PORTFOLIO CHOICE AND EQUILIBRIUM IN CAPITAL MARKETS WITH SAFETY-FIRST INVESTORS

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This paper develops optimal portfolio choice and market equilibrium when investors behave according to a generalized lexicographic safety-first rule. We show that the mutual fund separation property holds for the optimal portfolio choice of a risk-averse safety-first investor. We also derive an explicit valuation formula for the equilibrium value of assets. The valuation formula reduces to the well-known two-parameter capital asset pricing model (CAPM) when investors approximate the tail of the portfolio distribution using Tchebychev's inequality or when the assets have normal or stable Paretian distributions. This shows the robustness of the CAPM to safety-first investors under traditional distributional assumptions. In addition, we indicate how additional information about the portfolio distribution can be incorporated to the safety-first valuation formula to obtain alternative empirically testable models.

1. Introduction

This paper studies portfolio choice and market equilibrium when investors behave according to a generalized lexicographic form of the safety-first principles introduced by Roy (1952) and Telser (1955). The literature on this subject is limited. Notably, Pyle and Turnovsky (1970, 1971) studied the portfolio problem under safety-first preferences when the assets are normally distributed. Chipman (1971) gave a clear statement of the axiomatic characteristics of safety-first in his study of lexicographic preferences. More recently, Bawa (1976a,b) studied the use of safety-first rules to obtain stochastically undominated portfolios. The analysis of this paper, which is mostly distribution-free, focuses on the equilibrium value of assets. Specifically, we study the implications of behavior represented by the following lexicographic form of the safety-first principle:

$$\max\,(\pi, \mu), \tag{1}$$

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where

$$\begin{aligned} \pi &= 1, \quad &&\text{if} \quad P = \Pr\{Y \leqq s\} \leqq \alpha, \\ &= 1-P, \quad &&\text{otherwise,} \end{aligned}$$

and

$$\mu = E(Y).$$

Y is the random value of final wealth in a single-period choice situation, E is the expectation operator, s is the critical level of wealth, and α is the admissible probability of failure. (1) orders any two assets lexicographically according to π and μ, that is, the asset with the higher π is the preferred one. If π is the same for both assets, the order is based upon μ. Two assets with the same π and μ are equally preferred.

The reason for studying the lexicographic form (1) instead of the original criteria proposed by Roy (1952) and Telser (1955) is that these original criteria fail to order risky assets which are unambiguously ordered by the principle of absolute preference.[1] It is easy to verify that (1) produces a complete ordering of all risky assets with finite mean and satisfies absolute preference. Moreover, (1) includes the lexicographic form of Roy's principle suggested by Chipman (1971) and, when the chance constraint is satisfied and $\pi = 1$, it becomes the original criterion of Telser.[2]

In the present formulation the investor is concerned about safety only when the probability of failure exceeds the admissible level α. In such a case (1) becomes Roy's principle and the investor minimizes the probability of failure. However, following Telser, we assume that the investor neglects what he considers to be admissible risks of failure and that, within this class, he maximizes expected wealth. In other words, whenever feasible, the investor maximizes expected wealth subject to the constraint that the probability of failure does not exceed the critical level. It is shown below that this choice criterion is reasonable in that it implies desirable attitudes toward risk according to the Arrow (1971) – Pratt (1964) theory of risk aversion.

[1]The principle of absolute preference [Massé and Morlat (1953)], a reasonable restriction on economic choice under uncertainty, says that a distribution G is preferred to a distribution F if $G(x) \leqq F(x)$ for all $x \in X$, and $G(x) < F(x)$ for some $x \in X$, where X is the set of possible monetary outcomes.

[2]In addition to these criteria, Pyle and Turnovsky (1970) considered a criterion originally proposed by Kataoka (1963) requiring the maximization of the α-fractile of the final wealth distribution (the α-fractile s is defined by $\Pr\{Y \leqq s\} = \alpha$). It can be verified that this criterion has the undesirable property that the individual, no matter how wealthy he is, will never buy any part of an actuarially favorable asset with negative α-fractile of return.

