firm's equilibrium strategy in the equilibria where the weak firm poaches. We find that the strong firm's optimal strategy can be either to retreat or to defend, depending on the values of α and B . In the *retreat* strategy, the firm spends *less* on search advertising spending (compared with the benchmark with no poaching) and moves the money to traditional advertising to obtain more customers who convert directly after being exposed to a traditional ad. This allows the

⁸ We note that if $R = 0$, i.e., the passive bidder bids zero, then only Regions D_2 and D_3 survive. That is, only the weak firm poaches partially or fully, and the strong firm does not poach.

firm to keep the price of search advertising low. On the other hand, in the *defend* strategy, the firm spends *more* on search advertising spending (compared with the benchmark with no poaching) and spends this money on its own keyword.

Figure 3 shows the equilibrium strategy of the strong firm in equilibria where the weak firm poaches. Note that, referring to Figure 2, the weak firm does not poach in equilibrium in Regions O, A, and B, and we mark the corresponding parts of Figure 3 with the symbol Φ ; only the parts corresponding to Regions C and D are of interest. Note also that Regions C and D_1 have multiple equilibria; for the purposes of this discussion, for Regions C and D_1 , we consider only the equilibria where the weak firm poaches.

When α is large for a given value of B , the strong firm uses the retreat strategy. This is because the negative effect of poaching (in terms of customers lost as a result of poaching) is small, and the strong firm focuses on direct sales through traditional advertising (rather than on saving the poached customers). On the other hand, for small values of α and large enough values of B , because the negative effect of poaching (in terms of customers lost as a result of poaching) is larger, the strong firm follows the defend strategy (to prevent losing too many customers to poaching). The strong firm's change of strategy from retreat to defend, as B increases or α decreases, is driven by two forces. First, saturation of the traditional advertising channel decreases the marginal utility gain from spending more on the traditional channel. Therefore, the strong firm benefits more from spending on search advertising, even though it leads to a price increase in search advertising. Second, as the degree of poaching increases, the strong firm's share of search advertising decreases. Therefore, the strong firm is less affected by an increase in the price of search advertising and

therefore is willing to defend by spending more on search advertising. This discussion is summarized in the following corollary to Proposition 1.

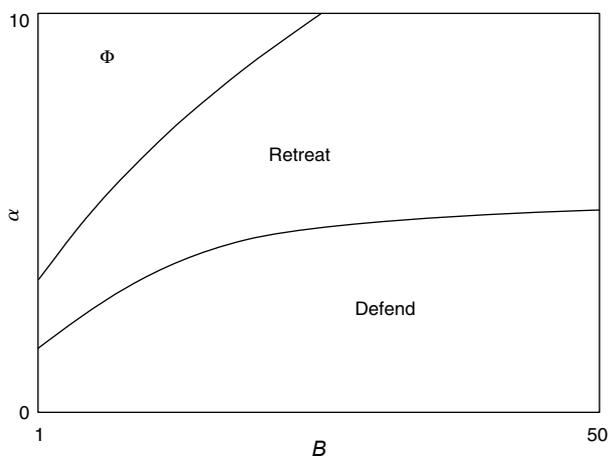
COROLLARY 4. *If α is small enough and B is large enough (as shown in Figure 3), the strong firm's strategy in equilibria where the weak firm poaches is to defend. Otherwise, the strong firm's strategy is to retreat.*

We now discuss the weak firm's equilibrium strategy in equilibria where the strong firm poaches. Note that, referring to Figure 2, the strong firm poaches on the weak firm in equilibrium only in Regions B and C. In Region B, the strong firm is only using the search advertising supply of the weak firm that is otherwise used by the third passive player, and the weak firm's strategy is not affected in any way by poaching by the strong firm. In Region C, the weak firm follows the retreat strategy; i.e., it allocates more budget to traditional advertising as a result of the strong firm's poaching.

We now consolidate the results in the discussion above to obtain an overall understanding of the firms' budget allocations to the different advertising options. The strong firm spends all of its budget on traditional advertising in Region O. In Region A, it spends the majority of its budget on traditional advertising and the remaining budget on its own keyword in sponsored search. In Region B, it spends the majority of its budget on traditional advertising, a significant fraction on its own keyword in sponsored search, and a small fraction on poaching in sponsored search. Region C has multiple equilibria; in different equilibria, either the strong or the weak firm may poach using a fraction of its advertising budget (as discussed in detail above). In Region D, the strong firm spends the majority of its budget on traditional advertising and the remaining budget on its own keyword in sponsored search and does not poach at all.

The weak firm spends all of its budget on traditional advertising in Regions O, A, and B. Region C has multiple equilibria; in different equilibria, either the strong or the weak firm may poach using a fraction of its advertising budget. In Regions D_1 and D_2 , the weak firm spends its budget on all three types of advertising; as the values of B and/or α increase, the fraction allocated to poaching increases while the fractions allocated to both traditional advertising and own keyword in sponsored search decrease; note that Region D_1 has multiple equilibria. In Region D_3 , the weak firm spends all of its budget on poaching.

Figure 3 The Strong Firm's Strategy in Equilibria Where the Competitor Poaches



4. Relevance Measures for Bids

All the major search engines, including Google, Yahoo!, and Bing, transform an advertiser's submitted bid into an *effective bid* before determining the outcome of the sponsored search auction. A multiplier

is typically used to compute the effective bid, and this multiplier depends on many parameters including the advertiser's past performance in terms of the click-through rate, the reputation of the advertiser's product or website, and the *relevance* of the keyword being bid on to the advertiser's ad. Our focus here is on the last parameter, which measures the alignment between what a user searches for and what is advertised to the user by an ad. For instance, Google uses the relevance of the ad copy and the relevance of the landing page of the ad to the user's search query to calculate such a relevance measure. In a similar way, Bing uses a "landing page relevance" score as such a relevance measure.⁹ We explain the key idea behind a relevance multiplier using the stylized example below.

Consider the keyword "iPad" and the two firms Apple and Samsung. Apple's website is much more relevant to this keyword than Samsung's because Apple produces the iPad; Samsung only sells a competing product, the Galaxy Tab, in the same category (electronic tablets). Therefore, if the relevance of Apple's website to the keyword "iPad" is 1 on a scale from 0 to 1, the relevance of Samsung's website to this keyword should be less than 1 and is, say, 0.5. For simplicity, assume that both firms have the same scores on other parameters used for calculating the effective bid (click-through rate, reputation of the company, ease of navigation of the landing page, etc.). Suppose that Apple bids \$1 per click and Samsung bids \$1.5 per click to be displayed in response to the keyword "iPad." It seems natural that the search engine should prefer to display Samsung instead of Apple in this case (assuming only one is displayed) as Samsung should generate more revenue than Apple for it. However, surprisingly, in a situation such as this, the search engine calculates Samsung's effective bid as $\$1.5 \times 0.5 = \0.75 and Apple's effective bid as $\$1 \times 1 = \1 ; Apple wins the keyword auction and has to pay only \$0.75 per click. Essentially, the search engine decides Apple is the winner because of the higher relevance of its ads to the keyword being bid on. In fact, Samsung will have to bid and pay at least $\$1/0.5 = \2 to win this auction.

One explanation for the existence of such relevance measures is that the search engine wants to improve

user experience. In other words, if a user searches a keyword and finds the ads and the associated landing pages to be closely relevant to the searched keyword, she will be more satisfied with the results presented by the search engine. Although this is a reasonable explanation, we argue that it is probably not the only explanation. We provide an additional, novel explanation—a search engine may use relevance measures to handicap poaching selectively to the extent it wants and increase its revenue in the process.¹⁰

We assume that if a firm wants to bid on the keyword of the other firm, its poaching bid will be multiplied by $0 \leq \gamma \leq 1$. If $\gamma = 1$, we are in the framework that we have been in so far; i.e., firms poach on each others' keywords without any handicap. On the other extreme, if $\gamma = 0$, firms cannot bid on each others' keywords, which effectively implies that bidding on trademarked keywords is not possible. To simplify and focus on the effect of relevance measures, we assume that both firms have the same values for other scores used to calculate the effective bid, such as click-through rate, website reputation, etc.; this assumption does not impact our results qualitatively.¹¹ All other components of the model are the same. We assume that the search engine announces the value of γ in stage I, i.e., when it announces the rules of the auction. We solve the model with this variation incorporated (details are provided in the appendix) and discuss the results and insights below. To illustrate the results and insights, as in §3, we assume, without loss of generality, that $B_1 = B \geq 1$, $B_2 = 1$, and $R = 1/2$. We obtain the following proposition.

PROPOSITION 2. (a) *For large enough α and B , the search engine can increase its revenue by handicapping poaching.*

(b) *The optimal degree of handicapping increases in B and decreases in α .*

We provide a proof of the proposition in the appendix. Part (a) of Proposition 2 states that for large enough α and B there exists a value less than 1 of γ that the search engine can choose such that its revenue is higher than it would be with $\gamma = 1$. In other words, the search engine can increase its revenue by handicapping poaching, i.e., by reducing competition in its own auctions.

⁹ More information for Google is available at <http://support.google.com/adwords/answer/2454010> and for Bing at http://advertise.bingads.microsoft.com/en-us/product-help/bingads/topic?market=en-US&project=adCenter_live_std&querytype=topic&query=MOONSHOT_CONC_MeasuringQuality-Score.htm (accessed August 30, 2013). It is clear from the information provided at these Web pages that search engines treat historical performance measures for the ad (such as click-through rates) and user experience at a landing page as different constructs from the relevance scores that we focus on.

