INTERNATIONAL ECONOMIC REVIEW
Vol. 30, No. 3, August 1989

INCOME INSURANCE WITH UNCERTAIN OUTPUT*

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This paper examines the properties of a market solution to the output uncertainty problem faced by unincorporated primary producers. An insurance contract is formulated and shown to provide income insurance to producers without exposing insurers to moral hazard. The equilibrium relative to this contract is shown to be equivalent to that effected by a stock market available to all producers.

1. INTRODUCTION

This paper examines the behavior of primary producers facing uncertain output and the properties of a market solution to their income insurance problem. The solution requires producers to be able to contract in an insurance market because in general, available commodity markets do not permit income insurance when output are uncertain. The shortcomings of available markets in dealing with output uncertainty is widely recognized in the literature (e.g., Arrow 1971, 1981; Diamond 1980). In particular, conventional futures markets permit simultaneous price and output hedging only under restrictive assumptions.[2] Similarly, commodity options cannot provide adequate output hedging whenever output uncertainty is not in one-to-one correspondence with price realizations. This lack of correspondence does occur in practice in the case of crops produced in several regions, each subject to its own weather and/or pest uncertainty.

In spite of its limitations (Stiglitz 1972, 1982; Drèze 1974), the stock market is the most effective institution for spreading risks. However, access to it is limited to large producers in countries with well developed security markets because of moral hazard and information costs such as those of monitoring the performance of agents in agriculture. Even in the U.S., a large part of agriculture is not incorporated.

In many countries, farmers do not bear all the risk of production. The U.S. government, for example, alleviates the effect of output fluctuations by giving farmers subsidized loans which decrease storage costs when output is high and finance planting for next year when output is low (low cost financing is a typical subsidy given to regions declared "disaster areas"). The farmer is also usually

* The author gratefully acknowledges the comments of two anonymous referees and financial support from the Faculty Research Fund of the Graduate School of Business of Columbia University.

[1] Paper received August 1987; revised May 1988.

[2] The output of all firms must be perfectly correlated for the producer to be able to eliminate income uncertainty in a conventional futures market. See McKinnon (1967) and Marcus and Modest (1984). Townsend (1978) demonstrated that non-contingent futures markets can effect efficient allocations in a pure exchange economy. Breeden (1984) has examined conditions under which continuous trading in non-contingent futures contracts or the introduction of commodity options can effect efficient allocations in a multi-period pure-exchange economy.

given a free option to sell his output to the government at a preannounced support price. In addition, in the U.S., Japan, Mexico and other countries, government-sponsored crop insurance programs permit farmers to insure a fraction of their normal output at subsidized premiums (Hazell et al. 1986). Thus, one should not be surprised by the lack of development of private insurance markets for dealing with output risk. Income support provided by the government, however imperfect, is essentially free or highly subsidized and may be responsible for the apparent failure of the market. Since government commodity programs can effect serious distortions to the allocation of resources,[3] it appears desirable to explore alternative means of providing income insurance to primary producers.

The problem of income instability of primary producers has a long tradition in economics. Most of the literature assumes market failure and deals with the effect and feasibility of some form of government managed buffer stock. We shall briefly mention the literature most directly related to the present paper. McKinnon (1967) recognized the problem created by output uncertainty and studied the extent to which a farmer can attain income stability in conventional futures markets by exploiting negative correlation between the crop price and his output. McKinnon perceptively observed that "the problem of determining what constitutes an 'optimal' futures contract is an interesting and important one. There should be no presumption that the current practice is adequate or optimal." The allocative implications of the lack of equality of the producers' marginal rates of substitution between uncertain and sure income across states of nature, itself a consequence of the lack of appropriate risk markets, were studied by Sandmo (1972) in his analysis of public investment under uncertainty. He examined the role of government production in the absence of a stock market. More recently, Diamond (1980) examined the role of taxes and government demand policy as an alternative to the inability of existing institutions to equate marginal rates of substitution of income across states of nature.

The insurance contract studied in this paper may be seen as a market implementation of the administered price scheme mentioned by Stiglitz (1980). Under such a scheme, the government would offer producers a price that is inversely related to the quantity produced within the region. It is shown in this paper that prices with precisely this property effect an efficient allocation of resources and that they are attained at the equilibrium of an insurance market. In addition, by the very nature of the insurance contract being considered, this market is not subject to arbitrage across regions or other problems that, as Stiglitz notes, are likely to affect the implementation of an administered price scheme.