We note that (1) is incompatible with the axiom of continuity and, when $\alpha > 0$, it is also incompatible with the axiom of independence.[3,4] This means that safety-first represents the preferences of investors other than expected utility maximizers. A review of the now classic pros and cons given on continuity and independence, which will not be repeated here [see Allais (1953), Arrow (1971, ch. 2), Chipman (1971), Savage (1954), and Thrall (1954)], reveals that they are not compelling restrictions on economic choices, and we find no sufficient *a priori* or empirical basis for preferring safety-first to expected utility or vice versa. Thus, it seems reasonable to consider both principles complementary, and to develop a theory of assets which allow for the existence of both types of investors.

2. Portfolio choice with safety-first preferences

2.1. The portfolio problem

Consider the single-period portfolio problem. Let V_j denote the initial market value of asset j, X_j the random final value, and $\bar{X}_j = E(X_j)$. The individual's initial wealth is denoted by W. He can buy any fractions $(\gamma_j) = (\gamma_1, \gamma_2, \ldots)$ of the risky assets (including short sales), and borrow any amount b at the constant interest rate $r-1$ ($b < 0$ represents lending).

As usual, a portfolio is defined as a combination of assets with return distribution independent of scale changes. The portfolio problem of a safety-first investor is

$$\max_{(\gamma_j),b} \quad (\pi, \mu) \quad \text{subject to} \quad \sum_j \gamma_j V_j - b = W, \tag{1'}$$

where

$$\begin{aligned} \pi &= 1, \qquad \text{if} \quad P = \Pr\left\{\sum_j \gamma_j X_j - br \leqq s\right\} \leqq \alpha, \\ &= 1-P \quad \text{otherwise,} \end{aligned}$$

[3] Here we refer to the standard axioms of utility theory as stated, for example, in Arrow (1971, ch. 2) or Chipman (1971). A choice criterion is said to satisfy or to be compatible with a given axiom when it fulfills the requirements imposed by the axiom in all possible choice situations.

[4] Incompatibility is shown by the following example: Let F_1, F_2, and F_3 denote risky assets, $X = \{r, t, u, v, w\}$ be a set of monetary outcomes such that $r < t < u < v < w$, $(w-u)/(u-t) > \alpha/(1-\alpha) > 0$, where u is the critical wealth level and α is the admissible probability of failure. Let F_1 generate r with probability one, F_2 generate u and v with probabilities α and $1-\alpha$, respectively, and F_3 generate t and w with probabilities α and $1-\alpha$, respectively. Now consider the random mixture $F_4 = [pF_1, (1-p)F_3]$. It can be verified that (1) implies $F_2 \succ F_1$, $F_2 \succ F_4$ for $p > 0$, and $F_5 = [pF_1, (1-p)F_2] \succ F_4$ for $p \leqq \alpha$, where $\succ$ denotes preference. This contradicts continuity which requires that $F_4 \succ F_2$ for some $p > 0$, and contradicts strong independence which requires that $F_4 \succ F_5$ for any $p < 1$. Weak independence is contradicted when $(w-u)/(u-t) = \alpha/(1-\alpha)$.

and

$$\mu = \sum_j \gamma_j \bar{X}_j - br.$$

Since risk aversion seems pervasive in financial markets, we are interested in the solution to (1′) for risk-averse safety-first (RASF) investors.

2.2. *Characterization of risk-averse safety-first behavior*

Following Arrow (1971, ch. 3), the choice among simple portfolios composed of a fraction of a single risky asset and a secure asset is studied in this section. Thus, subscripts can be omitted and the budget constraint becomes $\gamma V - b = W$.

An investor is said to exhibit risk aversion if he prefers a secure asset to an actuarially fair risky asset. It can be verified that the critical level of wealth of a RASF investor is smaller than his secure final wealth Wr.[5] Also, a RASF investor will be indifferent about holding a secure asset or a fair risky asset such that

$$\Pr\{\gamma X - br \leqq s\} \leqq \alpha, \tag{2}$$

since then $\pi = 1$ and $\mu = Wr$ for any $\gamma > 0$ satisfying (2). These assets represent negligible risks for RASF investors and are evaluated in the same way as secure assets, i.e., in terms of μ only. It is convenient to define a *material risk* as an asset not satisfying (2). Taking into account the budget constraint $\gamma V - b = W$ we can write

