Expectations-Based Reference-Dependent Preferences and Asset Pricing

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Abstract

This paper explores the quantitative asset-pricing implications of expectations-based reference-dependent preferences, as introduced by Koszegi and Rabin (2009), in an otherwise traditional Lucas-tree model. I find that the model easily succeeds in matching the historical equity premium and its variability when the preference parameters are calibrated in line with micro evidence. The equity premium is high because expectations-based loss aversion makes uncertain fluctuations in consumption more painful. Additionally, loss aversion introduces variation in returns because unexpected cuts in consumption are particularly painful, and the agent wants to postpone such cuts to let his reference point decrease. This variation causes strong predictability. However, it also generates counterfactually high volatility in the risk-free rate, which I address by allowing for variation in expected consumption growth, heteroskedasticity in consumption growth, time-variant disaster risk, and sluggish belief updating.

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1 Introduction

Several leading asset-pricing models assume reference-dependent preferences, which evaluate consumption relative to a reference point. Campbell and Cochrane (1999) assume habit-formation, Routledge and Zin (2010) and Bonomo et al. (2010) assume disappointment-aversion, and Benartzi and Thaler (1995), Barberis et al. (2001), and Yogo (2008) assume prospect-theory preferences. A common feature of these authors' models is that the reference point is backward-looking, that is, it depends on past consumption or returns. In this paper, I explore the asset-pricing implications of a class of reference-dependent preferences in which the reference point is forward-looking. More specifically, I adopt expectations-based reference-dependent preferences, which were introduced by Koszegi and Rabin (2006, 2007, 2009) and have since been shown to explain certain behavioral and experimental evidence.

I show that in an otherwise traditional Lucas-tree model, these preferences succeed in matching historical levels of the equity premium, the equity premium's volatility, and the degree of predictability in returns and excess returns. Moreover, I show that these preferences imply plausible risk attitudes toward small, medium, and large consumption gambles and wealth gambles, and thus take another step toward simultaneously explaining risk attitudes and toward matching asset-pricing moments. This key contribution to resolving the equity-premium puzzle was first made by Barberis and Huang (2008, 2009), who propose a preference specification that depends both on consumption and on the outcome of a narrowly-framed gamble, for instance, the stock market. I contribute to this literature by assuming a preference specification that has been shown to explain microeconomic evidence in domains other than monetary gambles, and that is based on consumption, which relaxes the framing assumptions to some extent and implies aversion to both consumption and wealth gambles.

Expectations-based reference-dependent preferences have two components. "Consumption utility" is determined by consumption and corresponds to the traditional model of utility. Contemporaneous and prospective "gain-loss utility" is determined by comparing current and future consumption with the reference point, and it corresponds to the prospect-theory model of utility. The latter component incorporates loss aversion: small losses are more painful than equal-sized gains are pleasurable. The reference point is stochastic and corresponds to the fully probabilistic rational beliefs about current and future consumption that the agent formed in the previous period. Thus, the agent compares the consumption utility of each possible outcome under his updated beliefs with the consumption utility of each possible outcome under his prior beliefs, and experiences a corresponding sensation of gain or loss. Accordingly, the agent derives gain-loss utility both from unexpected changes in present consumption and from revisions in expectations over future consumption. Therefore, gain-loss utility can be interpreted as utility over good and bad news.

This paper incorporates such "news-utility" preferences into an otherwise traditional asset-pricing model, and solves for the rational-expectations equilibrium in closed form. The model environment is a simple endowment economy with log-normal consumption growth in the spirit of Lucas (1979). The Mehra and Prescott (1985) model – which shows that constant relative risk aversion preferences are inconsistent with basic financial market moments – is preserved as a special case.

News-utility preferences predict an increased equity premium as well as variability in expected returns and excess returns. First, the equity premium is increased relative to the traditional model because uncertainty and expectations-based loss aversion introduce a first-order precautionary-savings motive. Uncertainty and loss aversion motivate precautionary savings because the agent expects that, on average, uncertain fluctuations in gain-loss utility will be painful. But these fluctuations are less painful on the less steep part of the concave utility curve, and this brings about a first-order precautionary motive to save. As precautionary savings are, by definition, caused by uncertainty, this motive increases the equity premium in general equilibrium.[1]

Second, expected returns and excess returns vary in contrast to the traditional model and despite the i.i.d. environment. Because the agent is loss averse relative to his expectations, he finds unexpected reductions in consumption more painful than expected reductions in consumption. Hence, in the event of an adverse shock, he wants to postpone the unexpected reduction in consumption until his expectations have decreased. More precisely, reducing future consumption is less painful than reducing present consumption because the future reference point is automatically reduced too, while the present reference point is fixed. If the agent desires to reduce future rather than present consumption in general equilibrium, expected returns have to increase. Intuitively, the agent sticks with low present consumption only if expected returns are high and make saving sufficiently valuable. Accordingly, in bad times, high future returns are predicted by a high consumption-price ratio such that the model generates predictability. Furthermore, the agent is willing to pay for current consumption not only in terms of future consumption but also in terms of future uncertainty. Thus, expected returns have a higher conditional variance, which increases the covariance with consumption growth and thus increase expected excess returns. Therefore, excess returns are predictable too.

As a quantitative exercise, I calibrate the news-utility preference parameters in line with microeconomic evidence, and I show that this calibration generates realistic attitudes toward small, medium, and large wealth gambles. Moreover, this calibration generates a log equity premium of approximately five percent with a standard deviation of nineteen percent, which roughly matches historical stock market data, even though I do not assume a separate process for dividends.[2] I also find variation in the consumption-wealth ratio around one percent and R^2s in the predictability regressions of approximately twenty percent. These values match the empirical findings of Lettau and Ludvigson (2001), who document the medium-term predictive power of the consumption-wealth ratio.[3] I show that such strong predictive power of the consumption-wealth ratio for the return and excess

[1]Koszegi and Rabin (2009) anticipate the precautionary-savings result in a two-period, two-outcome problem.

[2]The model's predicted equity premium and volatility are increasing in the simulation frequency, which thus constitutes a calibrational degree of freedom. Given the calibration of preference parameters, I choose a biannual frequency that matches both the historical equity premium and its volatility. At an annual frequency, which has been argued for by Benartzi and Thaler (1995) and Barberis et al. (2001), the model requires a coefficient of risk aversion around eight to match the historical risk-return tradeoff.

[3]Furthermore, Lustig et al. (fthc) and Hirshleifer and Yu (2011) document the volatility of the consumption-wealth ratio and the return on the aggregate consumption claim.

return on the aggregate consumption claim is not generated by other leading asset-pricing models.

Unfortunately, this variation results in too much negative autocorrelation in returns, which cannot be found in the data. Moreover, the consumption-wealth ratio is not characterized by any persistence, as the preferences feature full belief updating in every period, which is not true in the data. Finally, the model counterfactually predicts strong variation in the risk-free rate, which is known from habit-formation models.[4] The expected risk-free rate of return is increased in the event of adverse shock realizations because the agent dislikes immediate reductions in consumption and is unwilling to substitute intertemporally. Although it is not reflected in the aggregate data, this underlying time-variation in substitution motives is not implausible in practice. Indeed, because people are sometimes unwilling to substitute intertemporally, they use credit cards and payday loans, thus borrowing at high interest rates.

To address the model's shortcomings, I first try not to change the evidence-based utility function; rather, I take the variation in substitution motives seriously, and explore three model-environment extensions in which the strong intertemporal-substitution effects on the risk-free rate are partly offset by other forces. More precisely, I assume variation in expected consumption growth, as in Bansal and Yaron (2004), and variation in consumption growth volatility, i.e., heteroskedasticity in the consumption process, as in Campbell and Cochrane (1999). I also add disaster risk to the consumption process, that is, the agent expects a small probability of suffering a large loss in consumption, as in Barro (2006, 2009). I find that news-utility preferences amplify disaster risk, because they feature "left-skewness aversion," as prospect-theory preferences that assume overweighting of small probabilities. The addition of heteroskedasticity or disaster risk introduces variation in the strength of the precautionary-savings motive, which partly offsets the effects of the variation in substitution motives on the risk-free rate, adds variation in the price of risk, and generates long-horizon predictability. Nevertheless, these extensions only partially succeed in smoothing the risk-free rate. Therefore, I additionally explore the implications of sluggish belief updating, which turns out to generate a realistic set of moments.

After a literature review, I present the preferences, the model environment, and the Markovian rational-expectations equilibrium in Section 2. Then, I explain the model's predictions about the consumption-wealth ratio. In Section 3, I discuss the model's asset-pricing implications and calibrate the model to gauge its quantitative implications. In Section 4, I extend the model to allow for time-variant expected consumption growth, time-variant volatility, disaster risk, and sluggish beliefs updating. Finally, Section 5 concludes and discusses the model's welfare implications.

Comparison to the Literature Expectations-based reference-dependent preferences have already found a wide range of applications in microeconomics. See, for example, Heidhues and Koszegi (2008, 2014), Herweg and Mierendorff (2012), and Rosato (2012) on consumer pricing and Herweg et al. (2010) on principal-agent problems. Furthermore, a number of experimental studies appear to provide support for these preferences. See,

[4] Additionally, the term structure of expected equity returns is upward sloping, opposite to the evidence in Binsbergen et al. (2012).

for example, Abeler et al. (2011) on labor-supply decision making, Gill and Prowse (2012) on real-effort tournaments, Meng (2013) on the disposition effect, and Ericson and Fuster (2011) on the endowment effect.[5] I contribute to this literature by incorporating these preferences into the canonical asset-pricing context and evaluating its empirical performance, and I show that the preferences are tractable in a multi-period, continuous-outcome framework; this is not readily apparent given their high level of complexity.

Expectations-based reference-dependent preferences were developed to discipline several degrees of freedom associated with prospect theory. In particular, they are based on consumption, their reference points are endogenous, and tight ranges exist for all preference parameters. However, as noted in the introduction, a degree of freedom emerges in dynamic models, which does not receive much attention in static applications: the length of each time period. Reducing the length of a time period or simulating the model at a higher frequency increases the equity premium because the agent is loss averse, or first-order risk averse. First-order risk aversion implies time diversification, that is, the investment is preferred if its horizon is increased. Increasing the investment's horizon implies that its risk increases with the square root of the horizon while its return increases linearly with the horizon, which makes the investment overall more favorable. I calibrate the preferences in line with microeconomic evidence and choose a biannual frequency that matches both the equity premium and the equity premium's volatility, i.e., the historical risk-return trade off.

The pioneering prospect-theory asset-pricing papers, Barberis et al. (2001) and Benartzi and Thaler (1995), specify gain-loss utility directly over fluctuations in financial wealth. In so doing, the authors make an assumption about narrow framing.[6] In Barberis et al. (2001), variation in the coefficient of loss aversion introduces predictability, whereas the additively separable gain-loss component over financial wealth yields a constant consumption-wealth ratio and risk-free rate. Yogo (2008), however, argues that fluctuations in consumption rather than financial wealth are the relevant measure of risk. The author's preferences are a mixture of habit formation and prospect theory, which yields a high equity premium, while variation in the risk-free rate is mitigated by persistence in the habit process. Moreover, Yogo (2008) shows that his preferences exhibit reasonable attitudes towards small and large wealth gambles. The preference formulation of Andries (2013) features a kink in the value function at its expected value. Since the value function is approximately linear, and the author calibrates the kink in line with the coefficient of loss aversion estimated by Kahneman and Tversky (1979), the model is consistent with observed attitudes toward small and large wealth gambles. However, it does not generate predictability, because the agent is not loss averse over present consumption and the consumption-wealth ratio is constant. In contrast, Andries (2013) focuses on validating the cross-sectional asset-pricing implications and the implications for the security market line.

Campanale et al. (2010) assume disappointment-aversion preferences in the spirit of Gul (1991) in a production economy. In this model, the excessive volatility of the risk-free rate can be reduced by assuming a high intertemporal elasticity of substitution. How-

[5]On the endowment effect, see, however, the contradicting evidence in Heffetz and List (fthc).

[6]Narrow framing refers to the phenomenon in which people evaluate an offered gamble in isolation, rather than mixing it with existing risk and considering its implications for consumption rather than for financial wealth.

ever, the variation in returns is acyclical by construction, which rules out predictability.[7] Routledge and Zin (2010) develop generalized disappointment-aversion preferences. The authors assume that the disappointing outcome corresponds to a fraction of the certainty equivalent of consumption. The authors show that these preferences are consistent with basic financial market moments. Bonomo et al. (2010) extend their model to long-run risk and show that it matches predictability patterns in returns. These two models generate variability in returns by variation in risk aversion, that is, the agent is highly risk averse in low-consumption situations where he is likely to be disappointed.[8] Thus, the preferences feature low risk aversion in high-consumption situations, high risk aversion for medium gambles in low-consumption situations, but low risk aversion for small gambles in low-consumption situations, as the reference point corresponds to a fraction of the certainty equivalent. Thus, they are not necessarily consistent with risk attitudes toward gambles. Campbell and Cochrane (1999) show that habit formation matches a range of asset-pricing moments, and also emphasize one of the present paper's main predictions: the variation in the agent's willingness to substitute intertemporally. However, the authors exactly offset the variation in intertemporal-substitution motives by a habit process that features variation in the agent's precautionary-savings motive. Like disappointment aversion, the agent's effective risk aversion is high in bad states and becomes the main variability-driving mechanism.

2 The Model

The model environment. I consider a Lucas (1979) tree model in which the sole source of consumption is an everlasting tree that produces C_t units of consumption each period t. I assume that consumption growth is log-normal, following Mehra and Prescott (1985). Thus, the endowment economy's exogenous consumption process is given by

$$log(\frac{C_{t+1}}{C_t}) = \mu_c + \varepsilon_{t+1} \text{ with } \varepsilon_{t+1} \sim N(0, \sigma_c^2). \tag{1}$$

The price of the Lucas tree in each period t is P_t. Moreover, there exists a risk-free asset in zero net supply with return R^f_{t+1}. The period $t+1$ return of holding the Lucas tree is thus $R_{t+1} = {}^{P_{t+1}+C_{t+1}}/_{P_t}$. Each period t, the agent faces the price of the Lucas tree P_t and the risk-free return R^f_{t+1} and, acting as a price taker, optimally decides how much to consume C_t^* and how much to invest in the risky asset α_t^*.

Expectations-based reference-dependent preferences. The agent's instantaneous utility function depends on both consumption and "beliefs" about consumption, which I explicitly define first.

[7] Epstein and Zin (1989) preferences are able to rationalize the equity premium with the addition of long-run risk or heterogeneous agents as shown by Bansal and Yaron (2004). Epstein and Zin (1989) preferences feature a constant elasticity of intertemporal substitution that can be chosen as an additional parameter in the model.

[8] Strong variation in effective risk aversion has trouble matching the evidence on risk attitudes towards wealth gambles and is controversial given household-level data on portfolio choice (Donaldson and Mehra (2008) and Brunnermeier and Nagel (2008)).

Definition 1. Let I_t denote the agent's information set in some period $t \leq t + \tau$. Then, the agent's probabilistic beliefs about consumption in period $t + \tau$, conditional on period t information, are denoted by $F^t_{C_{t+\tau}}(c) = Pr(C_{t+\tau} < c|I_t)$ and $F^{t+\tau}_{C_{t+\tau}}$ is degenerate.

I assume rational expectations such that the agent's beliefs about any of the model's variables equal the objective probabilities determined by the economic environment. In the Lucas-tree equilibrium, the agent's consumption is determined by the exogenous market-clearing consumption process, such that $F^t_{C_{t+\tau}} = log\text{-}N(log(C_t) + \tau\mu_c, \tau^2\sigma_c^2)$ for any $t \in [0, \infty)$ and any $\tau > 0$ as $I_t = \{C_t, P_t, \varepsilon_t\}$.

I now move on to the instantaneous utility function, which is the sum of consumption utility and gain-loss utility. The latter component consists of contemporaneous gain-loss utility about current consumption and prospective gain-loss utility about the entire stream of future consumption. More formally, total instantaneous utility in period t is given by

$$U_t = u(C_t) + n(C_t, F^{t-1}_{C_t}) + \gamma \sum_{\tau=1}^{\infty} \beta^\tau \boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}}). \tag{2}$$

The first term on the right-hand side of equation (2) corresponds to consumption utility in period t, which is a power-utility function $u(c) = \frac{c^{1-\theta}}{1-\theta}$. To understand the remaining terms in equation (2), first note that the reference point in period t is the stream of fully probabilistic beliefs about consumption in period t and all future periods $t + \tau$, given the information available in period $t - 1$. According to Definition 1, the agent's beliefs formed in period $t - 1$ about period $t + \tau$ consumption are denoted by $F^{t-1}_{C_{t+\tau}}$. Thus, the second term in equation (2), $n(C_t, F^{t-1}_{C_t})$, corresponds to gain-loss utility in period t over contemporaneous consumption. Gain-loss utility is determined by a piecewise-linear value function $\mu(\cdot)$ with slope η and a coefficient of loss aversion λ, that is, $\mu(x) = \eta x$ for $x > 0$ and $\mu(x) = \eta\lambda x$ for $x \leq 0$. The parameter $\eta > 0$ weights the gain-loss utility component relative to the consumption utility component and $\lambda > 1$ implies that losses are weighed more heavily than gains: the agent is loss averse. Because the agent compares his actual contemporaneous consumption with his prior beliefs, he experiences gain-loss utility over "news" about contemporaneous consumption as follows

$$\begin{aligned} n(C_t, F^{t-1}_t(C^{t-1}_t)) &= \int_0^\infty \mu(u(C_t) - u(c)) dF^{t-1}_{C_t}(c) \\ &= \eta \int_0^{C_t} (u(C_t) - u(c)) dF^{t-1}_{C_t}(c) + \eta\lambda \int_{C_t}^\infty (u(C_t) - u(c)) dF^{t-1}_{C_t}(c). \end{aligned} \tag{3}$$

The third term on the right-hand side of equation (2), $\gamma \sum_{\tau=1}^{\infty} \beta^\tau \boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}})$, corresponds to prospective gain-loss utility in period t over the entire stream of future consumption. Prospective gain-loss utility about period $t+\tau$ consumption, $\boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}})$, depends on $F^{t-1}_{C_{t+\tau}}$, the beliefs with which the agent entered the period, and on $F^t_{C_{t+\tau}}$, the agent's updated beliefs about period $t + \tau$ consumption. $F^{t-1}_{C_{t+\tau}}$ and $F^t_{C_{t+\tau}}$ are correlated distribution functions, because future uncertainty is contained in both prior and updated beliefs about $C_{t+\tau}$. Thus, there exists a joint distribution, which I denote by $F^{t,t-1}_{C_{t+\tau}} \neq F^t_{C_{t+\tau}} F^{t-1}_{C_{t+\tau}}$.

Because the agent compares his new beliefs with his prior beliefs, he experiences gain-loss utility over "news" about future consumption

$$\boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}}) = \int_0^\infty \int_0^\infty \mu(u(c) - u(r))dF^{t,t-1}_{C_{t+\tau}}(c, r). \tag{4}$$

Both contemporaneous and prospective gain-loss utility correspond to an outcome-wise comparison, as assumed in Koszegi and Rabin (2006, 2007).[9] In addition, the agent discounts prospective gain-loss utility exponentially by β, the traditional agent's consumption utility discount factor; and prospective gain-loss utility is subject to another discount factor, γ, relative to contemporaneous gain-loss utility, so that the agent puts a total weight $\gamma\beta^\tau < 1$ on prospective gain-loss utility about consumption in period $t + \tau$.