¹⁰ Assessing the relative importance of different drivers in explaining the phenomenon of interest is an empirical question that can be answered once plausible theoretical explanations are available. Our aim is to provide one such novel explanation in the poaching context.

¹¹ Note that this assumption favors the poaching firm. We show that even if the ad from the poaching firm is as good as the ad from the firm being poached, in terms of click-through rate and other measures of quality, the search engine still benefits from handicapping poaching.

We note that, as the value of γ is reduced below 1, if the value is above a threshold value, the game has a unique equilibrium. However, if the value of γ is below the threshold value, the game has multiple equilibria. Part (a) of Proposition 2 is an existence result (that handicapping can benefit the search engine) and it can be shown while restricting ourselves to the region with unique equilibrium. For part (b) of the proposition, which is about the optimal degree of handicapping, we consider two extreme cases of equilibrium selection—when the weak firm is the least aggressive in poaching and when the weak firm is the most aggressive in poaching—and show that the statement holds for both these extremes. In the least aggressive-poaching case, we select the equilibrium in which the weak firm’s poaching amount is the lowest among all equilibria. In the most aggressive-poaching case, we select the equilibrium in which the weak firm’s poaching amount is the highest among all equilibria.

Figure 4 shows the optimal values of the relevance multiplier (at which the search engine’s revenue is maximized) for different values of B and α under the two extreme equilibrium selection rules. Panel (a) shows this value when the least aggressive-poaching equilibrium selection rule is used, and panel (b) shows this value when the most aggressive-poaching equilibrium selection rule is used. It is clear that the two panels show qualitatively the same patterns for the optimal value of the relevance multiplier.

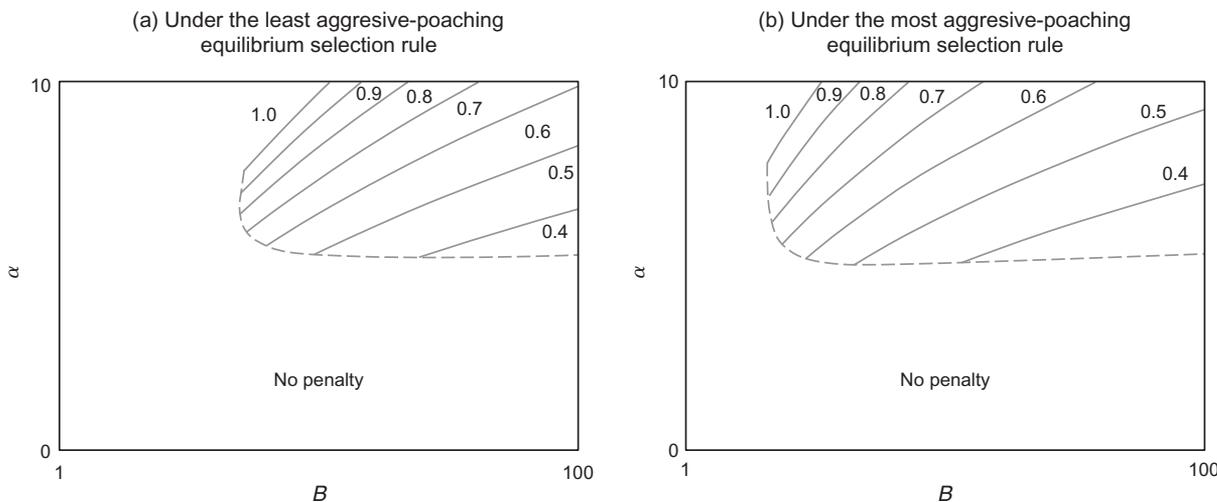
If B is small, the search engine does not penalize poaching. Also, if B is large enough and α is small enough, the search engine does not penalize poaching. In this case, the weak firm spends all of its budget for poaching on the strong firm’s keyword, and the strong firm uses the defend strategy in response and spends a large part of its budget on its own keyword.

Therefore, the search engine benefits from poaching and does not penalize it. On the other hand, if α is large enough, the strong firm’s response to poaching by the weak firm is to retreat. Since in the retreat strategy the strong firm lowers its search advertising spending, the search engine benefits from penalizing poaching and making the poaching bid less effective. In the situation where the weak firm fully poaches, the penalty is set to the highest value at which the weak firm still spends all of its budget for poaching. By doing so, the search engine still collects all of the weak firm’s budget but moderates the effect of poaching on the strong firm’s response. The penalty decreases as α increases or as B decreases because the weak firm’s incentive to poach decreases, and therefore, the weak firm poaches only if the penalty is small enough.

The higher-level intuition behind using a relevance multiplier is the following. In the relevant region, the strong firm does both traditional and sponsored search advertising, whereas the weak firm does only sponsored search advertising. In other words, the strong firm is creating the sponsored search market (through the overflow from its traditional advertising) but the weak firm is only free riding on this market. The free riding hurts the strong firm because not only is the sponsored search market smaller but the strong firm also obtains a smaller fraction of this market. In equilibrium, this may induce the strong firm to spend less on sponsored search, which may be sub-optimal for the search engine, and the search engine therefore penalizes free riding by the weak firm.

The policy that search engines such as Google, Yahoo!, and Bing follow regarding poaching is somewhat perplexing—they *allow* poaching by competitors on trademarked keywords (such as brand and company names) *yet still handicap* poaching by employing

Figure 4 Optimal Values of the Relevance Multiplier Under the Two Extreme Equilibrium Selection Rules



ad relevance measures. Interestingly, our result in Proposition 2 suggests that this should be exactly the policy of search engines because it gives them the flexibility to control poaching to the optimal degree to make it most beneficial to them. Furthermore, we find from the model that there are cases in which the weak firm practices poaching and benefits from it, the strong firm is hurt from poaching, and the search engine benefits from poaching by the weak firm. These results support the observation that some leading firms in their respective industries (e.g., Rosetta Stone, Louis Vuitton) sued search engines in an effort to prevent them from following a policy of allowing bidding on trademarks by competitors (Mullin 2010, 2011). The search engines won these lawsuits and have continued to allow poaching on trademarked keywords; at the same time, they continue to use ad relevance scores to handicap poaching. For these examples, the predictions from our model are in close agreement with the actual behavior of the strong firms, the weak firms, and the search engines.

To summarize, our basic model shows that firms in an industry, especially firms with relatively smaller advertising budgets, have the incentive to poach in sponsored search. Under certain conditions, the firms being poached accommodate poaching and reduce their spend in sponsored search (which is the retreat strategy); under other conditions, they protect their own keyword by investing more in sponsored search (which is the defend strategy). Surprisingly, even though poaching leads to more competition in the search engine's auctions, search engines have the incentive to handicap poaching to protect larger firms so that they do not reduce their investment in sponsored search.

5. Extensions to the Basic Model

5.1. Endogenous Budget

In the main model, we made the assumption that the firms' advertising budgets, B_i , are exogenously specified. To check the robustness of our results to this assumption, in this section, we relax this assumption and endogenously determine how much each firm will spend on advertising. We find our results to be qualitatively unchanged.

We assume that each firm has a certain valuation per customer that represents how much the firm gains from each new customer. This can represent price minus cost, i.e., how much the firm profits from selling the product to each customer. Let v_i be the valuation per customer of Firm i , which is common knowledge. Without loss of generality, we assume that $v_1 \geq v_2 > 0$ and call Firm 1, which is the firm with the (weakly) higher valuation per customer, the stronger

firm and Firm 2 the weaker firm. Firms endogenously decide how much to spend on advertising based on values v_i and equilibrium costs of advertising. We assume the following sequence of actions and solve for the subgame-perfect equilibria of the game. In the first round, the firms decide how much to spend on advertising; i.e., Firm i decides B_i , $i \in \{1, 2\}$. In the second round, each firm decides how much to spend on traditional advertising, on search advertising of its own keyword, and on search advertising of the competitor's keyword. In the third round, customers make purchase decisions and firms collect profits. We note that the model is analytically intractable with this extension, so we resort to numerical analysis. More details are available in §A.4 of the appendix.

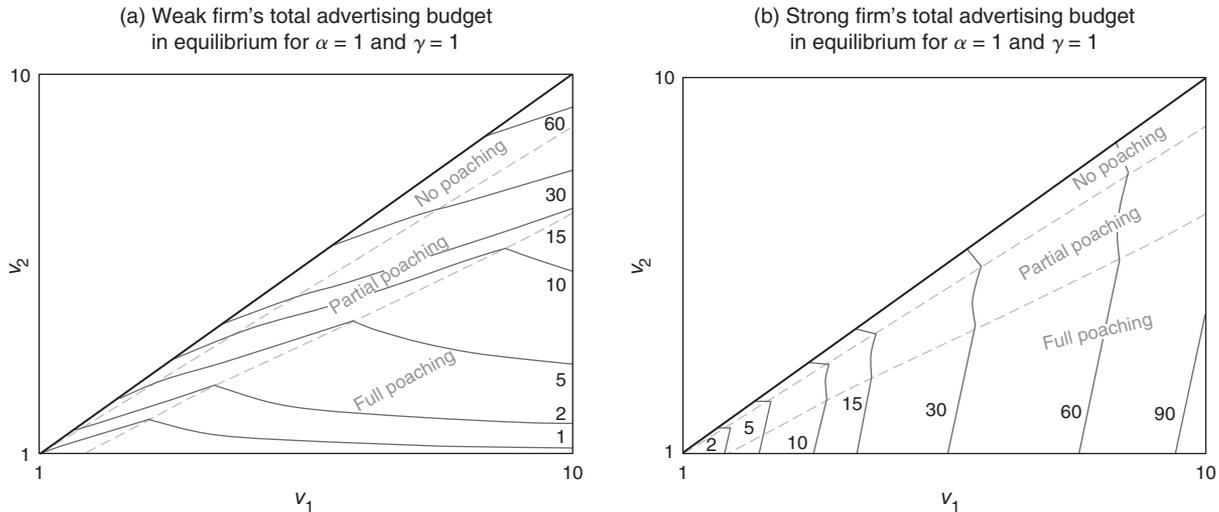
Figure 5 shows equilibrium advertising spendings of firms as functions of their valuations per customer. In this figure, we present the results for $v_1, v_2 \in [1, 10]$ and $v_1 \geq v_2$. This is a representative example, and the results are qualitatively unchanged for other ranges of valuations.