$$\Pr\{\gamma X - br \leqq s\} = \Pr\left\{\frac{X}{V} \leqq \frac{s+br}{W+b}\right\} = \Pr\left\{R \leqq r + \frac{s-Wr}{W+b}\right\},$$

where $R = X/V$ is the return on the risky asset. Thus, letting $q_\alpha(R)$ denote the α fractile of the return distribution (i.e., $\Pr\{R \leqq q_\alpha(R)\} = \alpha$) we have the following equivalent, but more useful characterization of a material risk:

$$q_\alpha(R) < r + \frac{s-Wr}{W+b}. \tag{3}$$

The definition of a material risk implies that RASF investors will never buy an actuarially favorable material risk. However, investors can make the risk

[5] In fact, a RASF investor will not hold a fair risky asset that is too risky for him. More precisely, when $s < Wr$, $P = \Pr\{\gamma X - br \leqq s\} = 0$ for $\gamma = 0$ and $b = -W$. Therefore, pure lending will be preferred to a portfolio having any fraction γ of a fair risky asset ($\bar{X}/V = r$) such that $P > \alpha$. On the other hand, when $s \geqq Wr$ pure lending cannot exceed the critical wealth level ($P = 1$) and the investor will prefer to hold a fair risky asset giving $P < 1$.

of any divisible asset negligible by investing sufficiently little on it. Thus, we have proved the following:

Lemma. A RASF investor will always buy some part of a divisible favorable risk and the amount bought will be the maximum satisfying $(W+b)q_\alpha(R)-br = s$, *or*[6]

$$W+b = \frac{s-Wr}{q_\alpha(R)-r}. \tag{4}$$

This is the counterpart of Arrow's (1971, ch. 3) result for risk-averse expected-utility maximizers. It says that a RASF individual invests in a favorable risky asset up to the point where it becomes a material risk. It then follows that favorable risks cannot be negligible for all investment amounts, for otherwise $W+b = \infty$. This rules out $q_\alpha(R) \geqq r$ for favorable risks. Otherwise their market values would increase up to the point where they become fair assets, indistinguishable from the riskless asset.

It follows from (4) that RASF investors exhibit decreasing absolute risk aversion [Arrow (1971, ch. 3), Pratt (1964)]. In fact, (4) implies that the amount invested in favorable risks increases with wealth. We note that the relative risk aversion of a RASF investor, which can be measured by the proportion of his wealth $(W+b)/W$ invested in risky assets, can be increasing ($s < 0$), constant ($s = 0$), or decreasing ($s > 0$).[7]

2.3. *The separation property*

We now consider the portfolio problem with several risky assets formulated in subsection 2.1. The return on the risky part of the portfolio is

$$R = \left(\sum_j \gamma_j X_j\right)\Big/\left(\sum_j \gamma_j V_j\right), \tag{5}$$

with $\bar{R} = E(R)$. Also, using (5) and the budget constraint $\sum_j \gamma_j V_j = W+b$, we can write the final value of the portfolio as

$$\sum_j \gamma_j X_j - br = (W+b)R - br = Wr + (W+b)(R-r).$$

[6]The investor will buy a positive amount of a favorable material risk if and only if $q_\alpha(R) > -\infty$, which is a minor restriction on the class of risky assets. It excludes assets unbounded from below when $\alpha = 0$.

[7]This distribution-free result generalizes Pyle and Turnovsky's (1971, pp. 220-222) conclusion that Telser investors exhibit decreasing relative aversion when $s > 0$.

When favorable assets are available, the solution to the portfolio problem (1′) can be obtained from

$$\max_{(\gamma_j),b} \mu = Wr+(W+b)(\bar{R}-r),$$

among those portfolios exceeding the critical level of wealth with probability $1-\alpha$, i.e., those satisfying (4). Using (4), the optimization problem reduces to

$$\max_{(\gamma_j),b} \mu = Wr-(s-Wr)\left(\frac{\bar{R}-r}{r-q_\alpha(R)}\right). \tag{6}$$

Thus, a RASF investor can solve his portfolio problem in two stages. First, he maximizes the ratio of the risk premium to the return opportunity loss that he can incur with probability α,

$$\max_{(\gamma_j)} \frac{\bar{R}-r}{r-q_\alpha(R)}, \tag{7}$$

and determines the optimal fractions (γ_j) up to scale constant. Note that (7) is independent of proportional changes in (γ_j) because R and, therefore, $\bar{R}$ and q are zero-homogenous functions of (γ_j). In a second stage the investor finds the scale of the risky part of his portfolio from (4), and the amount to be borrowed from the budget constraint.