Because both contemporaneous and prospective gain-loss utility are experienced over news, the preferences are referred to as "news utility".

The model's equilibrium. Because the agent fully updates his beliefs each period and the consumption process is i.i.d., I look for an equilibrium price and risk-free return process that is "Markovian" in the sense that the price-consumption ratio depends on the current shock only.

Definition 2. The price process $\{P_t\}_{t=0}^\infty$ and risk-free return process $\{R^f_{t+1}\}_{t=0}^\infty$ are Markovian if, in each period t, the price-consumption ratio P_t/C_t and the risk-free return R^f_{t+1} depend only on the realization of the shock ε_t, such that $P_t/C_t = p(\varepsilon_t)$ and $R^f_{t+1} = r(\varepsilon_t)$ with the functions $p(\cdot)$ and $r(\cdot)$ being independent of calendar time t and endowment C_t.

Facing prices and returns, the agent's maximization problem in period t is given by

$$max_{C_t}\{u(C_t) + n(C_t, F^{t-1}_{C_t}) + \gamma \sum_{\tau=1}^{\infty} \beta^\tau \boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}}) + E_t[\sum_{\tau=1}^{\infty} \beta^\tau U_{t+\tau}]\}. \tag{5}$$

The agent's wealth in the beginning of period t, W_t, is determined by his portfolio return R^p_t, which in turn depends on the risky return realization R_t, the risk-free return R^f_t, and the previous period's optimal portfolio share α_{t-1}. The budget constraint is

$$W_t = (W_{t-1} - C_{t-1})R^p_t = (W_{t-1} - C_{t-1})(R^f_t + \alpha_{t-1}(R_t - R^f_t)). \tag{6}$$

In each period t, the agent optimally decides how much to consume C^*_t, how much to invest $W_t - C^*_t$, and how much to invest in the risky asset α^*_t. In equilibrium, the price

[9] The outcome-wise comparison of Koszegi and Rabin (2006, 2007) has been generalized to an ordered comparison in Koszegi and Rabin (2009), because the agent would otherwise experience gain-loss disutility over future uncertainty even if no update in information takes place. I circumvent this problem by explicitly noting that prior and new beliefs about consumption are correlated, i.e., I generalize the gain-loss formula of Koszegi and Rabin (2006, 2007)

$$n(F_c, F_r) = \int_0^\infty \int_0^\infty \mu(u(c) - u(r))dF_r(r)dF_c(c) \quad \text{to} \quad \boldsymbol{n}(F_{c,r}) = \int_0^\infty \int_0^\infty \mu(u(c) - u(r))dF_{c,r}(c, r).$$

The ordered comparison yields qualitatively and quantitatively similar results, but the model's solution is not as tractable.

of the tree $P_t = W_t - C_t$ adjusts so that the single agent in the model always chooses to hold the entire tree, i.e., $\alpha_t^* = 1$ for all t, and to consume the tree's entire payoff $C_t^* = C_t$ for all t as determined by the endowment economy's exogenous consumption process (1). In the following, I derive the "Markovian rational-expectations equilibrium" recursively. It corresponds to the preferred-personal equilibrium, as defined in Koszegi and Rabin (2006).[10]

Definition 3. The Markovian rational-expectations equilibrium consists of a Markovian price process $\{P_t = C_t p(\varepsilon_t)\}_{t=0}^{\infty}$ and a risk-free return process $\{R_{t+1}^f = r(\varepsilon_t)\}_{t=0}^{\infty}$ such that the solution $\{C_t^*, \alpha_t^*\}_{t=0}^{\infty}$ of the price-taker's maximization problem (5), subject to the budget constraint (6), satisfies goods-market clearing $\{C_t^* = C_t\}_{t=0}^{\infty}$ and asset-market clearing $\{\alpha_t^* = 1\}_{t=0}^{\infty}$.

Proposition 1. *A Markovian rational-expectations equilibrium exists.*

This and the following propositions' proofs can be found in Appendices B.1 to B.5.

The equilibrium has a very simple structure and can be derived in closed form. In each period t, optimal consumption C_t^* is a fraction of current wealth W_t such that $C_t^* = W_t \rho_t$. As Appendix B.2 shows, the consumption-wealth ratio ρ_t is

$$\rho_t = \frac{C_t^*}{W_t} = \frac{1}{1 + \frac{Q + \Omega + \gamma Q \Omega + \gamma Q(\eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t)))}{1 + \eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t))}}. \tag{7}$$

Here, $F(\cdot)$ denotes the cumulative normal distribution function $N(0, \sigma_c)$ and Q and Ω are determined by exogenous parameters. Thus, ρ_t varies with the realization of ε_t, is i.i.d., and is independent of calendar time t and of the current endowment C_t. The price-consumption ratio is $P_t/C_t = 1-\rho_t/\rho_t$. The agent's value function is proportional to the power utility of wealth $V_t = u(W_t)\Psi_t$. Ψ_t varies with the realization of ε_t, is i.i.d., independent of calendar time t, and the current endowment C_t. I now explain the news-utility agent's first-order condition in detail to build intuition for Q and Ω and to clarify why and how ρ_t varies with ε_t.

2.1 Predictions about the consumption-wealth ratio

Before turning to the model's asset pricing implications, I describe the agent's first-order condition in order to provide intuition for two predictions about the agent's consumption-wealth ratio, which are formalized in Propositions 2 and 3 and illustrated in Figure 1. Although the first-order condition appears complicated, the terms can be easily understood one component at a time. First, for $\eta = 0$, the model collapses to the traditional consumption-based asset-pricing model with constant relative risk aversion and log-normal

[10] The personal-equilibrium solution concept introduced by Koszegi and Rabin (2006) is the family of credible state-contingent plans, which the agent's beliefs are rationally based on. Moreover, among all credible state-contingent plans, the agent chooses the plan that maximizes expected reference-dependent utility going forward; this is preferred-personal equilibrium. Because the agent's plan is credible, his behavior is time consistent. The first-order condition is derived under the premise that the agent enters period t, takes his beliefs as given, and optimizes with respect to consumption. Moreover, he rationally expects to behave like this in the future so that behavior maps into correct beliefs and vice versa.

consumption growth studied by Mehra and Prescott (1985) among many others. The first-order condition becomes

$$C_t^{-\theta} = (\frac{\rho^s}{1-\rho^s})^{1-\theta}(W_t - C_t)^{-\theta}Q \tag{8}$$

and results in a constant consumption-wealth ratio $\rho^s = \frac{1}{1+Q}$. Let me return to news utility and henceforth assume that $\eta > 0$ and $\lambda > 1$. The agent's consumption-wealth ratio ρ_t, equation (7), results from the model's first-order condition

$$C_t^{-\theta}(1 \underbrace{+\eta F(\varepsilon_t) + \eta\lambda(1-F(\varepsilon_t))}_{\textbf{contemporaneous gain-loss}}) = (\frac{\rho_t}{1-\rho_t})^{1-\theta} \underbrace{(W_t - C_t)^{-\theta}(Q+\Omega+\gamma\Omega Q}_{=-\frac{d\beta E_t[u(W_{t+1})\Psi_{t+1}]}{dC_t}} + \underbrace{\gamma Q(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))}_{\textbf{prospective gain-loss}})). \tag{9}$$

In the following, I describe equation (9) in detail to provide an intuition for why, in contrast to the traditional model, the consumption-wealth ratio is shifted down and is not constant.

The shift in the consumption-wealth ratio. The left-hand side of the first-order condition, equation (9), is simply determined by marginal consumption and gain-loss utility over contemporaneous consumption. Marginal gain-loss utility is given by the states that would have promised less consumption $F_{C_t}^{t-1}(C_t)$, weighted by η, or more consumption $1-F_{C_t}^{t-1}(C_t)$, weighted by $\eta\lambda$, i.e., $\partial n(C_t, F_{C_t}^{t-1})/\partial C_t = u'(C_t)(\eta F_{C_t}^{t-1}(C_t) + \eta\lambda(1-F_{C_t}^{t-1}(C_t)))$. A key technical insight here allows me to simplify the marginal gain-loss utility term: In the Lucas-tree model, equilibrium consumption is determined by the realization of the shock ε_t, which allows me to simplify $F_{C_t}^{t-1}(C_t) = F(\varepsilon_t)$.

Let me turn to the right-hand side of equation (9). The first term represents the marginal value of savings $-d\beta E_t[u(W_{t+1})\Psi_{t+1}]/dC_t = u'(W_t - C_t)(Q + \Omega + \gamma\Omega Q)$ with Q and Ω determined by exogenous parameters. In the traditional model, the marginal value of savings is given by $u'(W_t - C_t)Q$. Thus, Q represents the discounted stream of future consumption utility. In contrast, Ω represents expected gain-loss utility; such that, the marginal value of savings is determined by $Q + \Omega + \gamma\Omega Q$. This is the sum of expected consumption utility, expected contemporaneous gain-loss utility, and expected prospective gain-loss utility discounted by γ. Because expected gain-loss disutility is positive, $\Omega > 0$, the marginal value of saving increases relative to the traditional model. The underlying intuition is that the agent expects to experience gain-loss utility over resolving consumption uncertainty that is proportional to marginal consumption utility. Expected gain-loss disutility is thus less painful on the less steep part of the utility curve, and the agent has an additional incentive to increase savings. Moreover, it can be shown that the additional precautionary-savings motive is first-order, that is, $\partial\Omega/\partial\sigma_c|_{\sigma_c=0} > 0$, because it depends on the concavity of the utility curve rather than on prudence as in the traditional model. As precautionary savings depend on uncertainty, this motive increases the model's equity premium.

However, if the agent discounts news about the future, $\gamma < 1$, he has an additional reason to consume more today, because positive news about contemporaneous consumption is overweighted. Thus, the additional precautionary-savings motive results in the consumption-wealth ratio being lower than in the traditional model as long as the agent does not discount future news too highly $\gamma > \bar{\gamma}$. These ideas are formalized in the following proposition.

Proposition 2. *If $\theta > 1$ and $\gamma > \bar{\gamma}$ with $\bar{\gamma} = {}^{\eta\lambda - \Omega/Q}/_{\Omega+\eta\lambda} < 1$ then, for all realizations of ε_t, the consumption-wealth ratio in the news-utility model is lower than in the traditional model $\rho_t < \rho^s$. Moreover, $\bar{\gamma}$ is decreasing in the news-utility parameters $\partial\bar{\gamma}/\partial\lambda, \partial\bar{\gamma}/\partial\eta \leq 0$.*[11]

This result reflects the finding by Koszegi and Rabin (2009) that news utility introduces an additional first-order precautionary-savings motive in a two-period, two-outcome model. This finding carries over to my setting for $\theta > 1$ only because I consider multiplicative instead of additive shocks. Multiplicative shocks imply that savings will increase the absolute value of tomorrow's wealth bet, which the news-utility agent dislikes. For $\theta < 1$, this effect dominates the desire for intertemporal smoothing. For log utility $\theta = 1$, the two motives exactly offset each other and $\Omega = 0$. Thus, if $\theta = 1$ and $\gamma = 1$, the news-utility model becomes observationally equivalent to the traditional model.[12]

Variation in the consumption-wealth ratio. Let me move on to the second part of the right-hand side of the first-order condition (9), which represents marginal prospective gain-loss utility. In the absence of both expected gain-loss disutility and prospective gain-loss discounting, $\Omega = 0$ and $\gamma = 1$, marginal contemporaneous and prospective gain-loss utilities would cancel out. Then, I would be back in the traditional model with a proportional response of consumption to wealth. However, contemporaneous marginal utility is driven above future marginal utility by the additional marginal value of savings, $\Omega > 0$, so that $Q + \Omega + \gamma Q\Omega \neq \gamma Q$. Thus, the consumption-wealth ratio ρ_t varies with the realization of ε_t.

Moreover, the consumption-wealth ratio is decreasing for $\theta > 1$; to explain this I will first outline a simplified intuition. Because unexpected losses are particularly painful, the agent consumes relatively more of his wealth in the event of an adverse shock. If the agent encounters an adverse shock, decreasing his consumption below expectations today is more painful than decreasing consumption tomorrow when the reference point will have decreased. If the agent encounters a positive shock, he experiences less painful gain-loss fluctuations today relative to tomorrow when the reference point will have increased. Thus, the agent wants to delay the consumption response to shocks, which makes the consumption-wealth ratio vary. As explained for Proposition 2, the desire to raise consumption unexpectedly and $\gamma < 1$ increases the consumption-wealth ratio in both good and bad times. But moreover, the utility gain from delaying adverse shocks is larger than the utility gain from overconsuming favorable shocks, which brings about variation in the consumption-wealth ratio.

[11] If $\theta > 0$ and ${}^{\eta - \Omega/Q}/_{\Omega+\eta} < \gamma < \bar{\gamma}$ then ρ^s and ρ_t cross at $\varepsilon_t = \bar{\varepsilon}_t$ and $\bar{\varepsilon}_t$ is decreasing in the news-utility parameters $\partial\bar{\varepsilon}_t/\partial\lambda, \partial\bar{\varepsilon}_t/\partial\eta \leq 0$.

[12] This result is analogous to a result for quasi-hyperbolic discounting obtained by Barro (1999).

More formally, in equation (9), $\Omega + \gamma Q\Omega$ corresponds to expected marginal gain-loss utility that is constant because the future reference point adjusts to the present shock. Thus, a positive share of future marginal utility is inelastic to the present shock, which implies that future marginal utility is less sensitive to changes in consumption than is present marginal utility. Future marginal gain-loss utility remains constant following an adverse shock, whereas present marginal gain-loss is high; thus, the agent wants to consume relatively more today and relatively less tomorrow.[13] The following proposition formalizes this idea.

Proposition 3. *If $\theta \neq 1$, news utility introduces variation in the consumption-wealth ratio $\partial\rho_t/\partial\varepsilon_t \neq 0$. Moreover, for $\theta > 1$, the consumption-wealth ratio is decreasing $\partial\rho_t/\partial\varepsilon_t < 0$.*

This variation in the consumption-wealth ratio introduces variation in expected returns and amplifies the variability of returns. This is because news utility decreases the price of the Lucas tree even further when the consumption process has paid out badly.

These predictions are illustrated in Figure 1, which displays the consumption-wealth ratio ρ_t as a function of the shock to consumption growth, and contrasts it with the traditional agent's ratio for two levels of σ_c.[14] Figure 1 illustrates that ρ_t is smaller than ρ^s and that, for a small increase in σ_c, the downward shift in ρ_t is larger than the downward shift in ρ^s. The latter results from the additional precautionary-savings motive being a first-order effect, $\partial\rho_t/\partial\sigma_c|_{\sigma_c\to 0} > 0$, while the traditional precautionary-savings motive is second order. Furthermore, ρ_t is decreasing in ε_t while ρ^s is constant. The shape of ρ_t is driven by marginal gain-loss utility, which depends on the shock distribution $\eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t)) \in [\eta, \eta\lambda]$. As ε_t is characterized by a bell-shaped distribution, the variation in the consumption-wealth ratio is bounded. The agent experiences gain-loss utility over all other previously expected realizations of consumption, weighted by their probabilities. For extreme realizations of ε_t, the consumption-wealth ratio approaches a limit because the states near these realizations have very low probabilities.

[13] This prediction about consumption is different from a result in Koszegi and Rabin (2009) predicting that the agent consumes entire small unexpected gains but delays entire small unexpected losses. The prediction in Koszegi and Rabin (2009) results from the assumption that $1/\lambda < \gamma < 1$ and that small gains and losses are coming so unexpectedly that the agent initially planned a certain consumption path absent any gain-loss utility. The prediction would turn into my result, that is, where the agent consumes relatively less in the event of a good shock and relatively more in the event of a bad shock, if the gain or loss in wealth came expectedly.

[14] The calibration is displayed in Table 2 and discussed in Section 3.1.

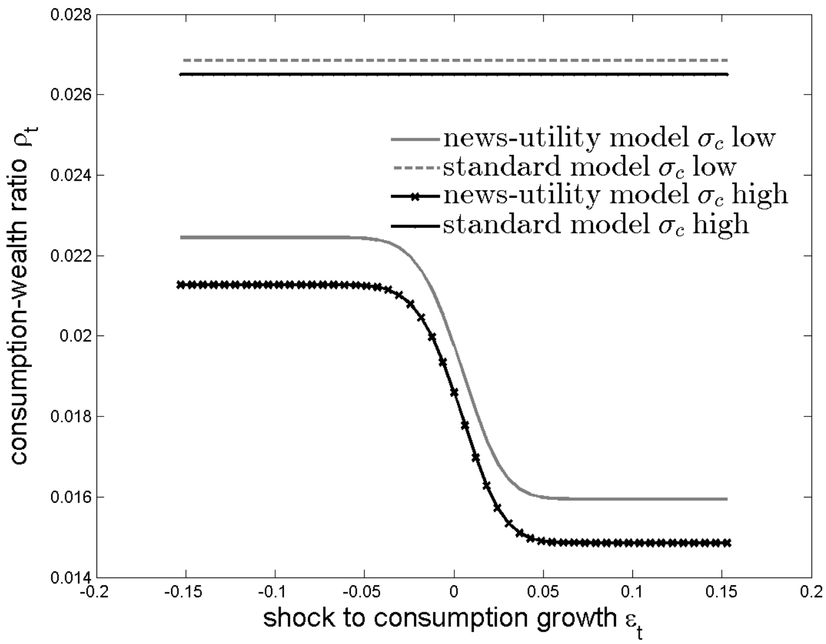

Figure 1: Consumption-wealth ratio ρ_t in the news-utility and traditional models.

3 Asset Pricing

Now I turn to the model's asset-pricing implications. First, I derive the expected risky return, the risk-free return, and the equity premium. Second, I illustrate the model's main asset-pricing predictions, namely the variation in expected returns and in the equity premium and the model's predictability properties that are formalized in Proposition 4. I aim to build intuition for these asset-pricing results by connecting them back to my prior theoretical results about the consumption-wealth ratio. In Section 3.1, I calibrate the model to gauge its quantitative performance and then compare its predictions to those of other models.

Expected returns and the equity premium. The return of holding the entire Lucas tree is $R_{t+1} = {}^{P_{t+1}+C_{t+1}}/_{P_t}$. I can rewrite the expected risky return in terms of the consumption-wealth ratio ρ_t and consumption growth ${}^{C_{t+1}}/_{C_t}$ by taking expectations and noting that $P_t = W_t - C_t = C_t{}^{1-\rho_t}/_{\rho_t}$

$$E_t[R_{t+1}] = \frac{\rho_t}{1-\rho_t} E_t[\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}}]. \tag{10}$$

Note that the second part of equation (10), $E_t[{}^{C_{t+1}}/_{C_t \rho_{t+1}}]$, is constant because consumption growth ${}^{C_{t+1}}/_{C_t} = e^{\mu_c + \varepsilon_{t+1}}$ and next-period's consumption-wealth ratio ρ_{t+1} are i.i.d., as reported in Definition 2 such that ${}^{P_{t+1}}/_{C_{t+1}} = p(\varepsilon_{t+1}) = {}^{1-\rho_{t+1}}/_{\rho_{t+1}}$. However, $E_t[R_{t+1}]$ varies with the consumption-wealth ratio ρ_t.