The competitive insurance market differs from typical government sponsored crop insurance programs in a number of respects. The latter offers insurance of a fraction of the producer's normal output at a subsidized premium. The proposed insurance market permits producers to buy or sell insurance contingent on the output of any region in the economy and for amounts that are optimal according to their preferences, beliefs and endowments, attaining in that way the benefits of portfolio diversification available in stock market economies. Moreover, contract

[3] See, for example, Johnson (1973).

prices are those that result from a competitive equilibrium associated with an efficient allocation of resources and risk bearing.

2. THE MODEL

The model to be used in this paper is a two-date one-good economy similar to that of Diamond (1967) and Sandmo (1972). The good is available at dates 0 and 1. It is the input in the production of itself for date 1. Date-1 output is subject to uncertainty.

The economy has I producers, indexed $i = 1, \ldots, I$. Each producer has a preference ordering over date-0 (c_{0i}) and date-1 (c_{1i}) consumption which is representable by $U^i(c_{0i}, c_{1i})$. Each producer starts at date 0 with a positive endowment of the good (w_i) and a farm which produces output for date 1. The farm is located in one of J different regions (indexed j or $m = 1, \ldots, J$), and is represented by the production function $K_i g_i(x_i)\Phi_j(s)$. g_i gives the expected yield per acre as a function of the amount of date-0 input (x_i) applied to each acre. K_i is the size of the farm in acres. Without loss of generality, we let $K_i = 1$ for all farms in the economy. $\Phi_j(s)$ is a multiplicative random factor common to region j which is a function of the state of the world parameter s. Uncertainty is resolved at date 1 by the occurrence of one of the S mutually exclusive states of the world, indexed $s = 1, \ldots, S$.[4] Each farm is small relative to its region in the sense that changes in its input per acre have a negligible effect on the aggregate output of the region. U^i is a twice continuously differentiable, strictly concave function with positive partial derivatives and $U^i(c_{0i}, c_{1i}) > U^i(0, 0)$ iff $c_{0i} > 0$ and $c_{1i} > 0$. g_i is an increasing, twice continuously differentiable strictly concave function of x_i. The farmers are Von Neumann-Morgenstern expected utility maximizers. Though their probability beliefs need not be the same, they do agree about the expected yield of each region. Hence, without loss of generality we assume that g_i is normalized such that $E_i[\Phi_j(s)] = 1, \forall\, i, j$. E_i is the expectation operator applied with respect to the i^{th}'s producer subjective probability distribution of s.

Producers are not incorporated. They have access to a bond market toward which they behave competitively. At date 0, producer i can borrow b_i at the default-free interest rate r ($b_i < 0$ denotes lending). At date 1 the producer repays (collects) the loan plus interest. In addition, there is a market for contingent-delivery contracts.

Insurance Market. The producer can insure his date-1 income by selling z_{ji} contracts committing him to deliver, at a known price p_j per contract, a quantity z_{ji} times an adjustment factor $\Phi_j(s)$ to be determined ex-post. That is, at date 1, the producer collects $z_{ji}p_j$ and delivers $z_{ji}\Phi_j(s)$ units in fulfillment of his commitment. The adjustment factor $\Phi_j(s)$ is the ratio of actual to expected yield per acre in region

[4] This specification of technological uncertainty is consistent with the econometric evidence on agricultural production functions. See, for example, Wolfson (1958).

j.[5] When $z_{ji} < 0$, the producer buys $-z_{ji}$ contracts committing him to accept delivery of $-z_{ji}\Phi_j(s)$ for the known price $-z_{ji}p_j$. As many different contracts as there are production regions are traded. Producers are assumed to be price takers in this market. Note that these contracts are not subject to moral hazard. This is so because, since farms are small relative to their regions, each producer has a negligible effect on the actual yield per acre of the region and, therefore, contractual contingent deliveries are independent of his own production decisions.