Therefore, we have obtained the following *separation property*:

Theorem 1. The portfolio problem of a RASF investor can be separated into two problems: (1) The choice of the optimal risky asset proportions, which are independent of wealth and borrowing, and (2) the choice of the scale of the risky portfolio and the amount borrowed.

Theorem 1 extends the class of preferences admitting separability in portfolio allocation. Of course, this extension can only be made outside the domain of expected utility theory for which the results of Hakansson (1969) and Cass and Stiglitz (1970) are definitive.

A unique solution to (7) implies that under homogeneous beliefs the risky portfolios will be identical up to a scale constant for all investors with the same α. These are strong but not unreasonable requirements. Observe that investors focusing on the same probability of failure α do not have to have the same preferences. Individual variation is allowed for by wealth and critical levels. α uniquely determines the α-fractile of each portfolio, $q_\alpha(R)$, which is an intrinsic characteristic of the portfolio distribution and, as such, is independent of individual preferences. Furthermore, the α-fractile is intuitively a better measure of risk than the second moment since it is based upon leftward deviations only.

Alternative to the assumption that investors focus on the same α-fractile is to assume that

$$q_\alpha(R) = \bar{R} - g(\alpha)h((\gamma_j), S), \tag{8}$$

where $g > 0$, $g' < 0$, $h > 0$, and S is the parameter set of the return distribution of the risky assets. In such a case (7) is equivalent to

$$\max_{(\gamma_j)} \frac{\bar{R}-r}{h}, \tag{9}$$

which, under homogeneous belief, implies that (γ_j) is identical up to a scale constant for all investors no matter what probability of failure they focus on. The appeal of (8) is reinforced by the fact that it holds exactly in two important cases: (1) when the portfolio distribution (F) is normal, stable with common characteristic exponent in the open interval $(1, 2)$ and common skewness parameter not necessarily zero, or Student's t, and (2) when investors obtain $q_\alpha(R)$ using Tchebychev's inequality, as suggested by Roy and Telser. $g = F^{-1}(\alpha)$ in the first case, and $g = \alpha^{-1}$ in the second case [see (16) in subsection 4.1 below]. h equals the dispersion of the portfolio in both cases.

3. Equilibrium in a safety-first market

Let us now consider the value of assets in a market of RASF investors focusing on the same α and holding homogeneous beliefs. A new subscript i is added to distinguish among investors. Thus, the risky part of the portfolio of investor i will be characterized by its expected value $\sum_j \gamma_{ij}\bar{X}_j$ and its α-fractile $Q^i = Q^i(\gamma_{i1}, \gamma_{i2}, \ldots)$, where γ_{ij} is the fraction of asset j held by investor i and Q^i is defined by

$$\Pr\left\{\sum_j \gamma_{ij}X_j \leqq Q^i\right\} = \alpha.$$

According to (7) each investor selects its risky portfolio to maximize $(\bar{R}^i-r)/(r-q^i)$, where now

$$\bar{R}^i = \left(\sum_j \gamma_{ij}\bar{X}_j\right)\Big/\left(\sum_j \gamma_{ij}V_j\right),$$

and

$$q^i = Q^i(\gamma_{i1}, \gamma_{i2}, \ldots)\Big/\Big(\sum_j \gamma_{ij} V_j\Big),$$

and, therefore,

$$\frac{\bar{R}^i - r}{r - q^i} = \Big(\sum_j \gamma_{ij}(\bar{X}_j - rV_j)\Big)\Big/\Big(r \sum_j \gamma_{ij} V_j - Q^i\Big). \tag{10}$$