I can rewrite the first-order condition as $1 = E_t[M_{t+1}R_{t+1}]$, which gives rise to the agent's stochastic discount factor M_{t+1} derived in Appendix B.2. The risk-free return is the inverse of the conditional expectation of the stochastic discount factor

$$R^f_{t+1} = \frac{1}{E_t[M_{t+1}]} = \frac{\rho_t}{1-\rho_t}(Q + \Omega + \gamma\Omega Q)E_t[\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{-\theta}\Psi_{t+1}]^{-1}. \tag{11}$$

Note that the second part of equation (11), $E_t[\beta(C_{t+1}/C_t\rho_{t+1})^{-\theta}\Psi_{t+1}]^{-1}$, is constant because consumption growth $C_{t+1}/C_t = e^{\mu_c+\varepsilon_{t+1}}$, the next period's consumption-wealth ratio ρ_{t+1}, and the value function's proportionality factor Ψ_{t+1} are i.i.d. However, R^f_{t+1} varies with the consumption-wealth ratio ρ_t. The equity premium

$$E_t[R_{t+1}]-R^f_{t+1} = -\frac{Cov_t(M_{t+1},R_{t+1})}{E_t[M_{t+1}]} = \underbrace{-\frac{\sigma_t(M_{t+1})}{E_t[M_{t+1}]}\frac{Cov_t(M_{t+1},R_{t+1})}{\sigma_t(M_{t+1})\sigma_t(R_{t+1})}}_{\text{constant price of risk}}\underbrace{\sigma_t(R_{t+1})}_{\text{quantity of risk}} \tag{12}$$

$$= \frac{\rho_t}{1-\rho_t}(E_t[\frac{1}{\rho_{t+1}}\frac{C_{t+1}}{C_t}] - (Q+\Omega+\gamma\Omega Q)E_t[\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{-\theta}\Psi_{t+1}]^{-1}) \tag{13}$$

is characterized by a constant price of risk. The price of risk and the conditional Sharpe ratio $S_t = E_t[R_{t+1}-R^f_t]/\sigma_t(R_{t+1})$ are constant, because the agent holds the entire stock market and thus faces the same risk each period. However, the quantity of risk $\sigma_t(R_{t+1})$ varies with the consumption-wealth ratio ρ_t. Variation in the conditional variance of returns translates into variation in the conditional covariance of returns and excess returns with consumption growth, as the risk-free rate is constant. This translates into variation in the expected equity premium.

The news-utility implications about the location and shape of the consumption-wealth ratio ρ_t, which are formalized in Propositions 2 and 3, directly carry over to the expected return, the risk-free return, and the equity premium. The decrease in the consumption-wealth ratio due to the additional precautionary-savings motive depends on uncertainty and is thus reflected in a high equity premium. The news-utility agent perceives uncertain fluctuations in consumption to be much more painful than the traditional agent does. I now turn to the variation in the consumption-wealth ratio that generates variation in returns, variation in the equity premium, and predictability.

Variation in expected returns and predictability. I have shown that the expected risky return, the risk-free return, and the equity premium vary with the consumption-wealth ratio ρ_t. The variation in the expected risky return is driven by variation in the agent's willingness to substitute intertemporally, as reflected by variation in ρ_t. In bad states of the world, the agent would like to delay adjustments in consumption to let his reference point adjust. To induce the agent to consume his endowment, the price of the Lucas tree must be low and expected returns have to be high. Thus, despite the i.i.d. environment, the expected risky return varies to make the agent willing to hold the entire tree each period. Moreover, the variation in the consumption-wealth ratio generates return predictability. In particular, the realization of ε_t predicts the one-period-ahead return R_{t+1}. If ε_t is low, then ρ_t the consumption-wealth ratio is high and the one-period-ahead return is high; hence, the consumption-wealth ratio positively predicts one-period ahead returns. This mechanism also generates predictability in excess returns. Bad states predict high future returns, and this implies that the standard deviation of returns is also high and that the expected equity premium varies with ε_t. By the same argument as above, the realization of ε_t then predicts the one-period-ahead excess return $R_{t+1} - R^f_{t+1}$.[15]

The intuition for the variation in excess returns is not trivial. Equation (12) decomposes the expected excess return into the price of risk, which is constant, and the quantity of risk, $\sigma_t(R_{t+1})$, which varies with ${}^{\rho_t}/_{1-\rho_t}$. In the event of a bad shock, the consumption-wealth ratio is high, which increases not only expected returns but also the standard deviation of expected returns. The reason is that the agent wants to consume more today and is willing to pay for such consumption by giving up future consumption as well as future consumption uncertainty: that is, his willingness to take risks has increased. This intuition can be most easily understood by looking at an approximation of the agent's optimal portfolio share in a partial-equilibrium model that is given by

$$\alpha_t = \frac{\mu - r^f + \frac{(1-\beta+\gamma\beta)E_t[\eta(\lambda-1)\int_{r_{t+1}}^{\infty}(r_{t+1}-\tilde{r})dF_r(\tilde{r}))]}{1+\gamma(\eta F_{C_t}(C_t)+\eta\lambda(1-F_{C_t}(C_t)))}}{\sigma^2}.$$

As can be seen if $F_{C_t}(C_t)$ is low, then the agent's optimal portfolio share is higher, that is, he is willing to take on relatively more of the risky asset. The basic intuition for this variation in the optimal portfolio share is that, upon a favorable return realization, the agent wants to realize the good news about consumption and liquidates his risky asset holdings. In contrast, upon an adverse return realization, the agent prefers not to realize all the bad news associated with future consumption. Rather, he wants to keep the bad news in future consumption more uncertain and thus increases his portfolio share. This allows him to effectively delay the realization of bad news until the next period, by which point his expectations will have decreased. In a general-equilibrium model, $\alpha_t = 1$ for all t such that, if the agent wants to take on more risk, returns have to become more risky, which increases the compensation to hold the Lucas tree.[16] When the investor demands more risk the conditional variance of the risky return increases, which necessarily increases the covariance of the risky return and excess return with consumption growth because the risk-free rate is constant. When the covariance of excess returns with consumption growth is increased, the expected equity premium is higher.

The following proposition formalizes the model's implications for variation and predictability in returns and the equity premium.

Proposition 4. *If $\theta > 1$, the realization of the shock ε_t negatively impacts the expected risky return ${}^{\partial E_t[R_{t+1}]}/_{\partial \varepsilon_t} < 0$, risk-free return ${}^{\partial R^f_t}/_{\partial \varepsilon_t} < 0$, and equity premium ${}^{\partial (E_t[R_{t+1}]-R^f_{t+1})}/_{\partial \varepsilon_t} <$*

[15] Because the consumption-price ratio has a similar shape to the consumption-wealth ratio, the rates of return also correspond. Accordingly, the variation is bounded because gain-loss utility is bounded for a bell-shaped shock distribution. Moreover, expected returns are negatively skewed due to the skewness in the variation of the consumption-wealth ratio. In contrast, realized gross returns are positively skewed.

[16] That the partial-equilibrium portfolio share increases does not necessarily imply that the demand for shares of the risky asset increases because the agent decides to consume more and invest less into both the risky and risk free assets; overall, the increase in the consumption-wealth ratio ρ_t may therefore offset the increase in the portfolio share α_t such that overall demand for the risky asset $(1-\rho_t)W_t\alpha_t$ decreases in bad times.

0. *This implies predictive power of the period t consumption-wealth ratio ρ_t for the period $t+1$ return R_{t+1} and excess return $E_t[R_{t+1}] - R^f_{t+1}$.*

For illustration, Figure 2 in Appendix A compares the annualized news-utility return and equity premium with those of the traditional model under the calibration in Table 2 with $\eta(\lambda - 1) = 2$.[17] The expected equity premium amounts to approximately seven percent for low values of ε_t and three percent for high values of ε_t. But the figure also illustrates how the model fails to predict reality: the risk-free return varies considerably, a phenomenon not observed in aggregate data.

3.1 Basic model: Calibration and moments

I now calibrate the model to gauge its quantitative performance. Before assessing the model's ability to match asset-pricing moments, I show that the preferences imply plausible attitudes toward small and large wealth gambles.

Risk attitudes over small and large stakes. I now illustrate which news-utility parameter values, i.e., η, λ, and γ, are consistent with existing micro evidence on risk preferences over small and large stakes and time preferences. I first show that the news-utility model does not generate high equity premia by curving the value function to generate high effective risk aversion. On the contrary, the news-utility model retains a value function with constant curvature because it is proportional to the power utility of wealth, i.e., $V_t = u(W_t)\Psi_t$ such that $RRA_t = -{}^{W_t V_t''}/_{V_t'} = \theta$.[18]

In Table 1, I illustrate the risk preferences over gambles of various stakes of the traditional, news-utility, habit-formation (Campbell and Cochrane (1999)), and long-run risk (Bansal and Yaron (2004)) agents. In particular, I analyze a range of 50-50 win G or lose L gambles at an initial wealth level of $\bar{W}_t$ in the spirit of Rabin (2001), Chetty and Szeidl (2007), and Barberis and Huang (2008, 2009). I elicit the agents' risk attitudes by assuming that each of them is presented with the gamble after the shock to period t consumption growth has been realized and all consumption C_t in period t has taken place. Thus, the news-utility agent will experience prospective gain-loss utility over wealth gambles and contemporaneous gain-loss utility over immediate-consumption gambles. In Appendix B.6, I show that the news-utility agent is just indifferent to a wealth gamble if

$$(Q + \Omega + \gamma Q\Omega)u(\bar{W}_t) = \gamma(0.5\eta(u(\bar{W}_t + G) - u(\bar{W}_t))Q + \eta\lambda 0.5(u(\bar{W}_t - L) - u(\bar{W}_t))Q) \\ + (Q + \Omega + \gamma Q\Omega)(0.5u(\bar{W}_t + G) + 0.5u(\bar{W}_t - L)). \quad (14)$$

The first part of the right-hand side of equation (14) represents prospective gain-loss utility, while the second part represents the same value comparison as made by the traditional

[17] If $\theta < 1$ and γ not too low, the realization of the shock ε_t positively impacts the expected risky return ${}^{\partial E_t[R_{t+1}]}/_{\partial\varepsilon_t} > 0$, risk-free return ${}^{\partial R^f_t}/_{\partial\varepsilon_t} > 0$, and equity premium ${}^{\partial(E_t[R_{t+1}] - R^f_{t+1})}/_{\partial\varepsilon_t} > 0$. This implies predictive power of the period t consumption-wealth ratio ρ_t for the period $t+1$ return R_{t+1} and excess return $R_{t+1} - R^f_{t+1}$.

[18] The intertemporal elasticity of substitution is disentangled and exhibits variation. Such disentanglement is a feature of a broad range of non-time-separable utility functions, such as habit formation.

agent, i.e., $u(\bar{W}_t) \lesseqgtr 0.5u(\bar{W}_t + G) + 0.5u(\bar{W}_t - L)$. Thus, if γ were zero, the news-utility agent's risk attitudes over wealth gambles would be exactly the same as those of the traditional agent. Moreover, if L and G are small but $G > L$, this second part will certainly be positive, as $u(\cdot)$ is almost linear, but the first part will induce prospect-theory risk preferences over future consumption. Although λ alone determines the sign of prospective gain-loss utility, there are restrictions on the other parameters, because the positivity of the second part may dominate the negativity of the first part if γ is small. γ implies attitudes towards intertemporal consumption tradeoffs that resemble those of a hyperbolic-discounting agent whose hyperbolic-discounting coefficient is equal to γ. The hyperbolic-discounting coefficient has been estimated in a variety of contexts to be between 0.7 and 0.8 (e.g., Laibson et al. (fthc)). The experimental and field evidence on agent's attitudes towards intertemporal consumption tradeoffs thus dictates a choice of $\gamma \approx 0.7$ when $\beta \approx 1$.

The model should simultaneously match risk attitudes towards gambles about immediate consumption, which are determined solely by η and λ, because it can be reasonably assumed that utility over immediate consumption is linear. The laboratory evidence on loss aversion over immediate consumption, i.e., the endowment effect literature (Kahneman et al. (1990)), dictates $\eta(\lambda - 1) \approx 2$.[19] More precisely, $\eta(\lambda - 1) \approx 2$ implies that the equivalent Kahneman and Tversky (1979) coefficient of loss aversion is around 2, because the news-utility agent experiences consumption and gain-loss utility, whereas classical prospect theory consist of gain-loss utility only, and consumption utility works in favor of any small-scale gamble.

In Table 1, I calculate the required G for each value of L to make each agent just indifferent between accepting or rejecting a 50-50 win G or lose L gamble at wealth level $\bar{W}_t = 300,000$ for $\eta = 1$ and $\lambda = 3$. It can be seen that the news-utility agent's risk attitudes take reasonable values for small, medium, and large stakes.[20] In contrast, the traditional and long-run risk agents are risk neutral for small stakes and almost risk neutral for medium stakes. The habit-formation agent is risk neutral for small stakes,

[19] Let me take a concrete example from Kahneman et al. (1990), assuming that utility over mugs, pens, and small amounts of money is linear. Kahneman et al. (1990) hand out mugs to half their subjects, and ask those who did not receive one about their willingness to pay and those who received one about their willingness to accept when selling the mug. The authors observe that the median willingness to pay for the mug is \$2.75 whereas the willingness to accept is \$5.25. Accordingly, I can infer $(1+\eta)u(mug) = (1+\eta\lambda)2.25$ and $(1+\eta\lambda)u(mug) = (1+\eta)5.25$ which implies that $\lambda \approx 3$ when $\eta \approx 1$. For the pen experiment I also obtain $\lambda \approx 3$. Unfortunately, so far I can only jointly identify η and λ. However, $\eta = 1$ and $\lambda \approx 3$, so that $\eta(\lambda - 1) \approx 2$, seem to be reasonable choices and have also been typically used in the literature for the static preferences.

[20] While the news-utility agent's risk preferences over contemporaneous consumption exactly match the findings of Kahneman et al. (1990), his required gain for small gambles about future consumption is somewhat lower than the estimates obtained by Tversky and Kahneman (1992), even though the authors consider monetary gambles and thus future consumption. But the news-utility model also predicts that people consume entire small gains when surprised by them (Koszegi and Rabin (2009)). Thus, the contemporaneous consumption results might be applicable even for monetary gambles. Moreover, in a setting which does explicitly consider gambles over future consumption, Andreoni and Sprenger (2012) find significantly less small-scale risk aversion toward those gambles. In any case, I do not aim to perfectly match the experimental evidence here. I simply want to demonstrate that the model explains small- and large-scale risk aversion reasonably well.

Table 1: **Risk attitudes over small and large wealth gambles**

Loss (L)	traditional	news-utility contemp.	news-utility prospective	habit-formation	long-run risk
10	10	20	14	10	10
200	200	402	283	200	201
1000	1010	2041	1434	1362	1035
5000	5357	11119	76742	19749	6002
50000	163000	415480	189580	∞	303499
100000	∞	∞	∞	∞	∞

For each loss L the table's entries show the required gain G to make each agent indifferent between accepting and rejecting a 50-50 gamble win G or lose L at wealth level 300,000.

reasonably risk averse for medium stakes, but unreasonably risk averse for large stakes. Campbell and Cochrane (1999) also discuss this finding and indicate that the curvature of the habit-formation agent's value function is approximately 80 at the steady-state surplus-consumption ratio; thus, the habit-formation agent behaves similarly to a traditional agent with $\theta = 80$. The long-run risk agent behaves similarly to a traditional agent with $\theta = 10$, the choice of Bansal and Yaron (2004). The disappointment-aversion model (Routledge and Zin (2010) and Bonomo et al. (2010)) does not robustly match risk attitudes toward small and large wealth gambles. Because the agent is loss averse with respect to a fraction of the gamble's certainty equivalent, he is not necessarily "at the kink" in high or low-consumption situations. The asset-pricing theories based on prospect theory (Barberis et al. (2001), Benartzi and Thaler (1995), Yogo (2008), and Andries (2013)) imply plausible attitudes towards small and large wealth gambles. However, Barberis et al. (2001), Benartzi and Thaler (1995), and Andries (2013) do not feature loss aversion over immediate consumption and are thus inconsistent with attitudes towards consumption gambles and the endowment-effect evidence.

Calibration. Table 2 displays the calibration and the resulting moments of the news-utility and traditional models. I assume a traditional Lucas-tree model environment in which consumption equals dividends, so that the model environment is fully calibrated by μ_c and σ_c. I follow Bansal and Yaron (2004) and choose $\mu_c = 1.89\%$ and $\sigma_c = 2.7\%$ in annualized terms. I then choose the well-known preference parameters β and θ are then chosen to roughly match the level of the mean risky return, the mean risk-free return, and the risky return volatility, as done by Bansal and Yaron (2004). I simulate the model at a biannual frequency and then annualize moments. The news-utility equity premium increases in the model's frequency. The reason is that the news-utility agent dislikes fluctuations in beliefs about consumption. As news disutility is a first-order effect, it is proportional to the standard deviation of the consumption process. News disutility thus increases with the square root of the investment's horizon while the investment's return increases linearly, which makes the investment less favorable over shorter horizons. Therefore, the required compensation for bearing the risk of the Lucas tree increases. The model's frequency can be interpreted as the frequency with which the agent learns about the realization of the stock market, observes his wealth, and reoptimizes his consumption plans. The empirical evidence on how often people look up and trade in their brokerage accounts is mixed (refer to Bonaparte and Cooper (2009), Calvet et al. (2009), Karlsson et al. (2009), Alvarez et al. (2012), and Brunnermeier and Nagel (2008) among others). For instance, Bonaparte and Cooper (2009) find that investors rebalance their portfolios approximately 1.2 times per year, while Alvarez et al. (2012) find that people trade approximately twice per year but observe their brokerage accounts approximately once per month. Thus, I conclude that a frequency between one and twelve months is reasonable.

The simulation frequency thus constitutes a calibrational degree of freedom in the news-utility model. At a monthly frequency, θ has to be close to one to match the historical equity premium. To have a bit more space in picking θ, I choose a biannual frequency, $\theta = 4$, and $\beta = 0.999$ to roughly match the historical equity premium, the equity premium's volatility, and the mean risk-free rate. Simulating the model at an annual frequency requires a somewhat higher coefficient of risk aversion θ and consumption volatility σ_c. But these are not unusual in the literature. For instance, with $\sigma_c = 3.79\%$, as in Barberis et al. (2001), and $\theta = 10$, as in Bansal and Yaron (2004), the annualized news-utility model would roughly match the historical equity premium and its volatility. More specifically, at an annual frequency the calibration used here, i.e., $\theta = 4$ and $\eta(\lambda - 1) = 2$, would result in an equity premium of 2.3% with a volatility of 11%.

The news-utility parameters are calibrated as standard in the prospect-theory literature: $\eta = 1$ and $\eta(\lambda - 1) \in [1.6; 2.3]$ to match the large array of experimental evidence on loss aversion and to induce reasonable risk attitudes over small and large stakes, as can be seen in Table 1. $\eta(\lambda - 1) \approx 2$ implies that the equivalent Kahneman and Tversky (1979) coefficient of loss aversion, or those used in Benartzi and Thaler (1995), Barberis et al. (2001), and Andries (2013), is around 2. The reason is that the news-utility agent experiences consumption and gain-loss utility, whereas classical prospect theory consists of gain-loss utility only, and consumption utility works in favor of any small-scale gamble. Not surprisingly, similar values have also been used in the existing prospect-theory asset-pricing literature: Benartzi and Thaler (1995) assume a coefficient of loss aversion of 2.5 and Barberis et al. (2001) and Andries (2013) assume a mean coefficient of loss aversion of approximately 2.25. Moreover, because $\gamma = 0.8$ implies attitudes towards intertemporal consumption tradeoffs similar to those implied by a hyperbolic-discounting factor of 0.7, I choose to follow the estimates of the hyperbolic-discounting literature (Angeletos et al. (2001) and Laibson (1997)) in choosing this value. I argue that the existing experimental literature suggests fairly tight ranges for all the news-utility parameters, η, λ, and γ, as well as for the traditional preference parameters θ and β. Thus, news utility does not allow for large parameter ranges that could be used at my discretion, and there is no need to scale the gain-loss utility component, as it is based on consumption. However, the simulation frequency constitutes a more worrisome degree of freedom, because it has been ignored in static applications of Koszegi and Rabin (2006, 2007) preferences.