As expected, each firm's spending on advertising increases as its valuation per customer increases. We also note that Firm 1 always has a weakly higher total advertising budget than Firm 2. After deciding the total advertising budgets, the firms' decisions regarding how to split the budget between the different advertising options are exactly the same as those in §3. The dashed lines show the regions in which the weak firm poaches on the strong firm's keyword. When the valuations are close, the firms' advertising budgets are comparable, and they do not poach on each others' keywords. However, larger asymmetry in valuations between the firms leads to larger asymmetry in advertising budgets. Consequently, as in §3, when the budget asymmetry is large enough, the firm with the smaller advertising budget spends all or part of its budget for poaching on the competitor's keyword. Furthermore, as in §3, the strong firm's strategy when the weak firm poaches could be the retreat or the defend strategy, depending on the degree of poaching. In the retreat case, the search engine can benefit from partially penalizing poaching.

5.2. Multiple Search Advertising Slots per Keyword

In the main model, we made the assumption that only one firm is displayed in the sponsored links section after a keyword search. In this section, we assume that there are two advertising slots available for each keyword, and both firms could be displayed. Following the typical assumption in the literature, we assume that the second slot has lower probability of click than the first slot. In particular, we assume that the second slot gets θ times as many clicks as the first

Figure 5 Firms' Total Advertising Allocation as Functions of Per-Customer Valuation



slot, where $\theta < 1$. There are two passive players who always bid R . Therefore, Firms 1 and 2 have to bid at least R in order to be displayed.

Changing the number of slots per keyword affects the result of Lemma 1. In particular, assuming that R is small enough, if the budget of one firm is much larger than that of the other firm for a keyword, the weaker firm's share from the supply of that keyword would be very small in case of a single slot. However, when there are two advertising slots available, the weaker firm can always have at least $\theta/(1 + \theta)$ share of the total supply by bidding R and always getting the second slot.

For two firms with two advertising slots per keyword, the result of Lemma 1 can be generalized to the following.

LEMMA 2. Suppose that n queries are made for a specific keyword and the two slots for each query are sold using a generalized second-price auction. Suppose that the click-through rate of the second slot is $\theta < 1$ times that of the first slot and the total number of clicks per search is normalized to 1; i.e., the average number of clicks on the first slot is $1/(1 + \theta)$ and on the second slot is $\theta/(1 + \theta)$. Assume that two bidders, Bidder 1 and Bidder 2, with budgets L_1 and L_2 (where $L_1 + L_2 \geq nR$ and $L_i \geq \theta nR/(1 + \theta)$ for $i \in \{1, 2\}$), are participating in the auctions, and each bidder wants to maximize its total number of clicks. In any subgame-perfect equilibrium of the game, depending on the tie-breaking rule used by the search engine for equal bids, Bidder i wins the first slot t times (out of n times), where

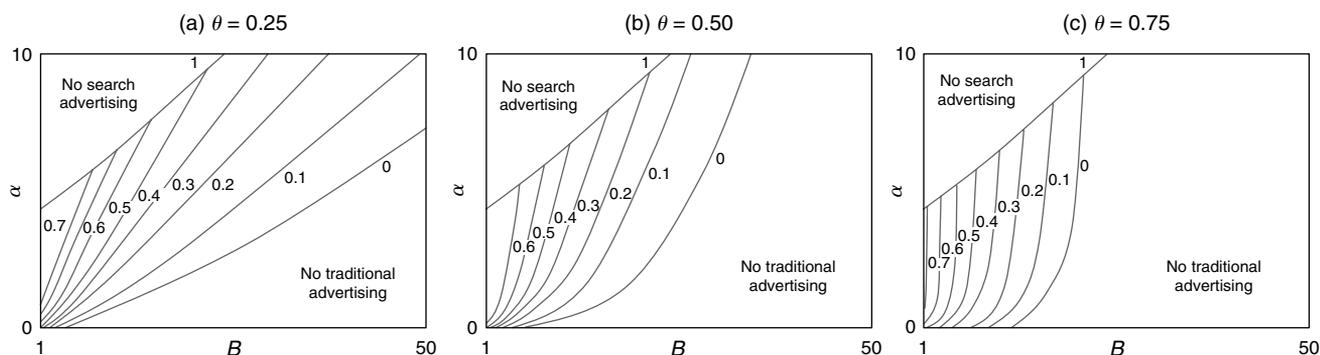
$$t \in \left\{ \left\lfloor \frac{n((1 + \theta)L_i - \theta nR)}{L_i + L_j + \theta(-2nR + L_i + L_j)} \right\rfloor, \left\lceil \frac{n((1 + \theta)L_i - \theta nR)}{L_i + L_j + \theta(-2nR + L_i + L_j)} \right\rceil \right\}.$$

Note that if a bidder wins the first slot t times, its expected number of clicks is $t/(1 + \theta) + (n - t)\theta/(1 + \theta)$. Also note that if $L_1 + L_2 < nR$, supply exceeds demand, and each bidder's best strategy is to always bid R and collect $\lfloor L_i/R \rfloor$ clicks. Similarly, if $L_i < \theta nR/(1 + \theta)$, Bidder i 's best strategy is to always bid R and collect $\lfloor L_i/R \rfloor$ clicks; the competitor could bid anything larger than R in response.

Lemma 2 is a generalization of Lemma 1. In particular, when $\theta = 0$, the number of Bidder i 's clicks simplifies to $\min(\lfloor L_i/R \rfloor, \lfloor nL_i/(L_1 + L_2) \rfloor)$ or $\min(\lfloor L_i/R \rfloor, \lceil nL_i/(L_1 + L_2) \rceil)$, depending on tie-breaking rules, which is equivalent to the result of Lemma 1. Since in practice the value of n is relatively large, we use $t = n((1 + \theta)L_i - \theta nR)/(L_i + L_j + \theta(-2nR + L_i + L_j))$ instead of $\lfloor n((1 + \theta)L_i - \theta nR)/(L_i + L_j + \theta(-2nR + L_i + L_j)) \rfloor$ and $\lceil n((1 + \theta)L_i - \theta nR)/(L_i + L_j + \theta(-2nR + L_i + L_j)) \rceil$ to simplify the formulation.

Using Lemma 2, we can calculate the number of clicks that each firm gains for any given set of search advertising budgets. Therefore, using the same techniques as in §3, we can calculate how firms allocate their budgets to traditional and search advertising channels in equilibrium. We note that the model is analytically intractable with this extension, so we resort to numerical analysis. More details are available in §A.4 of the appendix.

In Figure 6, we present the results for representative sets of values of the parameters; the results are qualitatively unchanged for other ranges of values. Figure 6 shows how much the weak firm spends on search advertising in equilibrium for different values of θ . (For this figure, as before, we normalize the budget of the weak firm to 1 and denote the budget of the strong firm using $B \geq 1$.) We see that while θ affects some details of budget allocation, the main insights from §3 hold regarding budget allocation strategies.

Figure 6 Weak Firm's Fraction of Budget Spent on Traditional Advertising in Equilibrium

In particular, the weak firm only poaches on the strong firm's keyword if B is large enough and α is small enough. Furthermore, the percentage of budget allocated to poaching increases with B and decreases with α . If the strong firm uses the retreat strategy when the weak firm poaches, the search engine benefits by partially penalizing poaching.

Not surprisingly, poaching happens more often when there are two advertising slots available. This is because the second slot is always available at price R which motivates the other firm to poach. If there is no poaching penalty (the relevance multiplier is 1), under certain conditions, the weak firm chooses to get the second slot for both keywords while the strong firm gets the first slot.

6. Conclusions and Discussion

In this paper, we study poaching by firms in sponsored search advertising. A firm can spend on traditional channels of advertising such as television, print, and radio to create awareness, attract customers, and increase the search volume of its keyword at a search engine. Alternatively, a firm may limit its awareness-creating activities and spend its budget on stealing the potential customers of its competitor by advertising on the competitor's keyword in sponsored search, which we call poaching. Using a game theory model, we study the poaching behavior of firms and the search engine's policy on poaching.