The first-order conditions for the maximum of (10) are

$$\frac{\partial}{\partial \gamma_{ij}} (\bar{R}^i - r)/(r - q^i) = 0, \qquad j = 1, 2, \ldots. \tag{11}$$

Since the separation property (Theorem 1) and the assumption of homogeneous beliefs imply $\gamma_{ij}/\gamma_{ik} = \gamma_{lj}/\gamma_{lk}$ for any two investors i and l, and at equilibrium $\sum_l \gamma_{lk} = \sum_l \gamma_{lj} = 1$, we have $\gamma_{ij} = \gamma_i$ for all j and i. This property allows us to reduce (11) after some simple manipulations to

$$\frac{\bar{X}_j - rV_j}{\bar{X}_j - \partial Q^i/\partial \gamma_{ij}} = \frac{\bar{X}_m - rV_m}{\bar{X}_m - Q^i/\gamma_i}, \qquad j = 1, 2, \ldots, \tag{12}$$

where

$$\bar{X}_m = \sum_j \bar{X}_j \quad \text{and} \quad V_m = \sum_j V_j.$$

It follows from $\gamma_{ij} = \gamma_i$ and the definition of Q^i that Q^i/γ_i is the same for all investors and equals the α-fractile of the total market value, that is,

$$\Pr\Big\{\sum_j \gamma_{ij} X_j \leqq Q^i\Big\} = \Pr\Big\{\sum_j X_j \leqq Q^i/\gamma_i\Big\} = \alpha.$$

Therefore, we can write $Q_m = Q^i/\gamma_i$ for all i. Also, since q^i is zero homogeneous, Q^i is linearly homogeneous and, therefore, $\partial Q^i/\partial \gamma_{ij}$ is zero-homogeneous and invariant to scale changes. Thus, we can write $Q_j = \partial Q^i/\partial \gamma_{ij}\big|_{(\gamma_{ij}) = (\gamma_i)}$ for all i. Furthermore, Q_j can be interpreted as the contribution of asset j to the α-fractile of the total market value. That is, from the linear homogeneity of Q^i and $\gamma_{ij} = \gamma_i$, we have

$$Q_m = \frac{Q^i}{\gamma_i} = \frac{1}{\gamma_i} \sum_j \gamma_i \frac{\partial Q^i}{\partial \gamma_{ij}}\bigg|_{(\gamma_{ij}) = (\gamma_i)} = \sum Q_j.$$

Taking into account these properties we can rewrite (12) to obtain the equilibrium market value of asset j,

$$V_j = \frac{1}{r}\left[\bar{X}_j - \frac{\bar{X}_m - rV_m}{\bar{X}_m - Q_m}(\bar{X}_j - Q_j)\right], \tag{13}$$

and the equilibrium expected return,

$$\bar{R}_j = r + \frac{\bar{R}_m - r}{r - q_m}(r - q_j), \tag{14}$$

where

$$\bar{R}_j = \bar{X}_j / V_j \quad \text{and} \quad \bar{R}_m = \bar{X}_m / V_m,$$

$$q_j = Q_j / V_j \quad \text{and} \quad q_m = Q_m / V_m.$$

The results of this section are summarized by the following:

Theorem 2. The value of a risky asset in a RASF market is equal to its certainty equivalent future value discounted at the riskless rate of interest. The certainty equivalent adjustment is equal to the spread between the contribution of the asset to the expected value of the market and its contribution to the α-fractile of the market multiplied by the market premium per unit spread.

4. Testable specifications of the valuation formula

4.1. A two-parameter specification

In order to derive empirically testable specifications of the safety-first valuation formula it is necessary to specify the α-fractile of the portfolio in terms of estimable characteristics of the risky assets. For example, when $\alpha > 0$ the Tchebychev inequality can be used to approximate the tail of the risky portfolio distribution from above. That is,

$$\Pr\left\{\sum_j \gamma_{ij} X_j \leqq Q^i\right\} \leqq \sigma_i^2 \Big/ \left(Q^i - \sum_j \gamma_{ij} \bar{X}_j\right)^2 = \alpha, \tag{15}$$

where

$$\sigma_i^2 = \sum_{j,k} \gamma_{ij} \gamma_{ik} \sigma_{jk},$$

and σ_{jk} is the covariance between the final values of assets j and k. For $Q^i < \sum_j \gamma_{ij}\bar{X}_j$, (15) gives