3. PROPERTIES OF THE EQUILIBRIUM

Let c, x, z and b denote the sets of consumption, production, insurance and lending plans and (r, p) denote the price system of the insurance market economy. The equilibrium in this economy is defined as follows:

DEFINITION 1. *A* competitive equilibrium *in the* insurance market economy *relative to initial endowments of private farm ownership and* $\{w_i\}_i$, *is an array* (c, x, b, z, r, p) *such that (a) for each i:* c_i, x_i, b_i *and* $\{z_{mi}\}_m$ *maximize* $E_i[U^i(c_{0i}c_{1i})]$ *subject to*

$$c_{0i} + x_i \leqq w_i + b_i, \tag{1}$$

$$c_{1i} \leqq g_i(x_i)\Phi_j(s) - (1 + r)b_i + \Sigma_m z_{mi}p_m - \Sigma_m z_{mi}\Phi_m(s), \quad \forall s \tag{2}$$

$$c_{0i},\ c_{1i},\ x_i \geqq 0, \tag{3}$$

and (b) the bond and insurance markets clear:

$$\Sigma_i b_i = 0 \tag{4}$$

$$\Sigma_i z_{mi} = 0, \quad \forall m. \tag{5}$$

Producers' Decisions. The i^{th} producer's optimal behavior satisfies the following first-order conditions:

$$\frac{E_i[U_0^i]}{E_i[U_1^i]} = 1 + r, \tag{6}$$

$$\frac{E_i[U_1^i\Phi_j(s)]}{E_i[U_1^i]}\, g_i'(x_i) = 1 + r, \tag{7}$$

$$\frac{E_i[U_1^i\Phi_m(s)]}{E_i[U_1^i]} = p_m, \quad \forall m, \tag{8}$$

where $E_i[U_0^i] = E_i[\partial U^i/\partial c_{0i}]$, etc. and $g_i'(x_i) = \partial g_i(x_i)/\partial x_i$.

Conditions (6) and (7) have familiar interpretations. (6) says that producers

[5] Note that the effective price received by the producer is $p_j/\Phi_j(s)$. That is, the insurance market yields a price that is inversely related to the output of the region.

equate their marginal rate of time preference to the rate of interest. (7) equates the risk adjusted marginal productivity of investment to the rate of interest. The left side of (7) is the marginal rate of substitution (MRS) between uncertain and sure date-1 consumption, multiplied by the expected marginal productivity of investment. (8) gives the producer's insurance portfolio as a function of his wealth, probability beliefs and attitude toward risk. In addition, it requires the equality of the MRS between uncertain and sure consumption across individuals. Substituting (8) into (7) yields

$$p_j g_i'(x_i) = 1 + r \tag{9}$$

which equates the value of the expected marginal productivity of farm i to the rate of interest. That is, the insurance market separates production and consumption decisions in the presence of output uncertainty and, as a consequence, equalizes the expected marginal productivities of all producers in the same region (i.e., $g_i'(x_i) = g_h'(x_h)$, $\forall\ i, h$ in region j).

Equivalence to Stock Market Economy. We now show that, under the stated assumption that uncertainty is multiplicative, the competitive equilibrium of the insurance market economy is equivalent to the competitive equilibrium of a stock market economy. In order to characterize the latter we need the following additional notation: d_i denotes lending in the stock market economy, α_{ki} denotes the shareholding of the i^{th} producer in the k^{th} farm such that $\Sigma_i \alpha_{ki} = 1\ \forall\ k$, and q_k is the market price of the k^{th} farm. The farms are the same as those available in the insurance market economy, that is, $k = 1, \ldots, I$, and are located in one of J different regions $m = 1, \ldots, J$, such that for some values of k, $m = 1$, for others $m = 2$, etc. As in the insurance market economy, each farm is initially owned by a farmer. d and α denote the set of lending and shareholding plans and (r, q) denotes the price system of the stock market economy.

DEFINITION 2. *A* competitive equilibrium *in the* stock market economy *relative to initial endowments of private farm ownership and* $\{w_i\}$ *is an array* (c, x, d, α, r, q) *such that* (*a*) *for each individual i:* c_i, d_i *and* $\{\alpha_{ki}\}_k$ *maximize* $E_i[U^i(c_{0i}, c_{1i})]$ *subject to*

$$c_{0i} + \Sigma_k \alpha_{ki} q_k \leqq w_i - d_i + q_i \tag{10}$$

$$c_{1i} \leqq \Sigma_k \alpha_{ki} \pi_k(s) + (1 + r) d_i, \ \ \forall\ \ s \tag{11}$$

$$c_{0i},\ c_{1i} \geqq 0, \tag{12}$$

and (*b*) *for each firm k:*

$$\pi_k(s) = g_k(x_k)\Phi_m(s) - (1 + r)x_k \tag{13}$$

and x_k *satisfies*

$$(q_k + x_k) g_k'(x_k)/g_k(x_k) = 1, \tag{14}$$

and (*c*) *the bond and stock markets clear:*

$$\Sigma_k x_k - \Sigma_i d_i = 0 \tag{15}$$

$$\Sigma_i \alpha_{ki} = 1, \ \forall \ k. \tag{16}$$

(14) is the condition for efficient production attained when the firm maximizes its market price q_k (see Diamond 1967).