We focus on the impact of two main factors: the fraction of customers whose purchase decisions are influenced by sponsored search after being exposed to a traditional ad and the degree of asymmetry between firms in terms of their advertising budgets. We find a number of interesting results. Specifically, we find interesting poaching outcomes when there is a significant overflow of customers into the sponsored search channel. In this case, the larger-budget firm creates awareness for its product through traditional advertising, whereas the smaller-budget firm has the incentive to poach on the larger-budget firm's keyword in sponsored search. The smaller-budget firm may

spend all or a part of its budget on poaching, depending on the degree of budget asymmetry. The larger-budget firm may do one of two things when it is poached: it can move more of its budget to traditional advertising to avoid head-on competition in the sponsored search channel (i.e., it may accommodate poaching by retreating from the sponsored search channel) or, in case the traditional channel is saturated (i.e., the amount of advertising in the traditional channel is such that allocating more money to it will only lead to reduced effectiveness of spending in this channel), the larger-budget firm can choose to defend its keyword against poaching in sponsored search. The larger-budget firm follows the defend strategy when its budget is significantly larger than that of the smaller-budget firm.

Poaching leads to higher competition in the search engine's auctions, and at first thought, it would appear that poaching should always be beneficial to the search engine. However, we find that, in some situations, the search engine has the incentive to limit poaching. This happens when the larger-budget firm follows a strategy of retreating from sponsored search in response to poaching. By handicapping poaching by the smaller-budget firm, the search engine reduces the incentive of the larger-budget firm to move its budget away from sponsored search. This offers a novel explanation for why search engines allow bidding on trademarked keywords by competitors yet still employ ad relevance measures to weaken bids of firms bidding on competitors' keywords.

We also consider extensions of the model that confirm the robustness of our results to key assumptions. Specifically, we find that making the total advertising budget of the firms endogenous and allowing multiple firms to be listed in response to a keyword search leads to qualitatively the same results. Introducing other variations to the model (not explicitly considered in the paper), such as allowing one firm to exogenously have a better reputation or higher popularity than the other (leading to more keyword searches on search engines without immediately preceding

awareness-generating advertising), allowing firms to bid on category-specific keywords (e.g., “tablet” in addition to “iPad” and “Galaxy Tab”), and assuming a reduced click-through rate when a competitor’s ad is displayed in response to a keyword search for a particular firm, will affect the extent of poaching but will lead to qualitatively the same results on firm advertising strategies and search engine response.

Our work sheds light on the poaching behavior of firms in a multichannel advertising setting. There are many other related problems that may be studied in future work. In particular, firms are not vertically differentiated in our model. Perhaps a joint model of our work and Desai et al. (2014) that allows differentiation in a multichannel advertising model would be an interesting direction for future work. Another interesting direction to consider is to understand the consequences of poaching among *channel partners* (e.g., Chiou and Tucker 2012). For example, online travel agencies such as Orbitz bid on keywords such as “Sheraton Hotel in San Francisco,” trying to win the potential customers of Sheraton and *resell* them back to Sheraton. This poaching not only decreases Sheraton’s profit from its own customers (because it has to share a part of the revenue with Orbitz for delivering this customer) but also increases the cost acquiring customers (because it increases the price of sponsored search advertising for Sheraton). It would be interesting to know how partners should react to such poaching behavior.

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Appendix

A.1. Proof of Lemma 1

Suppose that a seller wants to sell n units of an item, one by one, each in a second-price auction. We call this mechanism a *sequential second-price auction*. This mechanism captures the essence of the mechanism that search engines use to sell their advertising slots—whenever a consumer searches a keyword, the search engine runs a (generalized) second-price auction to sell the advertising slot. The seller can, instead, sell the n units using a *market-clearing-price mechanism*. In the market-clearing-price mechanism, the seller sets the highest price p at which demand meets supply. The following theorem proves that the two mechanisms lead to the same outcome.

THEOREM A1. *Suppose that n identical items are sold in a sequential second-price auction with reserve price R . Two bidders, Bidder 1 and Bidder 2, with budgets B_1 and B_2 are participating in the auctions, and each bidder wants to maximize the number of items that she wins. The outcome of any subgame-perfect equilibrium of the game is equivalent to the outcome of a market-clearing-price mechanism.*

PROOF. We first present the outline of the proof. Suppose that the market-clearing price is p . We prove that if Bidder i always bids p , he can always win at least $\lfloor B_i/p \rfloor$ items. We also show that if the other bidder plays optimally, Bidder i can never win more than $\lfloor B_i/p \rfloor + 1$ items. If both players play optimally, whether bidder i wins $\lfloor B_i/p \rfloor$ or $\lfloor B_i/p \rfloor + 1$ items would depend on the tie-breaking rules set by the auctioneer. In the following, we present the details of this proof.

First, suppose $\lfloor B_1/R \rfloor + \lfloor B_2/R \rfloor \geq n$; i.e., the market-clearing price is at least R . Let p be the market-clearing price; i.e., $\lfloor B_1/p \rfloor + \lfloor B_2/p \rfloor = n$. Note that if the first player bids p in all rounds, he can make sure that he wins at least $n - \lfloor B_2/p \rfloor = \lfloor B_1/p \rfloor$ items because his opponent has to pay p for every item that he wins. Similarly, if the second player bids p in all rounds, he can make sure that he wins at least $n - \lfloor B_1/p \rfloor = \lfloor B_2/p \rfloor$ items. Since $\lfloor B_1/p \rfloor + \lfloor B_2/p \rfloor = n$, we see that player i cannot win more than $\lfloor B_i/p \rfloor$ items, which means that he wins exactly $\lfloor B_i/p \rfloor$ items.

Now, consider the case where $\lfloor B_1/R \rfloor + \lfloor B_2/R \rfloor < n$. In this case, we know that if the larger bid in the auction is smaller than R , the item in that round will be left unallocated. Also, if the larger bid is at least R , but the smaller bid is less than R , the item will be allocated but at price R (instead of the second-highest bid). Given this information, bidding anything below R , in any round, is weakly dominated. Also, by bidding R , bidder i can make sure that he wins at least $\lfloor B_i/R \rfloor$ items. Since bidder i can never win more than $\lfloor B_i/R \rfloor$ items, in any subgame-perfect equilibrium, he wins exactly $\lfloor B_i/R \rfloor$ items.

In this proof, we made an implicit assumption that there exists p such that $\lfloor B_1/p \rfloor + \lfloor B_2/p \rfloor = n$. This assumption is not always true because $\lfloor B_i/p \rfloor$ is not a continuous function of p . In particular, if $B_1/p = \lfloor B_1/p \rfloor$, $B_2/p = \lfloor B_2/p \rfloor$, and $B_1/p + B_2/p = n + 1$, then we have $\lfloor B_1/(p + \epsilon) \rfloor + \lfloor B_2/(p + \epsilon) \rfloor = n - 1$, for any $\epsilon > 0$. In situations like this, the market-clearing-price mechanism sets the price to p and assigns the last item to one of the bidders using an arbitrary tie-breaking rule. Next, we prove the theorem for these cases.

Let p be the highest price at which $\lfloor B_1/p \rfloor + \lfloor B_2/p \rfloor \geq n + 1$. We know that $\lfloor B_1/(p + \epsilon) \rfloor + \lfloor B_2/(p + \epsilon) \rfloor < n$. Therefore, we must have $\lfloor B_i/p \rfloor = B_i/p = \lfloor B_i/(p + \epsilon) \rfloor + 1$ for $i \in \{1, 2\}$. Using the same argument as before, if bidder i always bids p , he will win at least $B_i/p - 1$ items. Furthermore, at least p of his budget will be left if he wins exactly $B_i/p - 1$ items. Therefore, in any subgame-perfect equilibrium, one of the bidders (suppose without loss of generality, Bidder 1) uses all of his budget and gets B_1/p items, whereas the other bidder gets $B_2/p - 1$ items. Consider the last item that is given to Bidder 1, and let it be item x (for $1 \leq x \leq n$). Note that when item x is being given to Bidder 1, assuming that Bidder 2 plays optimally, Bidder 2 has a budget of at least p and Bidder 1 has a budget of at most p . Therefore, if both players play optimally, they can both bid p and potentially

win item x . The auctioneer has to break ties to decide who wins item x . \square

Note that the subgame-perfect equilibrium of a sequential second-price auction is not unique, and there are many different optimal actions that the players may take in each period. However, they all eventually lead to the same outcome described in Theorem A1. The result above is also robust to different variations to the model. For instance, if all of the customers arrive at once, or if the firms cannot change the bids for each customer, or if the search engine uses a first-price auction instead of a second-price auction, we obtain the same outcome.

Furthermore, in the case of equal bids (as a result of equal valuations) by the two firms, we can show that if the search engine follows a rule such that it breaks the tie in favor of the bidder with larger leftover budget at that point in time, then there is no leftover budget with either firm at the end of the game. Below, we state this more formally as a lemma and provide a sketch of the proof.¹²

LEMMA. *Assuming that there is one unit of a divisible good, if, in the case of equal bids, the search engine breaks the tie in favor of the bidder with larger leftover budget at that point in time, then the total leftover budget is zero at the end of the sequential auction in any equilibrium.*

PROOF SKETCH. In the case of a divisible good, we assume that each differential unit is sold in a second price auction. When the item is divisible, we do not have to use $\lfloor \cdot \rfloor$ notation. If the market clearing price is p , Firm i can win B_i/p fraction of the item by always bidding p .