Risky and risk-free return moments. As can be seen in Table 2, the model roughly matches the historical mean equity premium, its volatility, and the mean risk-free rate elicited from CRSP return data. The news-utility model generates the historical equity

Table 2: **Calibration and moments of the basic model**

calibration						
μ_c	σ_c	β	θ	η	λ	γ
1.89%	2.7%	.999	4	1	2.6,3,3.4	0.7

moments	traditional and news-utility models				data
	$\eta = 0$	$\eta(\lambda - 1) = 1.6$	$\eta(\lambda - 1) = 2$	$\eta(\lambda - 1) = 2.4$	
$E[r_t - r_t^f]$	0.41	3.17	4.01	4.86	6.33
$\sigma(r_t - r_t^f)$	2.72	13.7	16.6	19.4	19.4
$E[r_t^f]$	7.07	3.07	1.99	0.95	0.86
$\sigma(r_t^f)$	0.00	11.3	14.2	16.9	0.97
$corr(\Delta c_t, r_t)$	0.87	0.68	0.66	0.48	0.4
$corr(\Delta c_t, r_{t+1})$	0.31	-0.43	-0.46	-0.16	0.09
$AR(r_t - r_t^f)$	0.03	0.036	0.04	0.04	0.01
$AR(r_t)$	0.03	-0.47	-0.48	-0.48	0.01
$E[c_t - p_t]$	-3.59	-3.86	-3.92	-3.97	-3.4
$\sigma(c_t - w_t)$	0.00	0.06	0.07	0.01	0.015
$AR(c_t - w_t)$	1.00	0.01	0.01	0.02	0.6
R^2	0.00	0.25	0.25	0.25	0.18

Return and consumption moments are annualized from value-weighted CRSP return data and BEA data on-real per-capita consumption of nondurables and services for the period 1929-2011. The first four rows of return moments are in percentage terms. The parameters μ_c, σ_c, and β are annualized. The annualized moments for the consumption-wealth ratio correspond to the annual data of Lettau and Ludvigson (2001, 2005) with the R^2 corresponds to a forecasting regression of quarterly stock returns on the quarterly consumption-wealth ratio $r_{t+1} = \alpha + \beta(c_t - w_t) + \delta r_t^f$.

premium volatility, despite the fact that consumption equals dividends in the basic Lucas-tree model. Thus, the model matches the historical risk-return trade-off with a Sharpe ratio of approximately 4. Unfortunately, the news-utility model completely mispredicts the risk-free rate volatility. Moreover, the risk-free rate is countercyclical in the model but procyclical in the data (Fama (1990)). The conditional variance of the stock market is countercyclical as in the data (Brandt and Kang (2004)), however, there is no empirical evidence that a high consumption-wealth ratio predicts high conditional variance.

The model's performance regarding other return moments is mixed, as can be seen in Table 2. The model matches the contemporaneous correlation of consumption growth with returns reasonably well, but overpredicts the one-period-ahead correlation.[21] Predicting too-high a correlation between returns and consumption growth is a common failure of leading asset-pricing models, as emphasized by Albuquerque et al. (2014) among others. But the variation in the consumption-wealth ratio in the news-utility model is a short-

[21] Many asset-pricing models overstate the contemporaneous correlation of consumption and returns, which can be reduced by introducing a separate dividend process. As I roughly match this value I conclude that a separate process for dividends is unnecessary in the basic news-utility model, though it will reduce the misprediction of the one-period-ahead correlation.

run phenomenon, and at longer horizons the correlation between consumption growth and asset returns is very low, thus matching the data. Additionally, while the autocorrelation of excess returns is matched quite well, the autocorrelation of returns is negative in the model but close to zero in the data. This is because, upon an adverse return realization, the news-utility agent underprices the stock market so that future returns are high.

The consumption-wealth ratio. The model's simulated consumption-wealth ratio reflects the prior theoretical results. First, the consumption-wealth ratio is lower than in the traditional model and exhibits variation. As consumption equals dividends in the traditional Lucas-tree model and there is no labor income, the values are difficult to compare with the data. However, the corresponding values in Lettau and Ludvigson (2001) are displayed as an illustration. Both the traditional and news-utility model roughly match the level of the consumption-price ratio, but the traditional model mispredicts its variation while the news-utility model's predicted variation is roughly in line with the data.[22] However, the news-utility consumption-wealth ratio is i.i.d., whereas Lettau and Ludvigson (2001) find relatively high persistence. Unfortunately, with full belief updating, there is no reason to expect persistence in the consumption-wealth ratio in the basic model.

With respect to the predictability properties of annual excess returns, the model yields R^2 values of approximately 25%. Lettau and Ludvigson (2001) emphasize the medium-run predictive power of the aggregate consumption-wealth ratio. The authors obtain R^2 values for annual excess returns of 18%. As noted by Lustig et al. (fthc) and Hirshleifer and Yu (2011), traditional leading asset-pricing models have difficulty matching the volatility of the consumption-wealth ratio and the return on the consumption claim, because they rely on a volatile dividend process, and the only variation in the consumption-wealth ratio stems from heteroskedasticity in consumption growth. I can confirm this finding; using the return on the consumption claim, the R^2 in the habit-formation model of Campbell and Cochrane (1999) is merely 1.6% and the R^2 in the long-run risk model of Bansal and Yaron (2004) is just 2.9%. The predictability properties of the price-consumption ratio are similar to those of the consumption-wealth ratio, as they exhibit the same variation.

In Figure 3 in Appendix A, I plot the simulated deviations of the consumption-wealth ratio in the news-utility, traditional, habit-formation, and long-run risk models, and compare these with the annual $\widehat{cay}$ data provided by Lettau and Ludvigson (2005). For the habit-formation and long-run risk models, I use the calibration of Campbell and Cochrane (1999) and Bansal and Yaron (2004) to then simulate the model at a monthly frequency and aggregate the consumption and wealth time series. For comparison purposes, I also simulate the news-utility model at a monthly frequency to then aggregate choosing lower values of θ and β to simultaneously match the equity premium and its volatility. Moreover, I feed in the deviations in log consumption growth $\Delta c - 12\mu_c$ supplied by the $\widehat{cay}$ data. The figure shows that news utility introduces considerably more rapid variation in the consumption-wealth ratio than does either the traditional model or the model augmented with long-run risk, but much less variation than the habit-formation model. While the long-run risk consumption-wealth ratio appears to be too smooth and the habit-formation

[22] Moreover, the consumption-wealth ratio cannot be used to forecast consumption growth, which is in line with the empirical findings in Lettau and Ludvigson (2001).

consumption-wealth ratio too variable, the variation in the news-utility consumption-wealth ratio matches the $\widehat{cay}$ data quite well. Although it is disputable to compare the $\widehat{cay}$ data to the simulated data of a Lucas-tree model, I conclude that the rapid variation is supported by the data.[23] Barberis et al. (2001) generate predictability via variation in the coefficient of loss aversion; however, the model's consumption-wealth ratio is constant. Similarly, Routledge and Zin (2010) generate predictability via variation in risk aversion, but the authors do not compare the properties of their model to the data given their two-outcome setting. In contrast, Bonomo et al. (2010) put special emphasis on matching the empirical return-predictability patterns. However, given that the variation in risk aversion and the variation in long-run risk are generating the predictability power of the price-dividend ratio, there is no reason to expect great predictability of the consumption-wealth ratio in their model.

At first blush, the model's asset pricing implications appear to be mixed. News utility raises the equity premium and its volatility to historical levels even though I omit a separate dividend process. Furthermore, the variation in substitution motives generates strong variation in the consumption-wealth ratio and predictability in returns, matching the data better than leading asset-pricing models. However, the model predicts excessive volatility in the risk-free rate. This shortcoming is addressed in the following section.

4 Extensions

4.1 Environment-based extensions

The news-utility model's most important shortcoming is the large predicted variation in the risk-free rate. Nevertheless, I want to take the predictions of the evidence-based utility specification seriously. People are unwilling to substitute consumption intertemporally in some states of the world; the most obvious evidence is credit-card borrowing and pay-day loans. However, there may be forces at work that offset the effects of this variation in substitution motives on the aggregate risk-free rate. Therefore, I ask: What would a consumption process look like that induces the risk-free rate to be less volatile? An adverse shock to contemporaneous consumption growth has to be associated with an adverse prediction about future consumption growth to keep the risk-free rate stable. More specifically, if low values of ε_t are associated with a decrease in μ_c or an increase in σ_c, the model's risk-free rate process will become more smooth. Variation in the agent's expected consumption growth μ_c has been exploited by Bansal and Yaron (2004) and termed long-run risk. Variation in the agent's expected volatility of consumption growth has been

[23] Greenwood and Shleifer (2014) compare a variety of survey data on stock market expectations with the predicted expected returns of leading asset-pricing models. The authors show that leading asset-pricing models' implied expected returns do not correlate highly with the survey evidence on expected returns. In particular, the $\widehat{cay}$ model of Lettau and Ludvigson (2001) fits the survey data better than the habit-formation and long-run risk models do. I replicate this finding using the American Association of Individual Investors Sentiment Survey, and I also find that the news-utility model is more positively correlated with the survey data than the habit-formation, long-run risk, models or the $\widehat{cay}$ data. However, this finding should not be overinterpreted, as the annual comparison includes the years 1987 to 2001 only.

exploited by Campbell and Cochrane (1999) and Bansal and Yaron (2004).[24]

I can reverse-engineer variation in expected consumption growth and its volatility to offset the effect of the variation in the agent's intertemporal smoothing incentives on the risk-free rate. An adverse shock to consumption growth today is then associated with low consumption growth but high volatility in the future. The empirical evidence on excess sensitivity suggests that there exists positive autocorrelation in consumption growth. Moreover, there exists empirical evidence for countercyclical variation in economic uncertainty, or consumption volatility.[25]

Unfortunately, it turns out that the variation in the agent's smoothing incentives requires variation in the agent's expected consumption growth that is too large to be consistent with aggregate consumption data. The reason is that the variation in consumption volatility appears to be too weak to significantly affect the strong first-order variation in the agent's risk-free rate.

As an alternative, I extend the model to account for time-variant disaster risk to smooth out the risk-free rate. Time-variant disaster risk is a very powerful device under news-utility preferences because, as I explain below, they feature left-skewness aversion: the news-utility agent dislikes the left tail and thus dislikes disaster risk more than the traditional agent. It turns out that time-variant disaster risk is powerful enough to successfully offset the variation in the risk-free rate. Moreover, Barro (2006) provides compelling evidence for the existence of a small probability of economic disaster.

It is important to note that introducing another source of variation does not eliminate the variation in substitution motives; it merely offsets the effects of that variation on the risk-free rate. Furthermore, the extended models feature two sources of variation: the news-utility variation in substitution motives and heteroskedasticity in consumption growth or time-variant disaster risk. While the first source of variation concerns intertemporal substitution, the latter works via variation in the price of risk.

Setup. A decrease in expected consumption growth μ_c or an increase in expected volatility σ_c makes the agent consume less and save more. Thus, if an adverse shock is associated with a decrease in expected consumption growth or an increase in expected volatility, the agent's intertemporal-substitution effects on the risk-free rate will be partially offset. Let consumption growth be given by $log(^{C_{t+1}}/_{C_t}) = \mu_t + \sigma_t \varepsilon_{t+1}$ with $\mu_{t+1} = \mu_c + \nu_\mu(\mu_t - \mu_c) + \tilde{\mu}(\varepsilon_{t+1}) + u_{t+1}$, $u_{t+1} \sim (0, \sigma_u^2)$, and $\tilde{\mu}(\varepsilon_{t+1}) = \bar{\mu}(log(^{1-\rho_{t+1}}/_{\rho_{t+1}}) - E[log(^{1-\rho_t}/_{\rho_t})])$. More-

[24]Campbell and Cochrane (1999) specify heteroskedasticity in consumption growth to make the risk-free rate exactly constant.

[25]Since French et al. (1987) it is well known that the volatility of stock returns fluctuates considerably over time. Moreover, Black (1976) was one of the first to document that stock returns are negatively correlated with future volatility, an empirical observation which has been referred to as the leverage or volatility-feedback effect. More recently, Lettau and Ludvigson (2004) document that the countercyclical and highly volatile Sharpe ratio is not replicated by leading consumption-based asset pricing models. The Sharpe ratio becomes both more countercyclical and more volatile if low returns imply high expected returns and low volatility, as I assume in the extended model. The authors find that the consumption-wealth ratio predicts stock market volatility and provide evidence for variation in aggregate consumption volatility. Furthermore, Tauchen (2011) connects the negative correlation in stock returns and volatility back to the consumption process underlying a standard Lucas-tree model. Finally, robust evidence for heteroskedasticity is provided by Bansal et al. (2005).

over, $\sigma_{t+1}^2 = \sigma_c^2 + \tilde{\sigma}(\varepsilon_{t+1}) + \nu_\sigma(\sigma_t^2 - \sigma_c^2) + w_{t+1}$, $w_t \sim (0, \sigma_w^2)$, and $\tilde{\sigma}(\varepsilon_t) = \bar{\sigma}(0.5 - F(\varepsilon_t))$. The variation in $\tilde{\sigma}(\varepsilon_{t+1})$ aims to reflect the variation in the homoskedastic consumption-wealth ratio, because heteroskedasticity is intended to offset the general-equilibrium impact on the risk-free rate. Note that σ_t is a Markovian process, increases in the event of an adverse shock and is characterized by a shape similar to the consumption-wealth ratio determined by the variation in intertemporal-substitution motives. Moreover, the conditional expectation of economic volatility is characterized by an AR(1) process with persistence ν_σ. If $\nu_\sigma > 0$, then a positive shock to economic volatility today implies high volatility in the future, because the heteroskedasticity process is autocorrelated. In that case, the size of the excess returns will be autocorrelated, and the model will be able to generate autocorrelation in the returns and long-horizon predictability.[26] μ_t is chosen to fine-tune the remaining variation in the risk-free rate. The functional form of $\tilde{\mu}(\varepsilon_t)$ is reverse-engineered such that, if $\bar{\mu} = 1$ and $\nu_\mu = 0$, the variation in the risk-free rate brought about by the variation in the price-consumption ratio will be exactly offset; as can be seen in equation (11). If $\nu_\mu > 0$, then the conditional expectation of consumption growth is characterized by an AR(1) process with persistence ν_μ.

Now, suppose that there exists a small probability of a disastrous consumption realization as in Barro (2006, 2009). An increase in the probability of disaster makes the agent value a unit of safe consumption more highly. Thus, if adverse shocks are associated with disaster risk, the risk-free rate smooths out. More specifically, suppose that in each period t, there is a probability p_t that a disaster occurs in period $t+1$ in which case consumption drops by d percent. Then, consumption growth is given by $log(C_{t+1}/C_t) = \mu_c + \varepsilon_{t+1} + v_{t+1}$ with $\varepsilon_{t+1} \sim N(0, \sigma_c^2)$ and $v_{t+1} = log(1-d)$ with probability p_t and zero otherwise. I assume that ε_{t+1} and v_{t+1} are independent. The simple process governing the variability in disaster risk is $p_{t+1} = p + \nu(p_t - p) + u_{t+1} + \tilde{g}(\varepsilon_{t+1})$ with $u_{t+1} \sim N(0, \sigma_u^2)$ and $\tilde{g}(\varepsilon_t) = p\bar{p}(0.5 - F(\varepsilon_t))$. Note that p_t is a Markovian process, increases in the event of an adverse shock, and has a similar shape to the consumption-wealth ratio determined by the variation in intertemporal-substitution motives. Moreover, the conditional expectation of disaster risk is characterized by an AR(1) process with persistence ν.

The news-utility agent is more affected by the probability of disaster than the traditional agent is, because the news-utility agent dislikes disaster risk more. The utility function's gain-loss component over news is inspired by prospect theory. Classical prospect theory assumes a value function of the form $v(c-r)$, defined over the actual consumption level c relative to the reference point r. Typically, the value function features a kink at the reference point r, concavity over gains where $c > r$, convexity over losses where $c < r$, and probability weighting. In contrast, Koszegi and Rabin (2009) specify gain-loss util-

[26]Koszegi and Rabin (2007) find that news utility causes variation in risk attitudes. In particular, the authors state that the agent becomes less risk averse when moving from a fixed to a stochastic reference point. With a stochastic reference point, a gamble does not appear as daunting, because some potential losses were previously expected. Thus, the equity premium in period t depends negatively on σ_{t-1}, because it is determined by the price of risk, i.e., $Cov_t(M_{t+1}, R_{t+1})/E_t[M_{t+1}]\sigma_t(R_{t+1})$, which varies with σ_t and σ_{t-1}. If high volatility is expected, ρ_t is less steep and thus less responsive to a shock to consumption growth, which tends to reduce the required equity premium. Hence, news-utility preferences introduce two sources of variation in the price of risk and thus in the required equity premium: The price of risk varies with economic volatility σ_t as in the traditional model. Furthermore, for any given σ_t, the price of risk varies inversely with the variability of beliefs determined by σ_{t-1}.

ity as the linear difference in utility values $\mu(u(c) - u(r))$ with $\mu(\cdot)$ being some type of prospect-theory value function. The authors note that diminishing sensitivity or probability weighting may be introduced via $\mu(\cdot)$. However, thus far I have followed the literature and will retain news-utility preferences in their most basic form, with $\mu(\cdot)$ being piecewise linear. Interestingly though, using a piecewise linear $\mu(\cdot)$ function results in left-skewness aversion: The news-utility agent hates the left tail. Because the agent assesses gain-loss utility as the linear difference in utility values $u(c) - u(r)$, the left tail, where $u(\cdot)$ becomes steep, is relatively overweighted. In classical prospect theory, left-skewness aversion can be caused only by low-probability overweighting. Thus, the basic form of Koszegi and Rabin (2009) preferences is likely to yield very interesting dynamics with respect to a small disaster probability.

Calibration and moments. Simulations reveal, unfortunately, that, if the variation in expected consumption growth smooths out 80% of the variability of the risk-free rate, the required variation in $\tilde{\mu}(\cdot)$ significantly changes the moments of the annualized consumption growth process which then fails to match the data even if lower levels for both μ_c and σ_c are chosen. For instance, the annualized standard deviation of the simulated consumption process should be at most 3.5% but reaches 9%.[27]

In contrast, if the variation in disaster risk smooths out 80% of the variability of the risk-free rate, the extended model yields a realistic set of moments using the parameters of disaster risk calibrated in Barro (2009). Moreover, a positive autocorrelation in the probability of disaster $\nu > 0$ will generate long-horizon predictability in returns and excess returns.[28]

4.2 Preference-based extensions

One of the important selling points of preference-based asset-pricing models is their ability to match moments even though the consumption process is simply i.i.d. Thus, it might be of interest to explore a modification of the preferences to improve the model's asset-pricing performance. In this section, I propose two preference-based extensions. The first one is minor: I simply assume that the length of the agent's upcoming time period varies with the consumption shock, in the spirit of the time-variant attention model of Andrei and Hasler (fthc). In the event of an adverse consumption realization, the upcoming period faced by the agent is shorter. A shorter upcoming period drives up the equity premium, which should smooth out the risk-free rate, because the agent can diversify across time. My second extension is more significant. Here I assume that the agent's reference point does not fully incorporate all information in each period, but rather that the reference point is characterized by gradual adjustment. More specifically, I assume that the reference point incorporates information with a more-than-one-period lag, and otherwise corresponds to a weighted average of past beliefs as suggested by Koszegi and Rabin (2009). In the event of a favorable shock today, the agent therefore expects, on average, positive gain-loss utility

[27]The simulation results are available on request. The model's simple structure is unaffected by variation in expected consumption growth and can be found in this paper's online Appendix.