$$Q^i = \sum_j \gamma_{ij}\bar{X}_j - \left(\frac{1}{\alpha}\sum_{j,k} \gamma_{ij}\gamma_{ik}\sigma_{jk}\right)^{1/2}, \tag{16}$$

$$Q_j = \left.\frac{\partial Q^i}{\partial \gamma_{ij}}\right|_{(\gamma_{ij})=(\gamma_i)} = \bar{X}_j - \frac{\operatorname{cov}(X_j, X_m)}{[\alpha \operatorname{var}(X_m)]^{1/2}}, \tag{17}$$

and

$$Q_m = \sum_j Q_j = \bar{X} - \left(\frac{\operatorname{var}(X_m)}{\alpha}\right)^{1/2}, \tag{18}$$

where $X_m = \sum_j X_j$.

Substituting (17) and (18) into (13) gives

$$V_j = \frac{1}{r}\left[\bar{X}_j - \frac{\bar{X}_m - rV}{\operatorname{var}(X_m)}\operatorname{cov}(X_j, X_m)\right], \tag{19}$$

which can be rewritten as

$$\bar{R}_j = r + \frac{\bar{R}_m - r}{\operatorname{var}(R_m)}\operatorname{cov}(R_j, R_m), \tag{20}$$

where $R_m = X_m/V_m$.

(19) and (20) are the valuation formulas derived in the two-parameter capital asset pricing literature assuming a quadratic utility function or normally distributed assets [Sharpe (1964), Lintner (1965), Mossin (1966)]. (19) and (20) also follow from (13) if the one-sided sharp extension of Tchebychev's inequality due to Cantelli [see Cramer (1946, p. 256)] is used instead. They follow exactly (not as approximations) when the assets are normally distributed. Also, it can be verified that the valuation equation derived by Fama (1971) for the Markowitz–Sharpe diagonal model with stable Paretian assets can be obtained from (13).

An important consequence of the results of this section is that the well-known two-parameter CAPM holds for a market of both RASF investors and risk-averse expected-utility maximizers: It holds exactly when the assets are normal or stable Paretian, and it holds approximately for arbitrary assets when Tchebychev inequality and quadratic utility or mean-variance type approximations are used.

4.2. More general specifications

Rather than arriving at the α-fractile of the portfolio through an approximation to the portfolio distribution, we may directly assume a plausible form for the α-fractile. One such a form is

$$Q^i = \sum_j \gamma_{ij}\bar{X}_j - g(\alpha_i)H((\gamma_{ij}), S), \tag{21}$$

which is of the form (8) and allows α_i to vary across investors. S is here the parameter set of the final value distribution of the risky assets. Taking (9) into account and following the derivation of section 3, we obtain

$$V_j = \frac{1}{r}\left[\bar{X}_j - \frac{\bar{X}_m - rV_m}{H_m(S)} H_j(S)\right], \tag{22}$$

where $H_j = \partial H/\partial\gamma_{ij}|_{(\gamma_{ij})=(\gamma_i)}$ and $H_m = \sum_j H_j$. For example, for

$$H = H(\sigma_i, s_i),$$

where

$$\sigma_i^2 = E\left[\left(\sum_j \gamma_{ij}(X_j - \bar{X}_j)\right)^2\right],$$

and

$$s_i^3 = E\left[\left(\sum_j \gamma_{ij}(X_j - \bar{X}_j)\right)^3\right],$$

(22) becomes

$$V_j = \frac{1}{r}\left[\bar{X}_j - \frac{\bar{X}_m - rV_m}{[\mathrm{var}\,(X_m]^{1/2} - \theta s_m}\left(\frac{\mathrm{cov}\,(X_j, X_m)}{[\mathrm{var}\,(X_m)]^{1/2}} - \theta\frac{\Delta_j(s_m^3)}{s_m^2}\right)\right], \tag{23}$$