First note that if both firms always bid p , the leftover budget will be zero. In particular, if both firms always bid p , the first $|B_1 - B_2|/p$ fraction of the item will be given to the firm with the larger budget. After that, the unit will be divided uniformly between the two firms. Both bidders' budgets will last until the end of the auction.

Next, assume for the sake of contradiction that Firm 1 uses a strategy other than always bidding p . Also, assume that Firm 1 wins B_1/p fraction of the item but has a positive leftover budget. Suppose that Firm 1's bid over the unit interval where the item is being auctioned is $f(\cdot)$. We only consider strategies where $f(\cdot)$ is measurable (the outcome of the auction is not well defined if $f(\cdot)$ is not measurable). Consider the set $S_{<} = \{x \mid f(x) < p\}$. Note that if $|S_{<}| > 0$, then the average price of Firm 2 for the item will be strictly less than p , which implies that Firm 2 buys strictly more than B_2/p fraction of the item. Measure $|S_{<}| = 0$ implies that at any point during the auction, the leftover budget of Firm 2 when Firm 1 uses strategy $f(\cdot)$ is greater than or

equal to Firm 2's leftover budget if Firm 1 was always bidding p . We know that if Firm 1 always bids p , the budget of Firm 2 will last until the end of the auction. Therefore, even if Firm 1 bids $f(\cdot)$, Firm 2's budget will last until the end. Therefore, Firm 1's average price will be p which shows that its leftover budget will be zero at the end of the auction. \square

A.2. Analysis for §3

Existence of a Pure-Strategy Nash Equilibrium. The profit of Firm i , as a function of T_i , is

$$\pi_i = \alpha \frac{T_i}{T_1 + T_2} \sqrt{T_1 + T_2} + \min\left(\frac{B_i - T_i}{R}, \frac{B_i - T_i}{B_1 + B_2 - T_1 - T_2} \sqrt{T_1 + T_2}\right).$$

Both functions, $(\alpha(T_i/(T_1 + T_2)))\sqrt{T_1 + T_2} + (B_i - T_i)/R$ and $(\alpha(T_i/(T_1 + T_2)))\sqrt{T_1 + T_2} + ((B_i - T_i)/(B_1 + B_2 - T_1 - T_2)) \cdot \sqrt{T_1 + T_2}$, are continuous and concave in T_i , and hence their minimum is also continuous and concave in T_i . Furthermore, the strategy of Firm i is defined by the closed interval $T_i \in [0, B_i]$. Therefore, by the Debreu-Glicksberg-Fan theorem, a pure-strategy Nash equilibrium exists.

Strategies. On taking the derivative of π_i^1 with respect to T_i , we get

$$-\frac{1}{R} - \frac{\alpha T_i}{2(T_i + T_j)^{3/2}} + \frac{\alpha}{\sqrt{T_i + T_j}}.$$

Setting the above equal to zero and solving for T_i , we get the value of T_i^1 as follows:

$$\begin{aligned} T_i^1 = & \frac{1}{12} [\alpha^2 R^2 - 12T_j + (\alpha^4 R^4 + 24\alpha^2 R^2 T_j) \cdot (\alpha^6 R^6 + 36\alpha^4 R^4 T_j \\ & + 216\alpha^2 R^2 T_j^2 + 24\sqrt{3}\sqrt{\alpha^4 R^4 T_j^3 (\alpha^2 R^2 + 27T_j)})^{1/3}]^{-1} \\ & + (\alpha^6 R^6 + 36\alpha^4 R^4 T_j + 216\alpha^2 R^2 T_j^2 \\ & + 24\sqrt{3}\sqrt{\alpha^4 R^4 T_j^3 (\alpha^2 R^2 + 27T_j)})^{1/3}. \end{aligned}$$

Similarly, by taking the derivative of π_i^2 with respect to T_i and setting it equal to zero, we get T_i^2 , which is the solution to the following equation:

$$\begin{aligned} & (\alpha(T_i + 2T_j) + ((T_i + T_j)(B_i^2 + B_i(B_j - 2T_i - T_j) + (T_i + T_j)(T_i + 2T_j) \\ & - B_j(3T_i + 2T_j)))/(B_i + B_j - T_i - T_j)^2) \cdot (2(T_i + T_j)^{3/2})^{-1} = 0. \end{aligned}$$

The value of T_i at which π_i^2 is maximized has a closed-form solution, but the expression is cumbersome to interpret and does not directly provide any insight. Therefore, we do not present the closed-form solution for T_i^2 .

Finally, T_i^3 is the solution to the following equation:

$$\frac{(B_i - T_i)\sqrt{T_i + T_j}}{B_i + B_j - T_i - T_j} = \frac{B_i - T_i}{R},$$

which gives us

$$T_i^3 = B_i + B_j + \frac{R^2}{2} - \frac{1}{2} R \sqrt{4(B_i + B_j) + R^2} - T_j.$$

Combining the three cases, we obtain

$$T_i^* = \begin{cases} T_i^1 & \text{if } T_i^1 > T_j^3, \\ T_i^2 & \text{if } T_i^2 < T_j^3, \\ T_i^3 & \text{otherwise.} \end{cases}$$

We do not allow values of T_i^* outside the interval $[0, B_i]$.

¹² Interestingly, the assumption of breaking a tie in favor of the larger leftover budget firm parallels the practice of "bid throttling" by search engines. Although search engines do not reveal complete details of their mechanisms, they claim that their mechanisms try to keep the budgets of the advertisers at nonzero until the end of the campaign, which is the essence of our assumption above. The advertisers benefit from bid throttling because their campaigns keep running until the end of the campaign, but this also implies that their nonwinning bids are being used against their competitors by the search engine; i.e., ensuring that no advertiser runs out of money too soon keeps the auction competitive for a longer time.

Analysis of Equilibria. To calculate the equilibrium strategies of the firms, we solve the following system of equations:

$$BR_1(T_2^e) = T_1^e \quad \text{and} \quad BR_2(T_1^e) = T_2^e.$$

The solution to T_i^e could be T_i^1 , T_i^2 , T_i^3 , 0, or B_i for each $i \in \{1, 2\}$. Therefore, we have up to 25 combinations, each of which gives us qualitatively different equilibrium behavior. However, many of these combinations cannot happen in equilibrium. In particular, if Firm i uses T_i^3 in equilibrium, the other Firm j cannot be using strategies T_j^1 or T_j^2 because strategy T_i^3 is relevant only if the price of search advertising is R and the third passive player has zero allocation. Similarly, if Firm i uses strategy T_i^2 , the other firm must be using either T_j^2 or 0 as a strategy. Finally, if Firm i uses strategy T_i^1 , the other firm can be using strategy T_j^1 or B_j . Therefore, we have the following five possible strategy pairs.

Strategy Pair 1 (B_1, B_2): In this case, both firms spend all of their budget on the traditional channel. This corresponds to *Region O* in Figure 2. Firms spend all of their budget on traditional advertising only if the marginal return on traditional advertising is at least $1/R$. In other words, if Firm i moves part of its budget from traditional advertising to search advertising, its payoff decreases. Therefore,

$$\frac{\partial}{\partial x} \left(\alpha \sqrt{B_i - x + B_j} \frac{B_i - x}{B_i - x + B_j} \right)$$

must be less than or equal to $1/R$ at $x = 0$. This simplifies to

$$\alpha \geq \frac{2(B_i + B_j)^{3/2}}{(B_i + 2B_j)R}, \quad (1)$$

which defines the condition under which Firm i spends all of its budget on traditional advertising.

Strategy Pair 2 (T_1^1, B_2): In this case, Firm 2 spends all of its budget on traditional advertising. There may be an excess of supply in the search advertising channel (i.e., the third passive player has nonzero allocation), and part of Firm 1's advertising budget overflows to search advertising. The price of search advertising is R . We divide this case into three subcases. In the first two subcases, there is an excess of search advertising supply. Firm 1's spending on traditional advertising does not change with changes in Firm 1's budget and is given by

$$\hat{B} = \frac{1}{12} [-12B_2 + \alpha^2 R^2 + [216\alpha^2 B_2^2 R^2 + 36\alpha^4 B_2 R^4 + \alpha^6 R^6 + 24\sqrt{3} \cdot \sqrt{\alpha^4 B_2^3 R^4 (27B_2 + \alpha^2 R^2)}]^{1/3} + \alpha^{4/3} (24B_2 R^2 + \alpha^2 R^4) [216B_2^2 R^2 + 36\alpha^2 B_2 R^4 + \alpha^4 R^6 + 24B_2^{3/2} \sqrt{81B_2 R^4 + 3\alpha^2 R^6}]^{-1/3}].$$

For $R = 1/2$ and $B_2 = 1$ in Figure 2, the spending of Firm 1 on traditional advertising reduces to

$$\hat{B}_{\text{Fig2}} = \frac{\alpha^2}{48} - 1 + \frac{\alpha^{4/3} (96 + \alpha^2)}{48(144\alpha^2 + \alpha^4 + 192(18 + \sqrt{3}\sqrt{108 + \alpha^2}))^{1/3}} + \frac{(\alpha^2(144\alpha^2 + \alpha^4 + 192(18 + \sqrt{3}\sqrt{108 + \alpha^2}))^{1/3})}{48}. \quad (2)$$

The difference between subcase 1 and subcase 2 is that in subcase 1, Firm 1 only advertises on its own keyword, whereas in subcase 2 Firm 1 also poaches on Firm 2's

keyword. Subcase 1 corresponds to Region A in Figure 2 and subcase 2 corresponds to Region B_1 . Transition from Region A to Region B_1 happens when Firm 1 starts using Firm 2's search advertising supply. In other words,

$$\frac{B - \hat{B}}{R} > \frac{\hat{B}}{\sqrt{1 + \hat{B}}} \quad (3)$$

defines the boundary between Region A and Region B_1 .