[28]The simulation results are available on request and the disaster-risk model's derivation can be found in this paper's online Appendix.

in the upcoming periods. While the first extension does little to correct the failures of the model, the second extension considerably improves its ability to match asset-pricing moments.

Setup. If the agent's beliefs about consumption are lower than actual consumption in response to a favorable shock, then he will consume more and save less. Thus, the agent's intertemporal-substitution effects on the risk-free rate will be partially offset. More precisely, consumption growth is given by $log(C_{t+1}/C_t) = \mu_c + \sigma_c \varepsilon_{t+1}$ and the agent's beliefs should correspond to it. But now suppose the agent's beliefs about consumption growth are given by $log(C_{t+1}/C_t) = \mu_t + \sigma_c \varepsilon_{t+1}$ with $\mu_{t+1} = \nu \mu_t + (1 - \nu)(\mu_c + \tilde{\mu}(\varepsilon_{t+1}))$. The variation in $\tilde{\mu}(\varepsilon_{t+1})$ should reflect the variation in the basic model's consumption-wealth ratio, because sluggish updating is intended to offset the general-equilibrium impact on the risk-free rate. Moreover, the conditional expectation of beliefs about consumption is roughly characterized by an AR(1) process with persistence ν. Thus, a positive shock today implies on average more positive gain-loss utility in the future. In turn, the size of the excess returns will be autocorrelated and the model is able to generate autocorrelation in the returns and long-horizon predictability. The model's simple structure is unaffected by variation in expected consumption growth and derived in Appendix C.

Calibration and moments. If I choose the basic model's calibration and the beliefs' autocorrelation to be 50%, the variation in expected consumption growth smooths out the majority of the variability in the risk-free rate. In turn, the extended model yields a realistic set of moments as displayed in Table 3. The model roughly matches the historical mean equity premium, its volatility, and the mean risk-free rate elicited from CRSP return data. Moreover, the extended model correctly predicts the risk-free rate volatility. The model's performance is thus comparable with those of Barberis et al. (2001), Yogo (2008), and Andries (2013). Moreover, the news-utility consumption-wealth ratio is no longer i.i.d., but reaches about a third of the persistence found in Lettau and Ludvigson (2005) but less than reported in Yogo (2008). In contrast, the consumption-wealth ratio Barberis et al. (2001) and Andries (2013) is constant. The model's performance regarding other return moments is still mixed. The contemporaneous correlation of consumption growth with returns is matched less well and the one-period-ahead correlation is still overpredicted. As before, the autocorrelation of excess returns is matched quite well, while the autocorrelation of returns is negative in the model but close to zero in the data. However, the positive autocorrelation in the agent's beliefs will also generate long-horizon predictability in returns and excess returns. With respect to the predictability properties of annual excess returns, the model yields R^2 values a bit lower than the value found in Lettau and Ludvigson (2001).[29]

5 Conclusion

In this paper, I incorporate expectations-based reference-dependent preferences into the canonical Lucas-tree model. In so doing, I contribute to the prospect-theory asset-pricing

[29]Lettau and Ludvigson (2001) report small-sample statistics, which might be biased upward.

Table 3: **Calibration and moments of the extended model**

	calibration						
	μ_c	σ_c	β	θ	η	λ	γ
	1.89%	2.7%	.965	3	1	2.6,3,3.4	0.7

moments	traditional and news-utility models				data
	$\eta = 0$	$\eta(\lambda - 1) = 1.6$	$\eta(\lambda - 1) = 2$	$\eta(\lambda - 1) = 2.4$	
$E[r_t - r_t^f]$	0.34	6.30	6.96	7.65	6.33
$\sigma(r_t - r_t^f)$	2.72	9.37	10.86	12.4	19.4
$E[r_t^f]$	8.91	2.72	1.75	0.75	0.86
$\sigma(r_t^f)$	0.0	2.30	3.77	5.23	0.97
$corr(\Delta c_t, r_t)$	0.87	0.80	0.79	0.77	0.4
$corr(\Delta c_t, r_{t+1})$	0.31	-0.20	-0.24	-0.26	0.09
$AR(r_t - r_t^f)$	0.02	-0.17	-0.11	-0.06	0.013
$AR(r_t)$	0.02	-0.33	-0.35	-0.36	0.011
$E[c_t - p_t]$	-3.31	-3.34	-3.37	-3.41	-3.4
$\sigma(c_t - w_t)$	0.00	0.04	0.06	0.07	0.015
$AR(c_t - w_t)$	1.00	0.14	0.14	0.14	0.6
R^2	0.00	0.03	0.06	0.09	0.18

Return and consumption moments are annualized from value-weighted CRSP return data and BEA data on-real per-capita consumption of nondurables and services for the period 1929-2011. The first four rows of return moments are in percentage terms. The parameters μ_c, σ_c, and β are annualized. The annualized moments for the consumption-wealth ratio correspond to the annual data of Lettau and Ludvigson (2001, 2005) with the R^2 corresponds to a forecasting regression of quarterly stock returns on the quarterly consumption-wealth ratio $r_{t+1} = \alpha + \beta(c_t - w_t) + \delta r_t^f$.

literature, pioneered by Benartzi and Thaler (1995) and Barberis et al. (2001) by assuming a generally-applicable utility function that is based on consumption, does not require a narrow-framing assumption, has an endogenous reference point, and has been shown to be consistent with behavior in a various micro domains. News utility has both desirable and undesirable implications. Most importantly, the preferences increase the equity premium and introduce considerable variation in excess returns, which match historical levels in spite of the fact that consumption equals dividends. Intuitively, reducing consumption below expectations is particularly painful in bad states of the world and the agent becomes unwilling to substitute present for future consumption – which, indeed, is likely to be true of people engaging in too much credit-card borrowing. However, in a general-equilibrium setup, this translates into large variability in the risk-free rate, a phenomenon not observed in aggregate data. I contribute to the asset-pricing literature by taking an additional step towards resolving the equity premium puzzle following Barberis and Huang (2008, 2009) and Yogo (2008): I show that the agent exhibits plausible risk attitudes towards small, medium, and large consumption and wealth gambles simultaneously.

Finally, I quickly describe the model's welfare implications. News utility increases the costs of business cycle fluctuations, in the spirit of Lucas (1978). For this paper's main calibration, the news-utility agent would be willing to give up approximately 43% of his consumption in exchange for a stable consumption path, whereas the traditional agent would give up merely 3%. Moreover, the first welfare theorem does not hold, because the preferences are subject to a time-inconsistent desire for immediate consumption. The agent behaves inconsistently because he takes today's beliefs as given when increasing today's consumption, but takes tomorrow's beliefs into account when increasing tomorrow's consumption. However, when he wakes up tomorrow, he will take tomorrow's beliefs as given, and will only consider the pleasure of increasing consumption above beliefs rather than increasing consumption and beliefs. As a result, the agent overconsumes relative to the optimal pre-committed consumption path that maximizes his expected utility.

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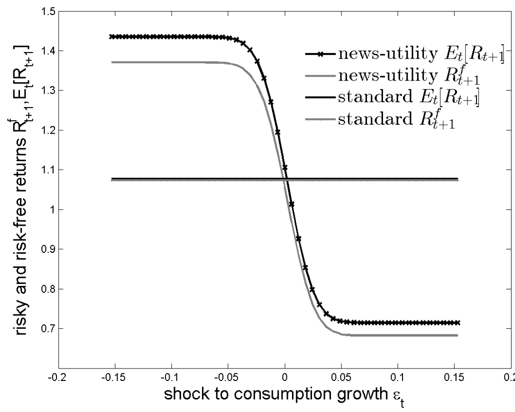
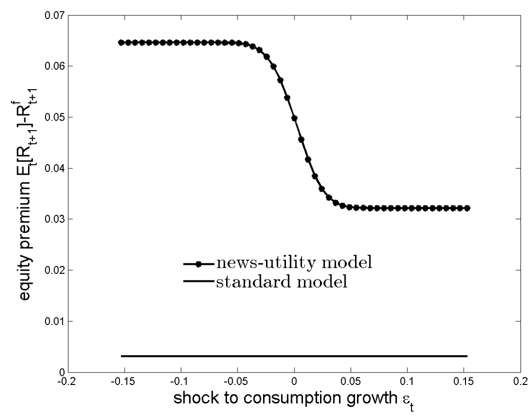
A More figures and tables



(a) Annualized expected risky $E_t[R_{t+1}]$ and risk-free returns R^f_{t+1} in the news-utility and traditional models



(b) Annualized equity premium $E_t[R_{t+1}] - R^f_{t+1}$ in the news-utility and traditional models.

Figure 2: Annualized returns and equity premium.

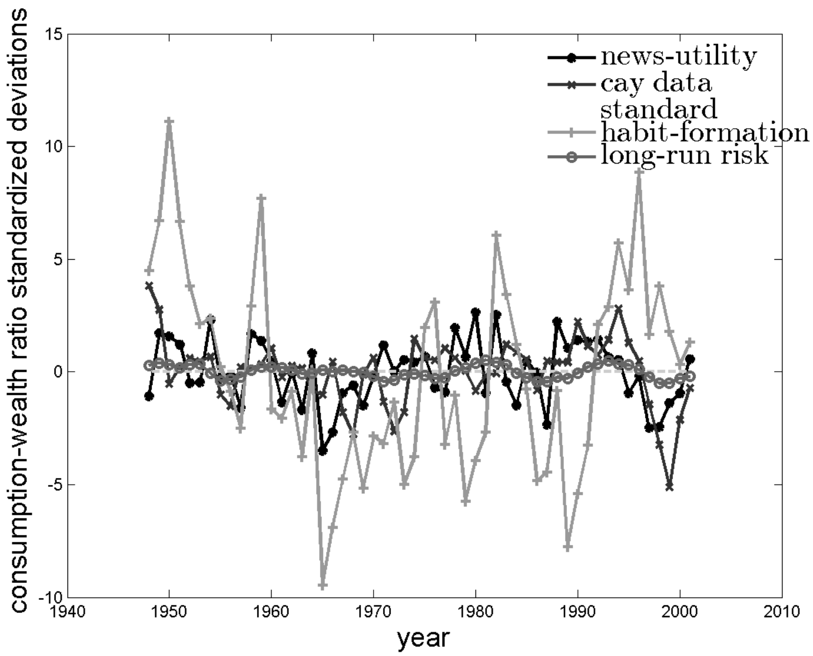


Figure 3: Simulated consumption-wealth ratio and comparison to the $\widehat{cay}$ data as provided by Lettau and Ludvigson (2005).

B Derivation and proofs

B.1 Proof of Proposition 1

In the following, I quickly guess and verify the model's equilibrium. In Section B.2, I derive the model's equilibrium in greater detail and more comprehensively. The exogenous consumption process is $C_{t+1}/C_t = e^{\mu_c+\varepsilon_{t+1}}$ and, in equilibrium, the agent beliefs about consumption are fully determined by it, i.e., $F^t_{C_{t+\tau}} = log\text{-}N(log(C_t) + \tau\mu_c, \tau^2\sigma_c^2)$. First, I define the following two constants determined by the exogenous parameters only

$$Q = E_t[\sum_{\tau=1}^{\infty}\beta^{\tau}(\frac{C_{t+\tau}}{C_t})^{1-\theta}] = E_t[\sum_{\tau=1}^{\infty}\beta^{\tau}(e^{\tau\mu_c+\sum_{j=1}^{\tau}\varepsilon_{t+j}})^{1-\theta}] = \frac{\beta e^{\mu_c(1-\theta)+\frac{1}{2}(1-\theta)^2\sigma_c^2}}{1-\beta e^{\mu_c(1-\theta)+\frac{1}{2}(1-\theta)^2\sigma_c^2}}$$

and

$$\psi = \beta e^{\mu_c(1-\theta)}E_t[(e^{\varepsilon_{t+1}})^{1-\theta} + (1+\gamma Q)(\eta\int_{-\infty}^{\varepsilon_{t+1}}((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta})dF(\varepsilon) + \\ + \eta\lambda\int_{\varepsilon_{t+1}}^{\infty}((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta})dF(\varepsilon)) + (e^{\varepsilon_{t+1}})^{1-\theta}\psi)].$$

The agent's maximization problem is

$$max_{C_t}\{u(C_t) + n(C_t, F^{t-1}_{C_t}) + \gamma\sum_{\tau=1}^{\infty}\beta^{\tau}\boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}}) + E_t[\sum_{\tau=1}^{\infty}\beta^{\tau}U_{t+\tau}]\}.$$

Now, it can be easily noted that $E_t[\sum_{\tau=1}^{\infty}\beta^{\tau}U_{t+\tau}] = u(C_t)\psi$ and $\gamma\sum_{\tau=1}^{\infty}\beta^{\tau}\boldsymbol{n}(F_{C_{t+\tau}}^{t,t-1})$ $= \gamma(\eta\int_{-\infty}^{C_t}(u(C_t)Q - u(c)Q)dF_{C_t}^{t-1}(c)) + \eta\lambda\int_{C_t}^{\infty}(u(C_t)Q - u(c)Q)dF_{C_t}^{t-1}(c))$ in equilibrium.

The agent is a price-taker. In the beginning of each period, the agent observes the realization of his wealth W_t and decides how much to consume C_t and how much to invest into the Lucas tree $P_t = C_t - W_t$. I guess the model's solution as $C_t = W_t\rho_t$ with ρ_t being i.i.d., independent of calender time t, or wealth W_t. Thus, next period's consumption is given by $C_{t+1} = (W_t - C_t)R_{t+1}\rho_{t+1}$ with $R_{t+1} = \frac{P_{t+1}+C_{t+1}}{P_t} = \frac{\rho_t}{1-\rho_t}\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}}$ so that $C_{t+1} = (W_t - C_t)\frac{\rho_t}{1-\rho_t}\frac{C_{t+1}}{C_t}$. From this consideration it can be easily seen that the agent's future value $u(C_t)\psi$ and $u(C_t)Q$ can be rewritten as $u(W_t - C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta}\psi$ and $u(W_t - C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta}Q$ whereby $\frac{\rho_t}{1-\rho_t}$ stems from the return and is thus taken as exogenous by the agent. In turn, the maximization problem can be rewritten as

$$\begin{aligned} max_{C_t}\{u(C_t) + \eta\int_{-\infty}^{C_t}(u(C_t) - u(c))dF_{C_t}^{t-1}(c)) + \eta\lambda\int_{C_t}^{\infty}(u(C_t) - u(c))dF_{C_t}^{t-1}(c) \\ + \gamma Q(\eta\int_{-\infty}^{C_t}(u(W_t - C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta} - u(c))dF_{C_t}^{t-1}(c)) \\ + \eta\lambda\int_{C_t}^{\infty}(u(W_t - C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta} - u(c))dF_{C_t}^{t-1}(c)) + u(W_t - C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta}\psi\} \end{aligned}$$

which yields the following first-order condition

$$\begin{aligned} & C_t^{-\theta}(1 + \eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t))) \\ & \quad = (W_t - C_t)^{-\theta}(\frac{\rho_t}{1-\rho_t})^{1-\theta}(\gamma Q(\eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t))) + \psi) \end{aligned}$$

as the agent takes his prior beliefs about consumption $F_{C_t}^{t-1}$ as given in the optimization and since $F_{C_t}^{t-1}(C_t) = F(\varepsilon_t)$ with $F \sim N(0, \sigma_c^2)$, because $C_t = C_{t-1}e^{\mu_c+\varepsilon_t}$. Rewriting the first-order condition allows me to verify the solution guess

$$\frac{C_t}{W_t} = \rho_t = \frac{1}{1 + \frac{\psi+\gamma Q(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))}{1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))}}.$$

B.2 Detailed derivation of the model's equilibrium

In the following I derive the model's equilibrium in greater detail. The agent optimally chooses his consumption C_t to maximize his life-time utility

$$max_{C_t}\{u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma\sum_{\tau=1}^{\infty}\beta^{\tau}\boldsymbol{n}(F_{C_{t+\tau}}^{t,t-1}) + E_t[\sum_{\tau=1}^{\infty}\beta^{\tau}U_{t+\tau}]\}. \tag{15}$$

The agent's wealth in the beginning of the period W_t is determined by the portfolio return $R_t^p = R_t^f + \alpha_{t-1}(R_t - R_t^f)$, which depends on the risky return realization R_t, the risk-free return R_t^f, and last period's optimal portfolio share α_{t-1}. I impose the equilibrium condition $\alpha_t = 1$ for all t to simplify the maximization problem. Now the agent's problem can be thought of as an infinite-horizon cake-eating problem with a single risky savings device. Thus, the budget constraint is

$$W_t = (W_{t-1} - C_{t-1})R_t \tag{16}$$

which results in the following first-order condition

$$u'(C_t)(1 + \eta F_{C_t}^{t-1}(C_t) + \eta\lambda(1 - F_{C_t}^{t-1}(C_t))) = u'(W_t - C_t)Q_t^0 + u'(W_t - C_t)\psi_t^0 \tag{17}$$

I explain each term in the first-order condition, equation (17), subsequently. The left hand side in equation 17 represents the agent's marginal utility due to consumption utility and gain-loss utility over contemporaneous consumption. Because the agent takes the reference point as given in the optimization and assuming optimal consumption is monotonically increasing in the return realization only the probability masses of states ahead and beneath remain to be considered. As an illustration, consider the following optimization

$$\begin{aligned}
&\frac{\partial}{\partial C_t}(\eta \int_{-\infty}^{C_t} (u(C_t) - u(c)))dF_{C_t}^{t-1}(c)) + \eta\lambda \int_{C_t}^{\infty} (u(C_t) - u(c)))dF_{C_t}^{t-1}(c)) \\
&\qquad = \eta \int_{-\infty}^{C_t} u'(C_t)dF_{C_t}^{t-1}(c) + \eta\lambda \int_{C_t}^{\infty} u'(C_t)dF_{C_t}^{t-1}(c) \\
&\qquad = u'(C_t)(\eta F_{C_t}^{t-1}(C_t) + \eta\lambda(1 - F_{C_t}^{t-1}(C_t)) \\
&\qquad = u'(C_t)(\eta F_{C_t}^{t-1}(C_t) + \eta\lambda(1 - F_{C_t}^{t-1}(C_t)) \\
&\qquad = u'(C_t)(\eta F_{R_t}^{t-1}(R_t) + \eta\lambda(1 - F_{R_t}^{t-1}(R_t)) = u'(C_t)(\eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t))
\end{aligned}$$

if C_t is monotonically increasing in the realization of R_t then $F_{R_t}^{t-1}(R_t) = F_{C_t}^{t-1}(C_t)$. In a preferred personal equilibrium the agent would know ex ante if the first-order condition induces him to "jump" realizations of R_t, and expectations over optimal consumption would adjust accordingly such that in equilibrium $F_{C_t}^{t-1}(C_t) = F_{R_t}^{t-1}(R_t)$ for each corresponding realization of C_t and R_t. Moreover, in general equilibrium the agent's beliefs have to match the model environment and hence $F_{R_t}^{t-1}(R_t) = F_{C_t}^{t-1}(C_t) = F(\varepsilon_t)$ for each corresponding realization of C_t, R_t, and ε_t such that both C_t and R_t are necessarily increasing in ε_t.