The relationship between the two economies is summarized in the following theorem:

THEOREM (*equivalence*). *To each equilibrium of the stock market economy there corresponds an equilibrium of the insurance market economy and vice versa.*

The proof is given in the Appendix.

From available results on the properties of the stock market economy we know that, under the assumption that short sales are bounded, a competitive equilibrium exists (Drèze 1974; and Grossman and Hart 1979). Furthermore, when output uncertainty is decomposable, every competitive equilibrium of the stock market economy is a Constrained Pareto Optimum (CPO) in the sense of Diamond and every CPO with $c_{0i} > 0 \ \forall \ i$ is a competitive equilibrium (Diamond 1967; and Drèze 1974). Therefore, under the assumptions that short sales are bounded, the equivalence theorem implies the following:

COROLLARY (*existence and optimality*). *There exists a competitive equilibrium of the insurance market economy. Furthermore, every competitive equilibrium of this economy is a CPO and every CPO with* $c_{0i} > 0 \ \forall \ i$ *is a competitive equilibrium.*

4. CONCLUDING REMARKS

Some of the results of this paper hold for the many-good economy. Introduction of an insurance market into an economy with many goods provides income insurance to producers and equates the expected marginal productivity of investment across producers. Yet, as in the case of the stock market economy with many goods, the insurance market economy is subject to the problem of imperfect coordination studied by Grossman (1977) and attains only what Grossman and Hart (1979) call a Production Social Nash Equilibrium which is a weaker notion of optimality than a CPO. A related issue is the welfare impact of opening an insurance market. For the single-good economy the answer is unambiguous: opening an insurance market results in a Pareto superior market structure. This is so because the allocations attainable in the absence of an insurance market are also attainable in the insurance market economy. Such a claim cannot be made for the many-good economy because of the incomplete coordination problem mentioned above. As Hart (1975) and Diamond (1980) have shown, the welfare effect of opening a new market or changing government policies in an incomplete market many-good economy depends on the nature of preferences and the distribution of wealth.

This paper has explored the ability of an insurance market to provide income insurance to producers subject to output uncertainty. The availability of such a market would make unnecessary government subsidies to producers affected by unfavorable weather or other natural phenomena since producers would be able to buy the amount of income insurance that is optimal for their wealth, probability beliefs and attitude toward risk. The equilibrium attained via an insurance market was shown to possess the same allocative properties as the equilibrium attained in the presence of a stock market available to all producers. However, while incorporation is subject to moral hazard or costly performance verification, the insurance market studied in this paper is free from such drawbacks. This is so because the insurance market can exploit a characteristic common in agriculture which is the existence of many production units within a homogeneous region. On the other hand, the insurance market would fail when production is dominated by few production units each affected by dissimilar risks. This is a situation more naturally handled by diversification via incorporation.

It should be emphasized that the implementation of the proposed insurance contract requires observation of realized output. Output observation may not be feasible or may be too costly in certain situations. In addition, implementation requires that the same technological uncertainty applies to a large number of producers. Hence, the proposed insurance contract is applicable mainly to agricultural commodities produced over large uniform areas. On the other hand, satisfaction of this requirement makes the contract amenable to a high degree of standardization permitting efficient trading in a futures market. This could be done via a straightforward generalization of the delivery concept of conventional futures markets in order to include delivery contingent on the occurrence of measurable natural phenomena such as the average yield of a production region.

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APPENDIX

The proof of the equivalence theorem of Section 3 is as follows: Let $\Omega = (c, x, d, \alpha, r, q)$ be the equilibrium of the stock market economy and $\Gamma = (c, x, b, z, r, p)$ be the corresponding equilibrium of the insurance market economy. Γ has the same values of c, x and r as Ω and, in addition,

$$b_i = x_i + q_i - d_i - \Sigma_k \alpha_{ki} q_k \tag{17}$$

$$z_{mi} = \delta_{mj} g_i(x_i) - \Sigma_k \alpha_{ki} g_k(x_k) \tag{18}$$

$$p_m = (1 + r)(q_k + x_k)/g_k, \text{ for any } k \text{ in region } m, \tag{19}$$

where δ_{mj} is the Kronecker delta ($\delta_{mj} = 1$ when $m = j$ and $\delta_{mj} = 0$, otherwise) and j denotes the region of the farm in the producer's initial endowment.