In the third subcase, there is no excess of search advertising supply. As Firm 1's budget increases in this region, the new budget is divided between traditional advertising and search advertising to increase search volume and keep the price of search advertising at R . This corresponds to Region B_2 in Figure 2. Firm 1's spending on traditional advertising in this region is

$$\tilde{B} = \frac{1}{2} (2B_i + R(R - \sqrt{4(B_j + B_i) + R^2})).$$

Regions B_1 and B_2 have qualitatively similar properties. In particular, Firm 1 poaches on Firm 2's keyword in both regions while Firm 2 spends all of its budget on traditional advertising. The boundary between the two regions is defined by

$$\hat{B} = \tilde{B}. \quad (4)$$

Transition from Region B to Region C happens when Firm 2's marginal return from the traditional channel becomes less than or equal to $1/R$. In other words,

$$\frac{\partial}{\partial x} \frac{\alpha(B_2 - x)}{\sqrt{B_2 - x + B_1}}$$

is less than or equal to $1/R$ at $x = 0$. The solution is given by

$$\alpha > \frac{(R(R - \sqrt{4(B_1 + B_2) + R^2}) + 2B_1 + 2B_2)^{3/2}}{\sqrt{2}R(R(R - \sqrt{4(B_1 + B_2) + R^2}) + 2B_1 + B_2)}, \quad (5)$$

which defines the boundary between Region B and Region C.

Strategy Pair 3 (T_1^3, T_2^3): In this case, each firm spends enough money on search advertising so as to purchase any search advertising supply not purchased by the other firm. Spending more on search advertising leads to an increase in the price of search advertising, which neither firm is willing to do. In this case, we obtain multiple equilibria. The following equation shows the relation between firms' spending on traditional advertising:

$$T_1 + T_2 = \frac{1}{2} (2B_1 + 2B_2 + R^2 - R\sqrt{4B_1 + 4B_2 + R^2}).$$

Although there are multiple equilibria in Region C, there is an upper bound U_i for each firm on how much it spends on traditional advertising. In other words, rather than spending more than U_i on the traditional channel, Firm i prefers to spend more on search advertising even if it leads to a higher price in search advertising. These upper bounds are given by¹³

$$U_1 = (-101 + 29\sqrt{17 + 16B} - 2B(4B(47 + 16B - 5\sqrt{17 + 16B}) - 35(-5 + \sqrt{17 + 16B})) + \alpha(-49 + 9\sqrt{17 + 16B} + 8B(-10 - 4B + \sqrt{17 + 16B}))) \cdot (2(-85 + 13\sqrt{17 + 16B}))$$

¹³ Since the expressions for U_i are cumbersome, we only present the expressions for the case of $R = \frac{1}{2}$, $B_2 = 1$, and $B_1 = B$.

$$+ \alpha(-9 - 8B + \sqrt{17 + 16B}) + 4B(-37 - 16B + 3\sqrt{17 + 16B}))^{-1}$$

and

$$U_2 = (-271 + 55\sqrt{17 + 16B} + 16B^2(-13 + \sqrt{17 + 16B}) + 68B(-7 + \sqrt{17 + 16B}) + \alpha(-49 + 9\sqrt{17 + 16B}) + 8B(-10 - 4B + \sqrt{17 + 16B})) \cdot (2(-85 + 13\sqrt{17 + 16B}) + \alpha(-9 - 8B + \sqrt{17 + 16B}) + 4B(-37 - 16B + 3\sqrt{17 + 16B}))^{-1}.$$

We can divide this case into two subcases representing Regions C and Region D₁. Both subcases have multiple equilibria, and the equilibrium price of search advertising is R in both. However, in the subcase corresponding to Region C, either firm may poach in equilibrium; in the subcase corresponding to Region D₁, only Firm 2 poaches in equilibrium. The degree of poaching of Firm 2 varies in different equilibria in Region D₁.

Let ω be the fraction defined as of the sum of budgets spent on traditional advertising divided by the sum of total budgets. Since the price of search advertising is R and all of search advertising supply is sold, ω is constant over different equilibria of Region C and is equal to

$$\omega = \frac{T_1 + T_2}{B_1 + B_2} = \frac{2B_1 + 2B_2 + R(R - \sqrt{4(B_1 + B_2) + R^2})}{2(B_1 + B_2)}.$$

If each firm spends exactly ω fraction of its budget on traditional advertising, then there is no poaching in equilibrium. If U₂ < ω, then we are in Region D₁. In other words, U₂ > ω defines the boundary between Region C and Region D₁, which is given by

$$\alpha > (2B_2(-R\sqrt{4(B_1 + B_2) + R^2} + 3B_1 + R^2 - 1) + (2B_1 - 1)(R(R - \sqrt{4(B_1 + B_2) + R^2}) + 2B_1) + 2B_2^2) \cdot ((2B_1 + 2B_2 - 1)(R(\sqrt{4(B_1 + B_2) + R^2} - R) + 2B_2(B_1 + B_2 - 1) - 2B_1))^{-1}. \tag{6}$$

Finally, the equilibrium price of search advertising increases from R when U₁ + U₂ < ω(B₁ + B₂). This defines the boundary between Region D₁ and Region D₂. This is given by

$$\alpha > \frac{\sqrt{4(B_1 + B_2) + R^2} - 2R}{3R}. \tag{7}$$

Strategy Pair 4 (T₁², T₂²): In this case, the price of search advertising is more than R and firms compete in both channels. This case corresponds to Region D₂ in Figure 2. Firm i's spending on traditional advertising is

$$\frac{(1 + 3\alpha)((4 + 9\alpha)B_i - 2B_j)}{6 + 33\alpha + 27\alpha^2}.$$

Firm 2's spending on traditional advertising becomes zero when

$$\alpha < \frac{2}{9} \left(\frac{B_1}{B_2} - 2 \right), \tag{8}$$

which defines the condition for separation between Region D₂ and Region D₃.

Table A.1 Mathematical Expressions for the Boundaries Between Regions in Figure 2

Boundary label	Locus
e ₁	$\alpha = \frac{4(B + 1)^{3/2}}{(B + 2)}$
e ₂	$2(B - \hat{B}_{Fig2}) = \frac{\hat{B}_{Fig2}}{\sqrt{1 + \hat{B}_{Fig2}}}$
e ₃	$\hat{B}_{Fig2} = \frac{1}{8} + B - \frac{1}{8}\sqrt{17 + 16B}$
e ₄	$\alpha = \frac{(9 + 8B - \sqrt{17 + 16B})^{3/2}}{\sqrt{2}(5 + 8B - \sqrt{17 + 16B})}$
e ₅	$\alpha = \frac{B(\sqrt{17 + 16B} - 1) - 1}{1 + 2B}$
e ₆	$\alpha = \frac{1}{3}(\sqrt{17 + 16B} - 2)$
e ₇	$\alpha = \frac{2}{9}(B - 2)$

Notes. Note that (i) expressions for the loci e₁, e₂, e₃, e₄, e₅, e₆, and e₇, respectively, are obtained by plugging in B₁ = B, B₂ = 1, and R = 1/2 in (1), and (3)–(8), respectively; and (ii) \hat{B}_{Fig2} is a function of α and B, as defined in (2).

Strategy Pair 5 (T₁², 0): As in the previous case, the price of search advertising is more than R. In this case, Firm 2 spends all of its budget for search advertising. Firm 1's spending on traditional advertising is

$$\frac{2B_1 + 2\alpha B_1 + 3B_2 + 2\alpha B_2 - \sqrt{B_2}\sqrt{8B_1 + 8\alpha B_1 + 9B_2 + 8\alpha B_2}}{2(1 + \alpha)}.$$

This case corresponds to Region D₃ in Figure 2.

Analysis of the Benchmark Scenario. In §3, we use the situation in which poaching is not possible (or allowed) as the benchmark to study the effect of poaching. Here, we provide the details of the analysis of the benchmark.

If poaching is not possible, Firm i's profit from spending T_i on the traditional channel, given that the other firm spends T_j on the traditional channel, is

$$\pi'_i = \alpha \frac{T_i}{T_1 + T_2} \sqrt{T_1 + T_2} + \min\left(\frac{B_i - T_i}{R}, \frac{T_i}{T_1 + T_2} \sqrt{T_1 + T_2}\right).$$

Using the same method as before, we have

$$\pi_i^1 = \alpha \frac{T_i}{\sqrt{T_1 + T_2}} + \frac{B_i - T_i}{R} \quad \text{and} \quad \pi_i^2 = \alpha \frac{T_i}{\sqrt{T_1 + T_2}} + \frac{T_i}{\sqrt{T_1 + T_2}}.$$

Similarly, we define T_i¹ and T_i² to be the values of T_i at which π_i¹ and π_i² are maximized, respectively. Also, we let T_i³ be the value of T_i at which

$$\frac{B_i - T_i}{R} = \frac{T_i}{\sqrt{T_1 + T_2}}.$$

Although we are using the same method as before, there are two important points regarding the solution for our benchmark analysis. First, π_i² is an increasing function of T_i. Therefore, it is always maximized at the boundary, which is either B_i or T_i³. Second, we have π_i¹ = π_i¹. Therefore, we also have T_i¹ = T_i¹. In other words, if a firm is using

strategy T_i^1 , it does not matter whether or not poaching is allowed. Intuitively, π_i^1 is only relevant when there is an excess of search advertising supply. Therefore, the possibility of poaching does not affect firms' strategies.