To explain the right hand side in equation 17 I guess and verify the equilibrium's structure. In each period t, the agent will consume a fraction ρ_t of his wealth W_t, i.e., $C_t = \rho_t W_t$. In the first-order condition, equation 17, the first term on the right hand side represents prospective gain-loss utility over the entire stream of future consumption. Note that, each future optimal consumption as a fraction of wealth can be iterated back to the current savings decision

$$C_{t+\tau} = (W_t - C_t)R_{t+\tau}\rho_{t+\tau} \prod_{j=1}^{\tau-1} R_{t+j}(1 - \rho_{t+j}).$$

Then, taking the reference point as given and assuming that optimal savings are monotonically increasing in the return realization results in

$$
\begin{aligned}
-(W_t - C_t)^{-\theta} Q_t^0 &= \frac{\partial \sum_{\tau=1}^{\infty} \beta^\tau \boldsymbol{n}(F_{C_{t+\tau}}^{t,t-1})}{\partial C_t} \\
&= \sum_{\tau=1}^{\infty} \beta^\tau \frac{\partial}{\partial C_t} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(u(c) - u(r)) dF_{C_{t+\tau}}^{t,t-1}(c, r) \\
&= -\sum_{\tau=1}^{\infty} \beta^\tau (W_t - C_t)^{-\theta} E_t[R_{t+\tau}^{1-\theta} \rho_{t+\tau}^{1-\theta} \prod_{j=1}^{\tau-1} R_{t+j}^{1-\theta} (1 - \rho_{t+j})^{1-\theta} \\
&(\eta F_{R_t}^{t-1}(R_t) + \eta\lambda(1 - F_{R_t}^{t-1}(R_t)))] = -(W_t - C_t)^{-\theta} E_t[\sum_{\tau=1}^{\infty} \beta^\tau R_{t+\tau}^{1-\theta} \rho_{t+\tau}^{1-\theta} \\
&\prod_{j=1}^{\tau-1} R_{t+j}^{1-\theta} (1 - \rho_{t+j})^{1-\theta}](\eta F_{R_t}^{t-1}(R_t) + \eta\lambda(1 - F_{R_t}^{t-1}(R_t))).
\end{aligned}
$$

Moreover,

$$
R_{t+1} = \frac{P_{t+1} + C_{t+1}}{P_t} = \frac{\rho_t}{1 - \rho_t} \frac{C_{t+1}}{C_t} \frac{1}{\rho_{t+1}}
$$

such that $R_{t+\tau}^{1-\theta} \rho_{t+\tau}^{1-\theta} = (\frac{C_{t+\tau}}{C_{t+\tau-1}} \frac{\rho_{t+\tau-1}}{1-\rho_{t+\tau-1}})^{1-\theta}$ and $R_{t+j}^{1-\theta}(1-\rho_{t+j})^{1-\theta} = (\frac{C_{t+j}}{C_{t+j-1}} \frac{\rho_{t+j-1}}{1-\rho_{t+j-1}} \frac{1-\rho_{t+j}}{\rho_{t+j}})^{1-\theta}$. Recall that, the model's exogenous consumption process implies $C_{t+\tau}/C_t = e^{\tau\mu_c + \sum_{j=1}^{\tau} \varepsilon_{t+j}}$. Because in a rational-expectations equilibrium, the agent's expectational terms have to match the model's specification $\partial \sum_{\tau=1}^{\infty} \beta^\tau \boldsymbol{n}(F_{C_{t+\tau}}^{t,t-1})/\partial C_t$ can be rewritten as

$$
\begin{aligned}
&-(W_t - C_t)^{-\theta} Q_t^0 \\
&= -(W_t - C_t)^{-\theta} (\frac{\rho_t}{1 - \rho_t})^{1-\theta} \gamma \underbrace{\frac{\beta e^{\mu_c(1-\theta) + \frac{1}{2}(1-\theta)^2 \sigma_c^2}}{1 - \beta e^{\mu_c(1-\theta) + \frac{1}{2}(1-\theta)^2 \sigma_c^2}}}_{=Q} (\eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t))) \\
&= -(W_t - C_t)^{-\theta} (\frac{\rho_t}{1 - \rho_t})^{1-\theta} \gamma Q (\eta F(\varepsilon_t) + \eta\lambda(1 - F(\varepsilon_t))).
\end{aligned}
$$

Returning to equation 17, the second term on the right hand side $-(W_t - C_t)^{-\theta} \psi_t^0$ refers to next period's marginal value, which turns out to be linear in the marginal utility of wealth. As above, iterating back next period's marginal utility, i.e., $\partial u(C_{t+1})/\partial C_t = (W_t - C_t)^{-\theta} R_{t+1}^{1-\theta} \rho_{t+1}^{1-\theta}$ and similarly for future consumption, for instance $\partial u(C_{t+2})/\partial C_t = (W_t -$

$C_t)^{-\theta} R_{t+1}^{1-\theta}(1-\rho_{t+1}^{1-\theta}) R_{t+2}^{1-\theta}\rho_{t+2}^{1-\theta}$, yields

$$\begin{aligned}
&(W_t - C_t)^{-\theta}\beta E_t[R_{t+1}^{1-\theta}\Psi_{t+1}] \\
&\quad = (W_t - C_t)^{-\theta}\beta E_t[R_{t+1}^{1-\theta}\rho_{t+1}^{1-\theta} + \eta \int_{-\infty}^{R_{t+1}\rho_{t+1}} (R_{t+1}^{1-\theta}\rho_{t+1}^{1-\theta} - (R\rho)^{1-\theta}) dF_{R\rho}(R\rho) + \\
&\qquad + \eta\lambda \int_{R_{t+1}\rho_{t+1}}^{\infty} (R_{t+1}^{1-\theta}\rho_{t+1}^{1-\theta} - (R\rho)^{1-\theta}) ddF_{R\rho}(R\rho) + \\
&\qquad + \gamma(\frac{\rho_{t+1}}{1-\rho_{t+1}})^{1-\theta} Q(\eta \int_{-\infty}^{R_{t+1}(1-\rho_{t+1})} (R_{t+1}^{1-\theta}(1-\rho_{t+1})^{1-\theta} - \\
&\qquad - (R(1-\rho))^{1-\theta}) dF_{R(1-\rho)}(R(1-\rho)) + \\
&\qquad + \eta\lambda \int_{R_{t+1}(1-\rho_{t+1})}^{\infty} (R_{t+1}^{1-\theta}(1-\rho_{t+1})^{1-\theta} - (R(1-\rho))^{1-\theta}) dF_{R(1-\rho)}(R(1-\rho)) + \\
&\qquad + \beta R_{t+1}^{1-\theta}(1-\rho_{t+1}^{1-\theta}) E_{t+1}[R_{t+2}^{1-\theta}\Psi_{t+2}]].
\end{aligned}$$

Now, let $\psi = \beta E_t[(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{1-\theta}\Psi_{t+1}] = \beta E_{t+1}[(\frac{C_{t+2}}{C_{t+1}}\frac{1}{\rho_{t+2}})^{1-\theta}\Psi_{t+2}]$ which is constant for any period t because C_{t+1}/C_t, ρ_{t+1}, and Ψ_{t+1} are all solely determined by the realization of ε_{t+1} and exogenous parameters. Then, the last term in the equation of $(W_t - C_t)^{-\theta}\beta E_t[R_{t+1}^{1-\theta}\Psi_{t+1}]$ is

$$\beta R_{t+1}^{1-\theta}(1-\rho_{t+1}^{1-\theta}) E_{t+1}[R_{t+2}^{1-\theta}\Psi_{t+2}] = R_{t+1}^{1-\theta}(1-\rho_{t+1}^{1-\theta})(\frac{\rho_{t+1}}{1-\rho_{t+1}})^{1-\theta}\psi = R_{t+1}^{1-\theta}\rho_{t+1}^{1-\theta}\psi.$$

And, moreover

$$\beta E_t[R_{t+1}^{1-\theta}\Psi_{t+1}] = \beta E_t[(\frac{C_{t+1}}{C_t}\frac{\rho_t}{1-\rho_t}\frac{1}{\rho_{t+1}})^{1-\theta}\Psi_{t+1}] = (\frac{\rho_t}{1-\rho_t})^{1-\theta}\psi$$

such that it follows for the first-order condition, equation 17, that $\psi_t^0 = (\rho_t/1-\rho_t)^{1-\theta}\psi$.

Plugging in $R_{t+1} = \frac{C_{t+1}}{C_t}\frac{\rho_t}{1-\rho_t}\frac{1}{\rho_{t+1}}$ in the equation for $E_t[R_{t+1}^{1-\theta}\Psi_{t+1}]$ and recalling that $C_{t+1}/C_t = e^{\mu_c+\varepsilon_{t+1}}$ or alternatively simply dividing next period's C_{t+1} terms by C_t allows to express ψ in much simpler terms

$$\begin{aligned}
(\frac{\rho_t}{1-\rho_t})^{1-\theta}\psi = (\frac{\rho_t}{1-\rho_t})^{1-\theta}\beta e^{\mu_c(1-\theta)} E_t[(e^{\varepsilon_{t+1}})^{1-\theta} + \\
+ (1+\gamma Q)(\eta \int_{-\infty}^{\varepsilon_{t+1}} ((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta}) dF(\varepsilon) + \\
+ \eta\lambda \int_{\varepsilon_{t+1}}^{\infty} ((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta}) dF(\varepsilon)) + (e^{\varepsilon_{t+1}})^{1-\theta}\psi)]
\end{aligned}$$

accordingly $\psi = Q + (1+\gamma Q)\Omega = Q + \Omega + \gamma\Omega Q$ with Ωgiven by

$$\begin{aligned}
\Omega = \beta e^{\mu_c(1-\theta)} E_t[\frac{(\eta \int_{-\infty}^{\varepsilon_{t+1}} ((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta}) dF(\varepsilon)}{1 - \beta e^{\mu_c(1-\theta) + \frac{1}{2}(1-\theta)^2\sigma_c^2}} + \\
+ \frac{\eta\lambda \int_{\varepsilon_{t+1}}^{\infty} ((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta}) dF(\varepsilon))}{1 - \beta e^{\mu_c(1-\theta) + \frac{1}{2}(1-\theta)^2\sigma_c^2}}]
\end{aligned}$$

$$\Omega = \frac{\beta e^{\mu_c(1-\theta)}\omega(\sigma_c)}{1-\beta e^{\mu_c(1-\theta)+\frac{1}{2}(1-\theta)^2\sigma_c^2}} \text{ with}$$

$$\omega(\sigma) = \int_{-\infty}^{\infty}(\eta\int_{-\infty}^{z}((e^z)^{1-\theta}-(e^\varepsilon)^{1-\theta})dF(\varepsilon)+\eta\lambda\int_{z}^{\infty}((e^z)^{1-\theta}-(e^\varepsilon)^{1-\theta})dF(\varepsilon))dF(z)$$

$$\text{and } z,\varepsilon \sim N(0,\sigma^2)$$

$$= \int_{-\infty}^{\infty}\{\eta F(z)e^{(1-\theta)z}-\eta e^{\frac{1}{2}(1-\theta)^2\sigma^2}(1-F(\frac{(1-\theta)\sigma^2-z}{\sigma}))$$

$$+\eta\lambda(1-F(z))e^{(1-\theta)z}-\eta\lambda e^{\frac{1}{2}(1-\theta)^2\sigma^2}F(\frac{(1-\theta)\sigma^2-z}{\sigma})\}dF(z)$$

In turn, the first-order condition can be rewritten as

$$u'(C_t)(1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))$$
$$= u'(W_t-C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta}(\gamma Q(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))+\psi).$$

And the general equilibrium consumption-wealth ratio is then given by

$$\frac{C_t}{W_t} = \rho_t = \frac{1}{1+\frac{Q+\Omega+\gamma Q\Omega+\gamma Q(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))}{1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))}}.$$

Now, the solution guess $C_t = \rho_t W_t$ and $W_t - C_t = (1-\rho_t)W_t$ can be verified. The agent's value function is given by $V_t(W_t) = u(W_t)\Psi_t$. Obviously, C_t, $W_t - C_t$, and R_t are all increasing in the realization of ε_t. Finally, note that solving the model using backward induction and taking it to its limit yields this exact same solution.

The stochastic discount factor can be inferred from the first-order condition

$$1 = E_t[M_{t+1}R_{t+1}]$$
$$= E_t[\frac{\beta u'(W_{t+1})\Psi_{t+1}}{u'(C_t)(1+\eta F(R_t)+\eta\lambda(1-F(R_t)))-E_t[u'(W_t-C_t)Q_t]}R_{t+1}]$$
$$\Rightarrow M_{t+1} = (1+(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))(1-\frac{\rho_t}{1-\rho_t}\gamma Q))^{-1}$$
$$(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{-1}\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{1-\theta}\Psi_{t+1}$$
$$= (\frac{\rho_t}{1-\rho_t}\psi)^{-1}(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{-1}\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{1-\theta}\Psi_{t+1}$$
$$\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{1-\theta}\Psi_{t+1}$$
$$= \beta e^{\mu_c(1-\theta)}\{(e^{\varepsilon_{t+1}})^{1-\theta}+(1+\gamma Q)(\eta\int_{-\infty}^{\varepsilon_{t+1}}((e^{\varepsilon_{t+1}})^{1-\theta}-(e^{\varepsilon})^{1-\theta})dF(\varepsilon)+$$
$$+\eta\lambda\int_{\varepsilon_{t+1}}^{\infty}((e^{\varepsilon_{t+1}})^{1-\theta}-(e^{\varepsilon})^{1-\theta})dF(\varepsilon))+(e^{\varepsilon_{t+1}})^{1-\theta}\psi\}$$

If $\eta(\lambda-1)>1$, the stochastic discount factor in the news utility model has a somewhat irritating feature: The existence of gain-loss utility generates negative values of the stochastic discount factor in particularly good states of the world. For any parameter choice, increasing the realization of ε_{t+1} will result in negative values of M_{t+1} at some point. The agent dislikes it if a return pays out in particularly good states of the world because he will experience adverse news-utility in all other states. Therefore, ex ante, the agent would prefer to burn consumption in those particularly pleasurable states. Although a negative stochastic discount factor implies arbitrage opportunities, non-satiated agents would not choose to buy consumption in these states at negative prices because they would experience adverse news utility in all other states. Therefore, the equilibrium is still valid. Moreover, the negativity of the stochastic discount factor in these states is unlikely to matter for the model's implications because, for reasonable parameter combinations, negativity only occurs in the range of four to five standard deviations from the mean. This positive probability of negative state prices is not new to the literature, Chapman (1998) elaborates on the possibility arising in habit-formation endowment economies and Dybvig and Ingersoll (1989) show how it arises in the CAPM.

Note that, for $\eta=0$ the model reduces to non-news or plain power utility in which the consumption-wealth ratio ρ^s is constant:

$$(\frac{\rho^s}{1-\rho^s})^{1-\theta}\psi=\beta E_t[R_{t+1}^{1-\theta}\Psi_{t+1}]$$

$$\Rightarrow\psi=\beta e^{\mu_c(1-\theta)}E_t[(e^{\varepsilon_{t+1}})^{1-\theta}+(e^{\varepsilon_{t+1}})^{1-\theta}\psi]\Rightarrow\psi=Q$$

$$1=E_t[M_{t+1}R_{t+1}]=E_t[\frac{\beta u'(W_{t+1})((\rho^s)^{1-\theta}+(1-\rho^s)(\frac{\rho^s}{1-\rho^s})^{1-\theta}\psi)}{u'(C_t)}R_{t+1}]$$

$$M_{t+1}=\beta(\frac{1}{\rho^s}\frac{C_{t+1}}{C_t})^{-\theta}(\rho^s)^{1-\theta}(1+\psi)=\beta(\frac{C_{t+1}}{C_t})^{-\theta}$$

B.3 Proof of Proposition 2

The marginal value of savings is given by $-d\beta E_t[u(W_{t+1})Q^0_{t+1}]/dC_t=u'(W_t-C_t)(Q+\Omega+\gamma\Omega Q)$ whereas in the traditional model $\eta=0\Rightarrow\Omega=0$ and the marginal value of savings is given by $u'(W_t-C_t)Q$. If $\eta>0$, $\lambda>1$ and $\theta>1$ then $\Omega>0$ such that $Q+\Omega+\gamma\Omega Q>Q$ because:

$$\Omega=\frac{\beta e^{\mu_c(1-\theta)}\omega(\sigma_c)}{1-\beta e^{\mu_c(1-\theta)+\frac{1}{2}(1-\theta)^2\sigma_c^2}}\text{ with }\omega(\sigma_c)>0\text{ for }\theta>1$$

$$\text{since }\omega(\sigma)=\int_{-\infty}^{\infty}(\eta\int_{-\infty}^{z}\underbrace{((e^z)^{1-\theta}-(e^{\varepsilon})^{1-\theta})}_{<0\text{ for }\theta>1}dF(\varepsilon)+\underbrace{\eta\lambda}_{>\eta}\int_{z}^{\infty}\underbrace{((e^z)^{1-\theta}-(e^{\varepsilon})^{1-\theta})}_{>0\text{ for }\theta>1}dF(\varepsilon))dF(z)$$

Therefore, news-utility introduces an additional precautionary-savings motive. Moreover, the consumption-wealth ratio is given by

$$\rho_t = \frac{1}{1+\frac{Q+\Omega+\gamma Q\Omega+\gamma Q(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))}{1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))}}$$

whereas in the traditional model $\rho = \frac{1}{1+Q}$.

Thus, the consumption-wealth ratio is unambiguously lower than in the traditional model for $\gamma = 1$ because $\frac{Q+\Omega+\gamma Q\Omega}{Q}+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))/1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)) > 1$. For $\gamma < 1$, the consumption-wealth ratio is lower if $\gamma > \bar{\gamma}$ with

$$\frac{Q+\Omega+\bar{\gamma}Q\Omega+\bar{\gamma}Q(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))}{1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))} = Q \Rightarrow \bar{\gamma} = \frac{\eta\lambda-\frac{\Omega}{Q}}{\Omega+\eta\lambda}.$$

As can be easily seen, $\bar{\gamma} < 1$. I chose $F(\varepsilon_t) = 1$ to obtain $\bar{\gamma}$ because $F(\varepsilon_t) = 1$ maximizes ρ_t if $\theta > 1$. Moreover, as can be easily seen $\partial\Omega/\partial\eta, \partial\Omega/\partial\lambda > 0$ if $\theta > 1$. Then

$$\frac{\partial\bar{\gamma}}{\partial\eta} = \frac{\partial\frac{\eta\lambda-\frac{\Omega}{Q}}{\Omega+\eta\lambda}}{\partial\eta} = \frac{(\lambda-\frac{1}{Q}\frac{\partial\Omega}{\partial\eta})(\Omega+\eta\lambda)-(\frac{\partial\Omega}{\partial\eta}+\lambda)(\eta\lambda-\frac{\Omega}{Q})}{(\Omega+\eta\lambda)^2} \leq 0 \text{ if } \Omega \leq \frac{\partial\Omega}{\partial\eta}\eta,$$

$$\frac{\partial\bar{\gamma}}{\partial\lambda} = \frac{\partial\frac{\eta\lambda-\frac{\Omega}{Q}}{\Omega+\eta\lambda}}{\partial\lambda} = \frac{(\eta-\frac{1}{Q}\frac{\partial\Omega}{\partial\lambda})(\Omega+\eta\lambda)-(\frac{\partial\Omega}{\partial\lambda}+\eta)(\eta\lambda-\frac{\Omega}{Q})}{(\Omega+\eta\lambda)^2} < 0 \text{ if } \Omega < \frac{\partial\Omega}{\partial\lambda}\lambda.$$

Additionally, by looking at Ω it is clear that $\partial\Omega/\partial\eta = \frac{\Omega}{\eta}$ and $\partial\Omega/\partial\lambda\lambda > \Omega$ if $\theta > 1$ so that the two conditions always hold.