If Ω is the equilibrium of the stock market economy, then Γ is the equilibrium of the insurance market economy. Γ satisfies the individual budget constraints of Definition 1. In fact, substituting (17) into (10) yields (1). Substituting (17) through (19) into (11) yields (2). Note that (10) and (11) are satisfied at Ω. Furthermore, the bond and insurance markets clear: Adding (17) and (18) over i and applying (15) and (16) yields (4) and (5). Finally, the individual optimum conditions are satisfied: It follows from Definition 2 that the optimum conditions for the producer in the stock market economy are

$$\frac{E_i[U_0^i]}{E_i[U_1^i]} = 1 + r, \tag{20}$$

$$q_k + x_k = \frac{E_i[U_1^i \Phi_m(s)]}{E_i[U_1^i]} \frac{g_k(x_k)}{1 + r}. \tag{21}$$

(20) is identical to (6) of the insurance market economy. Substituting (19) into (21) yields (8). In addition, substituting (19) into (14) yields (9).

The converse is proved in the same way with d_i, α_{ki} and q_k given by the solution of (17) through (19), and noting that the optimal stock portfolio is determined only up to the sum $\Sigma_k \alpha_{ki} q_k$, where k runs over the farms in the same region, because the output of the latter is perfectly correlated. Q.E.D.

REFERENCES

ARROW, K. J., "Insurance, Risk, and Resource Allocation," in K. J. Arrow, *Essays in the Theory of Risk Bearing* (Chicago: Markham, 1971), 134–143.

———, "Futures Markets: Some Theoretical Perspectives," *The Journal of Futures Markets* 1 (1981), 107–115.

BREEDEN, D. T., "Futures Markets and Commodity Options: Hedging and Optimality in Incomplete Markets," *Journal of Economic Theory* 32 (1984), 275–300.

DIAMOND, P. A., "A Stock Market in a General Equilibrium Model," *American Economic Review* 57 (1967), 759–776.

DRÈZE, J. H., "Investment under Private Ownership: Optimality, Equilibrium and Stability," in J. H. Drèze, ed., *Allocation under Uncertainty: Equilibrium and Optimality* (New York: Wiley, 1974), 129–166.

GROSSMAN, S. J., "A Characterization of the Optimality of Equilibrium in Stock Market Economies," *Journal of Economic Theory* 17 (1977), 1–15.

——— AND O. D. HART, "A Theory of Competitive Equilibrium in Stock Market Economies," *Econometrica* 47 (1979), 293–329.

HART, O. D., "On the Optimality of Equilibrium When the Market Structure is Incomplete," *Journal of Economic Theory* 11 (1975), 293–311.

HAZELL, P., C. POMAREDA AND A. VALDÉS, eds., *Crop Insurance for Agricultural Development* (Baltimore and London: The John Hopkins University Press, 1986).

JOHNSON, D. G., *Farm Commodity Programs* (Washington D.C.: American Enterprise Institute, 1973).

MARCUS, A. J. AND D. M. MODEST, "Futures Markets and Production Decisions," *Journal of Political Economy* 92 (1984), 409–426.

MCKINNON, R. I., "Futures Markets, Buffer Stocks, and Income Stability for Primary Producers," *Journal of Political Economy* 75 (1967), 844–861.

SANDMO, A., "Discount Rates for Public Investment under Uncertainty," *International Economic Review* 13 (1972), 287–302.

STIGLITZ, J. E., "On the Optimality of the Stock Market Allocation of Investment," *Quarterly Journal of Economics* 86 (1972), 25–60.

———, "Futures Markets and the Stabilization of Commodity Prices," in *Frontiers in Futures*, Proceedings of the First Annual Sponsor's Conference, Center for the Study of Futures Markets (New York: Columbia University, 1980), 72–76.

———, "The Inefficiency of the Stock Market Equilibrium," *Review of Economic Studies* 49 (1982), 241–261.

TOWNSEND, R. M., "On the Optimality of Forward Markets," *American Economic Review* 68 (1978), 54–66.

WOLFSON, R. J., "An Econometric Investigation of Regional Differentials in American Agricultural Wages," *Econometrica* 26 (1958), 225–257.