Given that T_i^2 is always on the boundary, the optimal allocation for traditional advertising is

$$T_i^* = \begin{cases} T_i^1 & \text{if } T_i^1 > T_i^3, \\ T_i^3 & \text{otherwise.} \end{cases}$$

Firm i allocates T_i^1 to traditional advertising if the supply of search advertising exceeds its demand. Otherwise, it allocates T_i^3 to traditional advertising. Therefore, equilibrium responses of the firms to each other can be one of the following cases.

Strategy Pair 1 (B_1, B_2): In this case, both firms spend all of their budget on traditional advertising. This is equivalent to the first case of the previous section.

Strategy Pair 2 (T_1^1, B_2): In this case, Firm 2 spends all of its budget on traditional advertising. However, since Firm 1 has a larger budget and is more affected by concavity of the advertising response function, it spends part of its budget on search advertising. No firm has any incentive to poach (even if it were possible). This case is the same as the second case in the previous section.

Strategy Pair 3 (T_1^3, B_2): This is similar to the previous case except that Firm 1 uses all of its search advertising supply. If poaching were possible, Firm 1 would be spending less on the traditional channel and more on search advertising.

Strategy Pair 4 (T_1^3, T_2^3): In this case, both firms keep a balance between traditional advertising and search advertising. They spend enough on search advertising to buy all the supply of their own keyword while keeping the price low at R . They spend the rest of their budget on traditional advertising. Note that this is equivalent to the third case of the previous section in terms of total search advertising and total traditional advertising of both firms. Furthermore, the equilibrium is the same as the nonpoaching equilibrium of the third case in the previous section.

A.3. Analysis for §4

In this section, we present analysis of the situation in which poaching is penalized by a multiplier $0 \leq \gamma \leq 1$. Multiplying the poaching firm's bid by γ implies that the poaching firm has to pay $1/\gamma$ of what it was paying (if poaching is not penalized) to get the same outcome when poaching. In other words, if a firm was spending x for poaching when poaching is not penalized, its *effective* poaching budget becomes γx when poaching is penalized.

We assume that Firm i is poaching on Firm j 's keyword. Let p_i and p_j be the equilibrium prices of the two keywords. Note that price p_i is always less than or equal to price p_j because Firm i is poaching on Firm j 's keyword. Furthermore, as long as both keywords have positive search volume, the price of keyword j cannot be more than $1/\gamma$ times of the price of keyword i . In other words, $\gamma p_j \leq p_i$; otherwise, Firm j would have incentive to move part of its budget from keyword j to keyword i .

Firm i poaches only if it has already bought the search advertising supply of its own keyword. Therefore, if Firm i poaches, it spends $(T_i/\sqrt{T_i + T_j})p_i$ on its own keyword. Since

T_i is spent on traditional advertising, the budget of Firm i for the competitor's keyword is

$$B_i - T_i - \frac{T_i}{\sqrt{T_i + T_j}}p_i.$$

Since poaching is penalized, the *effective* budget of Firm i on Firm j 's keyword is

$$\gamma \left(B_i - T_i - \frac{T_i}{\sqrt{T_i + T_j}}p_i \right).$$

In an equilibrium in which poaching happens, we have three cases for keyword prices p_i and p_j :

1. If $p_i = R$, then $\gamma p_j \leq R$.
2. If $p_i > R$, then $\gamma p_j = p_i$.
3. If search volume of keyword i is zero (Firm i spends all of its budget for poaching), then p_j is not bounded.

In the second and third cases above, we have

$$p_j = \frac{\sqrt{T_j + T_i}(B_j + B_i\gamma - T_j - \gamma T_i)}{T_j + \gamma^2 T_i},$$

and p_i is determined accordingly. In the first case, price $p_i = R$ and

$$p_j = \frac{B_j\gamma\sqrt{T_j + T_i} - \gamma R T_i + B_j\sqrt{T_i + T_j} - T_j\sqrt{T_j + T_i} - \gamma T_i\sqrt{T_j + T_i}}{T_j}.$$

Given the prices, we can calculate equilibrium profits of the firms. The profit of the poaching firm (Firm i) is

$$\pi_i = (1 + \alpha) \frac{T_i}{\sqrt{T_i + T_j}} + \gamma \frac{B_i - T_i - (T_i p_i)/\sqrt{T_i + T_j}}{p_j}.$$

Similarly, the profit of the firm being poached (Firm j) is

$$\pi_j = \alpha \frac{T_j}{\sqrt{T_i + T_j}} + \frac{B_j - T_j}{p_j}.$$

These profit functions are calculated assuming that, given T_i and T_j , firms optimally split their search advertising budget between the two keywords. In other words, using these expressions, each firm only has to optimize its traditional advertising spending (given the traditional advertising spending of the other firm) to maximize its profit. This makes the rest of the analysis very similar to the analysis in the previous section. We calculate T_1^* and T_2^* using the same method as in the previous section and solve for the best-response system of equations to calculate the equilibrium.

Next, we derive the expressions that we need for the proof of Proposition 2. Consider a full-poaching equilibrium in which the price of search advertising is $R = 1/2$. The profit of the weak firm in this equilibrium is $\gamma/R = 2\gamma$. The strong firm's best response to full poaching of the weak firm, given by the solution T_j to $\sqrt{T_j} = 2(B + \gamma - T_j)$, is to spend

$$B + \gamma + \frac{1}{8} - \frac{1}{8}\sqrt{16B + 16\gamma + 1} \quad (9)$$

on traditional advertising. This implies that the search engine's revenue in this equilibrium is

$$\frac{1}{8}(7 - 8\gamma + \sqrt{1 + 16B + 16\gamma}). \quad (10)$$

Next, we derive the conditions for the full-poaching equilibrium to exist and the conditions for this equilibrium to be unique. The full-poaching equilibrium is unique if and only

if the weak firm benefits from increasing its search advertising budget even if the increase leads to higher search advertising price. In other words, the derivative of the weak firm's profit with respect to its search advertising spending must be positive at the boundary (where the weak firm is fully poaching). The profit of the weak firm is given by

$$\pi_i = (1 + \alpha) \frac{T_i}{\sqrt{T_i + T_j}} + \gamma \frac{1 - T_i - T_j / (2\sqrt{T_i + T_j})}{(\sqrt{T_j + T_i}(B - \gamma T_i + \gamma - T_j) - \gamma R T_i / T_j)}.$$

If we take the derivative of this expression with respect to T_i and have it less than or equal to 0, we get

$$4\alpha + \frac{2\gamma(-B + \gamma(2T_j + \sqrt{T_j} - 1) + T_j)}{(B + \gamma - T_j)^2} - \frac{2\gamma(2T_j + \sqrt{T_j})}{B + \gamma - T_j} + 4 \leq 0,$$

and replacing T_j with the strong firm's strategy, given by (9), we get

$$\begin{aligned} & 1 + \alpha + (\gamma(-1 - 8B - 8\gamma + \sqrt{1 + 16B + 16\gamma}) \\ & - \sqrt{2}\sqrt{1 + 8B + 8\gamma - \sqrt{1 + 16B + 16\gamma}}) \\ & \cdot (-1 + \sqrt{1 + 16B + 16\gamma})^{-1} \\ & + (4\gamma(1 - \sqrt{1 + 16B + 16\gamma} + 2\gamma(1 + 8B + 8\gamma - \sqrt{1 + 16B + 16\gamma}) \\ & + \sqrt{2}\sqrt{1 + 8B + 8\gamma - \sqrt{1 + 16B + 16\gamma}})) \\ & \cdot ((-1 + \sqrt{1 + 16B + 16\gamma})^2)^{-1} \leq 0 \end{aligned} \quad (11)$$

as the condition for uniqueness of the full-poaching equilibrium.

For the full-poaching equilibrium to exist, the weak firm must not benefit from increasing traditional advertising spending if the price of search advertising is R and it is fully poaching. If the weak firm increases its traditional advertising to T_i , its profit increases by

$$\frac{(\alpha + 1)T_i}{\sqrt{T_j + T_i}} - 2\gamma \left(\frac{T_i}{2\sqrt{T_j + T_i}} + T_i \right).$$

For the full-poaching equilibrium to exist, we want the derivative of the above expression at $T_i = 0$ to be less than or equal to zero. This simplifies to

$$\alpha - 2\gamma\sqrt{T_j} - \gamma + 1 \leq 0.$$

Replacing T_j by the expression in (9) we get

$$\alpha - \frac{1}{2}\gamma \left(\sqrt{2}\sqrt{8B + 8\gamma + 1 - \sqrt{16B + 16\gamma + 1}} + 2 \right) + 1 \leq 0, \quad (12)$$

which is the condition for the existence of the full-poaching equilibrium. (There are other necessary conditions as well, but they do not affect this proof.)