If $\theta > 0$ then $\Omega > 0$ and $\partial\Omega/\partial\eta > 0$ and $\partial\Omega/\partial\lambda > 0$ such that $\partial\rho_t/\partial\eta < 0$ and $\partial\rho_t/\partial\lambda < 0$ for any ε_t. As $\partial\rho_t/\partial\varepsilon_t < 0$ and if $\eta-\frac{\Omega}{Q}/\Omega+\eta < \gamma < \bar{\gamma}$, such that ρ^s and ρ_t cross at some point $\varepsilon_t = \bar{\varepsilon}_t$ determined by $\rho_t = \rho^s$, then it can be easily inferred that $\bar{\varepsilon}_t$ is decreasing in the news-utility parameters $\frac{\partial\bar{\varepsilon}_t}{\partial\lambda}, \frac{\partial\bar{\varepsilon}_t}{\partial\eta} \leq 0$.

B.4 Proof of Proposition 3

The slope of the consumption-wealth ratio is given by

$$\frac{\partial\rho_t}{\partial\varepsilon_t} = -\rho_t^2\frac{(Q+\Omega+\gamma Q\Omega-\gamma Q)\eta f(\varepsilon_t)(\lambda-1)}{(1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))^2}.$$

Accordingly, $\partial\rho_t/\partial\varepsilon_t \neq 0$ iff $\lambda > 1$ and $Q+\Omega+\gamma\Omega Q \neq \gamma Q$, additionally, $\partial\rho_t/\partial\varepsilon_t < 0$ iff $\lambda > 1$ and $Q+\Omega+\gamma\Omega Q > \gamma Q$ which is necessarily true for $\theta > 1$ or for $\theta < 1$ if $\gamma < \tilde{\gamma}$ with $\tilde{\gamma} = Q+\Omega/Q(1-\Omega)$. Furthermore, if $\theta = 1$ and $\gamma = 1$ then $\partial\rho_t/\partial\varepsilon_t = 0$. If $\theta < 1$ and $\gamma > \tilde{\gamma}$ then $\partial\rho_t/\partial\varepsilon_t > 0$.

B.5 Proof of Proposition 4

The news-utility equity premium

$$E_t[R_{t+1}] - R^f_{t+1} = \frac{\rho_t}{1-\rho_t}(\underbrace{E_t[\frac{1}{\rho_{t+1}}\frac{C_{t+1}}{C_t}]}_{\text{constant}} - \underbrace{\psi E_t[\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{-\theta}\Psi_{t+1}]^{-1}}_{\text{constant}}).$$

Clearly, $E_t[R_{t+1}]$, R^f_{t+1}, and $E_t[R_{t+1}] - R^f_t$ vary with $\frac{\rho_t}{1-\rho_t}$ whereas the other terms are constant in an i.i.d. world. As $\frac{\partial \frac{\rho_t}{1-\rho_t}}{\partial \varepsilon_t} < 0$ for $\eta > 0$, $\lambda > 1$, and $\theta > 1$ so are $\frac{\partial E_t[R_{t+1}]}{\partial \varepsilon_t} < 0$, $\frac{\partial R^f_t}{\partial \varepsilon_t} < 0$, and $\frac{\partial E_t[R_{t+1}] - R^f_t}{\partial \varepsilon_t} < 0$.

B.6 Risk attitudes towards wealth gambles

Recall that $\beta E_t[V_{t+1}(W_{t+1})] = E_t[\sum_{\tau=1}^{\infty} \beta^\tau U_{t+\tau}] = u(C_t)\psi$ and $\gamma \sum_{\tau=1}^{\infty} \beta^\tau \boldsymbol{n}(F^{t,t-1}_{C_{t+\tau}}) = \gamma(\eta \int_{-\infty}^{C_t} (u(C_t)Q - u(c)Q) dF^{t-1}_{C_t}(c)) + \eta\lambda \int_{C_t}^{\infty} (u(C_t)Q - u(c)Q) dF^{t-1}_{C_t}(c))$ such that the news-utility agent will accept the gamble iff

$$\begin{aligned} &\gamma(0.5\eta(u((W_t + G)\rho_t)Q - u((W_t)\rho_t)Q) + \eta\lambda 0.5(u((W_t - L)\rho_t)Q - u((W_t)\rho_t)Q) \\ &\qquad + 0.5u((W_t + G)\rho_t)\psi + 0.5u((W_t - L)\rho_t)\psi > u(W_t\rho_t)\psi \\ &\Rightarrow \frac{\gamma(0.5\eta(u(W_t + G) - u(W_t)) + \eta\lambda 0.5(u(W_t - L) - u(W_t)))Q}{Q + \Omega + \gamma Q\Omega} + \\ &\qquad + 0.5u(W_t + G) + 0.5u(W_t - L) > u(W_t) \end{aligned}$$

whereas the traditional agent will accept the gamble iff

$$\begin{aligned} &0.5u((W_t + G)\rho^s)Q + 0.5u((W_t - L)\rho^s)Q > u(W_t\rho^s)Q \\ &\qquad \Rightarrow 0.5u(W_t + G) + 0.5u(W_t - L) > u(W_t). \end{aligned}$$

C Sluggish belief updating

To keep the model tractable, I simply assume that the agent's beliefs do not correspond to actual consumption growth any more, which is given by $log(C_{t+1}/C_t) = \mu_c + \sigma_c \varepsilon_{t+1}$. But rather I assume that the agent's beliefs about consumption growth are given by $log(C_{t+1}/C_t) = \mu_t + \sigma_c \varepsilon_{t+1}$ with $\mu_{t+1} = \nu\mu_t + (1-\nu)(\mu_c + \tilde{\mu}(\varepsilon_{t+1}))$ and $\tilde{\mu}(\varepsilon_{t+1}) = \bar{\mu}(E[R^{of}_{t+1}] - R^{of}_{t+1})$ with R^{of}_{t+1} being the risk-free rate in the basic model. The variation in $\tilde{\mu}(\varepsilon_{t+1})$ reflects the variation in the basic model's risk-free rate, because sluggish updating is intended to offset the general-equilibrium impact on the risk-free rate. The model's simple structure is unaffected, $C_t = \rho_t W_t$ and $V_t(W_t) = u(W_t)\Psi_t$ with ρ_t given by

$$\rho_t = \frac{1}{1 + \frac{\psi_t + \gamma Q_t(\eta F^b_{t-1}(\varepsilon_t) + \eta\lambda(1 - F^b_{t-1}(\varepsilon_t)))}{1 + \eta F^b_{t-1}(\varepsilon_t) + \eta\lambda(1 - F^b_{t-1}(\varepsilon_t))}}$$

with $F^b_{t-1}(\varepsilon_t) \sim N(\mu_{t-1}, 1)$.[30]

[30] Prospective marginal news utility is not determined by

$$F^{Qb}(\mu_{t-1}, \varepsilon_t) = E_t[\beta(e^{\mu_c + \sigma_c \varepsilon_{t+1}})^{1-\theta}](\eta F^b_t(\varepsilon_t) + \eta\lambda(1 - F^b_t(\varepsilon_t))) + E_t[\beta(e^{\mu_c + \sigma_c \varepsilon_{t+1}})^{1-\theta} F^{Qb}(\tilde{\mu}(\mu_{t-1}, \epsilon_t), \varepsilon_{t+1})]$$

because the agent considers uncertainty that has been realized only under the separated comparison and takes his beliefs as given in the optimization otherwise.

Fluctuations in beliefs about economic volatility make the exogenous parameters ψ_t and $Q_t^b = f^{Q^b}(\mu_t)$ variant, which makes the calculation of ψ_t and Q_t^b somewhat more complicated. By the same argument as above

$$\begin{aligned} Q_t &= E_t[\beta(e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta}] + E_t[\beta(e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta}Q_{t+1}] \\ &= e^{(1-\theta)\mu_c}(E_t[\beta(e^{\sigma_c\varepsilon_{t+1}})^{1-\theta}] + E_t[\beta(e^{\sigma_c\varepsilon_{t+1}})^{1-\theta}Q_{t+1}]) \\ Q_t^b &= E_t[\beta(e^{\mu_t+\sigma_c\varepsilon_{t+1}})^{1-\theta}] + E_t[\beta(e^{\mu_t+\sigma_c\varepsilon_{t+1}})^{1-\theta}Q_{t+1}^b]. \end{aligned}$$

And ψ_t is

$$\begin{aligned} \psi_t = \beta E_t[(e^{\mu_c+\sigma_c\varepsilon_{t+1}})^{1-\theta} &+ \eta(\lambda-1)\int_{-\infty}^{\varepsilon_{t+1}}((e^{\mu_c+\sigma_c\varepsilon_{t+1}})^{1-\theta} - (e^{\mu_t+\sigma_c x})^{1-\theta})dF(x)+ \\ &+ \eta(\lambda-1)\int_{-\infty}^{\varepsilon_{t+1}}((e^{\mu_c+\sigma_c\varepsilon_{t+1}})^{1-\theta}Q_{t+1}- \\ &- (e^{\mu_t+\sigma_c x})^{1-\theta}f^{Q^b}(\tilde{\mu}(\mu_t,x)))dF(x) + \beta(e^{\mu_c+\sigma_c\varepsilon_{t+1}})^{1-\theta}\psi_{t+1}]. \end{aligned}$$

Unfortunately, this model can no longer be solved analytically. But, thanks to the geometric-sum nature of Q_t^b and ψ_t they can be computed numerically using a simple interpolation procedure that iterates until convergence. The numerical solution procedure appears to be very robust and pricing errors in $1 = E_t[M_{t+1}R_{t+1}]$ are very small.

D Online Appendix

E Variation in consumption growth

The model's simple structure is unaffected, $C_t = \rho_t W_t$ and $V_t(W_t) = u(W_t)\Psi_t$ with ρ_t given by

$$\rho_t = \frac{1}{1 + \frac{\psi_t+\gamma Q_t(\eta F_{t-1}(\varepsilon_t)+\eta\lambda(1-F_{t-1}(\varepsilon_t)))}{1+\eta F_{t-1}(\varepsilon_t)+\eta\lambda(1-F_{t-1}(\varepsilon_t))}}.$$

Fluctuations in beliefs about economic volatility make the exogenous parameters $\psi_t = f^{\psi}(\mu_t,\sigma_t)$ and $Q_t = f^Q(\mu_t,\sigma_t)$ variant, and the calculation of ψ_t and Q_t thus becomes somewhat more complicated, by the same argument as above

$$Q_t = E_t[\beta(e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta}]+E_t[\beta(e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta}Q_{t+1}] = e^{(1-\theta)\mu_t}(E_t[\beta(e^{\sigma_t\varepsilon_{t+1}})^{1-\theta}]+E_t[\beta(e^{\sigma_t\varepsilon_{t+1}})^{1-\theta}Q_{t+1}]).$$

And ψ_t is

$$\psi_t = \beta E_t[(e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta} + \eta(\lambda-1)\int_{-\infty}^{\varepsilon_{t+1}}((e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta} - (e^{\mu_t+\sigma_t x})^{1-\theta})dF(x)+$$

$$+\eta(\lambda-1)\int_{-\infty}^{\varepsilon_{t+1}}((e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta}Q_{t+1}-(e^{\mu_t+\sigma_t x})^{1-\theta}f^Q(\tilde{\mu}(\mu_t,x),\tilde{\sigma}(\sigma_t,x)))dF(x)+\beta(e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta}\psi_{t+1}].$$

Note that in the traditional model $\rho_t = 1/1+\psi_t^s$ with

$$\psi_t^s = f^{\psi^s}(\mu_t,\sigma_t) = \beta E_t[(e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{1-\theta}] + \beta E_t[(e^{\mu_t+\sigma_t\varepsilon_{t+1}})^{(1-\theta)}\psi_{t+1}^s]$$

Unfortunately, the heteroskedasticity model can no longer be solved analytically. But, thanks to the geometric-sum nature of Q_t and ψ_t they can be computed numerically using a simple interpolation procedure that iterates until convergence. The numerical solution procedure appears to be very robust and pricing errors in $1 = E_t[M_{t+1}R_{t+1}]$ are very small.

F Disaster risk

The model's simple structure is unaffected by disaster risk in the consumption process, $C_t = \rho_t W_t$ and $V_t(W_t) = u(W_t)\Psi_t$, but ρ_t now depends on the probability of disaster p_t and if disaster happened v_t and is given by

$$\rho_t = \frac{1}{1 + \frac{\psi_t + \gamma Q_t(\eta F_{t-1}(\varepsilon_t, v_t) + \eta\lambda(1 - F_{t-1}(\varepsilon_t, v_t)))}{1 + \eta F_{t-1}(\varepsilon_t, v_t) + \eta\lambda(1 - F_{t-1}(\varepsilon_t, v_t))}}.$$

Note that $F_{t-1}(\varepsilon_t, 0) = p_{t-1}F(\varepsilon_t - log(1-d)) + (1-p_{t-1})F(\varepsilon_t)$ if a disaster does not occur with probability $1-p_{t-1}$ and $F_{t-1}(\varepsilon_t, log(1-d)) = p_{t-1}F(\varepsilon_t) + (1-p_{t-1})F(\varepsilon_t + log(1-d))$ if a disaster occurs with probability p_{t-1}.

If disaster risk is invariant, Q and ψ are constant, from the same arguments as above

$$Q = E_t[\sum_{\tau=1}^{\infty} \beta^\tau (e^{\tau\mu_c + \sum_{j=1}^{\tau} \varepsilon_{t+\tau} + \sum_{j=1}^{\tau} v_{t+\tau}})^{1-\theta}]$$

$$\text{with } E_t[e^{(1-\theta)\sum_{j=1}^{\tau} v_{t+j}}] = E_t[e^{(1-\theta)v_{t+1}}]^\tau = (1 - p + p(1-d)^{1-\theta})^\tau$$

$$\text{such that } Q = \frac{\beta e^{\mu_c(1-\theta) + \frac{1}{2}(1-\theta)^2\sigma_c^2}(1 - p + p(1-d)^{1-\theta})}{1 - \beta e^{\mu_c(1-\theta) + \frac{1}{2}(1-\theta)^2\sigma_c^2}(1 - p + p(1-d)^{1-\theta})}$$

$$\psi = Q + \Omega + \gamma Q\Omega \text{ and } \Omega = \frac{\beta e^{\mu_c(1-\theta)}((1-p)\omega(p) + p\omega_p(p))}{1 - \beta e^{\mu_c(1-\theta) + \frac{1}{2}(1-\theta)^2\sigma_c^2}(1 - p + p(1-d)^{1-\theta})} \text{ with}$$

$$\omega(p) = \int_{-\infty}^{\infty} \eta(\lambda - 1) \int_{z}^{\infty} (1-p)((e^z)^{1-\theta} - (e^\varepsilon)^{1-\theta}) + p((e^z)^{1-\theta} - (e^\varepsilon(1-d))^{1-\theta}) dF(\varepsilon)dF(z)$$

$$\omega_p(p) = \int_{-\infty}^{\infty} +\eta(\lambda - 1) \int_{e^z(1-d)}^{\infty} (1-p)((e^z(1-d))^{1-\theta} - (e^\varepsilon)^{1-\theta}) + p((e^z(1-d))^{1-\theta} - (e^\varepsilon(1-d))^{1-\theta}) dF(z)$$

For time-variation in disaster risk, $v_{t+1} \sim (p_t, d)$, $Q_t = f^Q(p_t)$ and $\psi_t = f^\psi(p_t)$ become variant with p_t

$$Q_t = \beta E_t[(e^{\mu_c+\sigma_c\varepsilon_{t+1}+v_{t+1}})^{1-\theta}]+\beta E_t[(e^{\mu_c+\sigma_c\varepsilon_{t+1}+v_{t+1}})^{1-\theta}Q_{t+1}] = E_t[\sum_{\tau=1}^{\infty}\beta^{\tau}(e^{\tau\mu_c+\sum_{j=1}^{\tau}\varepsilon_{t+j}+\sum_{j=1}^{\tau}v_{t+\tau}})^{1-\theta}]$$

$$= \beta(1-p_t+p_t(1-d)^{1-\theta})(E_t[(e^{\mu_c+\sigma_c\varepsilon_{t+1}})^{1-\theta}] + E_t[(e^{\mu_c+\sigma_c\varepsilon_{t+1}})^{1-\theta}Q_{t+1}])$$

$$\psi_t = \beta e^{(1-\theta)\mu_c}E_t[\{(1-p_t)(e^{\sigma_t\varepsilon_{t+1}})^{1-\theta} + p_t(e^{\sigma_t\varepsilon_{t+1}}(1-d))^{1-\theta}$$

$$+(1-p_t)\eta(\lambda-1)\int_{\varepsilon_{t+1}}^{\infty}((1-p_t)((e^{\sigma_t\varepsilon_{t+1}})^{1-\theta}-(e^{\sigma_t x})^{1-\theta})+p_t((e^{\sigma_t\varepsilon_{t+1}})^{1-\theta}-(e^{\sigma_t x}(1-d))^{1-\theta}))dF(x)$$

$$+p_t\eta(\lambda-1)\int_{log(e^{\varepsilon_{t+1}}(1-d))}^{\infty}((1-p_t)((e^{\sigma_t\varepsilon_{t+1}}(1-d))^{1-\theta}-(e^{\sigma_t x})^{1-\theta})+p_t((e^{\sigma_t\varepsilon_{t+1}}(1-d))^{1-\theta}-(e^{\sigma_t x}(1-d))^{1-\theta}))dF(x)$$

$$+\gamma(1-p_t)\eta(\lambda-1)\int_{\varepsilon_{t+1}}^{\infty}(1-p_t)((e^{\sigma_c\varepsilon_{t+1}})^{1-\theta}Q_{t+1}-(e^{\sigma_c x})^{1-\theta}f^Q(\tilde{p}(x,p_t)))+p_t((e^{\sigma_c\varepsilon_{t+1}})^{1-\theta}Q_{t+1}$$

$$-(e^{\sigma_c x}(1-d))^{1-\theta}f^Q(\tilde{p}(x,p_t)))dF(x)+\gamma p_t\eta(\lambda-1)\int_{e^{\sigma_t\varepsilon_{t+1}}(1-d)}^{\infty}((1-p_t)((e^{\sigma_t\varepsilon_{t+1}}(1-d))^{1-\theta}Q_{t+1}-(e^{\sigma_t x})^{1-\theta}$$

$$f^Q(\tilde{p}(x,p_t)))+p_t((e^{\sigma_t\varepsilon_{t+1}}(1-d))^{1-\theta}Q_{t+1}-(e^{\sigma_t x}(1-d))^{1-\theta})f^Q(\tilde{p}(x,p_t)))dF(x)$$

$$+(1-p_t+p_t(1-d)^{1-\theta})e^{(1-\theta)\sigma_c\varepsilon_{t+1}}\psi_{t+1}\}]$$

F.1 Beliefs-based present-bias

As can be seen in the first-order condition 9, the news discounting parameter γ is unambiguously positively related to the consumption-wealth ratio. For lower values of the news discounting parameter, the agent consumes more of his wealth because positive news about the present is overweighted. Therefore, the model induces overconsumption if the agent discounts news about future consumption. But, the preferences feature a more conceptual desire for time-inconsistent overconsumption. In equilibrium, the agent takes his beliefs as given and optimizes over consumption. In contrast, on some optimal pre-committed path the agent jointly optimizes over consumption and beliefs. The following proposition summarizes how the pre-committed consumption path differs from the time-consistent one.