where

$$\Delta_j(s_m^3) = E\,[(X_j - \bar{X}_j)(X_m - \bar{X}_m)^2],$$

$$s_m^3 = E\,[(X_m - \bar{X}_m)^3],$$

$$\theta = -(\partial H/\partial s_i)/(\partial H/\partial\sigma_i).$$

θ can be interpreted as the marginal rate of substitution of skewness for dispersion, for constant expected wealth and probability of failure. Homogeneous beliefs result in the same θ for all investors. $\Delta_j(s_m^3)/s_m^2$ can be interpreted as the contribution of asset j to the skewness of the market since $\sum_j [\Delta_j(s_m^3)/s_m^2] = s_m$. We note than when $\theta = 0$, (23) reduces to (19). Also, (23) can easily be expressed in terms of rates of return as in (14) and (20) and provides an alternative testable hypothesis.

5. Conclusions

This paper has shown that the safety-first approach implies attitudes toward risk, portfolio choices and market equilibrium which are comparable to those implied by the expected utility approach. As such, our results show the robustness of current theory to safety-first investors. Perhaps more important for the development of a positive theory of capital markets is that the safety-first valuation formula allows for different assumptions concerning the probabilistic information possessed by investors and, therefore, it can be used to obtain alternative empirically testable valuation models.

References

Allais, M., 1953, Le comportement de l'homme rationnel devant le risque: Critique des postulats et axiomes de l'école americaine, Econometrica 21, no. 4, 503–546.

Arrow, K.J., 1971, Essays in the theory of risk bearing (Markham, Chicago, IL).

Bawa, V.S., 1976a, Admissible portfolios for all individuals, Journal of Finance 31, no. 4.

Bawa, V.S., 1976b, Safety-first, stochastic dominance and optimal portfolio choice, Economic Discussion Paper no. 60 (Bell Laboratories, Holmdel, NJ).

Cass, D. and J.E. Stiglitz, 1970, The structure of investor preferences and asset returns, and separability in portfolio allocation: A contribution to the pure theory of mutual funds, Journal of Economic Theory 2, no. 1, 122–160.

Chipman, J.S., 1971, Non-Archimedean behavior under risk, in: J.S. Chipman, L. Hurwicz, M.R. Richter and H.S. Sonnenschein, eds., Preferences, utility and demand (Harcourt Brace Jovanovich, New York), 289–318.

Cramér, H., 1946, Mathematical methods of statistics (Princeton University Press, Princeton, NJ).

Fama, E.F., 1971, Risk, return and equilibrium, Journal of Political Economy 79, no. 1, 30–55.

Hakansson, N.H., 1969, Risk disposition and the separation property in portfolio selection, Journal of Financial and Quantitative Analysis 4, no. 4, 401–416.

Kataoka, S., 1963, A stochastic programming model, Econometrica 31, no. 1–2, 181–196.

Lintner, J., 1965, Security prices, risk and maximal gains from diversification, Journal of Finance 20, no. 4, 587–615.

Massé, P. and G. Morlat, 1953, Sur le classement économique des perspectives aleatoires, in: Econometrie (Centre National de la Recherche Scientifique, Paris), 165–199.

Mossin, J., 1966, Equilibrium in a capital asset market, Econometrica 34, no. 4, 768–783.

Pyle, D.H. and S.J. Turnovsky, 1970, Safety-first and expected utility maximization in mean-standard deviation portfolio selection, Review of Economics and Statistics 52, no. 1, 75–81.

Pyle, D.H. and S.J. Turnovsky, 1971, Risk aversion in chance-constrained portfolio selection, Management Science 18, no. 3, 218–225.

Pratt, J.W., 1964, Risk aversion in the small and in the large, Econometrica 32, no. 1–2, 122–136.

Roy, A.D., 1952, Safety-first and the holding of assets, Econometrica 20, no. 3, 431–449.

Savage, J.L., 1954, The foundations of statistics (Wiley, New York).

Sharpe, W.F., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, Journal of Finance 19, no. 3, 425–442.

Telser, L.G., 1955/6, Safety-first and hedging, Review of Economic Studies 23 (1), no. 60, 1–16.

Thrall, R.M., 1954, Applications of multidimensional utilities, in: R.M. Thrall, C.H. Coombs and R.L. Davis, eds., Decision processes (Wiley, New York), 181–186.