Next, we show that in a partial-poaching equilibrium, the search engine's revenue increases with γ . In other words, the search engine is willing to reduce the penalty to increase the poaching of the weak firm. In a partial-poaching equilibrium, the following system of equations gives us T_i and T_j , the amounts that the weak firm and the strong firm spend on traditional advertising, respectively:

$$\begin{aligned} \frac{T_j}{2\sqrt{T_i + T_j}} &= B + \gamma \left(-\frac{T_i}{2\sqrt{T_i + T_j}} - T_i + 1 \right) - T_j \quad \text{and} \\ (\alpha - \gamma + 1)(T_i + 2T_j) &= 4\gamma(T_i + T_j)^{3/2}. \end{aligned}$$

We define $\tau = T_i + T_j$ and solve the above system of equations for T_i and T_j to get

$$(\alpha - \gamma + 1) \left(\frac{\sqrt{\tau}(-2B - 2\gamma + 2\tau + \sqrt{\tau})}{(\gamma - 1)(2\sqrt{\tau} + 1)} + 2\tau \right) - 4\gamma\tau^{3/2} = 0.$$

The solution τ to the above equation is a decreasing function of γ . Therefore, the search engine's revenue, $1 + B - \tau$, is an increasing function of γ .

PROOF OF PROPOSITION 2. Note that for large enough α , large enough B , and large enough γ , the weak firm fully poaches and the price of search advertising is R . Using (10), the search engine's revenue is given by

$$\frac{1}{8}(7 - 8\gamma + \sqrt{1 + 16B + 16\gamma}),$$

which is strictly decreasing in γ . Therefore, the search engine benefits from decreasing γ below 1. (Note that unless γ is below a specific threshold, the equilibrium is unique.) This is sufficient to prove part (a) of Proposition 2.

As mentioned above, the search engine benefits from decreasing γ below 1. If γ is between 1 and a certain threshold value, the equilibrium involves full poaching. However, decreasing γ below this threshold creates multiple equilibria with full or partial poaching, and some of these equilibria could result in lower search engine profit. Let γ^* be the lowest level that γ can be set to for full-poaching equilibrium to be unique. In other words, decreasing γ below γ^* leads to the existence of multiple equilibria. Depending on equilibrium selection, the search engine may or may not benefit from decreasing γ even further. (Note, however, that search engine revenue at $\gamma = \gamma^*$ is higher than the revenue at $\gamma = 1$.) We consider two extreme cases of equilibrium selection: equilibria in which the weak firm is the most aggressive, and equilibria in which the weak firm is the least aggressive. In the least aggressive case, from the multiple equilibria, we select the equilibrium in which the weak firm's poaching amount is the lowest. In the most aggressive case, we select the equilibrium in which the weak firm's poaching amount is the highest.

If the equilibrium in which the weak firm is the least aggressive is selected, the search engine's revenue is maximized at γ^* . In other words, the search engine sets γ at the lowest level where full poaching is the unique equilibrium, and if γ is set any lower, the weak firm decreases its poaching amount which leads to lower search engine revenue. Therefore, γ^* is the optimum value of γ under this equilibrium selection rule.

Next, we show that γ^* decreases with B and increases with α . In other words, we want to show that as α decreases or B increases, the poaching firm is willing to accept a higher degree of handicapping (smaller γ) and still fully poach. Using (11), the full-poaching equilibrium is unique if

$$\begin{aligned} & 1 + \alpha + (\gamma(-1 - 8B - 8\gamma + \sqrt{1 + 16B + 16\gamma}) \\ & - \sqrt{2}\sqrt{1 + 8B + 8\gamma - \sqrt{1 + 16B + 16\gamma}}) \\ & \cdot (-1 + \sqrt{1 + 16B + 16\gamma})^{-1} \\ & + (4\gamma(1 - \sqrt{1 + 16B + 16\gamma} + 2\gamma(1 + 8B + 8\gamma - \sqrt{1 + 16B + 16\gamma}) \\ & + \sqrt{2}\sqrt{1 + 8B + 8\gamma - \sqrt{1 + 16B + 16\gamma}})) \\ & \cdot ((-1 + \sqrt{1 + 16B + 16\gamma})^2)^{-1} \leq 0. \end{aligned}$$

Using basic calculus, it can be shown that the left-hand side of the above inequality is an increasing function of α and a decreasing function of B and γ . Therefore, for the above inequality to hold, γ must increase if B decreases or if α increases. Consequently, γ^* decreases with B and increases with α .

If the equilibrium in which the weak firm is the most aggressive is selected, the full-poaching equilibrium is selected from among the multiple equilibria. In this case, the search engine benefits from decreasing γ until the full poaching equilibrium does not exist. This value of γ , say, γ^{**} , is the optimal value of the relevance multiplier under this equilibrium selection rule, and reducing the value of γ below this leads to partial-poaching equilibria with lower search engine revenue. For the full poaching equilibrium to exist, using (12), we have

$$\alpha - \frac{1}{2}\gamma\left(\sqrt{2}\sqrt{8B+8\gamma+1} - \sqrt{16B+16\gamma+1} + 2\right) + 1 \leq 0.$$

Using elementary calculus, we can show that the left-hand side of this inequality is an increasing function of α and a decreasing function of B and γ . Therefore, for the inequality to hold, γ^{**} must increase if B decreases or if α increases. Consequently, the optimum level of penalty decreases with B and increases with α . This optimum level of penalty is presented in Figure 4.

A.4. Analysis for §5

Extension with Endogenous Budget. In the previous sections, firms had exogenous advertising budget. Each firm tried to maximize “sales” using optimal splitting of the advertising budget across the channels. In this section, we assume that firms allocate their advertising budget endogenously. In particular, the profit of Firm i from selling each unit is v_i . Firm i wants to maximize $\pi_i v_i - B_i$, where π_i is the total number of units sold, v_i is the profit that the firm extracts from selling each unit, and B_i is the amount of budget allocated for advertising. Note that in the previous sections we had implicitly assumed that $v_i = 1$, and since the advertising budget is exogenous, firms were trying to maximize π_i .

In the first round, each firm decides how much to spend on advertising. In the second round, firms try to maximize π_i by optimal allocation of the advertising budget across the different channels. In the third round, customers make purchase decisions and firms collect profits. Note that after firms decide how much to spend on advertising, the game becomes equivalent to that of the previous sections. Therefore, we can use the same formulation of π_i as in the previous sections.

Let $\pi_i^e(B_i, B_j)$ be the equilibrium value of π_i when the firms use B_i and B_j for the advertising budget. Note that we have already calculated π_i^e in the previous sections. For given budget B_j , Firm i wants to maximize the following:

$$\pi_i^e(B_i, B_j)v_i - B_i.$$

For given v_i and v_j , assume that B_i^e and B_j^e are the equilibrium budget allocations. Using first-order conditions, we have

$$\frac{\partial \pi_i^e(B_i^e, B_j^e)}{\partial B_i} = \frac{1}{v_i}$$

for $i \in \{1, 2\}$. These equations allow us to calculate the equilibrium value of B_i^e for $i \in \{1, 2\}$. Although we calculate the partial differentiation analytically, the system of the equations cannot be solved analytically. Therefore, we numerically solve the system of equations above. In Figure 5, we present the results for $v_1, v_2 \in [1, 10]$ and $v_1 \geq v_2$. This is a representative example and the results are qualitatively unchanged for other ranges of values.

Extension with Multiple Search Advertising Slots per Keyword. The main difference from the previous sections is the use of Lemma 2, which replaces Lemma 1. Next, we prove Lemma 2.

First note that, assuming rationality, if a firm wins the first slot t times, it should have budget $(\theta(n-t)R)/(\theta+1)$, enough for winning the second slot at price R , for $n-t$ times. This is because clicks at lower slots, for lower price R , are more favorable for the firms. They only try to win the first slot if the total supply of the second slot is not enough for their budget.

PROOF OF LEMMA 2. We prove that if Firm i always bids $b^* = (L_i + L_j + \theta(L_i + L_j - nR))/n$, it wins the top slot at least $t_i = \lfloor n((1+\theta)L_i - \theta nR)/(L_i + L_j + \theta(-2nR + L_i + L_j)) \rfloor$ times. This is because $(n-t_i)b^*/(1+\theta) + \theta t_i R/(1+\theta) \geq L_j$, which means that, assuming rationality, Bidder j cannot afford to buy more than $n-t_i$ top slots. Furthermore, by bidding b^* , after winning the first slot for t_i times (at price at most b^*), Bidder i would have enough money to win the second slot, at price R , for $n-t_i$ times. In other words, we have $t_i b^*/(1+\theta) + \theta(n-t_i)R/(1+\theta) \leq L_i$. Similarly, if Bidder j uses the same strategy, he can win the first slot at least $t_j = \lfloor n((1+\theta)L_j - \theta nR)/(L_i + L_j + \theta(-2nR + L_i + L_j)) \rfloor$ times. Since we have

$$\frac{n((1+\theta)L_i - \theta nR)}{L_i + L_j + \theta(-2nR + L_i + L_j)} + \frac{n((1+\theta)L_j - \theta nR)}{L_i + L_j + \theta(-2nR + L_i + L_j)} = n,$$

Bidder i can always win the top slot at least $\lfloor n((1+\theta)L_i - \theta nR)/(L_i + L_j + \theta(-2nR + L_i + L_j)) \rfloor$ times, and it cannot win it more than $\lceil n((1+\theta)L_i - \theta nR)/(L_i + L_j + \theta(-2nR + L_i + L_j)) \rceil$ times. \square

We use the above lemma and solve the model numerically (as the analytical solution is intractable).

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