Proposition 5. *If there is uncertainty $\sigma_c > 0$, and $\theta \neq 1$ then the expected-utility-maximizing consumption path does not correspond to the Markovian rational-expectations equilibrium consumption path. In particular, for $\theta > 1$ the agent chooses a suboptimal overconsumption equilibrium path. The pre-committed consumption-wealth ratio is generally lower and the gap increases in good states:*

$$\rho_t < \rho_t^c \text{ and } \frac{\partial(\rho_t - \rho_t^c)}{\partial\varepsilon_t} > 0$$

Proof of Proposition 5 The optimal pre-committed and non-pre-committed consumption-wealth ratios are given by

$$\rho_t^c = \frac{1}{1+\frac{Q+\Omega+\gamma Q\Omega+\gamma Q\eta(\lambda-1)(1-2F(\varepsilon_t)))}{1+\eta(\lambda-1)(1-2F(\varepsilon_t))}} \text{ and } \rho_t = \frac{1}{1+\frac{Q+\Omega+\gamma Q\Omega+\gamma Q(\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t)))}{1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))}}.$$

For $\sigma_c = 0$ if $\gamma > \frac{1}{\lambda}$ then $\rho_t^c = \rho_t$, if $\gamma < \frac{1}{\lambda}$ then $\rho_t^c < \rho_t$.

For $\sigma_c > 0$, $\rho_t^c < \rho_t$ iff $\theta > 1$ as $\eta(\lambda-1)(1-2F(\varepsilon_t))) < \eta F(\varepsilon_t) + \eta\lambda(1-F(\varepsilon_t))$ for all $\varepsilon_t \sim N(0, \sigma_c^2)$ and $Q + \Omega + \gamma\Omega Q > \gamma Q$. Moreover, $\rho_t - \rho_t^c$ is increasing in ε_t because $\eta(1-F(\varepsilon_t)) + \eta\lambda F(\varepsilon_t)$ is increasing in ε_t, i.e., $\frac{\partial(\rho_t - \rho_t^c)}{\partial \varepsilon_t} > 0$. ■

Suppose the agent can pre-commit to an optimal history-dependent consumption path for each possible future contingency. When choosing the optimal pre-committed consumption in each state, the marginal gain-loss utility is no longer solely composed of the sensation of increasing consumption in that state $u'(C_t)(\eta F_{C_t}^{t-1}(C_t) + \eta\lambda(1 - F_{C_t}^{t-1}(C_t)))$. Additionally, the agent considers that in all other states of the world he experiences fewer feelings of gain and more feelings of loss due to increasing consumption in that contingency $-u'(C_t)(\eta(1-F_{C_t}^{t-1}(C_t)) + \eta\lambda F_{C_t}^{t-1}(C_t))$. Marginal gain-loss utility is then given by $\eta(\lambda-1)(1-2F(\varepsilon_t)) \in [-\eta(\lambda-1), \eta(\lambda-1)]$. Let me illustrate this derivation in greater depth.

Suppose the agent has the ability to pick an optimal history-dependent consumption path for each possible future contingency in period zero when he does not experience any gain-loss utility. The maximization problem can be represented in recursive format as above

$$max\{u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma \sum_{\tau=1}^{\infty} \beta^\tau \boldsymbol{n}(F_{C_{t+\tau}}^{t,t-1}) + \beta V(W_{t+1})\}$$

The crucial difference is that in period zero the agent chooses optimal consumption in period t in each possible contingency jointly with his beliefs, which of course coincide with the agent's optimal state-contingent plan. For instance, consider the joint optimization over consumption and beliefs for $C(W^*)$ when wealth W^* has been realized:

$$\begin{aligned}
&\frac{\partial}{\partial C(W^*)}\{\int\int \mu(u(C(W)) - u(C(W')))dF(W')dF(W)\} \\
=& \frac{\partial}{\partial C(W^*)} \int \eta \int_{-\infty}^{W} \{(u(C(W)) - u(C(W')))dF(W') + \eta\lambda \int_{W}^{\infty} (u(C(W)) - u(C(W')))dF(W')\}dF(W) \\
&= u'(C(W^*))(\eta F(W^*) + \eta\lambda(1 - F(W^*))) - u'(C(W^*))(\eta(1 - F(W^*)) + \eta\lambda F(W^*)) \\
&= u'(C(W^*))\eta(\lambda-1)(1-2F(W^*)) \text{ with } \eta(\lambda-1)(1-2F(W^*)) > 0 \text{ for } F(W^*) < 0.5
\end{aligned}$$

Consider the difference from the term in the initial first-order condition $u'(C_t)(\eta F(\varepsilon_t) + \eta\lambda(1-F(\varepsilon_t))$: When choosing the pre-committed plan, the additional utility of increasing consumption a little bit is no longer only composed of the additional step in the probability distribution. Instead, the two additional negative terms account for the fact that in all other states of the world, the agent experiences less feelings of gain and more feelings of loss due to increasing consumption in that contingency. The equation indicates that the marginal utility of state W^* will be increased by news utility if the realization is below the median. For realizations above the median, marginal utility will be decreased and the agent will consume relatively less. In general equilibrium, again the agent's expectational terms have to match the model's setup and the above expression becomes:

$$= C_t^{-\theta}(1 + \eta(\lambda-1)(1-2F(\varepsilon_t))) \text{ with } \eta(\lambda-1)(1-2F(\varepsilon_t)) \in [-\eta(\lambda-1), \eta(\lambda-1)]$$

Accordingly, by the same reasoning as above the first-order condition for the pre-committed consumption path is given by:

$$C_t^{-\theta}(1+\eta(\lambda-1)(1-2F(\varepsilon_t))) = (W_t - C_t)^{-\theta}(\frac{\rho_t}{1-\rho_t})^{1-\theta}(\gamma Q\eta(\lambda-1)(1-2F(\varepsilon_t))+\psi)$$

$$\rho_t^c = \frac{1}{1+\frac{\psi+\gamma Q\eta(\lambda-1)(1-2F(\varepsilon_t)))}{1+\eta(\lambda-1)(1-2F(\varepsilon_t))}} \text{ and } \frac{\partial \rho_t^c}{\partial \varepsilon_t} = -(\rho_t^c)^2\frac{(\psi-\gamma Q)\eta(\lambda-1)2f(\varepsilon_t)}{(1+\eta(\lambda-1)(1-2F(\varepsilon_t)))^2}$$

Not surprisingly, the agent's first-order condition has only changed with respect to present gain-loss utility over current and future consumption. In the non-pre-committed optimization, the agent took the beliefs he had as given, now he considers the true costs of increasing consumption on his gain-loss feelings in all other states of the world.[31]

Marginal pre-committed gain-loss utility is generally lower and thus the pre-committed agent consumes less in all states. Moreover, pre-committed marginal utility will only be increased by news utility if the realization is below the median. For realizations above the median marginal utility will be decreased. In contrast, on the non-pre-committed path $\eta F_{C_t}^{t-1}(C_t)+\eta\lambda(1-F_{C_t}^{t-1}(C_t)) \in [\eta, \eta\lambda]$ and marginal gain-loss utility is always positive, as the agent enjoys the sensation of increasing consumption in any state. Thus, in good states, the conceptual problem of beliefs-based present bias is more powerful: Pre-committed marginal gain-loss utility is negative, which never happens on the non-pre-committed path. Therefore, the degree of present bias is reference-dependent and increasing in good states.[32]

[31]Unfortunately, there is a problem that arises in the pre-commitment optimization problem that was absent in the non-pre-committed one: When beliefs are taken as given, the agent optimizes over two concave functions, consumption utility and the first part of gain-loss utility. Accordingly, the first-order condition specifies a maximum. In contrast, when the agent simultaneously chooses his beliefs and his consumption, he also optimizes over the second, convex part of gain-loss utility. The additional part determining marginal utility $-u'(C_t)(\eta(1-F(\varepsilon_t))+\eta\lambda F(\varepsilon_t))$ is largest in particularly good states of the world, as increasing consumption in these states implies additional feelings of loss in almost all other states of the world. It can be easily shown that the sufficient condition for the optimization problem holds if the parameters satisfy the following simple condition: $\eta(\lambda-1)(2F(\varepsilon_t)-1) < 1$. Accordingly, for $\eta(\lambda-1) > 1$, which is true for a range of commonly used parameter combinations, the first-order condition no longer specifies the optimum for favorable states $F(\varepsilon_t) = 1$. For the purposes of this paper, the pre-commitment case was merely meant to illustrate the agent's present bias. Hence, at this point, I am not going to pursue the issue of convexity in the pre-committed optimization further.

[32]The news-utility induced beliefs-based present-bias is not only conceptually very different from $\beta\delta-$preferences, but as well observationally distinguishable. In the Lucas-tree model $\beta\delta-$preferences, with a hyperbolic-discounting factor denoted by $b < 1$, would merely lead to an upward shift of the consumption-wealth ratio $\rho^b = \frac{1}{1+bQ}$ whereas in the traditional model $\rho^s = \frac{1}{1+Q}$ and the $\beta\delta-$agent would like to pre-commit to the traditional agent's path. Thus, there are three main differences between $\beta\delta-$preferences and news utility: First, news utility introduces an additional precautionary savings effect which is absent in the $\beta\delta-$model. Rather, uncertainty increases the future marginal propensity to consume, which increases the effective discount rate, so that the agent tends to consume more (Laibson (1998)). Secondly, the optimal pre-committed consumption path is time-variant. In contrast to $\beta\delta-$preferences, the agent does not have a universal desire to pre-commit himself and consume at his liquidity constraint each period (Laibson (1997)). With illiquid and liquid savings the news-utility agent would trade-off the benefits of smoothing consumption and news utility with his present-bias. Last but not least, news-utility preferences predict a state-dependent b, the agent's degree of present-bias varies. In particular, the agent is better behaved in bad times. In my opinion $\beta\delta-$preferences could be a reduced form of a more fundamental source of present-bias as introduced by news utility for instance.

F.2 Prospective gain-loss using the ordered comparison

Koszegi and Rabin (2009) assume that the decision-maker experiences prospective gain-loss utility by means of an ordered comparison of her prior and updated beliefs about the stream of future consumption. The ordered comparison is slightly different from the static comparison assumed in Koszegi and Rabin (2009). Rigorously applying the static comparison to prospective gain-loss utility would imply that the agent experiences gain-loss utility over risk, which has been priorly expected, but not resolved. I circumvent this problem by excluding future uncertainty from the static comparison. This captures a similar intuition but is not exactly the same as the ordered comparison. In the following, I outline the model solution under the assumptions of the ordered comparison. Prospective gain-loss about consumption in each future period $C_{t+\tau}$ is then given by:

$$\sum_{\tau=1}^{\infty}\beta^{\tau}N(F^{t}_{C_{t+\tau}},F^{t-1}_{C_{t+\tau}})=\sum_{\tau=1}^{\infty}\beta^{\tau}\int_{-\infty}^{\infty}\mu(u(C_{F^{t}_{C_{t+\tau}}}(p))-u(C_{F^{t-1}_{C_{t+\tau}}}(p)))dp$$

As above $C_{t+\tau}$ can be expressed as:

$$C_{t+\tau}=C_{t}e^{\tau\mu_c+\sum_{j=1}^{\tau}\varepsilon_{t+j}}=(W_{t-1}-C_{t-1})R_t\rho_t e^{\tau\mu_c+\sum_{j=1}^{\tau}\varepsilon_{t+j}}$$
$$=(W_{t-1}-C_{t-1})\frac{\rho_{t-1}}{1-\rho_{t-1}}\frac{C_t}{C_{t-1}}e^{\tau\mu_c+\sum_{j=1}^{\tau}\varepsilon_{t+j}}=(W_{t-1}-C_{t-1})\frac{\rho_{t-1}}{1-\rho_{t-1}}e^{(\tau+1)\mu_c+\sum_{j=0}^{\tau}\varepsilon_{t+j}}$$

Thus, I can write

$$\frac{\partial\sum_{\tau=1}^{\infty}\beta^{\tau}n(F^{t}_{C_{t+\tau}},F^{t-1}_{C_{t+\tau}})}{\partial C_t}=$$
$$=-(W_t-C_t)^{-\theta}(\frac{\rho_t}{1-\rho_t})^{1-\theta}\sum_{\tau=1}^{\infty}\beta^{\tau}\int_{-\infty}^{\infty}(e^{\tau\mu_c+\sum_{j=1}^{\tau}\varepsilon_{t+j}(p)})^{1-\theta}\mu_I(u(C_{F^{t}_{C_{t+\tau}}}(p))-u(C_{F^{t-1}_{C_{t+\tau}}}(p)))dp$$

with $\mu_I(x)=\eta$ if $x\geq 0$ and$\mu_I(x)=\eta\lambda$ if $x<0$. Moreover, the sum of expected consumption and gain-loss utility, ψ, looks slightly different. Recall, the agent's value

$V_{t+1}(W_{t+1}) = u(W_{t+1})\Psi_{t+1}$:

$$\beta E_t[V_{t+1}] = \beta E_t[u(W_{t+1})\Psi^{OC}_{t+1}] = u(W_t - C_t)\beta E_t[R^{1-\theta}_{t+1}\Psi^{OC}_{t+1}] = u(W_t - C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta}\psi$$

$$= u(W_t - C_t)(\frac{\rho_t}{1-\rho_t})^{1-\theta}\beta e^{\mu_c(1-\theta)}E_t[(e^{\varepsilon_{t+1}})^{1-\theta} + (\eta\int_{-\infty}^{\varepsilon_{t+1}}((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta})dF(\varepsilon)+$$

$$+\eta\lambda\int_{\varepsilon_{t+1}}^{\infty}((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta})dF(\varepsilon))]+$$

$$\gamma\beta(((W_t - C_t)\frac{\rho_t}{1-\rho_t})^{1-\theta})^{-1}\sum_{\tau=1}^{\infty}\beta^{\tau}\int_{-\infty}^{\infty}\mu(u(C_{F^t_{C_{t+\tau}}}(p)) - u(C_{F^{t-1}_{C_{t+\tau}}}(p)))dp$$

$$+(\frac{\rho_t}{1-\rho_t})^{1-\theta}\beta e^{\mu_c(1-\theta)}E_t[(e^{\varepsilon_{t+1}})^{1-\theta}\psi)]$$

since $C_{t+\tau} = (W_t - C_t)\frac{\rho_t}{1-\rho_t}e^{\tau\mu_c + \sum_{j=1}^{\tau}\varepsilon_{t+\tau}}$ with $(W_t - C_t)\frac{\rho_t}{1-\rho_t}$ known in period t

accordingly $\psi = Q + \Omega + \gamma\Omega^{OC}$ with Ω^{OC}given by

$$\Omega^{OC} = \frac{\beta\sum_{\tau=1}^{\infty}\beta^{\tau}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu((e^{\tau\mu_c+\varepsilon_{t+1}+\sum_{j=2}^{\tau}\varepsilon_{t+j}(p)})^{1-\theta} - (e^{\tau\mu_c+\sum_{j=1}^{\tau}\varepsilon_{t+j}(p)})^{1-\theta})dpdF(\varepsilon_{t+1})}{1-\beta e^{\mu_c(1-\theta)+\frac{1}{2}(1-\theta)^2\sigma_c^2}}$$

The stochastic discount factor is then given by:

$$1 = E_t[M_{t+1}R_{t+1}] = E_t[\frac{\beta u'(W_{t+1})\Psi^{OC}_{t+1}}{u'(C_t)(1+\eta F(R_t) + \eta\lambda(1-F(R_t))) - u'(W_t - C_t)Q^{OC}_t}R_{t+1}]$$

$$Q^{OC}_t = \gamma(\frac{\rho_t}{1-\rho_t})^{1-\theta}\sum_{\tau=1}^{\infty}\beta^{\tau}\int_{-\infty}^{\infty}(e^{\tau\mu_c+\sum_{j=1}^{\tau}\varepsilon_F(p)})^{1-\theta}\mu_I(u(C_{F^t_{C_{t+\tau}}}(p)) - u(C_{F^{t-1}_{C_{t+\tau}}}(p)))dp$$

$$\Rightarrow M_{t+1} = ((1+\eta F(\varepsilon_t)+\eta\lambda(1-F(\varepsilon_t))) - (\frac{1-\rho_t}{\rho_t})^{-\theta}Q^{OC}_t))^{-1}(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{-1}\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{1-\theta}\Psi^{OC}_{t+1}$$

$$\beta(\frac{C_{t+1}}{C_t}\frac{1}{\rho_{t+1}})^{1-\theta}\Psi^{OC}_{t+1} = \beta e^{\mu_c(1-\theta)}\{(e^{\varepsilon_{t+1}})^{1-\theta} + \eta\int_{-\infty}^{\varepsilon_{t+1}}((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta})dF(\varepsilon)+$$

$$+\eta\lambda\int_{\varepsilon_{t+1}}^{\infty}((e^{\varepsilon_{t+1}})^{1-\theta} - (e^{\varepsilon})^{1-\theta})dF(\varepsilon))+$$

$$+\sum_{\tau=1}^{\infty}\beta^{\tau}\int_{-\infty}^{\infty}\mu((e^{(\tau-1)\mu_c+\varepsilon_{t+1}+\sum_{j=2}^{\tau}\varepsilon_{t+j}(p)})^{1-\theta} - (e^{(\tau-1)\mu_c+\sum_{j=1}^{\tau}\varepsilon_{t+j}(p)})^{1-\theta})dp + (e^{\varepsilon_{t+1}})^{1-\theta}\psi\}$$

The term $\mu_I(u(C_{F^t_{C_{t+\tau}}}(p)) - u(C_{F^{t-1}_{C_{t+\tau}}}(p)))$ in the agent's first-order condition prevents an analytical solution. Instead I have to obtain the function for ρ_t by numerically finding a fixed point. The numerical procedures are very robust and pricing errors are very small. The results when using the ordered comparison are qualitatively and quantitatively very similar to the results under the static comparison excluding future uncertainty.

F.3 Comparison to the partial-equilibrium model

The Lucas-tree general-equilibrium setup simplifies the analysis considerably. In a partial-equilibrium model, in which R_t is i.i.d. and exogenous, the consumption-wealth ratio appears to be slightly more complicated

$$\rho_t = \frac{1}{1 + (\frac{\psi + \gamma Q(\eta F_{R_t}(R_t) + \eta\lambda(1 - F_{R_t}(R_t)))}{1 + \eta F_{R_t}(R_t) + \eta\lambda(1 - F_{R_t}(R_t))})^{\frac{1}{\theta}}}.$$

Q and ψ are constant but need to be solved simultaneously with ρ_t. Thus, the model needs to be solved with a simple fixed-point numerical procedure in an infinite-horizon model. In contrast, in general equilibrium Q and ψ depend on exogenous parameters, which gives rise to an analytical solution for ρ_t. Moreover, in the partial-equilibrium model, it has to be verified that consumption C_t and savings $W_t - C_t$ are increasing in the return realization R_t. In the Lucas-tree model, this is necessarily the case as consumption C_t, savings $W_t - C_t$, and returns are all increasing in ε_t.